$f(x,y) = x + y$, $3x^2 + y^2 = 3$ solution
$f(x,y) = 3x^2 + y^2$, $x + y = 1$. solution
$f(x,y) = \sin x + \cos y - x$, $x^2 + y^2 = 1$ solution
$f(x,y) = \sin x + \cos y - x$, $x + y = 2$. solution
$f(x,y) = xe^y$, $x^2 - 2y^2 = 4$ solution
$f(x,y,z) = xy + yz$, $x + y = 2$, $y + z = -1$ solution

Suppose a company makes a profit $p(x,y) = 7x^2 + 5$ and the production cost is $c(x,y) = 3x + 2y$. Suppose we want to keep $c(x,y)$ at \$500,000.00. Determine the maximum profit subject to the stated constraint. solution

Can we compute the determinant of

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}?$$

solution

Can we add the following two matrices? Can we multiply the two matrices?

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 9 \end{bmatrix}?$$

solution

What are the dimensions of the following products of matrices?
$AB$ and $BA$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 9 \end{bmatrix},$$

$$A = \begin{bmatrix} -3 & 0 & -1 \\ 0 & 7 & -9 \\ 4 & 5 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 & 2 \\ 0 & -4 \\ 5 & -1 \end{bmatrix}?$$

solution

What can you state about multiplication of matrices? Use technical terms and be in your answers.

Compute $\det A$ where $A$ is as follows

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

solution

Compute $\det A$ where $A$ is as follows

$$A = \begin{bmatrix} -1 & 1 & 5 \\ 1 & 3 & 2 \\ 0 & 4 & 6 \end{bmatrix}$$

solution

Compute $\det A$ where $A$ is as follows

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

solution

Compute $\det A$ where $A$ is as follows

$$A = \begin{bmatrix} 0 & 2 & 5 \\ 7 & 8 & 9 \\ 0 & 0 & -1 \end{bmatrix}$$

solution

Compute $\det A$ where $A$ is as follows

$$A = \begin{bmatrix} 1 & 2 & 5 & -3 \\ 9 & -2 & 0 & 1 \\ -7 & 0 & -3 & 4 \end{bmatrix}$$

solution

If $A$ is an $n \times n$ matrix, what would you expect the determinant of

$$A = \begin{bmatrix} a_{11} & a_{11} & \dots & a_{1,n} \\ 0 & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{n,n} \end{bmatrix}$$

to be? solution

Compute the Hessian of the following function.

$$f(x, y, z) = xyz$$

Solution

Compute the Hessian of the following function.

$$f(x, y, z) = \sin x + \sin y + \sin z$$

Solution

Compute the Hessian of the following function.

$$f(x, y) = \sin(xy)$$

Solution

Compute the Hessian of the following function.

$$e^y \ln x$$

Determine the critical points of the following function.

$$f(x, y, z) = xyz$$

Determine the critical points of the following function.

$$f(x, y, z) = \sin x + \sin y + \sin z$$

Determine the critical points of the following function.

$$f(x, y) = \sin(xy)$$

Determine the critical points of the following function.

$$f(x, y) = e^y \ln x$$

Determine whether or not the Hessian $hess_f(a)$ for each of the previous functions is positive definite, negative definite or neither.

Determine the critical points of each of the previous functions.

Using the general 2nd derivative test and the functions $f(\vec{x})$ from the previous problem, determine whether or not $f(\vec{a})$ is a local minimum or maximum for $f(\vec{x})$ or if the test is inconclusive for the critical point $\vec{a}$.

Define positive definite and negative definite.

Explain why $x^2 - y^2 s$ is not positive definite.

Explain how

$$A(a) = \begin{bmatrix} \frac{\partial^2 f(\vec{a})}{\partial x_1^2} & \frac{\partial^2 f(\vec{a})}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\vec{a})}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\vec{a})}{\partial x_2 \partial x_1} & \frac{\partial^2 f(\vec{a})}{\partial x_2^2} & \cdots & \frac{\partial^2 f(\vec{a})}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\vec{a})}{\partial x_m \partial x_1} & \frac{\partial^2 f(\vec{a})}{\partial x_m \partial x_2} & \cdots & \frac{\partial^2 f(\vec{a})}{\partial x_n^2} \end{bmatrix} \tag{1}$$

may be positive definite but $\vec{a}$ not a critical point of $f(\vec{x})$

Which of the following transformations are one-to-one?

- $T(u, v) = (u^2 - v^2, u + v)$
- $T(u, v) = (u^2 - v^2, u + v)$, $u \geq 0$, $v \leq 0$.
- $T(u, v) = (\sin(u + v), \cos(u - v))$
- $T(u, v) = (\sin(u + v), \cos(u - v))$, $\frac{\pi}{6} \leq u \leq \frac{\pi}{3}$ and $\frac{\pi}{6} \leq v \leq \frac{\pi}{3}$.

Determine whether or not the Jacobian vanishes at any points for the above transformations.

Does a change of variable given by a transformation $T(u, v) = (x(u, v), y(u, v))$ need to be a linear transformation?

Compute Jacobian of the following transformation

$T(u, v) = \left(\frac{u+1}{v-1}, u^2 - v^2\right)$

$T(u, v, w) = (w - u + v, uv, ve^u)$, $u, v, w \geq 0$.

Compute the following $\iint_A \sqrt{x + y}\, dxdy$ $A = [0, 1] \times [0, 1]$.

If $X = [0, 1] \times [0, 1]$, determine the region one gets under the following transformations. $T(u, v) = (u - v, u + v)$

$T(u, v) = (u - 2v, u + v)$

$T(u, v) = (2u - v, u + v)$

$T(u, v) = (u - v, 2u + v)$

$T(u, v) = (u - v, u + 2v)$

What can you say about the effect of $T(u, v) = (au - bv, cu + dv)$?

Define a rotation that will rotate a region in the plane by $\pi/6$ radians.

How does $(u - v, u + v)$ compare to

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

How does

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

compare to

$$\begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}?$$

What is the effect of

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$
$$T_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

and

$$T_3 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

on vectors $(u, v, w) \in \mathbb{R}^3$?