Type 1: Single-depot slow-charging e-Bus VSP

Table 1 A simple illustration of an electric vehicle transit schedule

Activity	Start time	Starting location	Ending location
Pull-out	5:00 am	Depot	A
Trip	6:00 am	A	B
Trip	6:40 am	B	A
Trip	7:15 am	A	B
Trip	8:00 am	B	A
Pull-in	8:40 am	A	Depot
……	…..	…..	…..
Trip	:15 pm	B	A
Pull-in	12 am	A	Depot

Mathematical modeling

Modeling assumption

(1) The re-charging operation is initiated when the remaining energy in battery electric buses is no less than the minimum energy.
(2) The electric buses are charged ahead to reserve energy for peak hours to operate according to the previously established timetable.
(3) The number of electric buses are fixed and all buses are standardized in terms of driving range, size and loading capacity.
(4) The energy consumed and energy recharged are proportional to the driving distance and charging duration respectively.
(5) The battery recharging is a continuous process and has a fixed duration.
(6) The depot location has enough capacity to accommodate all electric buses arriving for charging operation.

Formulation

Although the problem is a single depot VSP, for simplicity in notation the depot is represented as origin depot *'O'* and destination depot *'D'*. Let *'S'* be the set of scheduled trips to be served by an electric bus along a certain route. Let 'R' be the time required for charging an electric bus at a depot location. Deadheading time is the travel time of bus from the end point of a trip to depot location or from depot location to the starting point of a trip. Let d be the deadheading travel time from the end or starting point of trip $i \epsilon S$ to the depot location.

A mixed integer linear programming model formulated for single depot electric bus scheduling is as follows:

$$minC = \sum_{i \in S} (C_d d + C_0 + C_r R) Y_i \tag{1}$$

Subject to:

$$E_i + (\theta R - d) Y_i \leq g \; \forall \; i \in S, \tag{2}$$

$$E_{i+1} = E_i + (kR - d) Y_i - l \; \forall \; i \in S, \tag{3}$$

$$E_i \geq e_{min}, \forall i \in S \tag{4}$$

$$Y_i = \{0, 1\}, \forall i \; \in S \tag{5}$$

$$E_i = g_0, \forall i \; \in O \tag{6}$$

The objective function defined in Equation (1) minimizes the per day electric bus operating cost. The first part of objective function includes costs related to deadheading time and recharging costs. The second part of objective function includes cost of charging infrastructure at depot. The first constraint in equation (2) suggest that the maximum driving range of the electric bus is the sum of driving distance with remaining and recharged energy. Constraint in equation (3) brings system equilibrium ensuring the remaining energy after trip *i+1* is the summation of remaining and recharged energy after trip *i* less the energy consumed in the trip *i+1*. Constraint in equation (4) ensures that the remaining energy in an electric bus is no less than the minimum energy. Constraint in Equation (5) is a binary constraint and Equation (6) sets an initial range for electric bus at the depot.

Parameters	
g	maximum driving range for a fully-charged electric bus, in km;
g_0	the initial range for an electric bus at the depot, in km;
k	recharging rate, i.e., the extended driving distance using energy charged per minute, in km/min;
d	Dead-heading distance, in km;
e_{min}	extended driving distance using the minimum energy in an electric bus, in km;
C_d	cost of unit driving distance, in dollars/km;
C_0	fixed cost per recharging activity; refers to charger startup and operation in dollars;
C_r	variable recharging costs in unit time; refers to the electricity costs in dollars/min;
R	time required for charging an electric bus at a depot location, in min
Variable	
Y_i	=1 if the electric bus from trip $i \in S$ starts being recharged at depot; 0 otherwise