Scheduling of Electric Buses

Introduction and background

The economies of an urban bus transit system are critically governed by vehicle scheduling involving large scale deployment of resources. Vehicle scheduling problem (VSP) or blocking problem involves assigning vehicles to serve a set of predetermined trips associated with a timetable subjected to operational constraints (1). A daily schedule or block of a vehicle is identified by its pull-out time from the depot and pull-in time to the depot while serving a sequence of time-tabled trips (including dead-heading trips). Typically, the buses operating within urban areas initiate and terminate its daily operation at a single garage location, suggesting a simple single vehicle block assignment in VSP. However, the operation of electric buses includes a relatively complex multiple vehicle block assignment to accommodate continuous charging throughout the day, as its range is critically limited by the battery capacity (2). Essentially, the two types of charging strategy are implemented to regulate the maximum daily range requirement namely in-depot charging and on-route charging (both overhead conductive and wireless) (3). Although, the on-route charging offers higher operating range than in-depot charging, the commonly adopted pulse system of bus operation by transit agencies reduces the system efficiency. Typically, in a pulse system of operation, multiple buses serving different routes are scheduled to depart from the transit center at the same time; thus, suggesting overlapping of layover time or charging time available for buses. Consequently, inadequate number of chargers and improper coordination may result in increased delay in operation. Thus, considering the simplicity in operation this study focuses on development of an in-depot charging based e-bus scheduling problem for GRT agency.

The existing literature suggests vehicle scheduling as well-studied optimization problem implementing both heuristic and non-heuristic optimization techniques (4, 5, 6, 7). The classical VSP based on in-depot charging is categorized into two: Single-depot VSP (SDVSP) and Multi-depot VSP (MDVSP). SDVSP assigns vehicle trips from single-depot to multiple routes and MDVSP assigns vehicle trips from multiple-depots to multiple routes. The Single-depot VSP and Multi-depot VSP are generally mixed integer linear optimization problem. The commonly defined objective of a VSP is to minimize the fleet size (i.e., capital cost) and operating costs restricted to several constraints in operation. However, these optimizations are generally identified as NP hard problem and needs heuristic and non-heuristic algorithm to solve it (8, 9). Further, in addition to the classic constraints of vehicle scheduling, the electric bus scheduling models considers specific constraints for charging and the NP-hardness of an electric VSP is proven (10). Although a large number of studies have focused on optimizing an electric bus scheduling considering fast charging options on-route (11) or in-depot (12), no study has distinctly focused on slow charging Single-depot VSP. This study aims to develop a slow charging based single-depot VSP for 24 routes in GRT and further clarification to this consideration is provided in the section below. A core objective of GRT project is to assess the technical feasibility of deploying slow charging battery electric buses on specific routes of GRT system. The study methodology would enable us in defining a schedule of activity performed by each bus in the system for a given timetable and charging constraints. The nature (i.e., heuristic and non-heuristic) of optimization techniques to be adopted to solve the VSP would be decided based on a clearly defined objective functions and the constraints.

GRT route specifics

The GRT project includes modeling for electrification of 24 bus routes. The spatial distribution of the suggested routes for development and garage locations in GRT system is shown in the Figure below. It is evident from the figure that, all suggested route for electrification are spatially aggregated around the garage/depot in 250 Strasburg Road. Thus, suggesting for development of a Single-depot VSP.

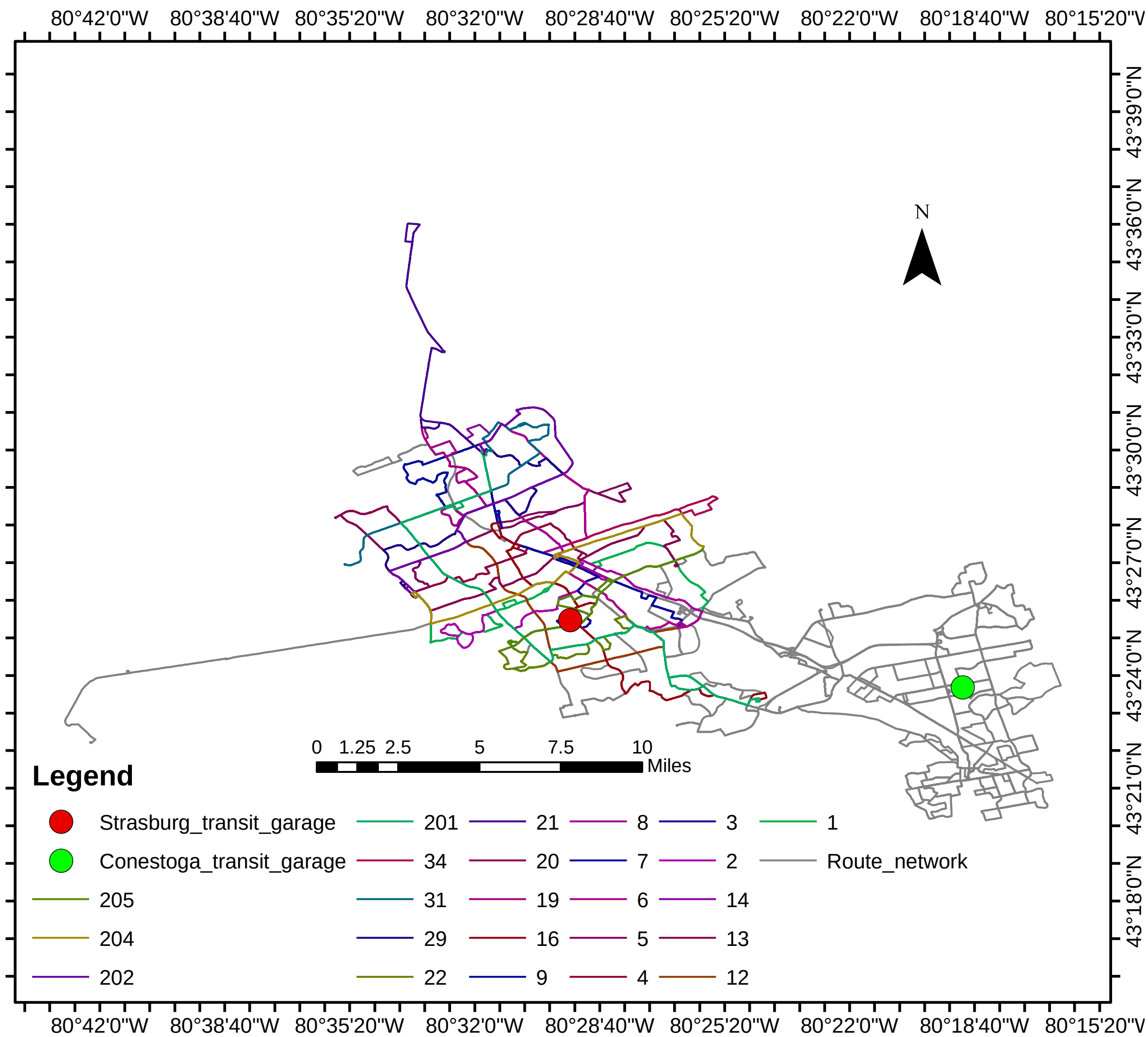

Figure A schematic representation of spatial distribution of routes and facilities of GRT system

Problem definition of electric vehicle scheduling

The electric bus scheduling problem is classified as a special case of classic VSP taking into consideration the additional objectives and constraints specific to electric buses in addition to the traditional VSP constraints (2). Given a set of timetabled trips with fixed departure and arrival times, start and end locations and operating time to reach terminal stations, the objective of a classic VSP is to assign trips to vehicles such that

- Each trip is covered exactly once
- Each vehicle is assigned to perform a sequence of timetabled trips
- The total cost is minimized

The total cost of bus transit operation is expressed as the sum of fixed costs (i.e., cost of vehicle acquisition and maintenance) and variable or operating cost (i.e., fuel and attrition). However, for an electric VSP the additional constraints considered are as follows.

- A lower limit of energy in the battery is defined to ensure arrival to the next charging station or depot.
- The time required for charging at the depot indicating a non-operational period for each bus. The consideration of fully charged battery replacement operation could also be implemented in the model as a function of time.

The consideration of cost of battery depreciation in the electric VSP could also be implemented in an advanced model. A simple illustration of a set of timetabled trips performed by a single bus in the route based on a given charging constraints is as shown in the table 1 below.

Table 1 A simple illustration of an electric vehicle transit schedule

Activity	Start time	Starting location	Ending location
Pull-out	5:00 am	Depot	A
Trip	6:00 am	A	B
Trip	6:40 am	B	A
Trip	7:15 am	A	B
Trip	8:00 am	B	A
Pull-in	8:40 am	A	Depot
……	…..	…..	…..
Trip	:15 pm	B	A
Pull-in	12 am	A	Depot

Mathematical modeling

Modeling assumption

(1) The re-charging operation is initiated when the remaining energy in battery electric buses is no less than the minimum energy.

(2) The electric buses are charged ahead to reserve energy for peak hours to operate according to the previously established timetable.
(3) The number of electric buses are fixed and all buses are standardized in terms of driving range, size and loading capacity.
(4) The energy consumed and energy recharged are proportional to the driving distance and charging duration respectively.
(5) The battery recharging is a continuous process and has a fixed duration.
(6) The depot location has enough capacity to accommodate all electric buses arriving for charging operation.

Formulation

Although the problem is a single depot VSP, for simplicity in notation the depot is represented as origin depot *'O'* and destination depot *'D'*. For *'S'* set of scheduled trips, let a_i and b_i be the start and end time of trip i. The capacity at a depot location at any time is evaluated for 'T' set of time nodes discretized into one time step (e.g., one minute). The 'T' set of time nodes is defined from the start time of the earliest trip to the end time of the latest trip. Let 'R' be the time required for charging an electric bus at a depot location. Thus, for each recharging operation, the starting time of recharging can be represented as 't' ($t \in T$) and the ending time is represented as $t + R$. Deadheading time is the travel time of bus from the end point of a trip to depot location or from depot location to the starting point of a trip. Let d be the deadheading travel time from the end or starting point of trip $i \in S$ to the depot location. Let P be the set of trip-pairs, $(i, j) \in P$, which means that trip $j \in S \cup D$ is served immediately after trip $i \in S \cup O$ by the same electric bus. For a trip pair $(i, j) \in P$, the trip interval is defined as the difference between start time of trip j and end time of trip i (i.e., $a_j - b_i$). Let A be the set of recharging activity at depot location, where $(i, j, t) \in A$, which means that the electric bus is recharged at depot during the trip interval between trip-pair $(i, j) \in P$, from time $t \in T$ to $t + R$, if $t \geq b_i + d$ and $t + R + d \leq a_j$. The two defined constraints guarantee no delay in the predefined time table. Q_{ijt} is the time spent by bus in queue for charging at the depot location. For an electric bus which serves trip i, the arrival time at the depot location should be $b_i + d$. If the bus starts recharging at time node $t \in T$, the time spent by bus in queue $Q_{ijt} = t - b_i - d, (i, j, t) \in A$.

A mixed integer linear programming model formulated for single depot electric bus scheduling is as follows:

$$minC = \sum_{i \in S} (C_d d + C_0 + C_r R)\bar{T}Y_i + \sum_{k=1}^{K} \sum_{(i,j,t) \in A} C_q Q_{ijt} \bar{T} X_{it}^k \tag{1}$$

Subject to:

$$E_i + (\theta R - d)Y_i \leq \beta \ \forall \ i \in S, \tag{2}$$

$$E_j = E_i + (\theta R - d)Y_i - l_j \ \forall \ (i, j) \in P, \tag{3}$$

$$E_i \geq e_{min}, \forall i \in S \tag{4}$$

$$\mathrm{Y_i} = \{0, 1\}, \forall \mathrm{i} \ \in \mathrm{S} \tag{5}$$

$$E_i = \beta_0, \forall i \ \in O \tag{6}$$

$$\sum_{k=1}^{K} \sum_{t:\,(i,j,t)\in A} X_{it}^{k} = Y_i \,, \forall i \in S \tag{7}$$

$$\sum_{i:(i,j,t')\epsilon A} \sum_{t'=t-R+1}^{t} X_{it'}^{k} \leq 1, \forall k = 1, 2, 3, 4, \ldots\ldots K, \forall t \in T \tag{8}$$

$$X_{it}^{k} = \{0, 1\},, \forall \mathrm{i} \ \in \mathrm{S}, \forall k = 1, 2, 3, 4, \ldots\ldots K, \forall t \in T \tag{9}$$

The objective function defined in Equation (1) minimizes the total annual electric bus operating cost. The first part of objective function includes costs related to deadheading time and recharging costs. The second part of objective function includes cost of charging infrastructure at depot. The first constraint in equation (2) suggest that the maximum driving range of the electric bus is the sum of driving distance with remaining and recharged energy. Constraint in equation (3) brings system equilibrium ensuring the remaining energy after trip *j* is the summation of remaining and recharged energy after trip *i* less the energy consumed in the trip *j*. Constraint in equation (4) ensures that the remaining energy in an electric bus is no less than the minimum energy. Constraint in Equation (5) is a binary constraint and Equation (6) sets an initial range for electric bus at the depot. Constraint in Equation (7) defines the relationship between variables *Y* and *X*. If an electric bus from trip *i* is recharged at the depot ($Y_i = 1$), there must be a *t* and a *k*, which enable $X_{it}^{k} = 1$; otherwise, all $X_{it}^{k} = 1$. Constraint in Equation (8) ensures that no more than one electric bus can be recharged on each individual charger at any time; in other words, the charging station capacity constraint has to be satisfied.

Parameters	
β	maximum driving range for a fully-charged electric bus, in km;
β_0	the initial range for an electric bus at the depot, in km;
θ	recharging rate, i.e., the extended driving distance using energy charged per minute, in km/min;
d	Dead-heading distance, in km;
e_{min}	extended driving distance using the minimum energy in an electric bus, in km;
C_d	cost of unit driving distance, in dollars/km;

C_0	fixed cost per recharging activity; refers to charger startup and operation in dollars;
C_r	variable recharging costs in unit time; refers to the electricity costs in dollars/min;
$\bar{T}$	number of operating days per year;
R	time required for charging an electric bus at a depot location, in min
C_q	cost of unit time spent in queue, in dollars/min;
Variable	
Y_i	=1 if the electric bus from trip $i \in S$ starts being recharged at depot at time t ϵ T; 0 otherwise
X_{it}^k	=1 if the electric bus from trip $i \in S$ starts being recharged on *k*th charger at charging station $n \in N$ at time t ϵ T, k=*1, 2, 3, 4,…..K*
Q_{ijt}	time spent by bus in queue for charging at the depot location

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