Project Dragonfly SNR Calculations

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v.1 - original document
v.2 - revised following detection of a bug by Karl Glazebrook. Also added a double check based on the NICMOS simulator.

Setup calculations

In[1]:=
```
Needs["Units`"]
```

I need to define a magnitude explicitly because *Mathematica* doesn't have it built-in.

In[2]:=
```
Magnitude::usage =
  "The magnitude is a dimensionless unit of brightness beloved by astronomers.";
```

Here is a table from Bessel (1979) giving the flux from a zeroth magnitude object at the top of the atmosphere. along with some useful quantities for converting to photon rates:

Band	λ_c	$\frac{\Delta\lambda}{\lambda}$	F_ν
–	(μm)	–	(Jy)
U	0.36	0.15	1810
B	0.44	0.22	4260
V	0.55	0.16	3640
R	0.64	0.23	3080
I	0.79	0.19	2250

The following relationships are useful:

1 Jy = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1}
1 Jy = 1.51×10^7 photon s^{-1} m^{-2}($\Delta\lambda/\lambda$)$^{-1}$

Here is the corresponding number of counts for an object with a given mean surface brightness μ subtending an angular size Ω seen through a telescope of aperture D for t seconds.

In[3]:=
```
counts[t_, μ_, D_, Ω_] := Convert[(10^(-0.4 × μ/(Magnitude/ArcSecond^2))) × (0.16) ×
  (3640 × 1.51 × 10^7 × Meter^-2 Second^-1 ArcSecond^-2) × π (D/2)^2 × Ω × t, 1]
```

For example, a 30th mag/arcsec^2 object subtending 10 square arcseconds will have the following counts in 1000 seconds in a six inch (15 cm) telescope:

In[4]:=
```
counts[1000 Second, 30 Magnitude / ArcSecond^2, 15 Centi Meter, 10 ArcSecond^2]
```

Out[4]=
```
1.55407
```

Double check

Radiometric calculations can be done using this web site:

http://www.stsci.edu/hst/nicmos/tools/conversion_form.html

For the situation above…

NICMOS Units Conversion Tool Results

```
INPUT FROM THE FORM IS
Input units = magnitude V
Output units = photons/cm2/s/A
Input flux =    30.0000 magnitude V
Temperature of the blackbody =    5500.00
INPUT wavelength =    0.550000 micron
OUTPUT wavelength =    0.550000 micron
Flux=9.72E-10 photons/cm2/s/A
```

According to this, a 30th mag/arcsec^2 object subtending 10 square arcseconds will have the following counts in 1000 seconds in a six inch (15 cm) telescope:

In[5]:=
```
sb = 9.72 × 10^-10 (Centi Meter)^-2 Second^-1 Angstrom^-1 ArcSecond^-2;
Ω = 10 ArcSecond^2;
aperture = 15 Centi Meter;
texp = 1000 Second;
Δλ = 1600 Angstrom;
mirrorArea = π (aperture/2)^2;
```

In[11]:=
```
referenceCounts = sb × mirrorArea × Ω × texp × Δλ
```

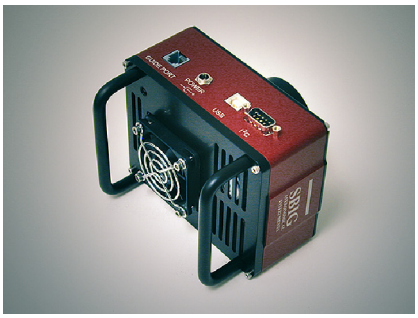
Out[11]=
```
2.74827
```

Note that the agreement is only good to within a factor of two, with the NICMOS calculator being more optimistic than my calculations.

SNR

Assumptions

In all the calculations below I'll assume the sky is 22.5 mag/arcsec^2 which is about right for a dark site. I'll explore a range of integration times from 1h to 50h. I've actually seen quite a few 50h integrations of pretty galaxies in Sky and Telescope, so I guess that's the sort of integration time a dedicated lunatic ought to be able to acquire. I assume we use a Canon 400mm f/2.8 lens and an SBIG ST-8300M camera.



In[12]:=
```
skySB = 22.5 Magnitude / ArcSecond^2;
aperture = 15 Centi Meter;
```

Note that 50h corresponds to 180,000s, and that if you can target an object for 5h/night one can get there in 20 nights.

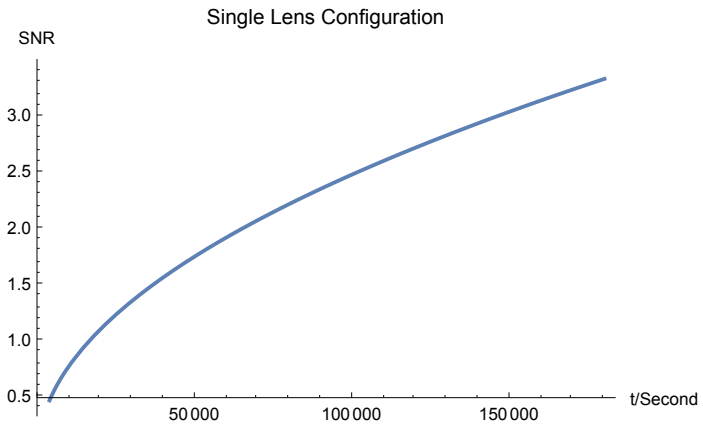
V=28 mag / arcsec^2 structure in a pixel area of 7.84 arcsec^2 (unbinned Canon 400 f/2.8 lens with Kodak 8300 detector)

In[14]:=
```
area = 10 ArcSecond^2;
streamSB = 28 Magnitude / ArcSecond^2;
```

1 lens configuration

In[16]:=
```
nlens = 1;
l1 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600},
  AxesLabel → {"t/Second", "SNR"}, PlotLabel → "Single Lens Configuration"]
```

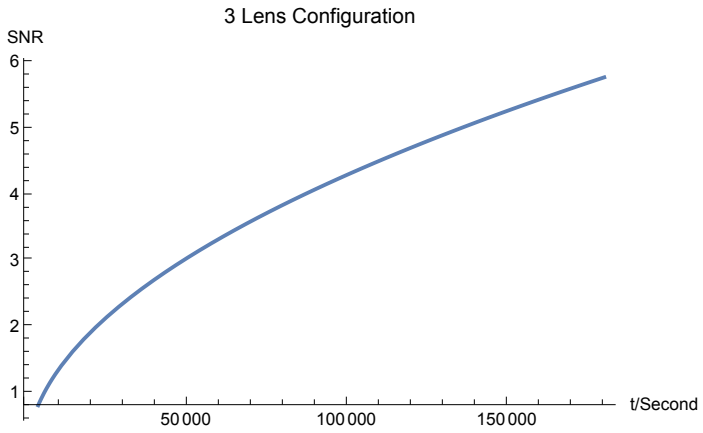
Out[17]=



3 lens configuration

In[18]:=
```
nlens = 3;
l3 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600},
  AxesLabel → {"t/Second", "SNR"}, PlotLabel → "3 Lens Configuration"]
```

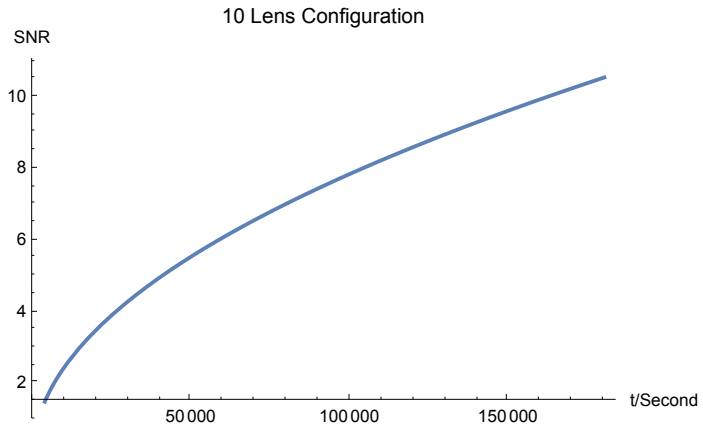
Out[19]=



10 lens configuration:

In[20]:=
```
nlens = 10;
l10 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600},
  AxesLabel → {"t/Second", "SNR"}, PlotLabel → "10 Lens Configuration"]
```

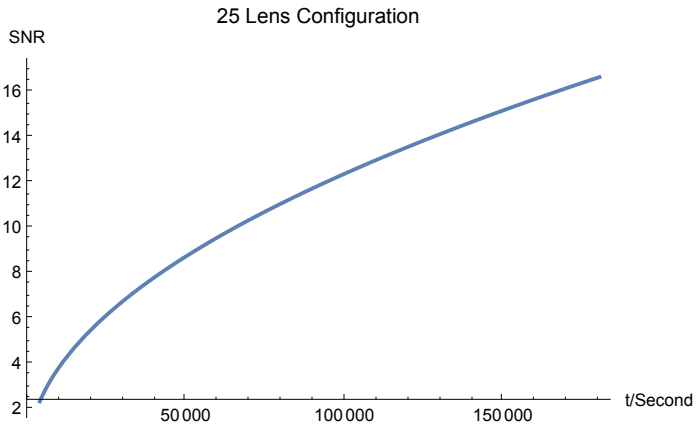
Out[21]=



25 lens configuration:

In[22]:=
```
nlens = 25;
l25 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600},
  AxesLabel → {"t/Second", "SNR"}, PlotLabel → "25 Lens Configuration"]
```

Out[23]=



All

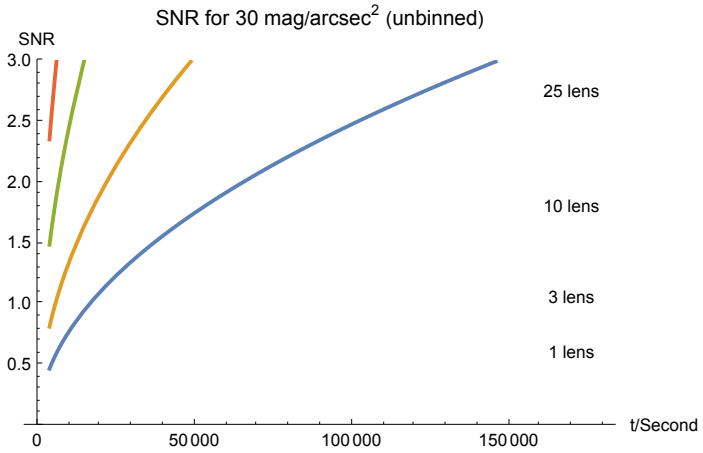
In[24]:=
```
Needs["PlotLegends`"]
```

General: PlotLegends` is now obsolete. The legacy version being loaded may conflict with current functionality. See the Compatibility Guide for updating information.

In[25]:=
```
lAll = Plot[
  {
    counts[t Second, streamSB, aperture, area]
   ----------------------------------------------,
   Sqrt[ counts[t Second, skySB, aperture, area]]

    3 counts[t Second, streamSB, aperture, area]
   ----------------------------------------------,
   Sqrt[ 3 counts[t Second, skySB, aperture, area]]
    10 counts[t Second, streamSB, aperture, area]
   -----------------------------------------------,
   Sqrt[ 10 counts[t Second, skySB, aperture, area]]
    25 counts[t Second, streamSB, aperture, area]
   -----------------------------------------------
   Sqrt[ 25 counts[t Second, skySB, aperture, area]]
  },
  {t, 3600, 50 × 3600 }, AxesLabel → {"t/Second", "SNR"},
  Epilog → {Text["1 lens", {170 000, 0.6}], Text["3 lens", {170 000, 1.05}],
    Text["10 lens", {170 000, 1.8}], Text["25 lens", {170 000, 2.75}]},
  PlotRange → {0, 3},
  PlotLabel → "SNR for 30 mag/arcsec^2 (unbinned)"]
```

Out[25]=



In[26]:=
```
Export["/Users/abraham/Desktop/dragonfly.pdf", lAll]
```

Out[26]=

/Users/abraham/Desktop/dragonfly.pdf

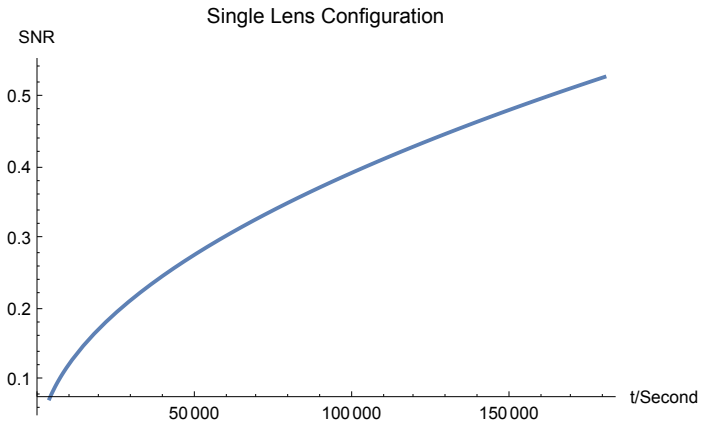
V=30 mag/arcsec^2 structure in a pixel area of 7.84 arcsec^2 (unbinned Canon 400 f/2.8 lens with Kodak 8300 detector)

In[27]:=
```
area = 10 ArcSecond^2;
streamSB = 30 Magnitude / ArcSecond^2;
```

1 lens configuration

In[29]:=
```
nlens = 1;
l1 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600}, AxesLabel → {"t/Second", "SNR"}, PlotLabel → "Single Lens Configuration"]
```

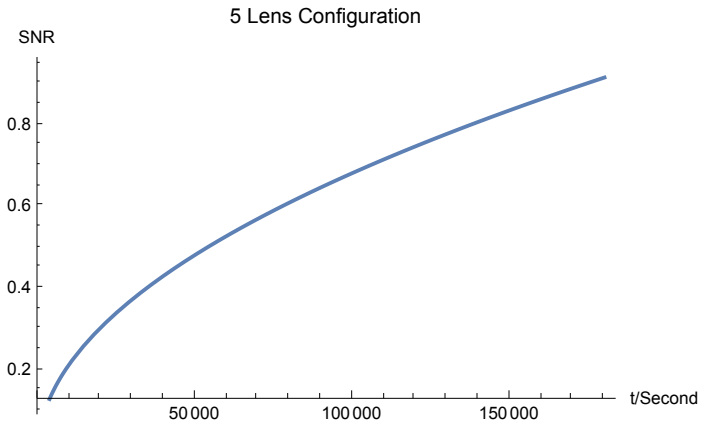
Out[30]=



3 lens configuration

In[31]:=
```
nlens = 3;
l3 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600}, AxesLabel → {"t/Second", "SNR"}, PlotLabel → "5 Lens Configuration"]
```

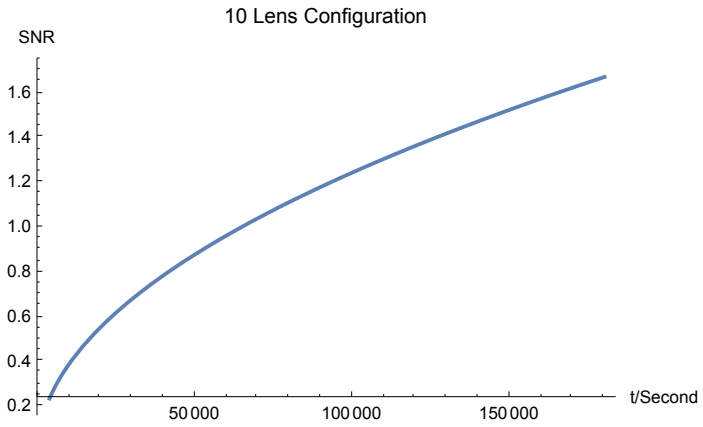
Out[32]=



10 lens configuration:

In[33]:=
```
nlens = 10;
l10 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600},
  AxesLabel → {"t/Second", "SNR"}, PlotLabel → "10 Lens Configuration"]
```

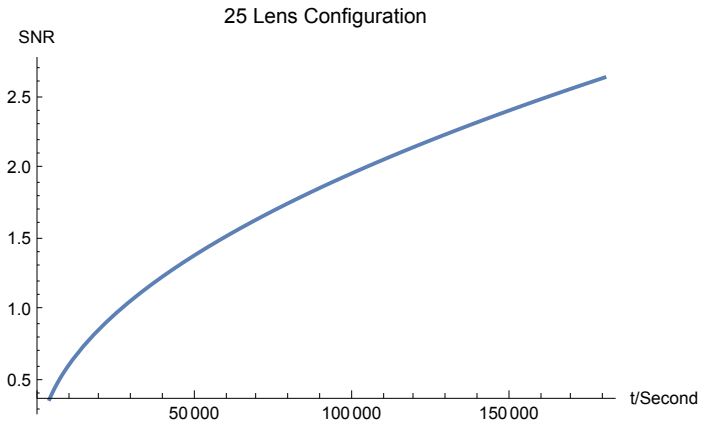
Out[34]=



25 lens configuration:

In[35]:=
```
nlens = 25;
l25 = Plot[nlens counts[t Second, streamSB, aperture, area] / Sqrt[nlens counts[t Second, skySB, aperture, area]], {t, 3600, 50 × 3600},
  AxesLabel → {"t/Second", "SNR"}, PlotLabel → "25 Lens Configuration"]
```

Out[36]=



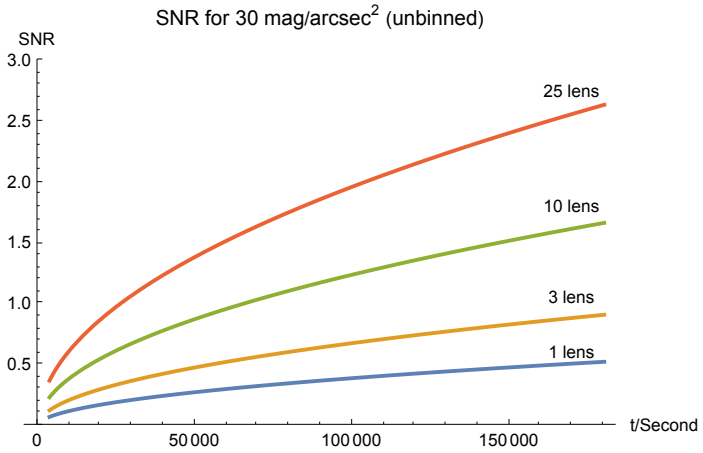
All

In[37]:=
```
Needs["PlotLegends`"]
```

In[38]:=
```
lAll = Plot[
  {
   counts[t Second, streamSB, aperture, area]
  ----------------------------------------------,
  Sqrt[ counts[t Second, skySB, aperture, area]]

   3 counts[t Second, streamSB, aperture, area]
  ----------------------------------------------,
  Sqrt[ 3 counts[t Second, skySB, aperture, area]]
   10 counts[t Second, streamSB, aperture, area]
  ----------------------------------------------,
  Sqrt[ 10 counts[t Second, skySB, aperture, area]]
   25 counts[t Second, streamSB, aperture, area]
  ----------------------------------------------
  Sqrt[ 25 counts[t Second, skySB, aperture, area]]
  },
  {t, 3600, 50 × 3600 }, AxesLabel → {"t/Second", "SNR"},
  Epilog → {Text["1 lens", {170 000, 0.6}], Text["3 lens", {170 000, 1.05}],
    Text["10 lens", {170 000, 1.8}], Text["25 lens", {170 000, 2.75}]},
  PlotRange → {0, 3},
  PlotLabel → "SNR for 30 mag/arcsec^2 (unbinned)"]
```

Out[38]=



In[39]:=
```
Export["/Users/abraham/Desktop/dragonfly.pdf", lAll]
```

Out[39]=

/Users/abraham/Desktop/dragonfly.pdf