# ATIVIDADE - SEMANA 2

(I) DETERMINAR A FAMÍLIA DE SOLUÇÕES E ENCONTRE O PLANO DE FASE PARA:

a) $y'' + 5y' + 4y = 0$

b) $y'' + 4y' + 4y = 0$

c) $y'' + 4y' + 5y = 0$

DADO QUE AS CONDIÇÕES INICIAIS SÃO:

$y(0) = 1 \quad ; \quad y'(0) = 3$

I. a) $y'' + 5y' + 4y = 0$

$$\lambda^2 + 5\lambda + 4 = 0$$

$$\lambda = \frac{-5 \pm \sqrt{25-16}}{2} \Rightarrow \begin{array}{l} \lambda_1 = -2,5 + 1,5 = -1 \\ \lambda_2 = -2,5 - 1,5 = -4 \end{array}$$

$$y = Ae^{-t} + Be^{-4t}$$
$$y' = -Ae^{-t} - 4Be^{-4t}$$

$$y(0) = A + B = 1 \Rightarrow B = 1 - A$$
$$y'(0) = -A - 4B = 3 \Rightarrow -A - 4 + 4A = 3 \Rightarrow 3A = 7$$
$$A = \frac{7}{3} \quad ; \quad B = 1 - \frac{7}{3} = \frac{-4}{3}$$

$$\boxed{y(t) = \frac{7}{3}e^{-t} - \frac{4}{3}e^{-4t}}$$

* PLANO DE FASE: $y'' + 5y' + 4y = 0$

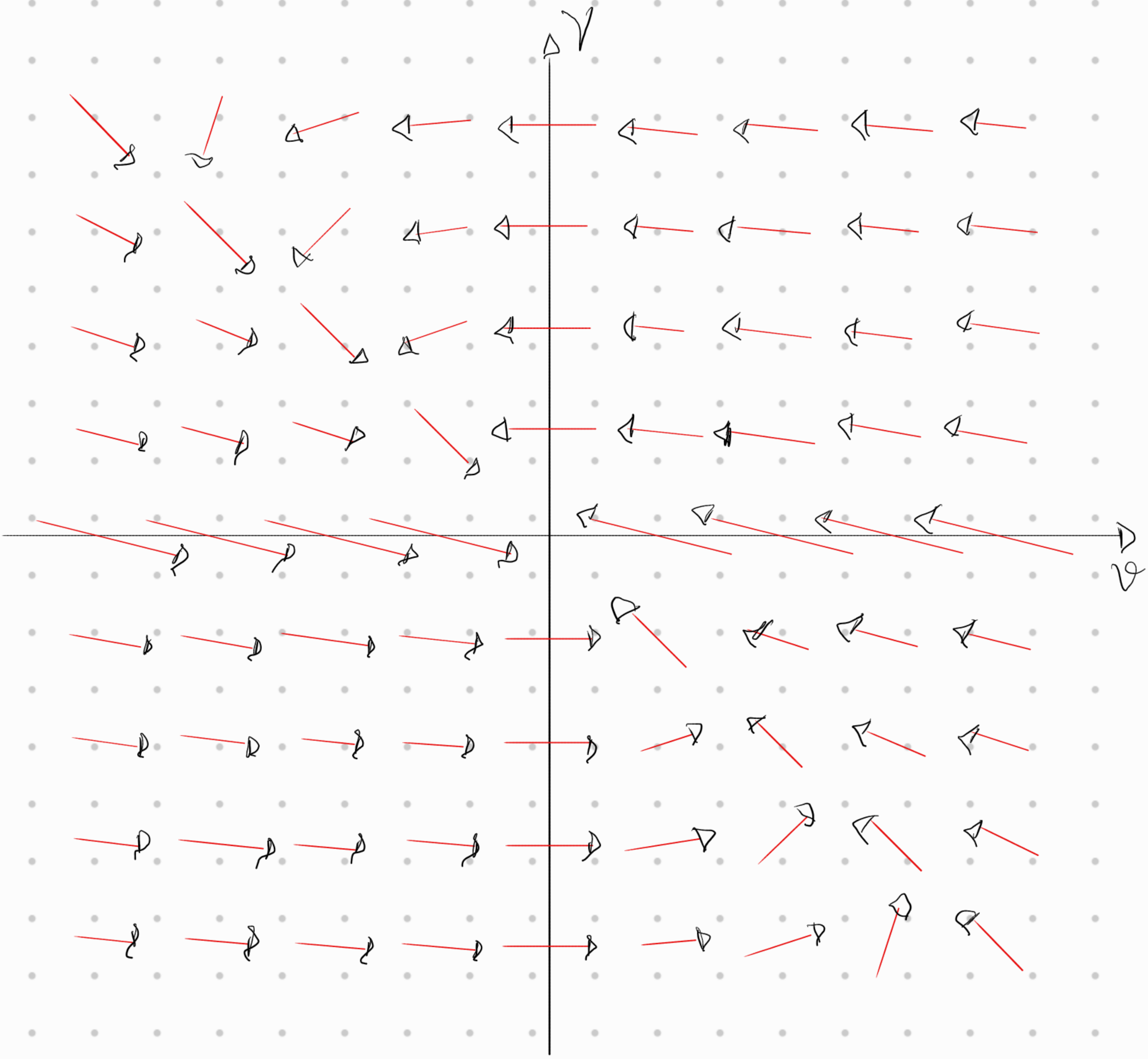
$$v = \frac{dy}{dt}$$

$$\frac{dv}{dt} = \frac{d^2y}{dt^2} = -5\frac{dy}{dt} - 4y = -5v - 4y$$

$$\frac{dy}{dv} = \frac{v}{-5v - 4y} \quad ; \quad \frac{5v}{4y} \neq -1$$

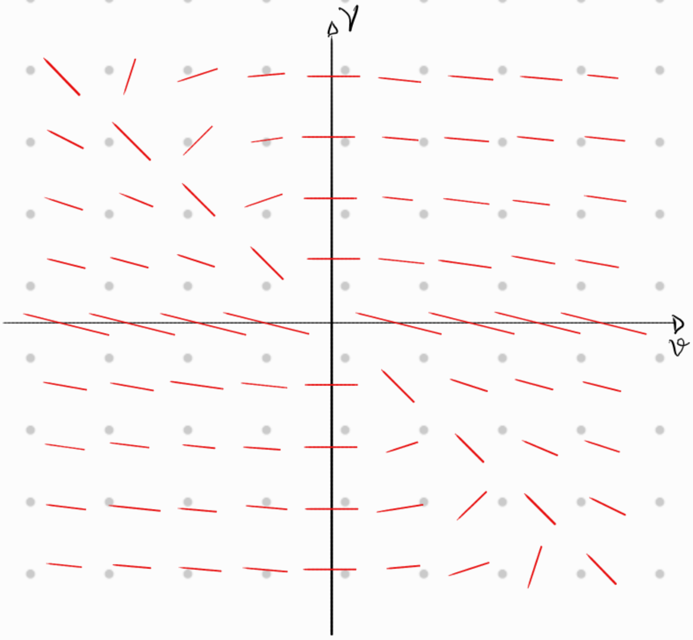
$$\frac{v}{y} \neq -\frac{4}{5} \neq -0{,}8$$

$$\varphi = \mathrm{ARG}\left(\operatorname{arctg}\left(\frac{dy}{dv}\right)\right)$$



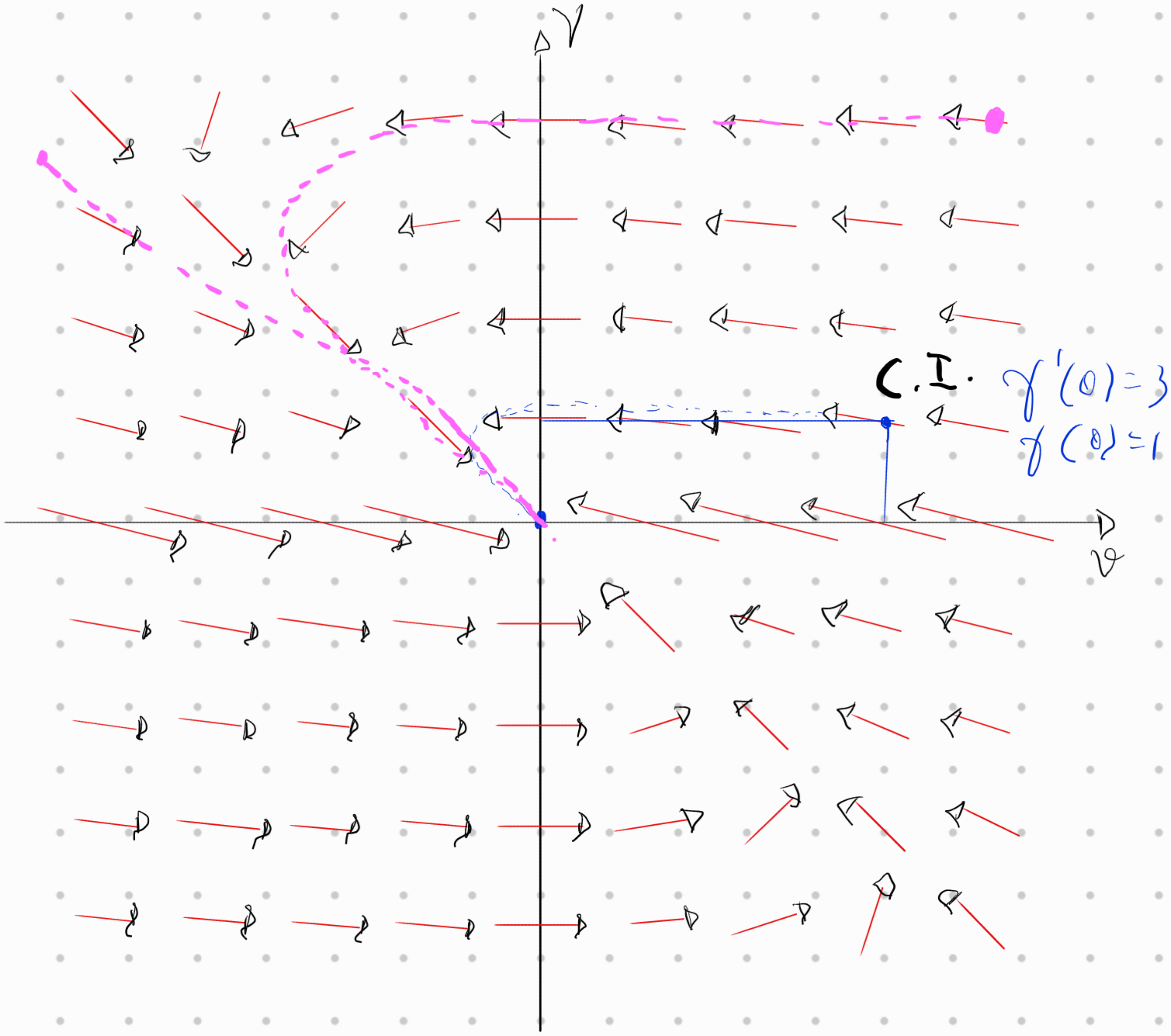
# Cálculos:

| $\vartheta$ | $Y$ | $\frac{dY}{d\vartheta}$ | $\varphi$ |
|---|---|---|---|
| $0^+$ | $Y \neq 0$ | 0 | 180° |
| $0^-$ | $Y \neq 0$ | 0 | 0° |
| $\vartheta \neq 0$ | 0 | $-\frac{1}{5}$ | −11,3° |
| 1 | 1 | −0,111 | −6,34° |
| −1 | 1 | −1 | −45° |
| 2 | 2 | −0,111 | −6,34° |
| −2 | 2 | −1 | −45° |
| 1 | −1 | −1 | −45° |
| −1 | −1 | −0,111 | −6,34° |
| −4 | −1 | −0,166 | −9,462 |
| −3 | −1 | −0,157 | −8,972 |
| −2 | −1 | −0,142 | −8,13 |
| 0 | −1 | 0 | 0° |
| 2 | −1 | −0,333 | −18,43 |
| 3 | −1 | −0,272 | −15,25 |
| 4 | −1 | −0,250 | −14,03 |
| −4 | 1 | −0,25 | −14,03 |
| −3 | 1 | −0,272 | −15,25 |
| −2 | 1 | −0,333 | −18,43 |
| 2 | 1 | −0,142 | −8,13 |
| 3 | 1 | −0,157 | −8,972 |
| 4 | 1 | −0,166 | −9,462 |
| −4 | 2 | | −18,43 |
| −3 | 2 | | −23,19 |
| −2 | 2 | | −45° |
| −1 | 2 | | 18,43 |
| 0 | 2 | | 0 |




$$Y'' + 5Y' + 4Y = 0$$

| | | | |
|---|---|---|---|
| 1 | 2 | | −4,398 |
| 2 | 2 | | −6,34 |
| 3 | 2 | | −7,431 |
| 4 | 2 | | −8,13 |



γ
ϑ
C.I.
γ'(0)=3
γ(0)=1

(I) b) $y'' + 4y' + 4y = 0$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{16-16}}{2} = -2$

Portanto:

$y(t) = Ae^{-2t} + B.te^{-2t}$

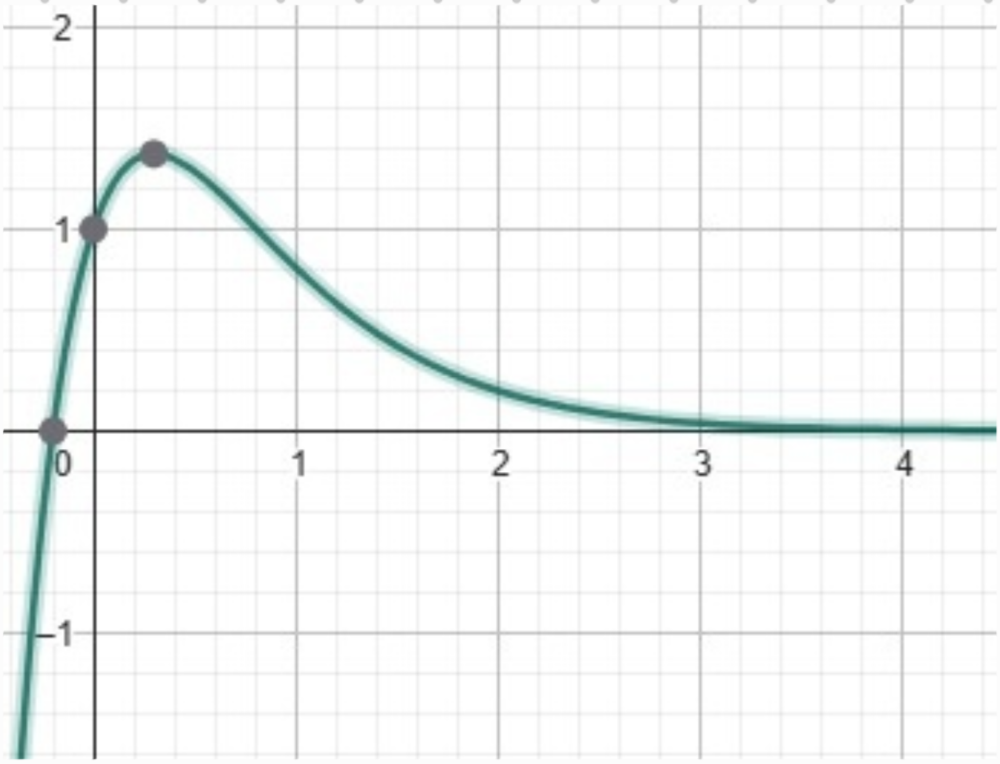
$y'(t) = -2Ae^{-2t} - 2Bte^{-2t} + Be^{-2t} = e^{-2t}(-2A + B - 2Bt)$

$y(0) = A + B.0 = 1 \Rightarrow A = 1$

$y'(0) = (-2.1 + B - 2.B.0) = 3$

$3 = (-2 + B) \Rightarrow B = 5$

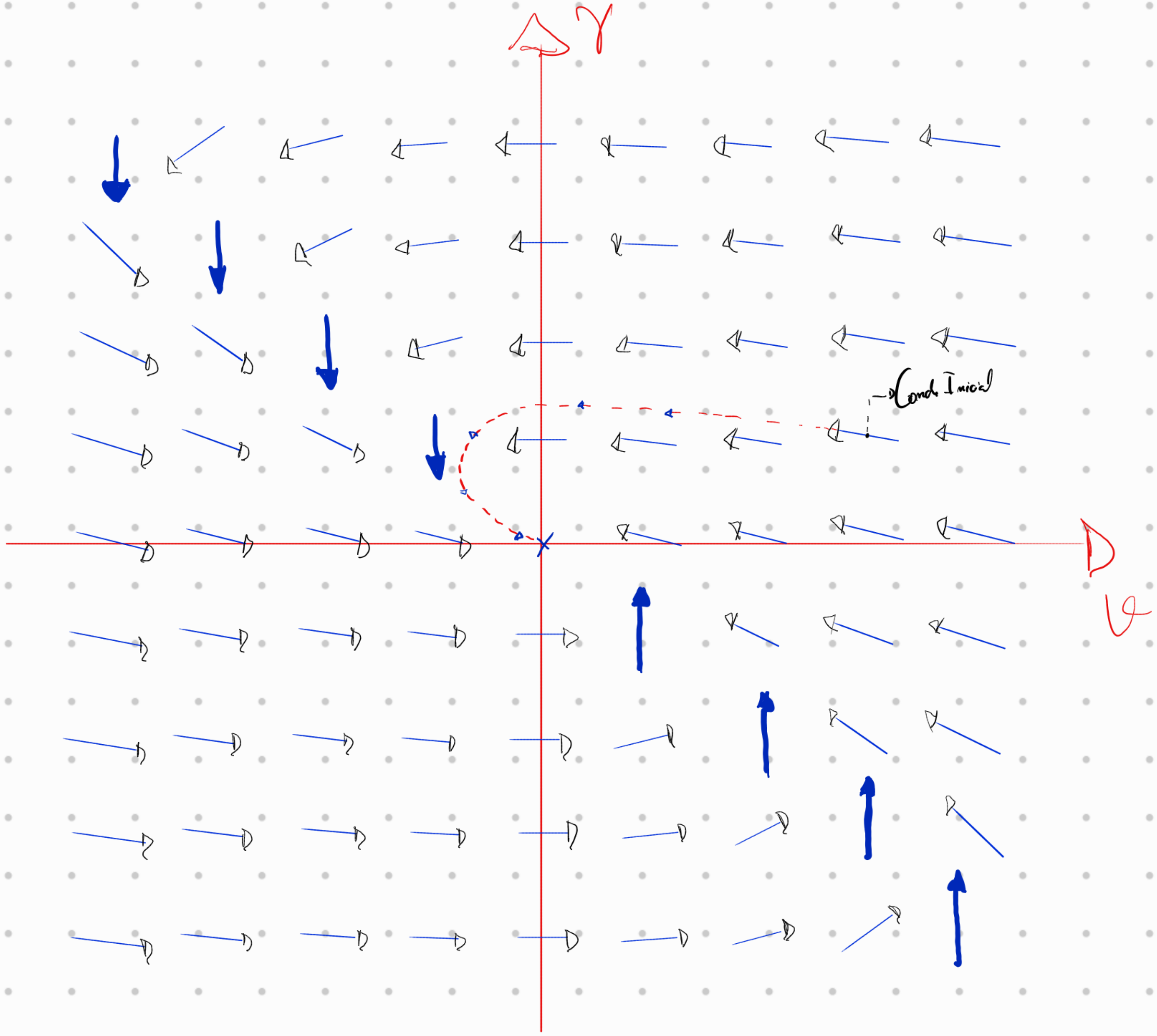
$y(t) = e^{-2t} + 5te^{-2t}$



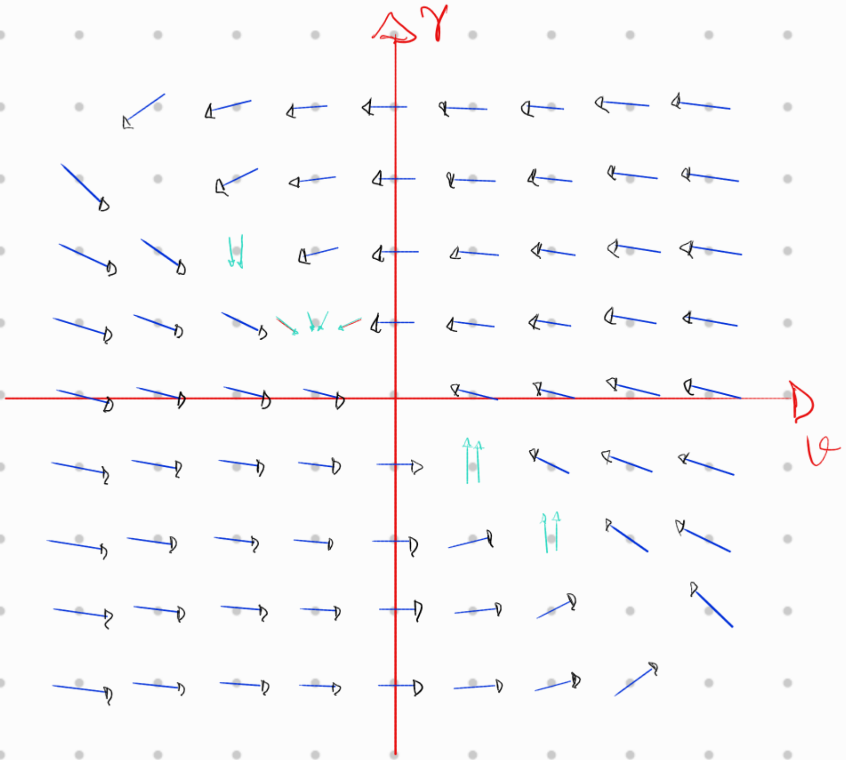
Plano de fase: $y'' + 4y' + 4y = 0$

$$v = y' = \frac{dy}{dt}$$
$$v' = y'' = -4y' - 4y = -4v - 4y = \frac{dv}{dt}$$

$$\frac{dy}{dv} = \frac{v}{-4v - 4y} \quad ; \quad \boxed{v \neq -y}$$



| $v$ | $\gamma$ | $\frac{dv}{d\gamma}$ | $\theta$ |
|---|---|---|---|
| $0^+$ | $\gamma \neq 0$ | 0 | 180° |
| $0^-$ | $\gamma \neq 0$ | 0 | 0° |
| -4 | 1 | | -18 |
| -3 | 1 | | -20 |
| -2 | 1 | | -26 |
| -1 | 1 | × | × |
| 1 | 1 | | -7 |
| 2 | 1 | | -9 |
| 3 | 1 | | -10 |
| 4 | 1 | | -11 |
| -4 | 2 | | -26 |
| -3 | 2 | | -36 |
| -2 | 2 | × | × |
| -1 | 2 | | 14 |
| 1 | 2 | | -4 |
| 2 | 2 | | -7 |
| 3 | 2 | | -8 |
| 4 | 2 | | -9 |
| ⋮ | ⋮ | | ⋮ |



(I) c) $y'' + 4y' + 5y = 0$

$\lambda_1 = -2 + j$

$\lambda_2 = -2 - j$

$y(t) = A e^{(-2+j)t} + B e^{(-2-j)t}$

$y(t) = A \cdot e^{-2t} e^{+jt} + B e^{-2t} e^{-jt}$

$y(t) = e^{-2t} \left(A \cdot (\cos(t) + j\,\text{sen}(t)) + B(\cos(t) - j\,\text{sen}(t))\right)$

$A \cos t + A j\,\text{sen}\,t + B \cos t - B j\,\text{sen}(t)$

$e^{-2t} ((A+B) \cdot \cos t + (A-B)\,\text{sen}\,t)$

$y(t) = e^{-2t} (K_0 \cos t + K_1\,\text{sen}\,t)$

$y'(t) = -2e^{-2t} (K_0 \cos t + K_1\,\text{sen}\,t)$
$\quad + e^{-2t} (K_0 \cdot -\text{sen}\,t + K_1 \cos t)$

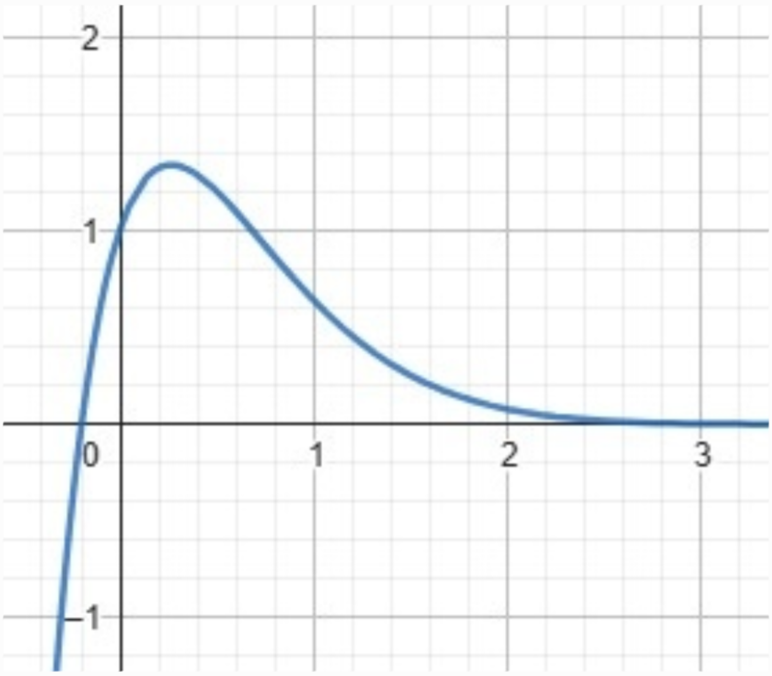
$y(0) = 1 = K_0$

$y'(t) = -2e^{-2t} (\cos t + K_1\,\text{sen}\,t)$
$\quad + e^{-2t} (\text{sen}\,t + K_1 \cos t)$

$y'(0) = 3 = -2(1) + 1(K_1)$

$K_1 = 3 + 2 = 5$

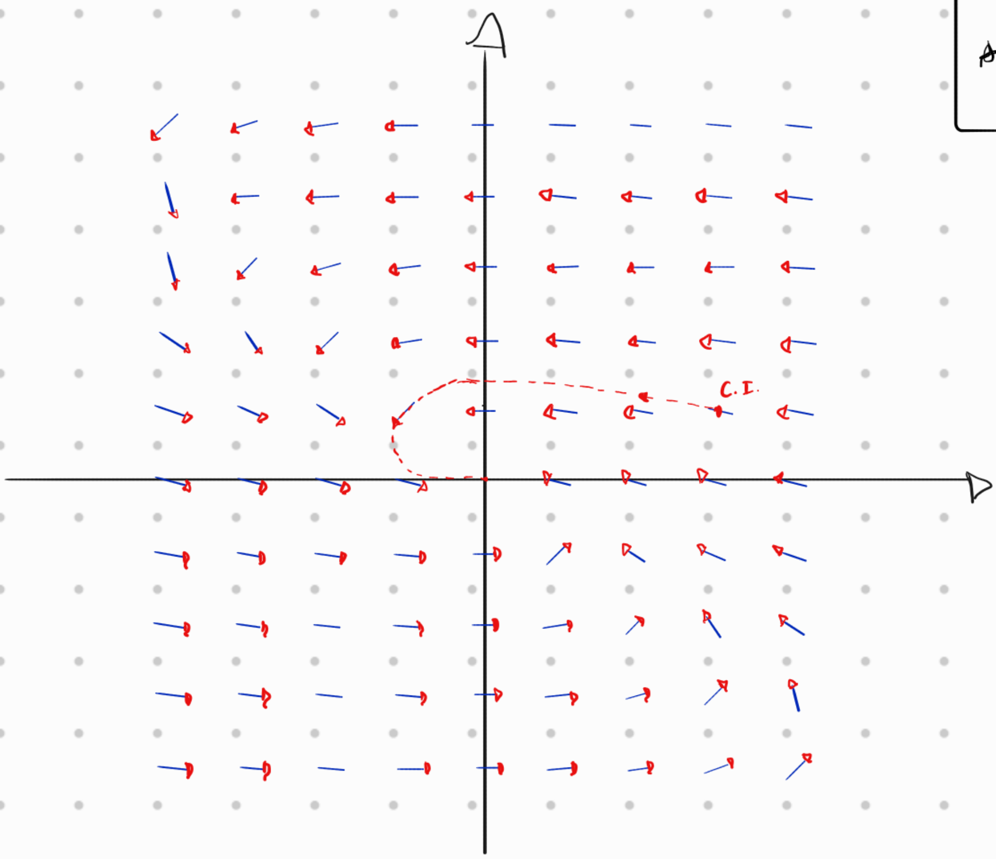
$$\boxed{y(t) = e^{-2t} (\cos t + 5\,\text{sen}\,t)}$$

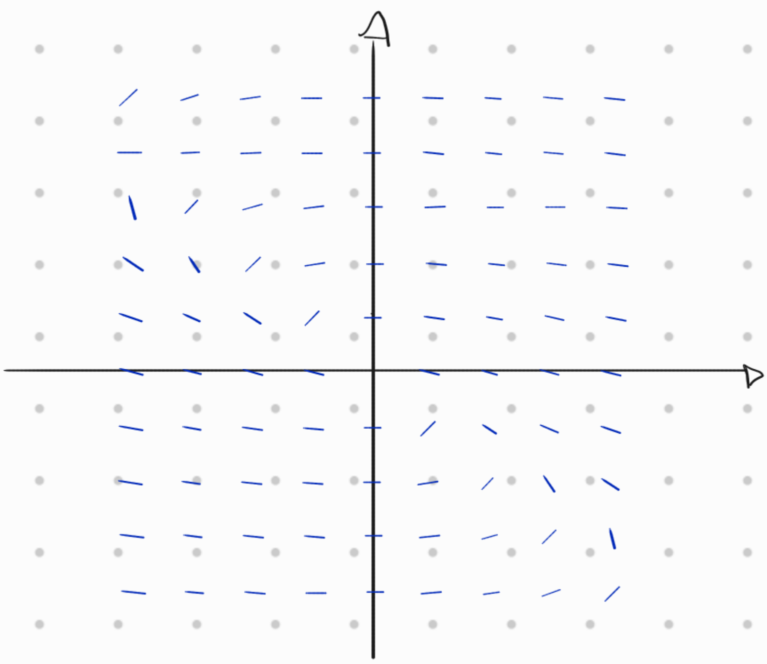


Plano de fase: $y'' + 4y' + 5y = 0$

$$\frac{dv}{dy} = \frac{v}{-4v - 5y}$$

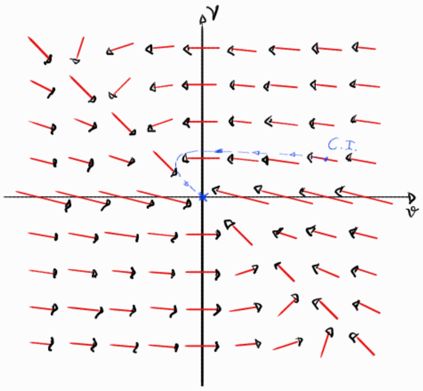
$\boxed{\text{* Calcular no excel}}$

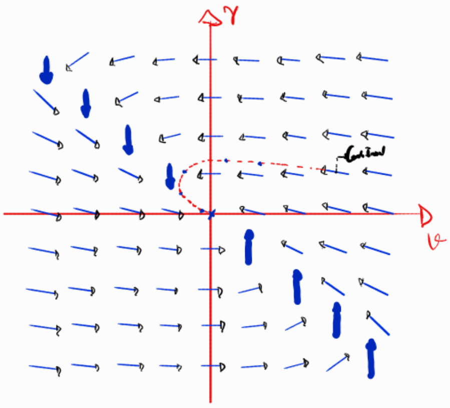


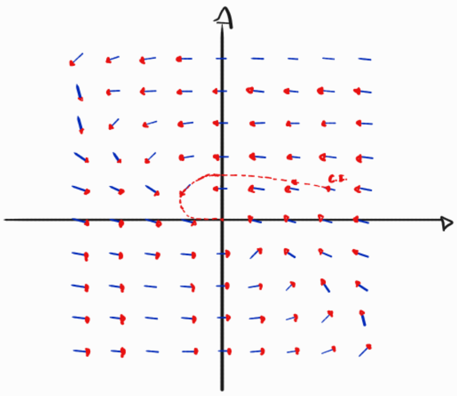



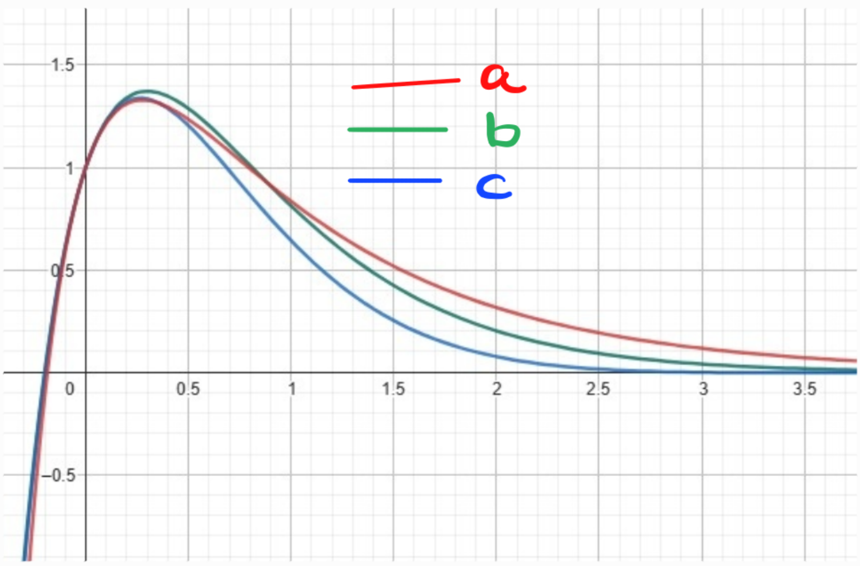
Resultado :

a) $y'' + 5y' + 4y = 0$ | b) $y'' + 4y' + 4y = 0$ | c) $y'' + 4y' + 5y = 0$

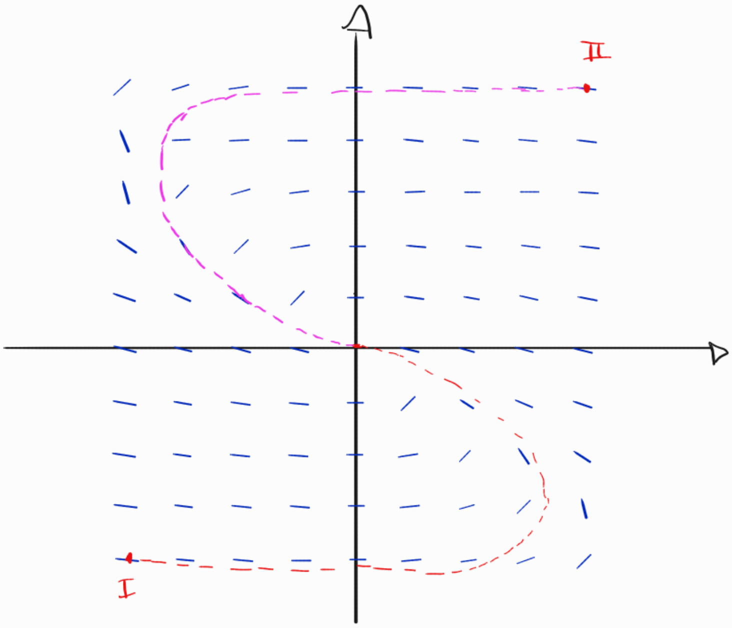




# Outras soluções p/ $y'' + 4y' + 5y = 0$




- Considerando:

(I) $\begin{cases} y'(0) = -4 \\ y(0) = -4 \end{cases}$

(II) $\begin{cases} y'(0) = 4 \\ y'(0) = 4 \end{cases}$

$y(t) = e^{-2t}(K_0 \cos t + K_1 \sin t)$

$y'(t) = -2e^{-2t}(K_0 \cos t + K_1 \sin t) + e^{-2t}(-K_0 \sin t + K_1 \cos t)$

$y(0) = -4 = 1(K_0 \cdot 1)$

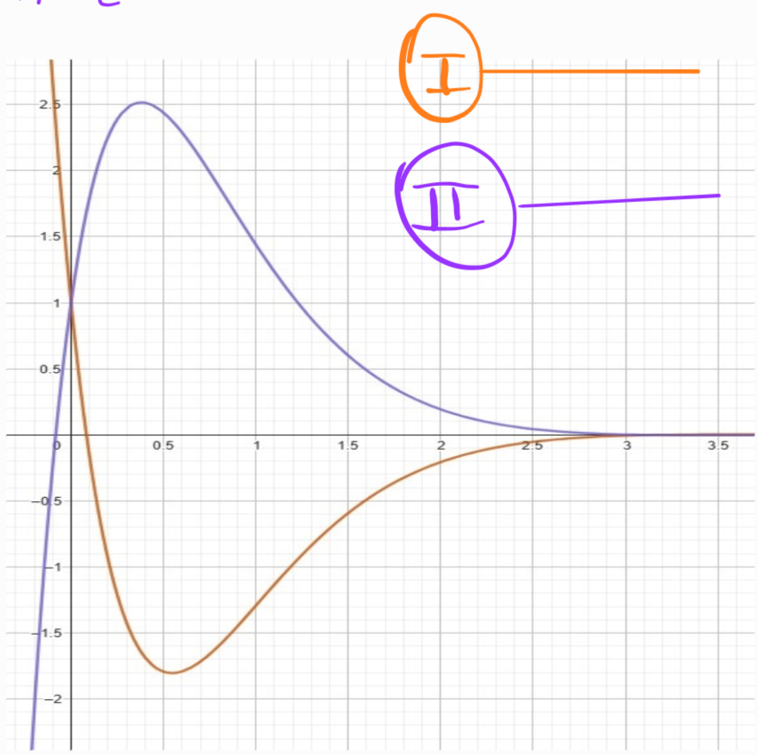
$y'(0) = -4 = -2((-4)) + 1(K_1) \Rightarrow K_1 = -4 - 8 = -12$

(I) $y(t) = e^{-2t}(-4\cos t - 12 \sin t)$

$y(0) = 4 = K_0$

$y'(0) = 4 = -2 \cdot 4 + K_1 \Rightarrow K_1 = 4 + 8 = 12$

(II) $y(t) = e^{-2t}(4\cos t + 12 \sin t)$



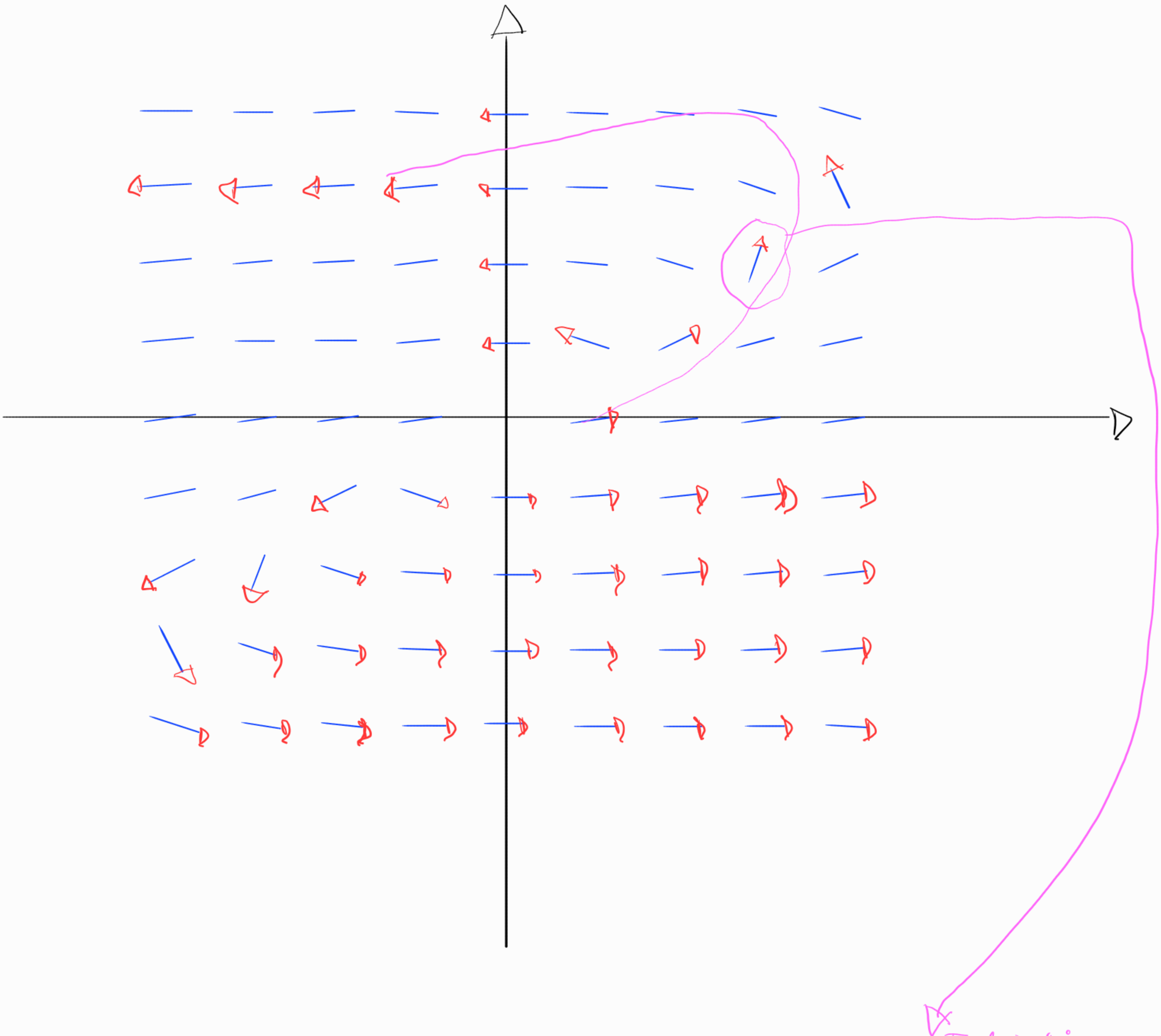
$$y'' - 7y' + 10y = 0 \qquad \Rightarrow y'' = 7y' - 10y$$

$$\frac{dy}{dt} = v$$

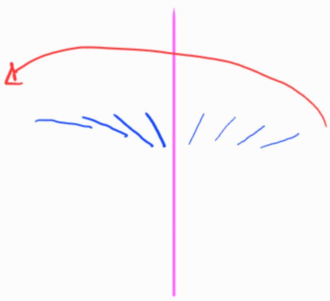
$$\frac{d^2y}{dt^2} = \frac{dv}{dt}$$

$$\frac{dy}{dv} = \frac{v}{d^2y/dt^2} = \frac{v}{7v - 10y}$$

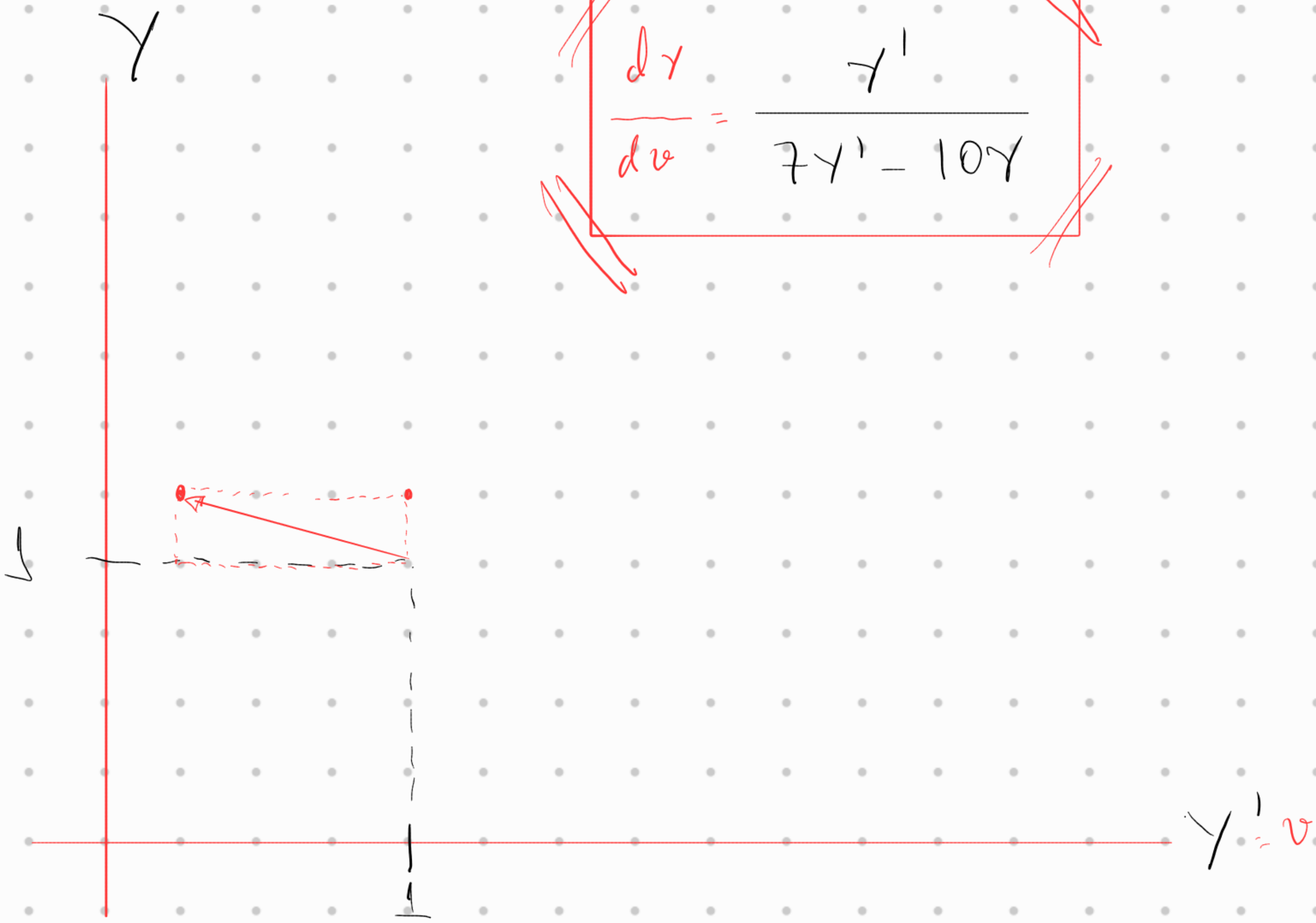
$y \neq 0{,}7v$



Tende à 90°



$$\boxed{\frac{dY}{dv} = \frac{Y'}{7Y' - 10Y}}$$




$1 = Q \to$