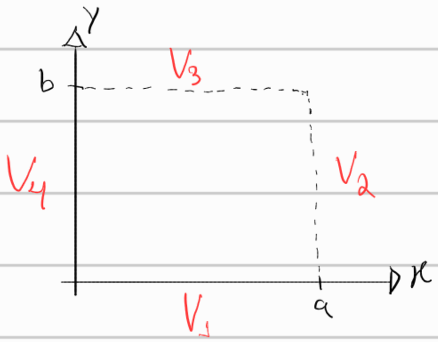
* A TENSÃO EM UMA SUPERFÍCIE DEPENDE DA POSIÇÃO, $V = V(x, y)$, E PARA A REGIÃO: $0 \leq x \leq a$, $0 \leq y \leq b$, TEMOS AS SEGUINTES CONDIÇÕES DE CONTORNO:



$$V(x, 0) = V_1 \qquad (1.1)$$

$$V(a, y) = V_2 \qquad (1.2)$$

$$V(x, b) = V_3 \qquad (1.3)$$

$$V(0, y) = V_4 \qquad (1.4)$$

ALÉM DISSO, $\nabla^2 V = 0$ (2)

DESEJA-SE OBTER UMA REPRESENTAÇÃO GRÁFICA PARA A TENSÃO EM UMA REGIÃO CONTIDA NAS CONDIÇÕES DE CONTORNO.

* SOLUÇÃO:

DA EQ. (2), $$\frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} = 0 \qquad (3)$$

ASSUMINDO QUE $V(x,y) = X(x) \cdot Y(y)$ (4)

$$\frac{\partial^2}{\partial x^2} XY = \frac{\partial^2}{\partial y^2} - XY \quad \therefore \quad X^{-1} X'' = -Y^{-1} Y'' \qquad (5)$$

A EQ. (5) É VERDADEIRA PARA $X^{-1}X'' = -Y^{-1}Y'' = \lambda$ (6)

EXISTEM INFINITAS CONSTANTES λ QUE GARANTEM INFINITAS SOLUÇÕES PARA $X(x)$ e $Y(y)$, DE MODO QUE A COMBINAÇÃO LINEAR DE X_m e Y_m SEJA SOLUÇÃO PARA $V(x,y)$.

$$V(x,y) = \sum_m a_m X_m Y_m \qquad (6.0)$$

$$X'' - \lambda X = 0 \qquad (6.1)$$

$$Y'' + \lambda Y = 0 \qquad (6.2)$$

PARA RESOLVER 6.1, VAMOS SUPOR QUE $X(x)$ POSSUI O SEGUINTE FORMATO:

$$X(x) = A_0 \cdot e^{rx} \qquad (7)$$

APLICANDO $\frac{d^2}{dx^2}$:

$$X'(x) = A_0 r e^{rx}$$

$$X''(x) = A_0 r^2 e^{rx}$$

SUBSTITUINDO EM (6.1):

$$A_0 r^2 e^{rx} - \lambda A_0 e^{rx} = 0$$

$$A_0 r^2 - \lambda A_0 = 0 \quad \therefore \quad r^2 = \frac{\lambda A_0}{A_0} = \lambda$$

$$r = \pm\sqrt{\lambda}$$

PARA GARANTIR A EQ 7, $\lambda \neq 0$.

(a) SE $\lambda > 0$, $r = \pm\sqrt{|\lambda|}$.

(b) SE $\lambda < 0$, $r = \pm j\sqrt{|\lambda|}$

PARA A CONDIÇÃO (a),

$$X(x) = A_0 e^{\sqrt{|\lambda|}\, x} + B_0 e^{-\sqrt{\lambda}\, x}$$

MANIPULANDO A_0 E B_0

$$A_0 = \frac{K_0}{2} + \frac{K_1}{2} \quad , \quad B_0 = \frac{K_0}{2} - \frac{K_1}{2}$$

$$X(x) = \frac{K_0 e^{\sqrt{|\lambda|}x} + K_0 e^{-\sqrt{\lambda}x}}{2} + \frac{K_1 e^{\sqrt{|\lambda|}x} - K_1 e^{-\sqrt{|\lambda|}x}}{2}$$

$$X(x) = K_0 \cosh\left(\sqrt{|\lambda|}\,x\right) + K_1 \operatorname{senh}\left(\sqrt{|\lambda|}\,x\right) \qquad (8)$$

PARA A CONDIÇÃO (b),

$$X(x) = A \cdot e^{j\sqrt{|\lambda|}x} + B e^{-j\sqrt{|\lambda|}x}$$

$$A = \frac{K_0}{j2} + \frac{K_1}{j2} \quad , \quad B = \frac{K_0}{j2} - \frac{K_1}{j2}$$

$$X(x) = \frac{K_0 e^{j\sqrt{|\lambda|}x} + K_0 e^{-j\sqrt{|\lambda|}x}}{j2} + \frac{K_1 e^{j\sqrt{|\lambda|}x} - K_1 e^{-j\sqrt{|\lambda|}x}}{j2}$$

$$X(x) = K_0 \cdot \cos\left(\sqrt{|\lambda|}\,x\right) + K_1 \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\,x\right) \qquad (9)$$

PARA RESOLVER 6.2 REPETIMOS O MESMO PROCEDIMENTO.
NOTE QUE OS FORMATOS APENAS INVERTEM.

- $\lambda \neq 0$
- P/ $\lambda > 0$:

$$Y(y) = K_0' \cdot \cos\left(\sqrt{|\lambda|}\,y\right) + K_1' \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\,y\right) \qquad (10)$$

- P/ $\lambda < 0$:

$$Y(y) = K_0' \cdot \cosh\left(\sqrt{|\lambda|}\,y\right) + K_1' \cdot \operatorname{senh}\left(\sqrt{|\lambda|}\,y\right) \qquad (11)$$

SABENDO QUE A SOLUÇÃO DE $V(x,y)$ É COMPOSTA PELA COMBINAÇÃO DE FUNÇÕES LINEARMENTE INDEPENDENTES, AS CONDIÇÕES DE CONTORNO PODEM SER SIMPLIFICADAS PARA A SUPERPOSIÇÃO DE 4 NOVAS CONDIÇÕES CARACTERIZADAS POR APENAS UMA SUPERFÍCIE RESTRINGIR SOLUÇÕES NÃO TRIVIAIS PARA Y E X.

O PROBLEMA SERÁ RESOLVIDO PARA QUATRO CONDIÇÕES DE CONTORNO:

$$C1: \begin{cases} V(x,0) = V_1 \\ V(a,y) = 0 \\ V(x,b) = 0 \\ V(0,y) = 0 \end{cases} \qquad C_2: \begin{cases} V(x,0) = 0 \\ V(a,y) = V_2 \\ V(x,b) = 0 \\ V(0,y) = 0 \end{cases}$$

$$C3: \begin{cases} V(x,0) = 0 \\ V(a,y) = 0 \\ V(x,b) = V_3 \\ V(0,y) = 0 \end{cases} \qquad C4: \begin{cases} V(x,0) = 0 \\ V(a,y) = 0 \\ V(x,b) = 0 \\ V(0,y) = V_4 \end{cases}$$

EM SEGUIDA AS QUATRO SOLUÇÕES SERÃO SOBREPOSTAS.

C_1:

$$V(x,0) = V_1 \qquad (12.1)$$

$$V(a,y) = 0 \qquad (12.2)$$

$$V(x,b) = 0 \qquad (12.3)$$

$$V(0,y) = 0 \qquad (12.4)$$

De (12.2) e (12.4), temos que:

$$X(a)\,Y(y) = X(0)\,Y(y) = 0 \qquad (13)$$

ENTÃO, PARA $X(a) = X(0) = 0$, RESOLVEMOS, P/ $\lambda > 0$

$$(8) \to \quad X(x) = K_0 \cdot \cosh\left(\sqrt{|\lambda|}\, x\right) + K_1 \cdot \operatorname{senh}\left(\sqrt{|\lambda|}\, x\right)$$

$$X(0) = K_0 \cdot 1 + K_1 \cdot 0 = 0 \Rightarrow K_0 = 0$$

$$X(a) = 0 \cdot \cosh\left(\sqrt{|\lambda|}\, a\right) + K_1 \cdot \operatorname{senh}\left(\sqrt{|\lambda|}\, a\right) = 0$$

* PARA GARANTIR SOLUÇÕES NÃO TRIVIAIS, $A_1 \neq 0$

ASSIM $\operatorname{senh}\left(\sqrt{|\lambda|}\, a\right) = 0 \Rightarrow \sqrt{|\lambda|}\, a = m\pi$

$$|\lambda| = \left(\frac{m \cdot \pi}{a}\right)^2 , \quad X(x) = K_1 \cdot \operatorname{senh}\left(\frac{m\pi}{a} \cdot x\right) \qquad (14)$$

LEMBRANDO QUE $\lambda \neq 0$, ENTÃO $m \neq 0$.
PARA $m = 1$, $|\lambda| = \frac{\pi^2}{a^2}$. PARA $m = -1$, $|\lambda| = \frac{\pi^2}{a^2}$.

ENTÃO, $m = 1, 2, 3, 4, \dots$

$$\begin{cases} X(x)\,Y(0) = V_1 & (12.1) \\ X(x)\,Y(b) = 0 & (12.3) \end{cases}$$

$$(10) \to \quad Y(y) = A_0' \cdot \cos\left(\sqrt{\lambda}\, y\right) + A_1' \cdot \operatorname{sen}\left(\sqrt{\lambda}\, y\right)$$

SUBSTITUINDO $|\lambda| = \left(\frac{m\pi}{a}\right)^2$

$$Y(y) = K_0' \cdot \cos\left(\frac{m\pi}{a} y\right) + K_1' \cdot \operatorname{sen}\left(\frac{m\pi}{a} \cdot y\right) \qquad (15)$$

$$(9.3) \to X(x)\,Y(b) = 0 \quad \therefore \quad Y(b) = \frac{0}{X(x)} \quad \therefore \quad Y(b) = 0$$

$$Y(b) = K_0' \cdot \cos\left(\frac{m\pi}{a} \cdot b\right) + K_1' \cdot \operatorname{sen}\left(\frac{m\pi}{a} \cdot b\right) = 0 \quad (16)$$

PARA GARANTIR $Y(b) = 0$, FAREMOS UMA MANIPULAÇÃO:

$$A_0'(y) = \operatorname{sen}\left(\frac{m\pi}{a} y\right) \quad ; A_1'(y) = -\cos\left(\frac{m\pi}{a} y\right)$$

$$Y(y) = \operatorname{sen}\left(\frac{m\pi}{a} y\right) \cdot \underbrace{\cos\left(\frac{m\pi}{a} b\right)}_{CTE} - \cos\left(\frac{m\pi}{a} y\right) \cdot \underbrace{\operatorname{sen}\left(\frac{m\pi}{a} b\right)}_{CTE}$$

$$Y(y) = \operatorname{sen}\left(\frac{m\pi}{a} y - \frac{m\pi}{a} b\right) = \operatorname{sen}\left(\frac{m\pi}{a}(y-b)\right) \quad (17)$$

DE (6.0),

$$V(x,y) = \sum_m a_m X(x) Y(y) = \sum_{m=1}^{\infty} a_m \cdot A_1 \cdot \operatorname{senh}\left(\frac{m\pi}{a} x\right) \cdot \operatorname{sen}\left(\frac{m\pi}{a}(y-b)\right) \quad (18)$$

$$(9.1) \rightarrow V(x,0) = \sum_{m=1}^{\infty} a_m \cdot A_1 \cdot \operatorname{sen}\left(-\frac{m\pi}{a} b\right) \cdot \operatorname{senh}\left(\frac{m\pi}{a} x\right)$$

NOTE QUE P/ $\lambda > 0$ NÃO É POSSÍVEL DETERMINAR UMA LEI DE FORMAÇÃO PARA OS TERMOS CONSTANTES DA SÉRIE.

O OBJETIVO É OBTER UMA EQUAÇÃO PARA $V(x,y)$ QUE POSSUA O FORMATO DA SÉRIE DE FOURRIER, QUANDO V POSSUIR SOLUÇÕES NÃO TRIVIAIS.

CONSIDERANDO $\lambda < 0$:

$$X(x) = K_0 \cdot \cos\left(\sqrt{|\lambda|}\, x\right) + K_1 \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\, x\right) \quad (9)$$

$$Y(y) = K_0' \cdot \cosh\left(\sqrt{|\lambda|}\, y\right) + K_1' \cdot \operatorname{senh}\left(\sqrt{|\lambda|}\, y\right) \quad (11)$$

RESOLVENDO PARA $X(a) = X(0) = 0$:

$$X(0) = K_0 \cdot 1 + K_1 \cdot 0 = 0 \Rightarrow K_0 = 0$$

$$X(a) = K_1 \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\, a\right) = 0 \Rightarrow |\lambda| = \left(\frac{n\pi}{a}\right)^2 \quad , \; n = 1, 2, 3, \ldots$$

$$X(x) = K_1 \cdot \operatorname{sen}\left(\frac{n\pi}{a} x\right) \qquad (19)$$

PARA $Y(b) = 0$:

$$Y(b) = K_0' \cdot \cosh\left(\frac{n\pi}{a} b\right) + K_1' \cdot \operatorname{senh}\left(\frac{n\pi}{a} b\right)$$

MANIPULANDO AS CONSTANTES:

$$Y(y) = \operatorname{senh}\left(\frac{n\pi}{a} b\right) \cdot \cosh\left(\frac{n\pi}{a} y\right) - \cosh\left(\frac{n\pi}{a} b\right) \cdot \operatorname{senh}\left(\frac{n\pi}{a} y\right)$$

$$Y(y) = \operatorname{senh}\left(\frac{n\pi}{a}(b - y)\right) \qquad (20)$$

PARA $V(x,y) = \sum_{n=1}^{\infty} K_n \cdot \operatorname{sen}\left(\frac{n\pi}{a} x\right) \cdot \operatorname{senh}\left(\frac{n\pi}{a}(b-y)\right)$

$$V(x,0) = V_1 = \sum_{n=1}^{\infty} K_n \cdot \operatorname{senh}\left(\frac{n\pi}{a} b\right) \cdot \operatorname{sen}\left(\frac{n\pi}{a} x\right) \qquad (21)$$

* COMPARANDO COM A SÉRIE DE FOURRIER PARA OS SENOS:

$$f(x) = \sum_{n=1}^{\infty} c_n \cdot \operatorname{sen}\left(\frac{n\pi}{L} \cdot x\right)$$

$$c_n = \frac{2}{L} \cdot \int_0^L f(x) \cdot \operatorname{sen}\left(\frac{n\pi}{L} \cdot x\right) dx \qquad (23)$$

Fazendo $c_n = K_n \cdot \operatorname{senh}\left(\frac{n\pi}{a} \cdot b\right)$,

$$c_n = \frac{2}{a} \cdot \int_0^a V_1 \cdot \operatorname{sen}\left(\frac{n\pi}{a} x\right) dx \qquad (24)$$

$$C_n = \frac{2}{a} \cdot V_1 \cdot \left(\frac{a}{n\pi} \cdot \left(-\cos\left(\frac{n\pi}{a} x\right)\right)\Big|_0^a \right)$$

$$C_n = \frac{2}{a} \cdot V_1 \cdot \frac{a}{n\pi} \cdot \left(-\cos(n\pi) + \cos(0) \right)$$

$$C_n = \frac{2V_1}{n\pi} \cdot \left(-\cos(n\pi) + 1 \right)$$

$$\begin{cases} \text{Se } n \text{ for ímpar}, & C_n = \frac{2V_1}{n\pi} \cdot \left(-(-1) + 1\right) = \frac{4V_1}{n\pi} \\ \text{Se } n \text{ for par}, & C_n = \frac{2V_1}{n\pi} \cdot \left(-1 + 1\right) = 0 \end{cases} \quad (25)$$

Portanto, $\forall\ n$ ímpar: $\left(n = (2 \cdot n' + 1)\right)$, $n' = 0, 1, 2, 3, \ldots$

$n = 1, 3, 5, 7, \ldots$

$$K_n \cdot \operatorname{senh}\left(\frac{n\pi}{a} b\right) = \frac{4 \cdot V_1}{n \cdot \pi}$$

$$K_n = \frac{4 V_1}{n\pi \cdot \operatorname{senh}\left(\frac{n\pi b}{a}\right)} \quad (26)$$

ENTÃO, PARA CI:

$$V_{CI}(x, y) = \sum_{n'=0}^{\infty} \frac{4 \cdot V_1}{(2n'+1) \cdot \pi \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi b}{a}\right)} \cdot \operatorname{sen}\left(\frac{(2n'+1) \cdot \pi}{a} \cdot x\right) \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{a} (b - y)\right) \quad (27)$$

RESOLVENDO PARA C2:

$$C_2: \begin{cases} V(x,0) = 0 \\ V(a,y) = V_2 \\ V(x,b) = 0 \\ V(0,y) = 0 \end{cases}$$

CONSIDERANDO O DESENVOLVIMENTO ANTERIOR, BUSCAMOS UMA SÉRIE DE FOURRIER COM $V_2 = \sum C_n \cdot \operatorname{sen}(\sqrt{\lambda}\, y)$.

SABENDO QUE $Y(y)$ DEPENDE DE SEN E COS, CONSIDERAREMOS $\lambda > 0$

ENTÃO:

$$V_{C2}(x,y) = \sum_{n=1}^{\infty} K_n X(x) Y(y) \qquad (28)$$

$$X(x) = K_0 \cosh\left(\sqrt{|\lambda|}\, x\right) + K_1 \operatorname{senh}\left(\sqrt{|\lambda|}\, x\right) \qquad (29)$$

$$Y(y) = K_0' \cdot \cos\left(\sqrt{|\lambda|}\, y\right) + K_1' \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\, y\right) \qquad (30)$$

DE C2,

$$Y(b) = Y(0) = 0 = K_0' \cdot 1 + K_1' \cdot 0 \Rightarrow K_0' = 0$$

$$Y(y) = K_1' \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\, y\right) \qquad (31)$$

$$Y(b) = K_0' \cdot \operatorname{sen}\left(\sqrt{|\lambda|} \cdot b\right) = 0 \Rightarrow |\lambda| = \left(\frac{n\pi}{b}\right)^2$$

$$X(0) = 0 = K_0 \cdot 1 + K_1 \cdot 0 \Rightarrow K_0 = 0$$

$$X(x) = K_1 \cdot \operatorname{senh}\left(\frac{n\pi}{b} x\right) \qquad (32)$$

$$V_{C2}(x,y) = \sum_{n=1}^{\infty} K_n \cdot \operatorname{senh}\left(\frac{n\pi}{b} x\right) \cdot \operatorname{sen}\left(\frac{n\pi}{b} \cdot y\right) \qquad (33)$$

$$V_{C2}(a,y) = V_2 = \sum_{n=1}^{\infty} K_n \cdot \operatorname{senh}\left(\frac{n\pi a}{b}\right) \cdot \operatorname{sen}\left(\frac{n\pi y}{b}\right) \qquad (34)$$

$$C_n = K_n \cdot \operatorname{senh}\left(\frac{n\pi . a}{b}\right) \quad \text{Série de Fourrier!} \quad (35)$$

$$C_n = \frac{2}{b} \cdot \int_0^b V_2 . \operatorname{sen}\left(\frac{n\pi}{b} . y\right) dy = \frac{2}{b} \cdot V_2 \cdot \int_0^b \operatorname{sen}\left(\frac{n\pi}{b} y\right) dy \quad (36)$$

$$C_n = \frac{2 V_2}{b} \cdot \left(\frac{b}{n\pi} \cdot \left(- \cos\left(\frac{n\pi}{b} . y\right) \Big|_0^b \right) \right) = \frac{2 V_2}{n\pi} \cdot \left(- \cos(n\pi) + 1 \right)$$

$$\begin{cases} \text{Se } n \text{ for ímpar, } C_n = \dfrac{4 V_2}{n\pi} , \quad n = (2n'+1) \\ \text{Se } n \text{ for par, } C_n = 0 \end{cases} \quad (37)$$

$$V_{C2}(x,y) = \sum_{n'=0}^{\infty} \frac{4 V_2}{(2n'+1) . \pi . \operatorname{senh}\left(\frac{(2n'+1)\pi . a}{b}\right)} \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{b} x\right) . \operatorname{sen}\left(\frac{(2n'+1) . \pi}{b} y\right) \quad (38)$$

RESOLVENDO PARA C3:

$$C3: \begin{cases} V(x,0) = 0 \\ V(a,y) = 0 \\ V(x,b) = V_3 \\ V(0,y) = 0 \end{cases}$$

OBSERVANDO A METODOLOGIA ADOTADA, FICA CLARO QUE DEVEMOS ADOTAR $\lambda < 0$, FAZENDO $X(x)$ e $Y(y)$ CONFORME EQUAÇÕES 9 E 11.

$$X(x) = K_0 . \cos\left(\sqrt{|\lambda|}\, x\right) + K_1 . \operatorname{sen}\left(\sqrt{|\lambda|}\, x\right) \quad (9)$$

$$Y(y) = K_0' . \cosh\left(\sqrt{|\lambda|}\, y\right) + K_1' . \operatorname{senh}\left(\sqrt{|\lambda|}\, y\right) \quad (11)$$

De acordo com C3,

$$X(0) = X(a) = 0 \quad , \quad K_0 = 0 \quad , \quad \sqrt{|\lambda|} = \frac{n\pi}{a} \qquad (39)$$

$$X(x) = K_1 \cdot \operatorname{sen}\left(\frac{n\pi}{a} x\right)$$

$$Y(0) = 0 \quad , \quad K_0' = 0 \quad , \quad Y(y) = K_1' \cdot \operatorname{senh}\left(\frac{n\pi}{a} y\right) \qquad (40)$$

$$V_{C3}(x,y) = \sum_{n=1}^{\infty} K_n \cdot \operatorname{senh}\left(\frac{n\pi}{a} y\right) \cdot \operatorname{sen}\left(\frac{n\pi}{a} x\right) \qquad (41)$$

$$V_{C3}(x,b) = V_3 = \sum_{n=1}^{\infty} K_n \cdot \operatorname{senh}\left(\frac{n\pi}{a} \cdot b\right) \cdot \operatorname{sen}\left(\frac{n\pi}{a} \cdot x\right) \qquad (42)$$

$$C_n = K_n \cdot \operatorname{senh}\left(\frac{n\pi}{a} \cdot b\right) = \frac{2}{a} \cdot \int_0^a V_3 \cdot \operatorname{sen}\left(\frac{n\pi}{a} \cdot x\right) dx \qquad (43)$$

$$C_n = \frac{2 \cdot V_3}{a} \cdot \frac{a}{n\pi} \cdot \left(-\cos(n\pi) + 1\right) \qquad (44)$$

$$\begin{cases} \text{Se } n \text{ for ímpar}, \quad C_n = \dfrac{2V_3}{n\pi} \cdot 2 = \dfrac{4V_3}{n\pi} \quad , \quad n = 2m'+1 \\ \text{Se } n \text{ for par}, \quad C_n = 0 \end{cases} \qquad (45)$$

$$V_{C3}(x,y) = \sum_{m'=0}^{\infty} \frac{4V_3}{(2m'+1) \cdot \pi \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi \cdot b}{a}\right)} \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi}{a} y\right) \cdot \operatorname{sen}\left(\frac{(2m'+1)\pi}{a} x\right) \qquad (46)$$

PARA C4:

$$C4: \begin{cases} V(x,0) = 0 \\ V(a,y) = 0 \\ V(x,b) = 0 \\ V(0,y) = V_4 \end{cases}$$

$\lambda > 0$,

$$X(x) = K_0 \cosh\left(\sqrt{|\lambda|}\, x\right) + K_1 \operatorname{senh}\left(\sqrt{|\lambda|}\, x\right) \quad (8)$$

$$Y(y) = K_0' \cdot \cos\left(\sqrt{|\lambda|}\, y\right) + K_1' \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\, y\right) \quad (10)$$

$$Y(0) = 0 = K_0' \cdot 1 + K_1' \cdot 0 \Rightarrow K_0' = 0$$

$$Y(b) = 0 = K_1' \cdot \operatorname{sen}\left(\sqrt{|\lambda|}\, b\right) = 0 \Rightarrow \sqrt{|\lambda|} = \frac{m\pi}{b} \quad (47)$$

$$Y(y) = K_1' \cdot \operatorname{sen}\left(\frac{m\pi}{b} y\right) \quad (48)$$

$$X(a) = 0 = K_0 \cdot \cosh\left(\frac{m\pi}{b} \cdot a\right) + K_1 \cdot \operatorname{senh}\left(\frac{m\pi}{b} \cdot a\right) \quad (49)$$

$$X(x) = \operatorname{senh}\left(\frac{m\pi}{b} \cdot a\right) \cdot \cosh\left(\frac{m\pi}{b} \cdot x\right) - \cosh\left(\frac{m\pi}{b} \cdot a\right) \cdot \operatorname{senh}\left(\frac{m\pi}{b} \cdot x\right) \quad (50)$$

$$X(x) = \operatorname{senh}\left(\frac{m\pi}{b}(a - x)\right) \quad (51)$$

$$U_{C4}(x,y) = \sum_{m=1}^{\infty} K_m \cdot \operatorname{sen}\left(\frac{m\pi}{b} y\right) \cdot \operatorname{senh}\left(\frac{m\pi}{b}(a-x)\right) \quad (52)$$

$$U_{C4}(0,y) = V_4 = \sum_{m=1}^{\infty} K_m \cdot \operatorname{senh}\left(\frac{m\pi}{b} \cdot a\right) \cdot \operatorname{sen}\left(\frac{m\pi}{b} \cdot y\right) \quad (53)$$

$$C_m = K_m \cdot \operatorname{senh}\left(\frac{m\pi}{b} \cdot a\right)$$

$$C_n = \frac{2}{b} \cdot \int_0^b V_4 \cdot \operatorname{sen}\left(\frac{n\pi}{b} y\right) dy = \frac{2}{b} \cdot V_4 \cdot \frac{b}{n\pi} \cdot \left(-\cos(n\pi) + 1\right) \qquad (54)$$

$$\begin{cases} \text{Se } n \text{ for Ímpar,} & n = (2n'+1), \quad n' = 0, 1, 2, 3 \\ \quad C_n = \dfrac{4V_4}{n\pi} & \qquad\qquad 1, 3, 5, 7 \\ \text{Se } n \text{ for Par,} & \qquad (55) \\ \quad C_n = 0 & \end{cases}$$

$$V_{c4}(x,y) = \sum_{n'=0}^{\infty} \frac{4V_4}{(2n'+1)\cdot\pi\cdot\operatorname{senh}\left(\frac{(2n'+1)\cdot\pi\cdot a}{b}\right)} \cdot \operatorname{sen}\left(\frac{(2n'+1)\pi}{b} y\right) \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{b}(a-x)\right) \quad (56)$$

Portanto, $V(x,y) = V_{c1} + V_{c2} + V_{c3} + V_{c4}$

$$V(x,y) = \sum_{n'=0}^{\infty} \frac{4\cdot V_1}{(2n'+1)\cdot\pi\cdot\operatorname{senh}\left(\frac{n\pi b}{a}\right)} \cdot \operatorname{sen}\left(\frac{(2n'+1)\cdot\pi}{a}\cdot x\right) \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{a}(b-y)\right)$$

$$+ \sum_{n'=0}^{\infty} \frac{4V_2}{(2n'+1)\cdot\pi\cdot\operatorname{senh}\left(\frac{(2n'+1)\pi\cdot a}{b}\right)} \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{b} x\right) \cdot \operatorname{sen}\left(\frac{(2n'+1)\cdot\pi}{b} y\right)$$

$$+ \sum_{n'=0}^{\infty} \frac{4V_3}{(2n'+1)\cdot\pi\cdot\operatorname{senh}\left(\frac{(2n'+1)\pi\cdot b}{a}\right)} \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{a} y\right) \cdot \operatorname{sen}\left(\frac{(2n'+1)\pi}{a} x\right)$$

$$+ \sum_{n'=0}^{\infty} \frac{4V_4}{(2n'+1)\cdot\pi\cdot\operatorname{senh}\left(\frac{(2n'+1)\cdot\pi\cdot a}{b}\right)} \cdot \operatorname{sen}\left(\frac{(2n'+1)\pi}{b} y\right) \cdot \operatorname{senh}\left(\frac{(2n'+1)\pi}{b}(a-x)\right)$$

$$V(x,y) = \sum_{m'=0}^{\infty} \left[\frac{4 \cdot V_1}{(2m'+1) \cdot \pi \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi\, b}{a}\right)} \cdot \operatorname{sen}\left(\frac{(2m'+1) \cdot \pi}{a} \cdot x\right) \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi}{a}(b-y)\right) \right.$$

$$+ \frac{4 V_2}{(2m'+1) \cdot \pi \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi \cdot a}{b}\right)} \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi}{b} x\right) \cdot \operatorname{sen}\left(\frac{(2m'+1) \cdot \pi}{b} y\right)$$

$$+ \frac{4 V_3}{(2m'+1) \cdot \pi \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi \cdot b}{a}\right)} \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi}{a} y\right) \cdot \operatorname{sen}\left(\frac{(2m'+1)\pi}{a} x\right)$$

$$\left. + \frac{4 V_4}{(2m'+1) \cdot \pi \cdot \operatorname{senh}\left(\frac{(2m'+1) \cdot \pi \cdot a}{b}\right)} \cdot \operatorname{sen}\left(\frac{(2m'+1)\pi}{b} \cdot y\right) \cdot \operatorname{senh}\left(\frac{(2m'+1)\pi}{b}(a-x)\right) \right]$$