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16 July 2020

Project 5: Fourier Analysis

EE 3370.502

1. Waveform:

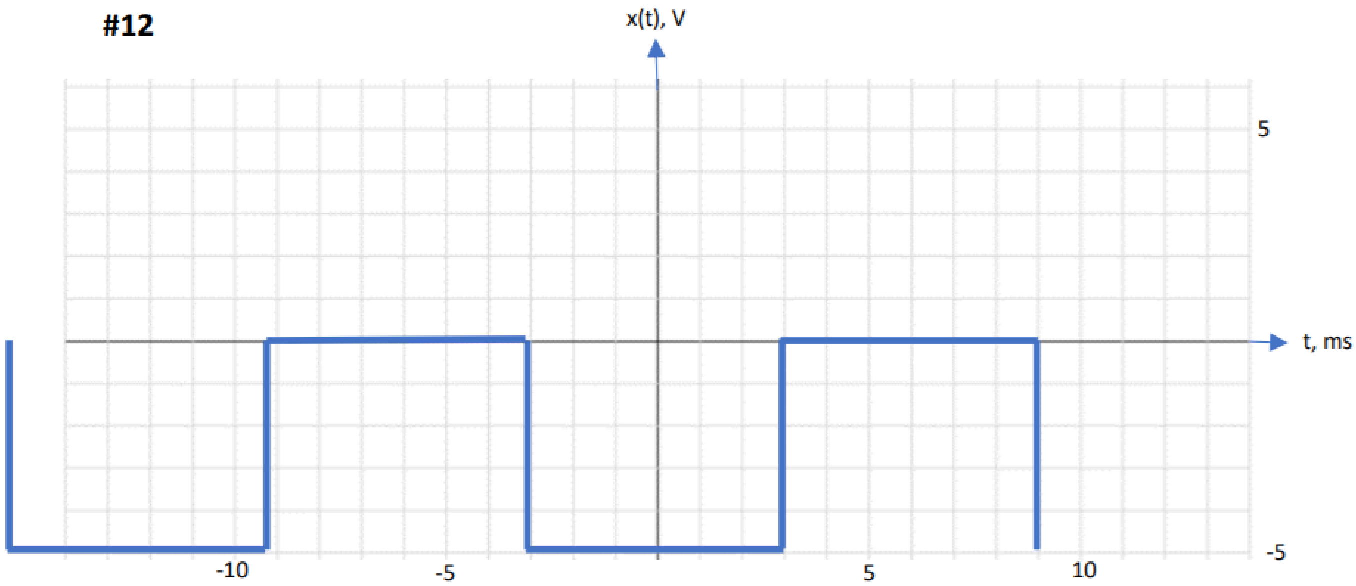


Fig. 1: The assigned waveform. We can see that the waveform has a period (T) of T = 12ms or 0.012s.

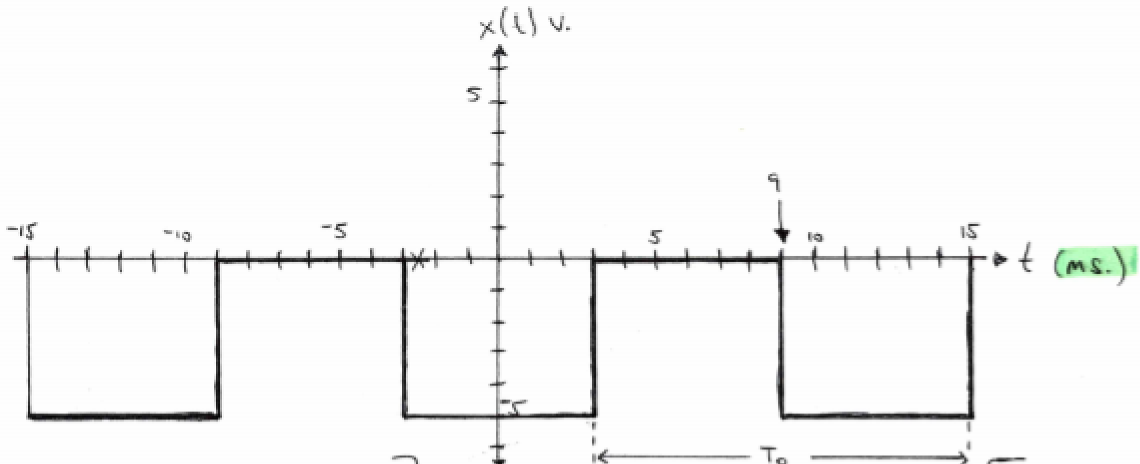
2. Derive Series:

2.1: Hand derived trigonometric series

EE 3370 Project 5 Derivation | Roberto Colon

-> ID: 205

-> Assigned Waveform: #12



$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{0.012\,s} = 166.67\pi = 523.6 \text{ RAD/S}$$

$$T_0 = 12_{ms} = 0.012\,s$$

$$f = \frac{1}{T} = 83.\bar{3}\text{ Hz}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt \qquad b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(n\omega_0 t)\,dt$$

$$a_0 = \frac{1}{0.012}\left[\cancel{\int_3^9 0\,dt}^{\,0} + \int_9^{15} -5\,dt\right] \Rightarrow \left(\frac{1}{0.012}\right)\int_9^{15} -5\,dt \Rightarrow \left(\frac{-5}{0.012}\right)\left[t\,\Big|_9^{15}\right] \rightarrow 15-9 = 6$$

$$a_0 = \left(\frac{-5}{0.012}\right)6 = \boxed{-2{,}500 = a_0}$$

$$a_n = \frac{2}{0.012}\left[\int_9^{15} -5\cos(n\omega_0 t)\,dt\right] = \left(\frac{-10}{0.012}\right)\left[\int_9^{15}\cos(n\omega_0 t)\,dt\right] \Rightarrow \left(\frac{-10}{0.012}\right)\left(\frac{1}{n\omega_0}\right)\sin(n\omega_0 t)\Big|_9^{15} \quad \left(\omega_0 = \frac{2\pi}{0.012}\right)*$$

$$a_n = \left(\frac{-10}{2\pi n}\right)\left[\sin\big[n\underbrace{(166.67\pi)15}_{2500.05\pi}\big] - \sin\big[n\underbrace{166.67\pi(9)}_{1500.03\pi}\big]\right]$$

$$\boxed{a_n = \left(-\frac{10}{2\pi n}\right)\left[\sin(2500.05n\pi) - \sin(1500.03n\pi)\right] \text{ VOLTS}}$$

Used By Sum to Product Formula: $a_n = \left(-\frac{10}{\pi n}\right)\left[\sin(500\pi n)\cos(2000\pi n)\right]$

* Work on Appendix Page (Last Page)

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ROBERTO COLON

$$b_n = \left(\frac{2}{0.012}\right)\left[\int_9^{15} -5\sin(n\omega_0 t)\,dt\right] \rightarrow \left(-\frac{10}{0.012}\right)\left(\frac{1}{n\omega_0}\right)\left[-\cos(n\omega_0 t)\Big|_9^{15}\right]$$

$$\left(\omega_0 = \frac{2\pi}{0.012}\right)$$

$$b_n = \left(\frac{10}{2\pi n}\right)\left(\cos[n\,166.67\pi(15)] - \cos[n\,166.67\pi(9)]\right)$$

$$\boxed{b_n = \left(\frac{10}{2\pi n}\right)\left[\cos(2500.05\,n\pi) - \cos(1500.03\,n\pi)\right]}$$

SUB. BY SUM TO PRODUCT RULES:

$$\boxed{b_n = \left(-\frac{10}{\pi n}\right)\left[\sin(2000\pi n)\sin(500\pi n)\right]} \; *$$

$$\boxed{x(t) = -2500 \text{ Volts} + \sum_{n=1}^{\infty} \underbrace{\left(\frac{-10}{2\pi n}\right)\left[\sin(2500.05n\pi) - \sin(1500.03n\pi)\right]}_{①}\cos(n\omega_0 t) \cdots \rightarrow + \underbrace{\left(\frac{10}{2\pi n}\right)\left[\cos(2500.03n\pi) - \cos(1500.03n\pi)\right]}_{②}\sin(n\omega_0 t)}$$

$$① \cancel{\left(\frac{-10}{2\pi n}\right)\left[\sin(2500n\pi)\cos(n\omega_0 t) - \sin(1500n\pi)\cos(n\omega_0 t)\right]} \quad \underline{NO}$$

USING EXCEL TO GET VALUES OF $\left[a_0, a_1, a_2, \ldots a_n \;\; \& \;\; b_0, b_1, b_2, \ldots, b_n\right]$

$\rightarrow a_0 \rightarrow a_{20} \;\; \& \;\; b_0 \rightarrow b_{20}$

$$x(t) = -2500 + (-0.0592)\cos(166.67\pi t) + (-0.0125)\sin(166.67\pi t) + (-0.0568)\cos(333.34\pi t) +$$

$$(-0.0249)\sin(333.34\pi t) + \cdots$$

$$\sin(x) \pm \sin(y) = 2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right)$$

$$\left.\begin{aligned} x &= 2500.05\pi n \\ y &= 1500.03\pi n \end{aligned}\right\} \Rightarrow 2\sin\left(\frac{2500.05\pi n - 1500.03\pi n}{2}\right)\cos\left(\frac{2500.05\pi n + 1500.03\pi n}{2}\right)$$

$$= 2\sin\left(\frac{1{,}000.02\pi n}{2}\right)\cos\left(\frac{4000.08\pi n}{2}\right)$$

$$= 2\sin(500.01\pi n)\cos(2000.04\pi n)$$

$$= \boxed{2\sin(500.01\pi n)\cos(2000.04\pi n)} \quad \underline{a_n}$$

TAKING THESE VALUES

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$= \boxed{-2\sin(2000.04\pi n)\sin(500.01\pi n)} \quad \underline{b_n}$$

Note: Above calculations are in units of milli. (a factor of 3 larger than actual). The units used to calculate the following were corrected and the cleaned up. The corrected calculations will be attached to the final report.

3. Reconstruction

3.1: SPICE network with all current sources.

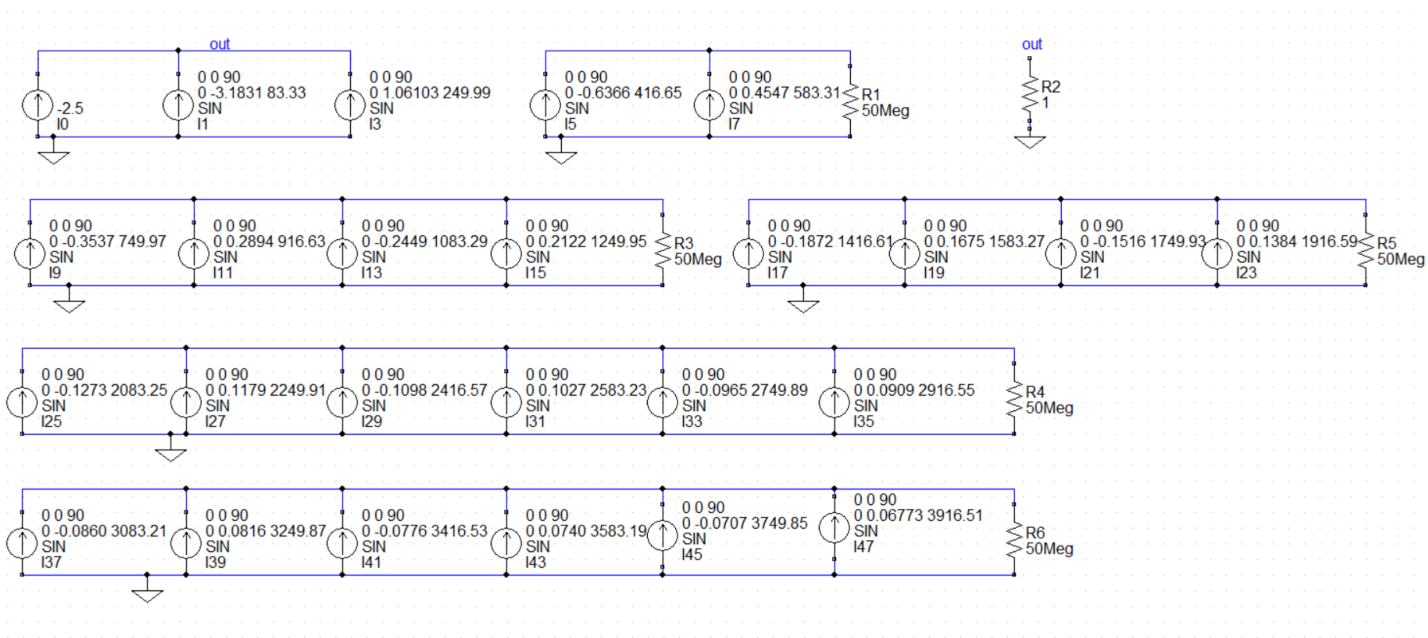

Fig. 2: SPICE network with all labeled current sources. The numbering convention is the number A_0 for each source. The information next to each source shows the phase, amplitude, then frequency respectively. Since the B_0 term was zero for all, only the cosine function was used hence a 90-degree phase shift for all sin sources.

3.2: SPICE reconstruction.

3.2.1: First Two Harmonics.

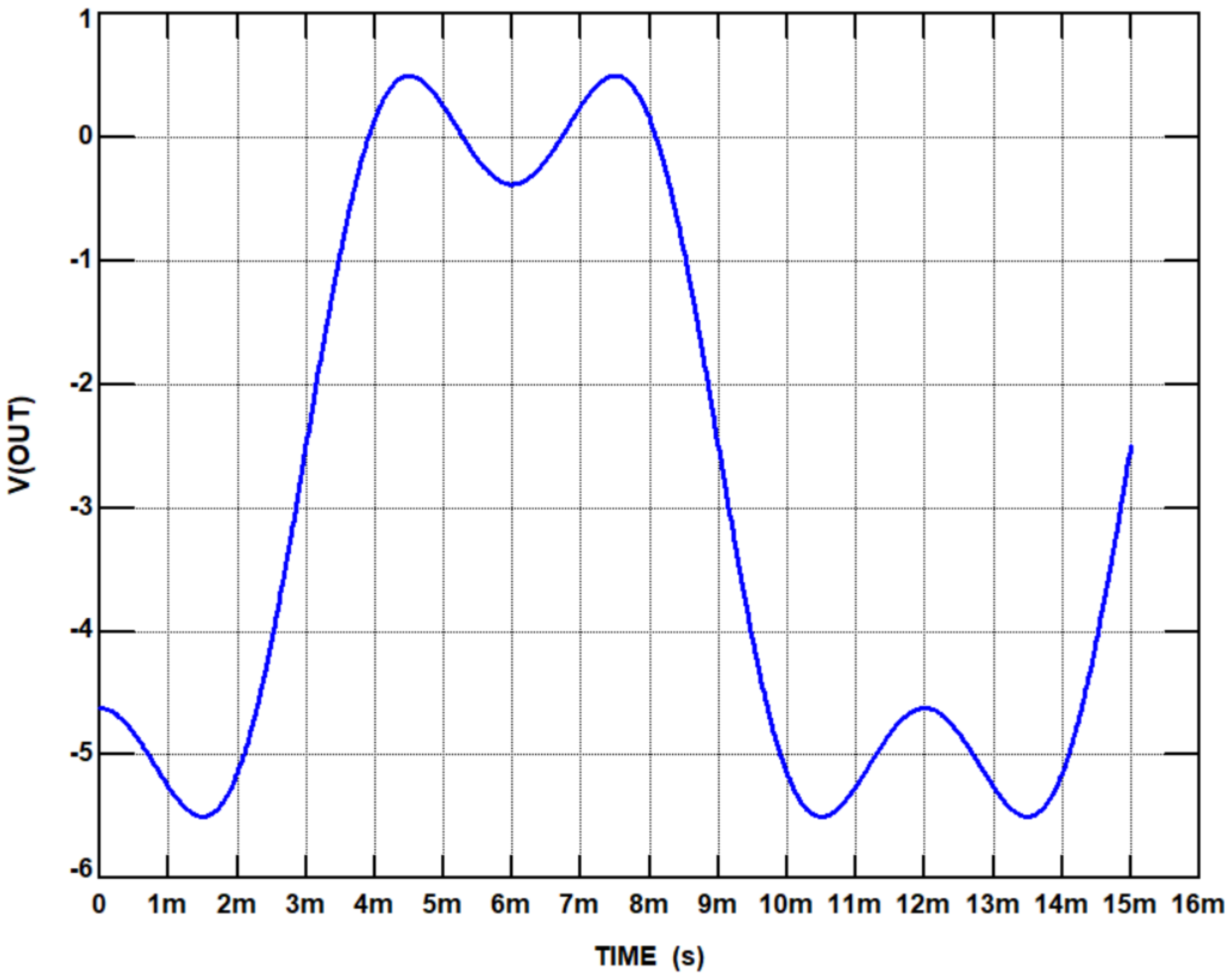

Fig. 3: The first two harmonics as well as A_0, which was calculated to be -2.5 Volts for the given signal.

3.2.2: The first four harmonics.

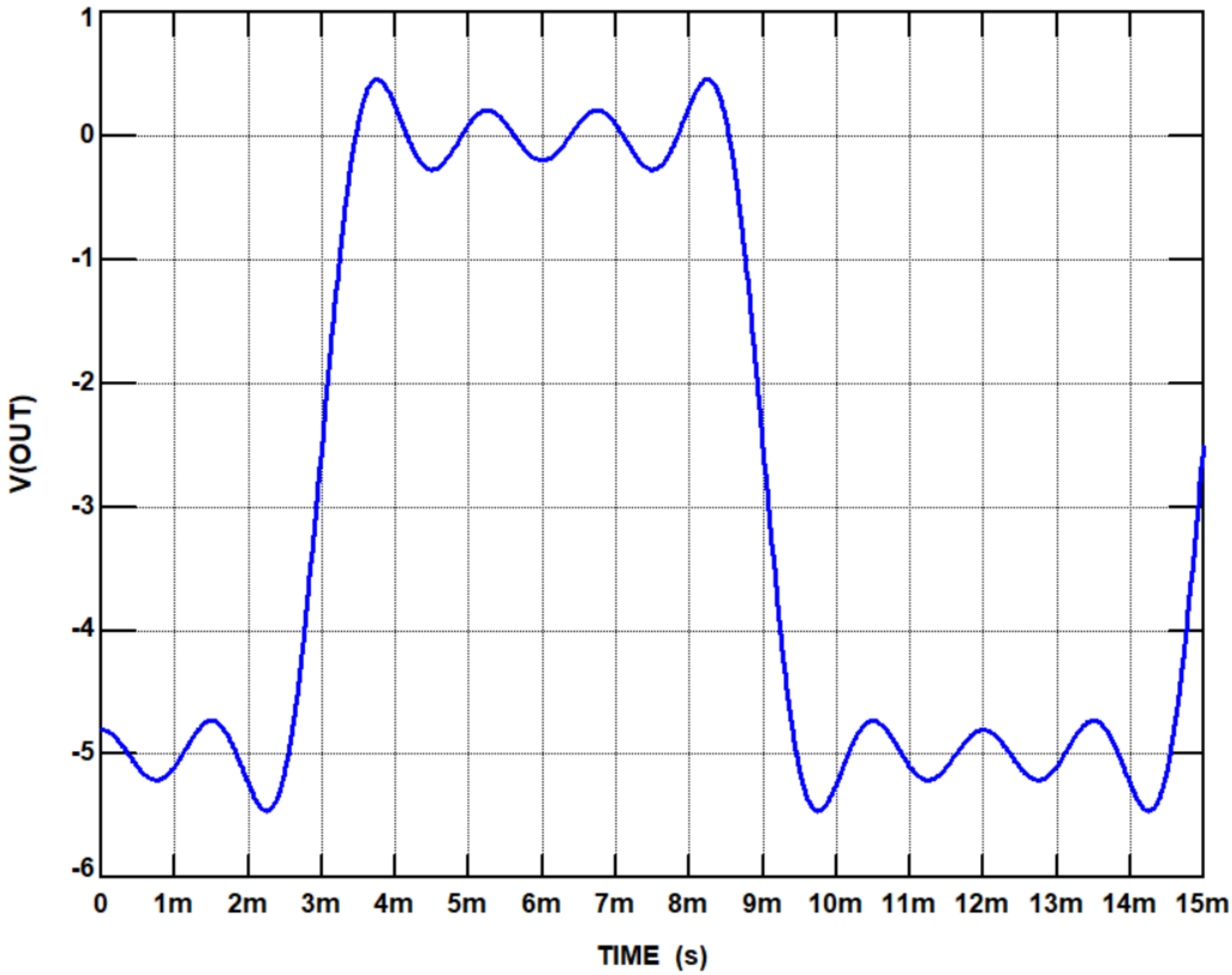

Fig. 4: The first four harmonics. We can see that the approximation is coming closer to the points of discontinuity from the given signal. These discontinuous points are located at t=3ms and t=9ms. As we add more harmonics, we should see this slope get more vertical but never completely vertical.

3.2.3: The first eight harmonics.

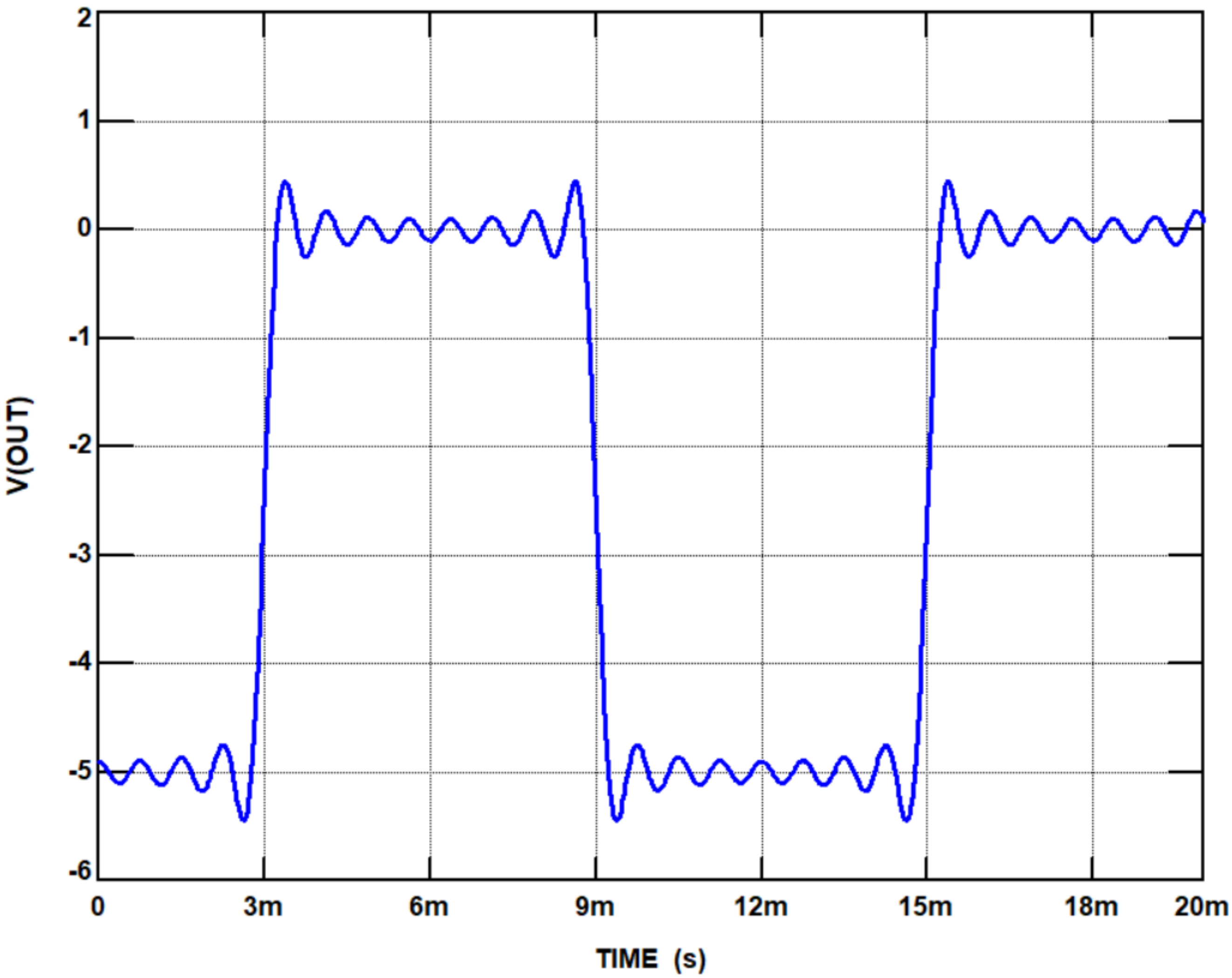

Fig. 5: The first eight harmonics. The approximation is getting closer but we are starting to see some peaking at the edges of the discontinuities. When referring to discontinuities, we are speaking of the original signal and not the Fourier signal approximation.

3.2.4: The first twelve harmonics.

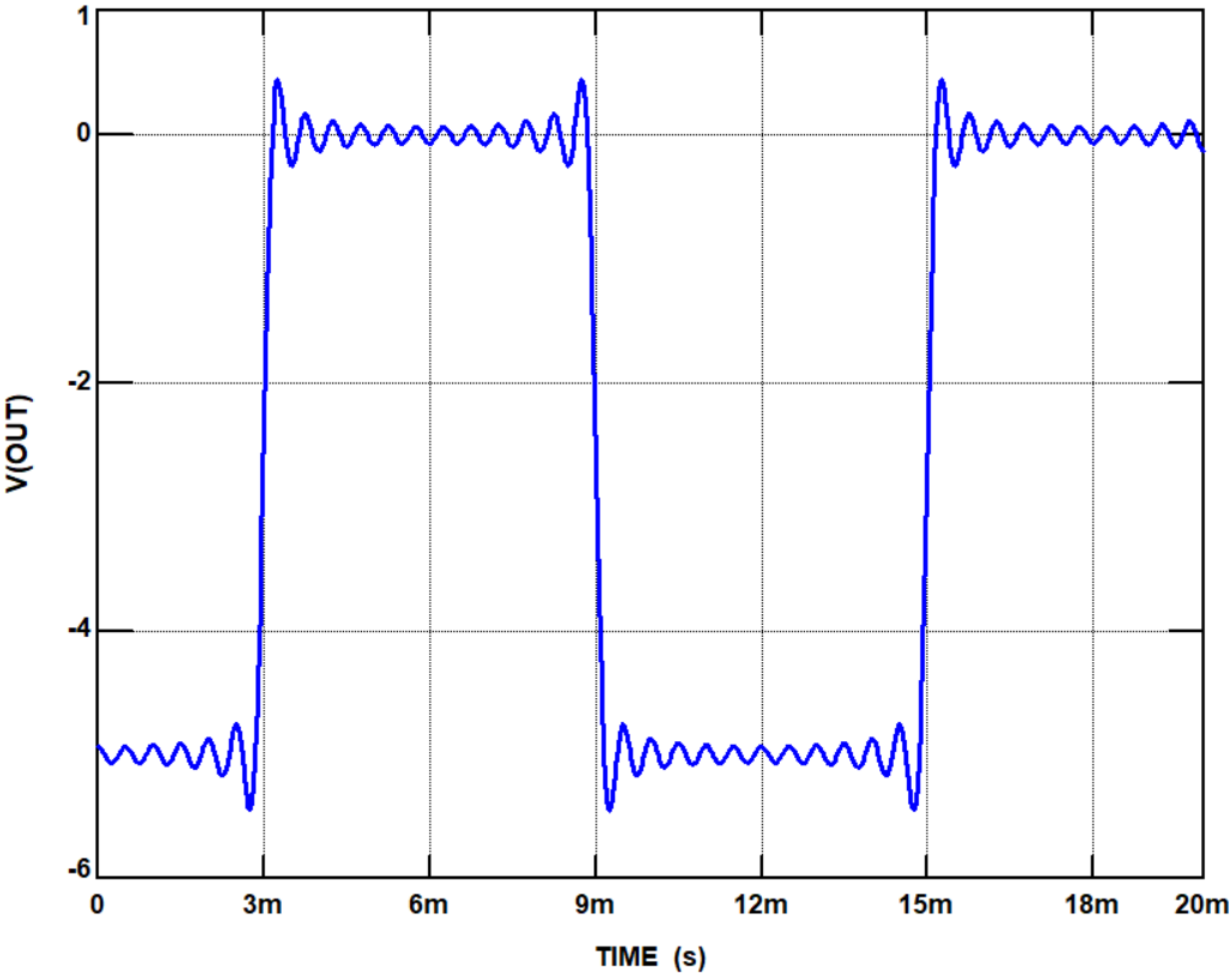

Fig. 6: The first twelve harmonics summed.

3.2.5: The first eighteen harmonics.

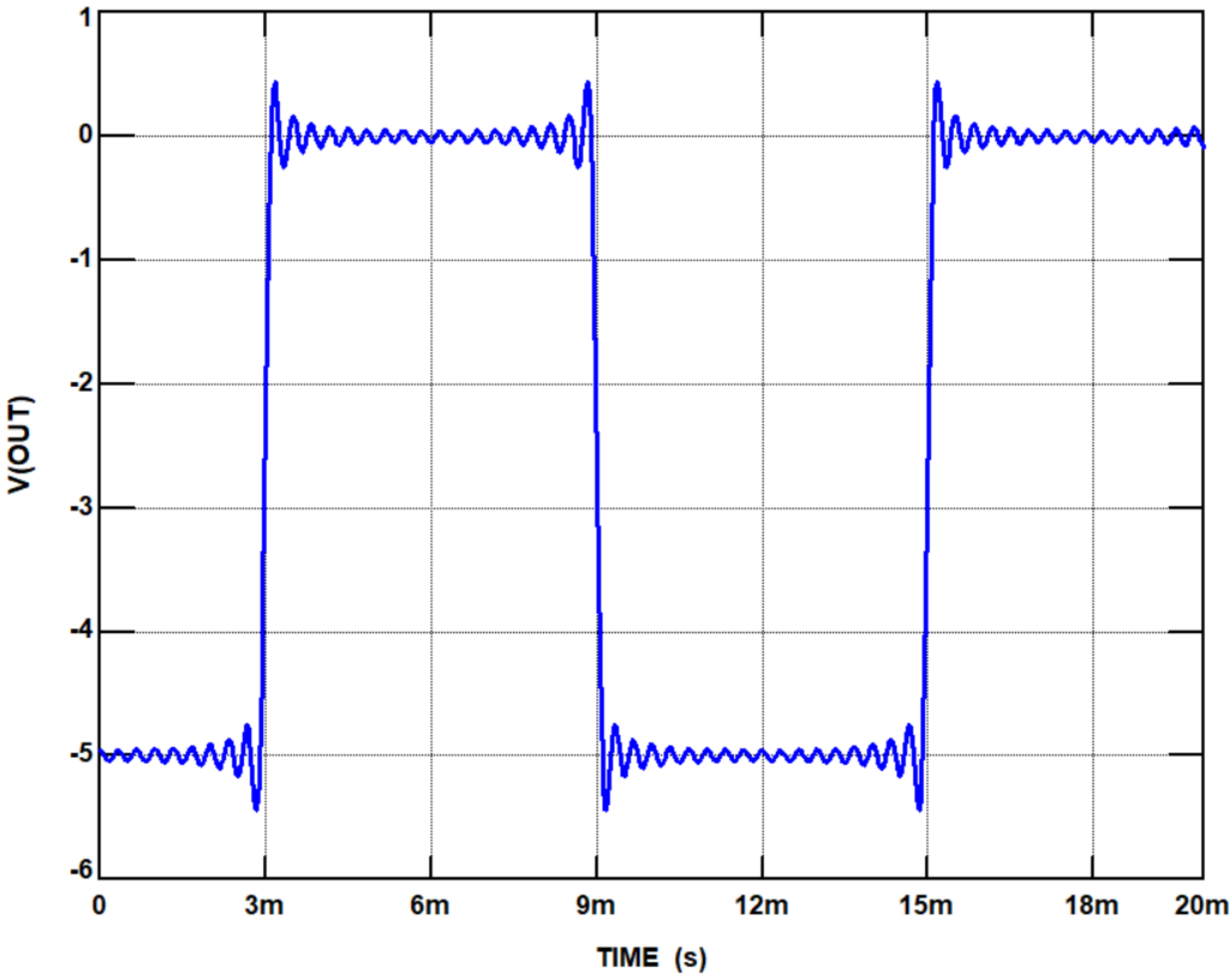

Fig. 7: The first eighteen harmonics. We are getting very close, and as we sum more harmonics, we see that the peaking near the inflection points stays at a very close constant 0.5 Volts.

3.2.6: All twenty-four harmonics.

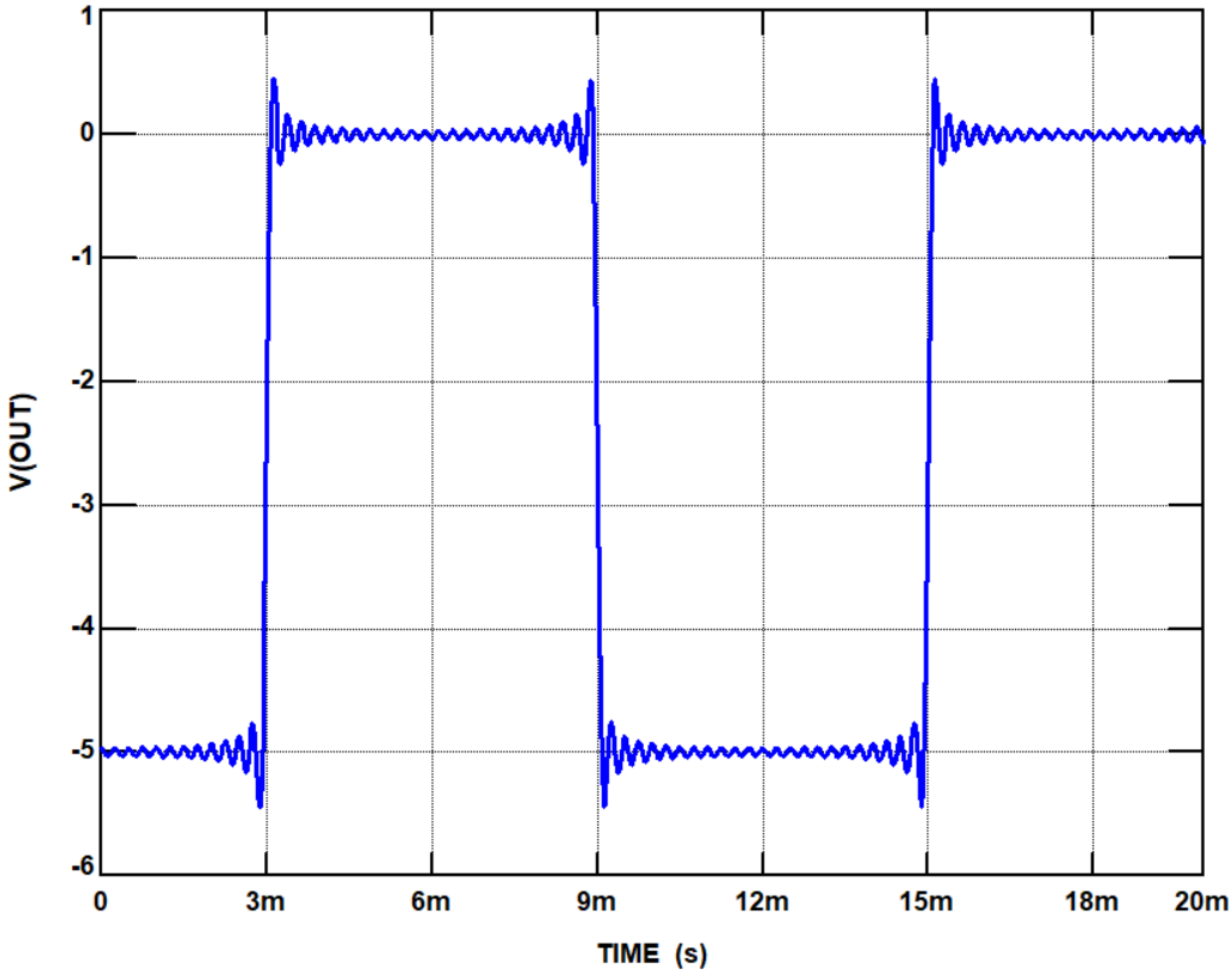

Fig. 8: All twenty-four harmonics summed. As we can see, we have a very close approximation to the original signal with just 0.5 V of peaking at the points of inflection.

4. Fourier Transform

4.1:

EE3370 P.5 TRANSFORM

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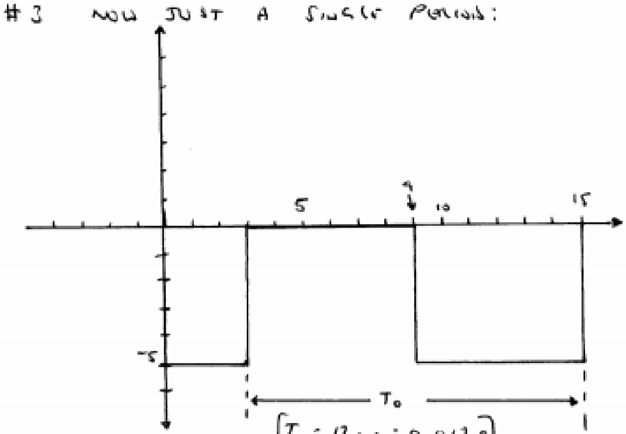
ID: #205

WAVEFORM: #3 NOW JUST A SINGLE PERIOD:



$\left[T_0 = 12ms = 0.012s\right]$

$\omega_0 = \frac{2\pi}{12} = \left[\frac{\pi}{6} = \omega_0\right]$

* in M.S

$$\left[X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt\right]$$

* ω is in MS (FULL UNIT) NOT in S. (SOU UNITS)

$$X(\omega) = \int_{\cancel{0.003}\ 3}^{\cancel{0.009}\ 9} -5\, e^{-j\pi/6 t}\, dt + \int_{\cancel{0.009}\ 9}^{\cancel{0.015}\ 15} 0\, e^{-j\pi/6 t}$$

$$X(\omega) = -5\int_3^9 e^{-j\pi/6 t} dt \Rightarrow \left(\frac{-5}{-j\pi/6}\right)\left[e^{-j\pi/6 t}\Big|_3^9\right]$$

$$\boxed{X(\omega) = \left(\frac{5}{j\pi/6}\right)\left[e^{-j\,3\pi/2} - e^{-j\,\pi/2}\right]} \Rightarrow \boxed{X(\omega) = \frac{5}{j\omega}\left[e^{-j9\omega} - e^{-j3\omega}\right]}$$

* DON'T REPL. OMEGA *

Fig. 9: Fourier Transform of given signal over a single period.

5. Final Report

5.1: A harmonic is a periodic signal which has a fundamental frequency. So, any signal, broken down to its fundamental or smallest period, is the first harmonic. From the fundamental, the subsequent harmonics come as integer multiples of the fundamental frequency of that first harmonic. An example would be if you had a signal which was sinusoidal with a period of 2π. The fundamental frequency, using the smallest period, would be 1 radian per second and any subsequent frequencies of this harmonic would just be an integer multiple of the fundamental frequency. Therefore, the second would be 2*1=2rad/s. the third would be 3*1=3 rad/s. and so on such that we could say that the subsequent harmonics of any fundamental are ($\boldsymbol{n}$*ω_o) where $\boldsymbol{n}$ is an integer and ω_o is the fundamental frequency that comes from the smallest period of the first harmonic of the signal. We can say that the harmonics are integer multiples or the process of repetitive addition of the fundamental frequency to itself. Both definitions are the same as multiplication is the process of repetitive addition. An interesting aspect of a harmonic, is that the higher harmonics always fit inside the fundamental harmonic period. So if we look at it as a graphic depiction, the fundamental harmonic would have the smallest frequency and the longest period, as we set the subsequent harmonics underneath our fundamental wave, we would see that because our frequency is integer multiplication, that now at the second harmonic, that twice the waves are present in the same space (period) as the fundamental, for the third, three times the waves are present in the same period as the fundamental, and the pattern continues. This is not hard to extrude since we defined the harmonic initially by the integer multiplication of its frequency. If we think about doubling a frequency, we can see how there are twice the waves in the span of the original!

5.2: During the addition of harmonics to the waveform that I was assigned, we could see that the number of waves inside a period increased as they appeared to ‘squeeze’ into the period of the fundamental harmonic which we were modeling. If we look at figure 3 in section 3.2.1 (attached a copy below this paragraph for convenience), we can see that this is the first two harmonics as labeled.

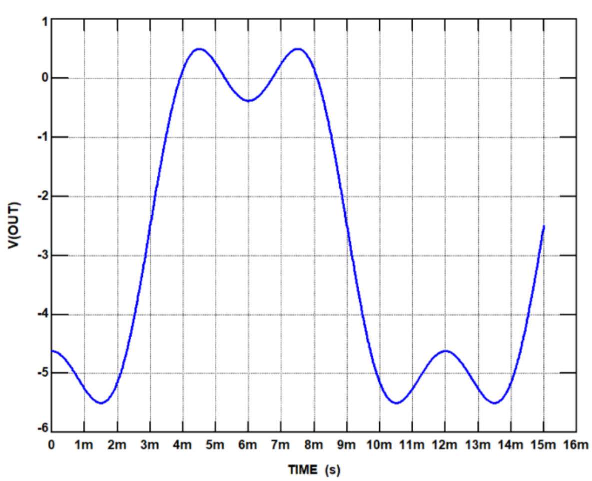


Fig. 3 copied for convenience. First two weighted harmonics.

But what is really happening here? If we observe, we can count waves crest to crest, ignoring that each wave needs to be an exact sinusoidal representation of itself and instead thinking of these waves as being scaled and fitted to each other in pieces in order to recreate a completely different signal which does not look exactly like a wave at all. Let us observe the beginning of the wave, which appears to be a cosine wave. At approximately 2.3 milliseconds. It looks like the wave ends and a completely new and alternately scaled wave picks up at an amplitude of approximately 4.5 Volts. Now we can see that at 4 milliseconds and at amplitude zero, it appears that a new, now sin wave, picks up for approximately 1.5 periods of itself before the mirror image of the prior wave with amplitude 4.5 volts. This is the shaping of waves to fit our signal, which is accomplished through the trigonometric Fourier series. Through this impressive piece of calculus, we are able to shape a series of weighted sinusoids to match our desired shape. So, as we add more harmonics, we are adding more and more summed and weighted sinusoids to shape the waves to our desired pattern and that is what is happening as we add these harmonics. It is not just the addition of higher harmonics, but the proper weighting of each which completes the task. As we scroll through the rest of the figures in section three, we can observe the squeezing of more waves (higher frequencies, more harmonics) into our desired shape thereby recreating the original signal as a very close approximation; and with only the first twenty-four harmonics.

5.3: The question is posed of passing a signal to a system which has implemented a low pass filter. By definition, a low pass filter denies all frequencies higher than a certain range, depending on the steepness of the slope of the filter and the cutoff frequency. This means, for our purposes of sending a signal which has been transformed using a Fourier transform or a Fourier trigonometric series, that the frequency which we are allowed to apply to our signal is limited to those which will pass through the low pass filter. In other words, we can only go so high in our harmonic range while staying inside the frequency range of the low pass filter. This means, by deduction, that we are limited to how ‘clearly’ or to how high a resolution we can use to transform our signal and thereby are limited to the precision of the representation of said signal. This means that compromises will have to be made on the precision of the signal and the amount of distortion which is acceptable.

5.4: The main difference between the Fourier trigonometric (trig) and Fourier transform is the use of the frequency domain and the signal given (periodic or aperiodic). Let me elaborate. If I have a periodic signal, I can easily find its period and then use calculus to find its transform using Fourier trig. This is nice and easy because I have been given a periodic signal which I can easily break down and analyze to transform. But the real world doesn’t always work in perfectly periodic ways. This is where the Fourier transform comes in. The Fourier transform allows the user to take a non-periodic signal and transform it into a frequency/amplitude representation of itself. This is very useful again, because the real world isn’t always periodic, and because it allows you to analyze what frequencies the given signal was made from, or what frequencies combined to give you this confusing aperiodic signal which you are trying to analyze. Each takes its use from what is given, allowing the user the ability to choose the representation that fits the situation.

It is obvious to say that one uses trigonometric identities which are weighted and summed while the other uses the complex exponential for frequency domain representation. This is too simplistic and obvious though. Also, as stated above, each has a use depending on whether the signal is periodic or aperiodic. I will say that the real power of each comes from what it gives the ability to do. The trig version allows you to take a periodic signal, and represent it using weighted and summed trig functions with the ability to choose the harmonic range for precision in representation. The transform though, allows you to take an aperiodic signal, and represent it by its frequency and amplitude over a chosen range. This means that we can use the transform to decompose a signal into its parent frequencies which were summed to make it in the first place. A very useful tool for many applications, including sound analysis!

5.5: The negative frequencies which result from a Fourier transform have no real physical significance, but instead are "place holders" and result from the implementation or use of complex numbers (the square root of -1). Since we are using the complex plane, we will have to deal with negative values being given. Our negative frequencies are merely a mirror representation of the actual frequencies resulting from the complex exponential in the transform. As an aside, the use of the exponential numbers is a matter of mathematical convenience. Our transforms could be done with only real numbers, but it would be much more mathematically rigorous in most applied situations.