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07 July 2020

Project 4

EE-3370.502

1. Network:

The assigned network to my student ID ending in ‘205’ is network number three from our list. The network is depicted below in Figure 1.

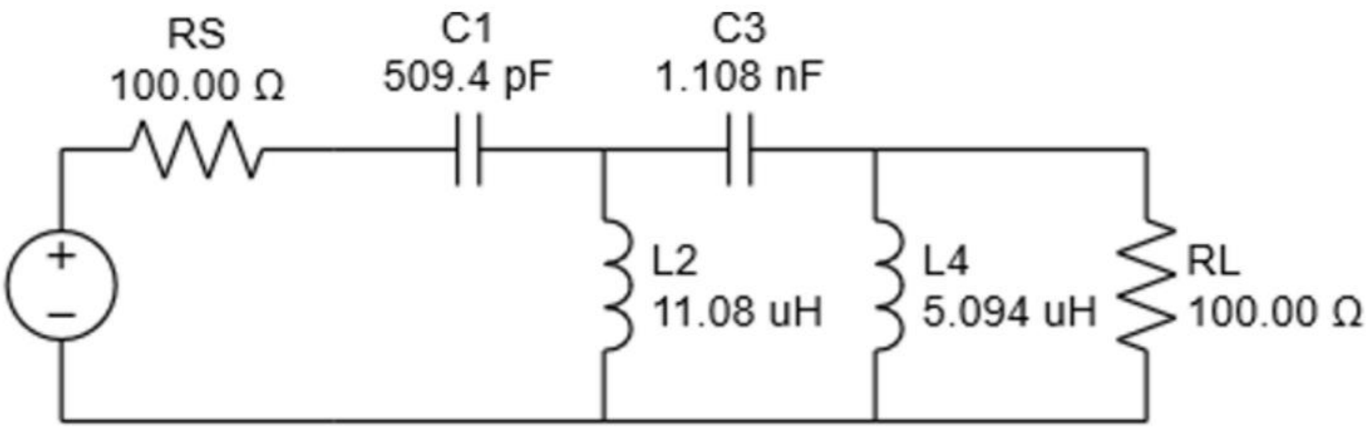

Fig. 1: The assigned Network with values for all elements (except source)

2. Transfer Function and MATLAB:

2.1: The hand derived transfer function is shown below in Figures 2, 3, & 4.

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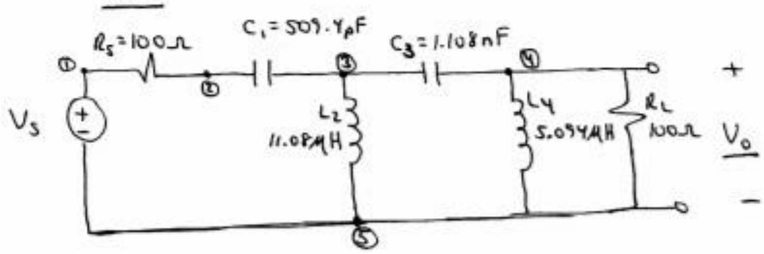
ROBERTO COLÓN

PART I:

STUDENT ID : AØ4973205

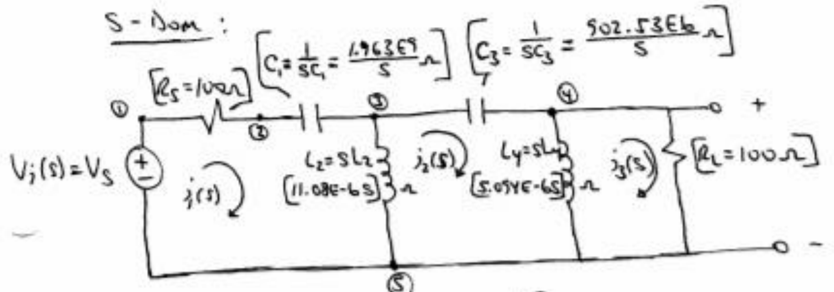
↳ 205 ⇒ NETWORK #3

#3 :



S-DOM :

DIAGRAM 1



$$\left\{H(s) = \frac{OUT}{IN} = \frac{V_o(s)}{V_i(s)}\right\}$$

$$\left[V_o(s) = j_3(s) R_L = 100\, j_3(s)\right]$$

OR

$$\left[V_o(s) = V_{n4} \quad B|C\right]$$ → RESIST. IN PARALLEL HAVE THE SAME VOLTAGE ACROSS

AND :

$$\left[j_{RL} = \frac{V_{n4}}{R_{EQ}}\right] = \frac{V_{n4}}{\left(\frac{sL_4 R_L}{sL_4 + R_L}\right)} = \left[\frac{V_{n4}(sL_4 + R_L)}{sL_4 R_L}\right]^{j_{RL}}$$

↳ NEED : V_{n4}

↳ NEED : V_{n3}

STRATEGY: COLLAPSE THE CIRCUIT TOWARD THE SOURCE BY COMBINING RESISTANCES, THEN EXPAND BACK TO THE RIGHT AS WE FIND V_{n3} THEN V_{n4}.

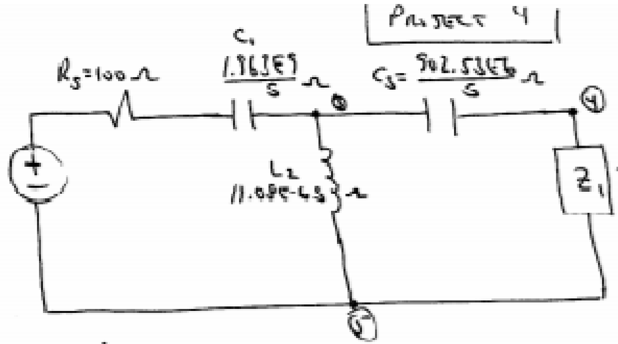
Fig. 2: Page one of the transfer function derivation

p.2

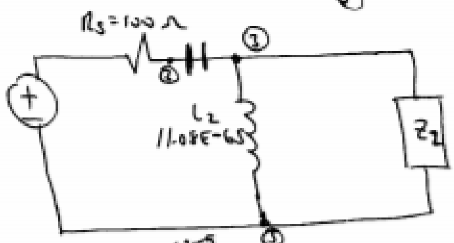
PROJECT 4

R. Colón

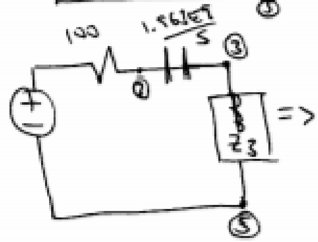
(2)



$$Z_1 = \left[\frac{R_L s L_4}{R_L + sL_4}\right]$$



$$Z_2 = \left[\left(\frac{R_L s L_4}{R_L + L_4 s}\right) + \frac{1}{C_3 s}\right]$$



$$\Rightarrow \left\{\frac{sL_2\left[\frac{sL_4R_L}{R_L+sL_4} + \frac{1}{C_3 s}\right]}{L_2 + Z_2}\right\}$$

$$\left[V_{35}(s) = \frac{V_i(s)\,Z_3}{R_s + \frac{1}{C_1 s} + Z_3}\right] \qquad \left[V_{45}(s) = \frac{V_{35}(s)\,Z_1}{Z_2}\right] = V_0(s)$$

$$V_{R_L} = V_0(s) = (V_{45})$$

$$\cancel{V_{35}(s) = V_i(s)\,\frac{sL_2\left[\frac{sL_4R_L}{R_L+sL_4} + \frac{1}{C_3 s}\right]}{}}$$

EXPANDED FORM

$$V_0(s) = V_{45}(s) = \frac{V_i(s)\left[\frac{sL_2\left(\frac{sL_4R_L}{R_L+sL_4} + \frac{1}{C_3 s}\right)}{L_2 + \left(\frac{R_L sL_4}{R_L+sL_4}\right) + \frac{1}{C_3 s}}\right]\left(\frac{R_L sL_4}{R_L + sL_4}\right)}{\left[\left(\frac{R_L sL_4}{R_L+sL_4}\right) + \frac{1}{C_3 s}\right]}$$

$$V_0(s) = V_i(s)\,\frac{R_L s L_2 s L_4\left(\frac{sL_4R_L}{R_L+sL_4} + \frac{1}{C_3 s}\right)}{\left(L_2 + \left(\frac{R_L sL_4}{R_L+sL_4}\right) + \frac{1}{C_3 s}\right)\left(R_L + sL_4\right)}$$

Fig. 3: Page two of the transfer function derivation

p.3

$$V_o(s) = \frac{V_i(s)\, R_L s^2 L_2 L_4 \left(\frac{s L_4 R_L}{s L_4 + R_L} + \frac{1}{s C_3}\right)}{\left(R_L + s L_4\right)\left(L_2 + \frac{s L_4 R_L}{s L_4 + R_L} + \frac{1}{s C_3}\right)}$$

$$V_o(s) = \frac{\frac{\left[\frac{V_i(s)\, s L_2\, s L_4}{R_s + s L_2 + \frac{1}{C_1 s}}\right]}{s L_4 + \frac{1}{C_3 s}} \cancel{\left(R_L + s L_4\right)}}{\cancel{R_L + s L_4}}$$

$$V_o(s) = \frac{V_i(s)\, s L_2\, s L_4}{\left(R + s L_2 + \frac{1}{s C_1}\right)\left(s L_4 + \frac{1}{s C_3}\right)}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{\cancel{V_i(s)}\, s L_2\, s L_4}{\left(R_s + s L_2 + \frac{1}{s C_1}\right)\left(s L_4 + \frac{1}{s C_3}\right)}}{\cancel{V_i(s)}}$$

$$H(s) = \left(\frac{s^2 L_2 L_4}{s L_4 R_s + \frac{R_s}{s C_3} + s^2 L_2 L_4 + \frac{L_2}{C_3} + \frac{L_4}{C_1} + \frac{1}{s^2 C_1 C_3}}\right) \cdot \left(\frac{s^2}{s^2}\right)$$

$$H(s) = \frac{s^4 L_2 L_4}{s^4 L_2 L_4 + s^3 L_4 R + \frac{s R}{C_3} + \frac{s^2 L_2}{C_3} + \frac{s^2 L_4}{C_1} + \frac{1}{C_1 C_3}}$$

$$H(s) = \frac{s^4 L_2 L_4}{s^4 (L_2 L_4) + s^3 (L_4 R) + s^2 \left(\frac{L_2}{C_3}\right) + s^2 \left(\frac{L_4}{C_1}\right) + s \left(\frac{R}{C_3}\right) + \left(\frac{1}{C_1 C_3}\right)}$$

$$\boxed{H(s) = \frac{(5.644E{-}11)\, s^4}{(5.644E{-}11)\, s^4 + (5.054E{-}4)\, s^3 + (2E4)\, s^2 + (8.03E10)\, s + 1.772E18}}$$

Fig. 4: Page three of the transfer function derivation

2.2: MATLAB BODE Plot

2.2.2: BODE Plot in MATLAB

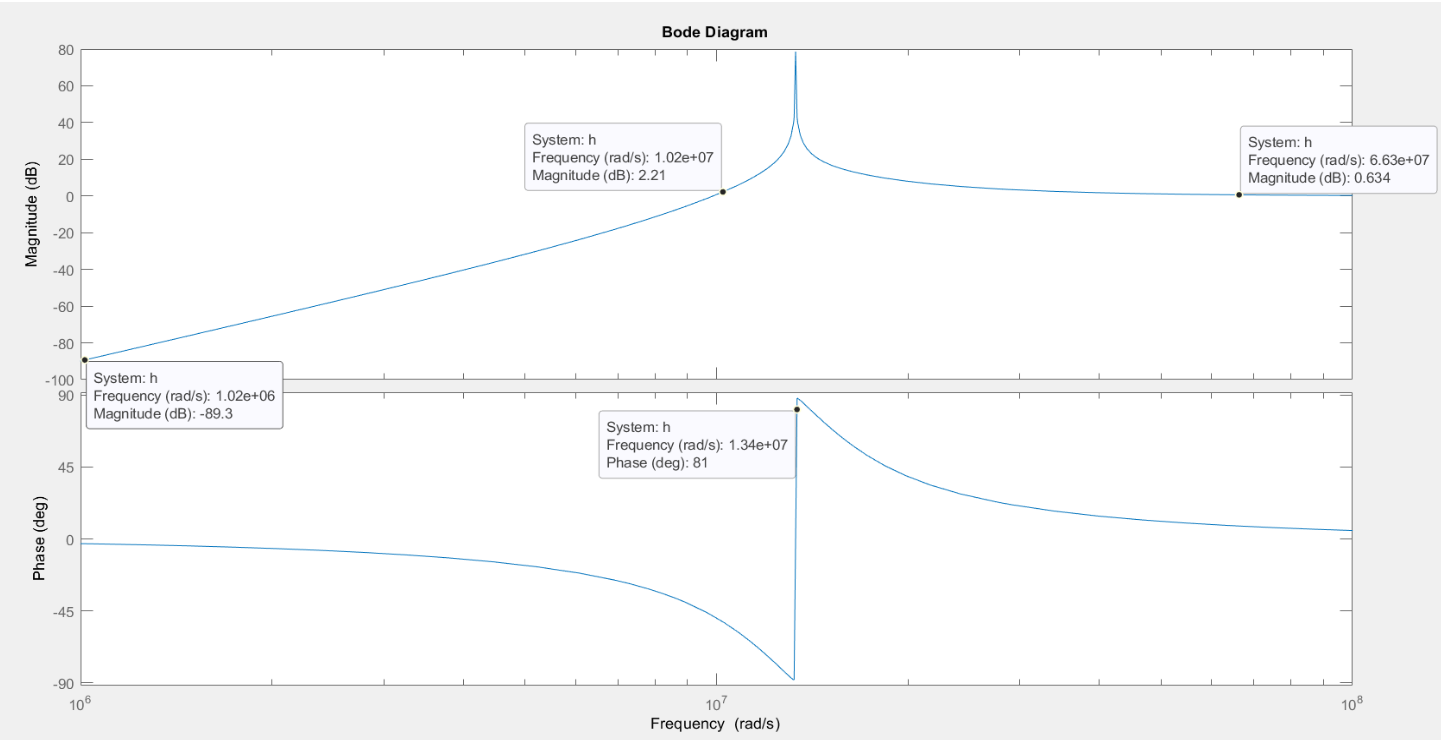


Fig. 5: Transfer function captured in MATLAB. We have data tips indicating the cutoff frequency in the center phase graph. We can see the 180-degree phase shift there. Also indicated are the start and end of a decade to indicate the slope of 90 dB per decade as well as the last marker on the right side of the magnitude graph indicating the constant magnitude as time frequency increases.

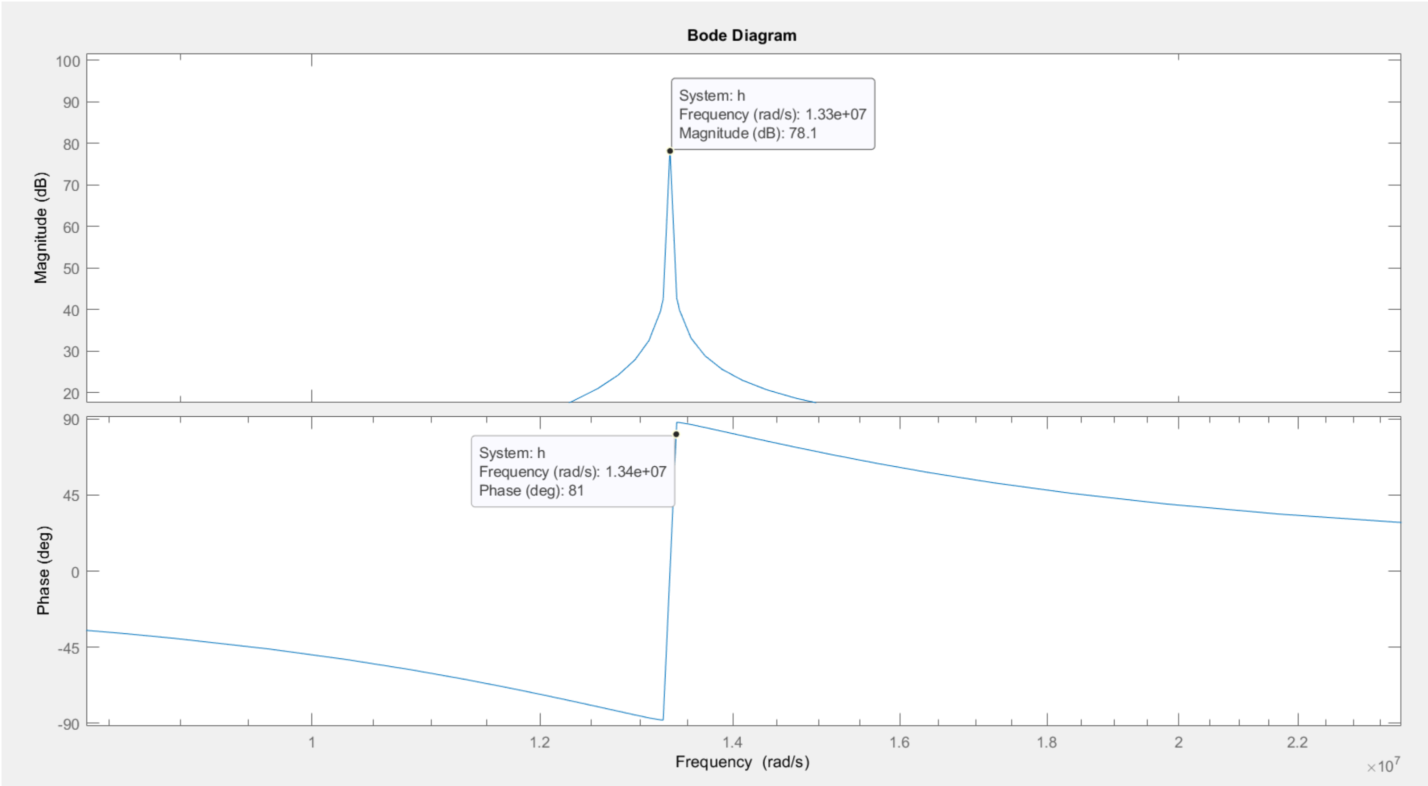


Fig. 6: A tighter view of the peak of the magnitude plot indicating the maximum at the peak of the notch filter. We can also see the phase shift line is not perfectly vertical at this magnification. The cutoff frequency is indicated in the phase plot.

2.2.3: MATLAB code: The code used to generate the BODE plot is shown below.

```
Command Window
>> syms s
>> s = tf('s')

s =

  s

Continuous-time transfer function.

>> h= ((5.644e-11)*s^4)/(5.644e-11*s^4+5.094e-4*s^3+2e4*s^2+9.03e10*s+1.772E18)

h =

                           5.644e-11 s^4
  ----------------------------------------------------------------
  5.644e-11 s^4 + 0.0005094 s^3 + 20000 s^2 + 9.03e10 s + 1.772e18

Continuous-time transfer function.

>> bode(h)
fx >>
```

3. P4 SPICE Bode Plot:

3.1: Network in SPICE (Top SPICE):

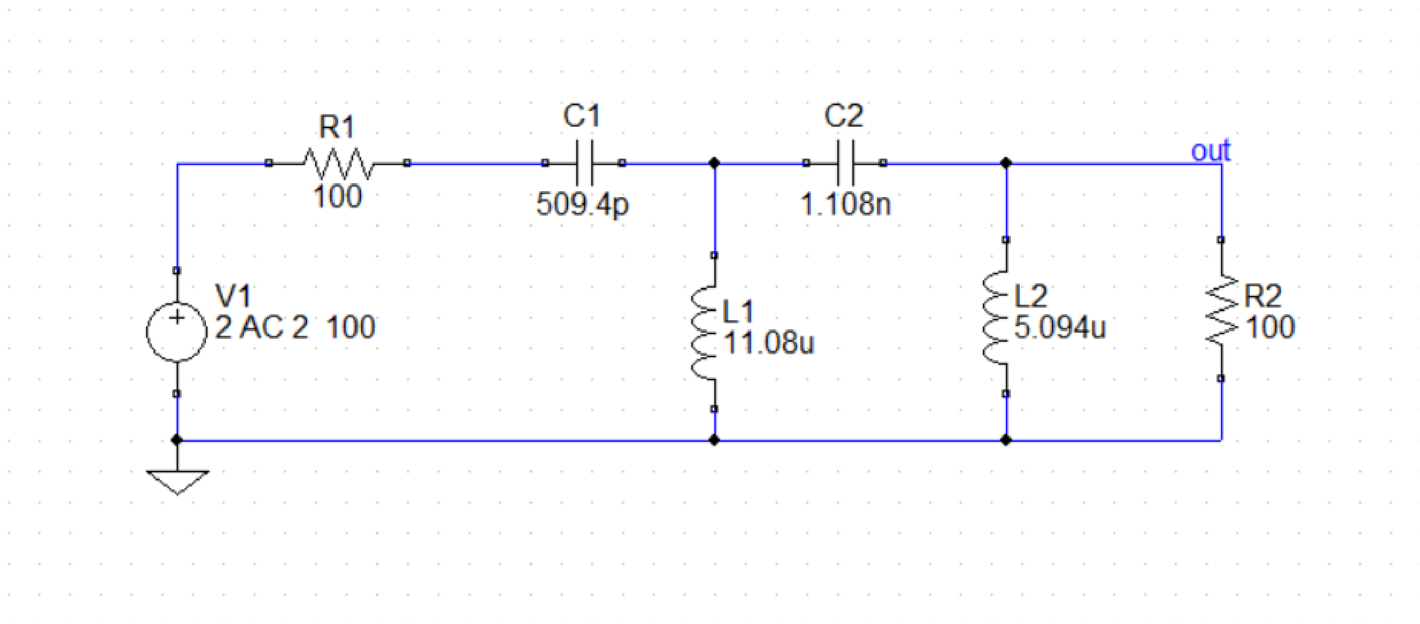


Fig. 7: The given network build in Top SPICE with all values. The selected output node for simulation is labeled 'out'.

3.2: SPICE Bode Plot:

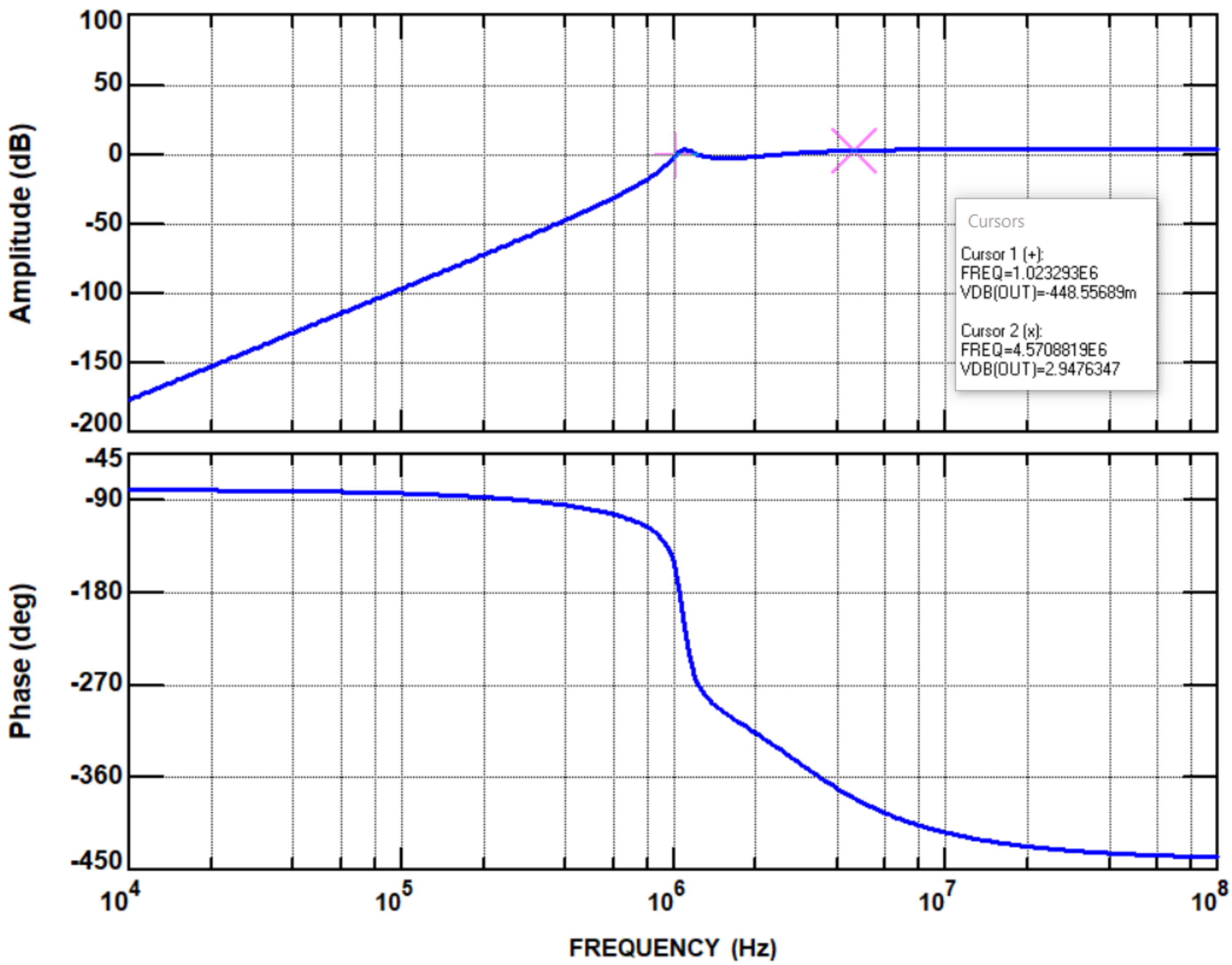


Fig. 8: The bode plot of our filter. There are two callouts with labeled values in the inset. The 'cross' is the -3dB point while the 'X' labels the constant value as the frequency is increased past the cutoff frequency.

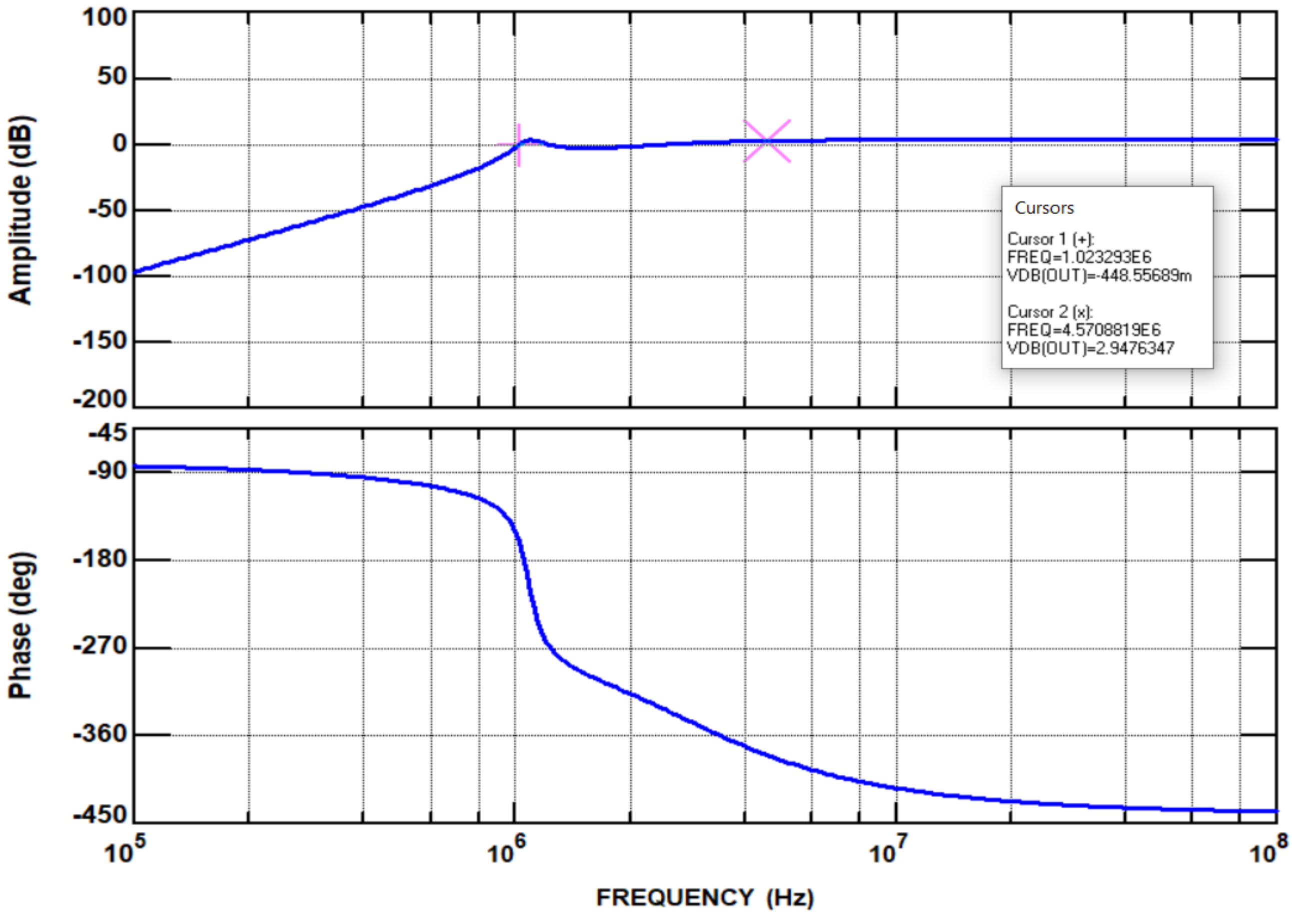


Fig. 9: This is a closer view of the bode plot for the given network. The lower limit was increased by a decade to bring out the smaller details of the plot near where the magnitude peaks as well as the detail of how the phase changes as frequency increases. The callout values are the same values as in figure 8.

The given network is a high-pass filter, but it appears that there is more than one term in the denominator which shows in the phase plot. The phase appears to have at least two terms affecting its slope as the frequency is increased. If there are not multiple terms affecting the phase shift, then the changes in slope may be attributed to the noise near the peak of the magnitude plot and the systems response as it attempts to level back out.

4. Final Report:

4.1:

In the most general mathematical description, a transfer function describes the ratio of a systems output to its input. With regard to systems and circuits, there are a few names to describe different types. The names are relative to the ‘gain’ described by the transfer function. For our network, our transfer function was a ration of the voltage output to the voltage input. This is a voltage gain transfer function and is mathematically described below. Consider V_{in} = X(t) and V_{out} = Y(t), then the transfer function is:

$$H(t) = \frac{Y(t)}{X(t)} = \frac{V_{out}}{V_{In}} \qquad \text{Eqn. 1}$$

Another way to describe the transfer function is by relating it to the first time one learns about functions when in precalculus or algebra. A function takes a variable into an equation and operates on it to produce a new value. With a transfer function, I like to think of it as adding a higher dimension to the original function description. Instead of putting just a single variable into our system, we are putting a function into it and getting a new function out. The new function is operated on by the system by scaling and phase shifting. And the new expression is a function which describes the input to output ratio mentioned above.

4.2:

My transfer function simulated in MATLAB did not match up with the SPICE simulation. My MATLAB plot made the transfer function appear as a notch filter (Band stop, Band reject). And the filter also looked inverted as I have only seen Notch filters with the open end facing up.

4.2.1:

I know where my error was during my transfer function derivation. The strategy that I decided to go with may have been what led me to this error as well as failure on subsequent attempts to derive the transfer function. My strategy was to conduct an s-domain analysis of the given network by reduction and then expansion exploiting voltage division equations to perform substitutions in order to get the input to output ratio in its ‘unclean’ form. This method chosen can be harkened to the ‘brute force’ method as it involved multiple pages of algebraic manipulation and simplification to reduce the transfer function into its most simplified and readable form which would deliver us the information with regards to zeros and poles.

The exact mistake I made is at the bottom of the second page of my derivation. During the step where the node voltage equation for V_{n3} was derived, I correctly stated that there was a reduced impedance Z_3 in the denominator for that node voltage equation. But when I went to write that node voltage equation into my transfer function, I accidentally replaced the impedance Z_3 with a capacitor impedance. This may have been a result of misreading, whereby the capacitor equation was written closely and mistakenly written or whereby the work was just a little sloppy and the wrong term was written. Either way, the mistake drastically changes the result. Cleanliness and organization are critical to this work. A second attempt was made once the mistake was found, but that iteration quickly turned into an exercise in algebraic manipulation with too many terms to expand during simplification and combination of denominators. A third attempt was considered with the plan to use KVL or KCL equations along with the use of a unit input voltage but that attempt has not come to fruition.

4.3: Each of these networks was a filter, state the filter response type that your filter displayed.

The filter response displayed by my filter is that of a high pass filter. The network is a fourth order network and only the combined output is displayed in the plot. This is a contrast to deriving the function by hand because when it is derived by hand and put input simplified form, the order of the numerator and denominator can be seen. The zeros and poles can then be analyzed and identified readily upon examination of the transfer function equation. When putting the network into SPICE, the bode plot is given but the actual mathematical expression of the transfer function is not displayed.

4.4:

The filter given is a fourth order filter, and this is confirmed not only by the number of independent energy storage elements, but also by the 20n dB/decade slope which can be seen in the filter magnitude plot where n= order of the filter. We can see a positive 80dB/dec. slope as the filter moves up to its cutoff frequency in the magnitude plot. As a filter's order is increased, there is a correlation to an increase in slope which is 20n dB/decade for our log graph. When using octaves, it is 6n dB/decade change (increase or decrease).

The filter type is a Chebyshev 4th order filter. I did some research on different types of filter designs to include, Butterworth, Chebyshev, Inverse Chebyshev, Elliptic, and Bessel. I chose Chebyshev as my filter design because it matches the layout of my capacitor/inductor parallel setup along with the output load resistor. Additionally, I was able to find a calculator which can calculate an output voltage based on the number of capacitor/inductor legs that are placed in series.