Selfish Behaviours in Resource Sharing for Mobile Wireless Systems

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***Abstract*—Resource sharing is an important problem in many different fields of application, from investments to water resource what we try to achieve is a fair and efficient way of splitting resources among different users. In this paper we analyse two easy ways of splitting resources in the context of channel allocation for wireless systems. We describe the outcomes of the different strategies and we look for those games in which a selfish behaviour arises.**

I. Introduction

The problem of sharing resources among different users is a topic of growing interest in many fields, also outside engineering. For example, the way in which we should share water resources [1] is a problem that affects our everyday life and its impact is growing with the recent climate changes. Similarly, the sharing of high-quality medical resources is a very important problem in many hospitals, specially in some regions of the world [2]. Inside the field of engineering and, in particular, of wireless communications the increasing number of devices and the arise of new technologies (such as Internet of Things [3]) has created a high need for an efficient way of splitting the available bandwidth.
In this paper we create a simple Bayesian game and we use it to analyse the selfish behaviours in the context of resource sharing. What we find, is that what favours the creation of selfish Nash Equilibria is the uncertainty present in the game. In fact, if the uncertainty grows over a certain threshold, the players are encouraged to stop cooperating and start acting only in their own interests.

II. Related Work

In wireless communications, in order to allow a reliable trasmission the avaible spectrum is divided into a set of disjoint and non-interfering channels. All such channels can be used simultaneously while maintaining an acceptable received signal [4]. In order to divide a given radio spectrum different techniques can be used, such as frequency division, time division, code division. A certain number of channels is then assigned to each user in the system. There exist three main schemes to perform channel allocation: fixed channel allocation (FCA), dynamic channel allocation (DCA) and hybrid channel allocation [5]. In fixed channel allocation schemes a set of channels is permanently allocated to a user for its exclusive use [4]. In dynamic channel allocation schemes, instead, channels are not assigned permanently, but are placed in a pool and assigned to new calls as needed [5]. Hybrid channel assignment schemes are a mixture of the FCA and DCA techniques.
In this paper we analyse what happens when the users are given the opportunity to choose which channels to use for transmission and we perform such analysis using the game theoretic framework.
The game theoretic framework is a tool widely utilized in the context of channel allocation for wireless communications. For wireless sensor networks, a protocol named GBCA has been designed to perform channel assignment using game theory [6]. The algorithm takes into account both network topology and routing information and finds the channel allocation solution minimizing the interference inside the network through a game among the Parent-Children Sets (PCS) [7]. There have also been proposed many algorithms that utilize game theory in order to perform channel allocation in cognitive radio networks [8]. In [9], a game theory model is proposed to improve the throughput of the CRN while solving the channel allocation problem considering the co-channel interference. In [10] a game theoretic model for channel allocation is proposed, where each secondary user knows about the pay-off values and strategies of each other. In [11], a model based on the auction of the bandwidth is proposed. Another possible approach is the Nash bargaining, this method is used in [12], where a multiuser bargaining algorithm is developed based on optimal coalition pairs among users.
In this paper, we develope a simple Bayesian game through which we first analyse what are the situations in which the users are more encouraged to play in a selfish manner when choosing the channel to transmit on. Then we compare this situation with the pareto optimal solution and we calculate the price of anarchy. Finally, we implemented some MATLAB simulations in order to draw some general conclusions.

III. System Model

We began our analysis considering a static game of complete information. We considered a two players game, both player 1 and player 2 can choose whether they want to transmit on channel 1 or on channel 2, therefore they both have two possible actions. Since channel 1 provides a better gain to both players, they both achieve a higher payoff (10) if they transmit on such channel. However, if they both transmit on the same channel they both cause interference to each other and, therefore their payoffs will be 0. The game described can be represented by the normal form game shown in Table I.

Player 1 \ Player 2	channel 1	channel 2
channel 1	0,0	10,1
channel 2	1,10	0,0

Table I: Static game of complete info, 2 channels

This game presents two pure Nash Equilibria, (ch1, ch2) and (ch2, ch1). In both NEs, the two players prefer to choose a worse channel rather than colliding and the result is that they transmit on two different channels. Therefore, all the available resource is used.
If instead, we consider the same game, but with different preferences for the two players, then the payoffs are:

- player 1 has a better gain on channel 1, therefore his payoff is 10 when transmitting alone on channel 1
- player 2 has a better gain on channel 2, therefore his payoff is 10 when transmitting alone on channel 2

This game can be represented by the normal form matrix in Table II.

Player 1 \ Player 2	channel 1	channel 2
channel 1	0,0	10,10
channel 2	1,1	0,0

Table II: Static game of complete info, 2 channels

In this case, the situation is very similar to the one in Table I. The game presents the same two NEs, (ch1, ch2) and (ch2, ch1). However, in this case the one of the two NEs, (ch1, ch2), represents the social optimum. In fact, the sum of the two players payoff is 20 in the (ch1, ch2) NE and is 2 in the (ch2, ch1) NE. Therefore, the avaiable resource is still completely used and a bad way of sharing the resource is still preferred to the collision case.
We then considered a game with a similar setup, but with a higher number of channels. We considered the case in which two players have to choose two channels to transmit on, among 4 possible channels. The payoffs when transmitting on the four different channels are distributed as follows:

- channel 1 has the highest gain for both players, therefore when trasmitting alone on this channel they both achieve a payoff of 100
- channels 2 and 3 have a worse gain than channel 1, but better than channel 4, the payoff for the two players when transmitting alone on those channels is 50
- channel 4 has the worst channel gain, their payoffs when transmitting alone on this channel is 10
- like in the previous considered cases, if they transmit on the same channel, they cause interference to each other and, therefore, they both achieve payoff equal to 0 The final payoff for the two players is the sum of the payoffs that they achieve in the two channels they decide to transmit on. The game described can be represented by the matrix in Table III.

	ch 12	ch 13	ch 14	ch 23	ch 24	ch 34
ch 12	0,0	50,50	50,10	100,50	100,10	150,60
ch 13	50,50	0,0	50,10	100,50	150,60	100,10
ch 14	10,50	10,50	0,0	110,100	100,50	100,50
ch 23	50,100	50,100	100,110	0,0	50,10	50,10
ch 24	10,100	60,150	50,100	10,50	0,0	50,50
ch 34	60,150	10,100	50,100	10,50	50,50	0,0

Table III: Static game of complete info, 4 channels

In this case, there are 6 pure NEs, that are: (ch 12, ch 34), (ch 13, ch 24), (ch 14, ch 23), (ch 34, ch 12), (ch 24, ch 13), (ch 23, ch 14). The NEs represent all the possible ways in which the two players can divide the two channels. Both players, in fact, prefer to transmit on worse channels than colliding in one or more channels. Our conclusion is that, when the players are completely informed on the game setup, they are induced to cooperate in order to use all the available resource.
If, instead, we consider a Bayesian setup for this game, the outcomes are different. We analysed the same game proposed in Table I and II in a Bayesian setup. We considered the case in which both the two players can have two different types:

- type A players have a higher channel gain (and therefore a higher payoff) when transmitting on channel 1
- type B players have a higher channel gain when transmitting on channel 2

In this setup there are 4 different possible payoff tables:

- if both players have type A (same game of Table I):

Pl 1 \ Pl 2	ch 1	ch 2
ch 1	0,0	10,1
ch 2	1,10	0,0

- if player 1 has type A and player 2 has type B (same game of Table II):

Pl 1 \ Pl 2	ch 1	ch 2
ch 1	0,0	10,10
ch 2	1,1	0,0

- if player 1 has type B and player 2 has type A:

Pl 1 \ Pl 2	ch 1	ch 2
ch 1	0,0	1,1
ch 2	10,10	0,0

- if both players have type B:

Pl 1 \ Pl 2	ch 1	ch 2
ch 1	0,0	1,10
ch 2	10,1	0,0

Since the players have two types and two possible actions there are four possible strategies for each player. In fact, every player needs to chose what action to play both if he is of type A and if he is of type B.
In the case in which the type distribution is uniform for both players the normal form of the game is the one shown in Table IV.

Pl 1 \ Pl 2	11	12	21	22
11	0,0	$\frac{1}{4}$(11,10)	$\frac{1}{4}$(11,2)	$\frac{1}{4}$(22,22)
12	$\frac{1}{4}$(20,11)	$\frac{1}{4}$(20,20)	$\frac{1}{4}$(20,2)	$\frac{1}{4}$(20,12)
21	$\frac{1}{4}$(2,11)	$\frac{1}{4}$(2,20)	$\frac{1}{4}$(2,2)	$\frac{1}{4}$(2,12)
22	$\frac{1}{4}$(22,22)	$\frac{1}{4}$(11,20)	$\frac{1}{4}$(11,2)	(0,0)

Table IV: Bayesian game, 2 channels

The payoffs in every cell are obtained taking the expected value over all the possible outcomes for the two considered strategies considering all the type distribution for the two players. For example, the payoff for player 1 in the (12, 11) case is obtained as:

$$
\begin{aligned}
u_1(12,11) &= \frac{1}{4} \cdot 0 \text{ (both players of type A)} \\
&= \frac{1}{4} \cdot 0 \text{ (pl 1 of type A, pl 2 of type B)} \\
&= \frac{1}{4} \cdot 10 \text{ (pl 1 of type B, pl 2 of type A)} \\
&= \frac{1}{4} \cdot 10 \text{ (both players of type B)} \\
&= \frac{20}{4}
\end{aligned}
$$

In this game formulation, there are three Bayesian Nash Equilibria:

- (11,22) and (22,11) are the equivalent of the previous games NEs. The players agree of transmitting on one channel and they cooperate to use all the achievable resource, even when this means trasmitting with a lower gain
- (12,12) a selfish BNE. Since there is uncertainty in the game, the two players are induced to transmit on their preferred channel, no matter what the other player does

We implemented the Bayesian game described above using MATLAB and we analysed what happens with different type distributions and with a higher number of channels. To perform some comparisons between the different cases we calculate the price of anarchy in this game.
We calculate the PoA considering the selfish BNE as the worst case scenario. For the social optimum scenario, instead, we consider a Stackelberg formulation of the game described above. In the Stackelberg formulation of the game, the players move alternately. Therefore, the player moving first will always pick the channel he prefers. The player moving second, instead, will choose the channels he prefers among the remaining ones, since this will still give him a better payoff than colliding. For example, for the stackelberg formulation of game described in Table I (Figure 1), the only NE of the game is (ch1, ch2). There is actually another NE (ch2, ch1), but we don't consider it since it contains a non credible threat by player 2.

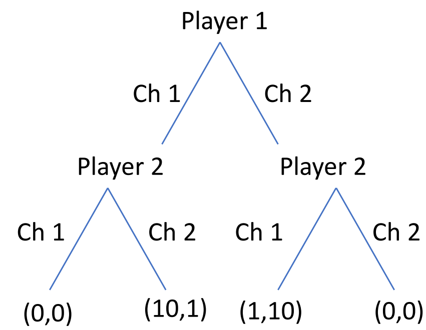

Figure 1: Stackelberg formulation for game in Tab I

For the Bayesian game described in Table IV, if player 1 is always the first to choose, he will get a payoff equal to 10, no matter what type is player 2. Player 2, instead, will get a payoff of 10 when his type is different from player 1's type and a payoff of 1 when their preferred channel is the same one. The average outcome of the game is $\frac{1}{4}(40, 22)$ and therefore

$$\text{PoA} = \frac{\text{total payoff social optimum}}{\text{total payoff worst case NE}} = \frac{40+22}{20+20} = 1.5$$

IV. Results

To draw our general conclusion we started from the case with two players contending two channel described in the previous section and we modified some parameters. In the first part of this section we expose these results. Each player has a preferred choice, which gives a best payoff, and a second preferred choice, which gives a worst payoff. Each player can also be of two different types, with respect to which kind of channel he prefers.
We introduced two parameters that we used to evaluate the outcomes of our simulations, what we call *Type distribution parameter* (*TDP*) is the ratio between the probability for a player of being of the first type and the probability of being of the second type. This means that for *TDP* $= 1$ the probabilities of the players of being of type 1 are equal to 0.5. We let this parameter vary in the interval $[1, 11]$.

$$TDP = \frac{P[\text{Pl.}_{\text{type}} = 1]}{P[\text{Pl.}_{\text{type}} = 2]} \in [1, 11]$$

The second parameter is the *Payoff parameter* (*PP*) defined as the ratio between the payoff given to the players when they transmit alone in their preferred channel and the payoff given to them when they transmitt alone on the worse channel. We used the same range of values for this parameter and the *TDP* parameter.

$$PP = \frac{\text{Payoff}_{\text{Better}}}{\text{Payoff}_{\text{Worse}}} \in [1, 11]$$

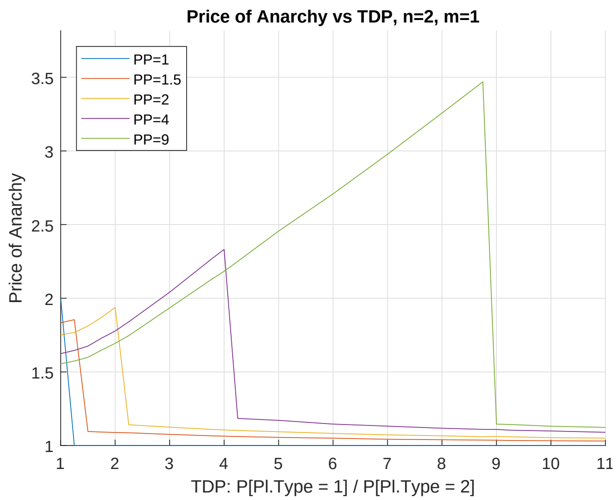

Figure 2: Price Of Anarchy vs. Type distribution parameter

In Figure 2 we show the behavior of the PoA as a function of the *TDP* parameter. It is possible to notice that all the curves grow almost linearly in the first part of the graph, then there is a vertical drop in the PoA value and from there on the value remains almost equal to 1.
The first part of the curve is explained with the fact that if the type probability grows and both players transmit on their preferred channel, more collisions occur on such channel channel. Therefore, when the *TDP* parameter grows, the social optimum solution becomes more and more convenient.
We analysed what happens in the game when there is the vertical drop in the PoA value and we found out that this is due to the fact that the selfish BNE is no longer a solution for the game. In fact, if the probabilities are too high, there is not enough uncertainty left in the game and the players prefer to cooperate, like in the static version of the game. For higher values of *TDP*, the only two BNEs in the game are the ones in which the two players agree on transmitting on two different channels and, therefore, the total payoff is equal to the social optimum one and the PoA is equal to 1.

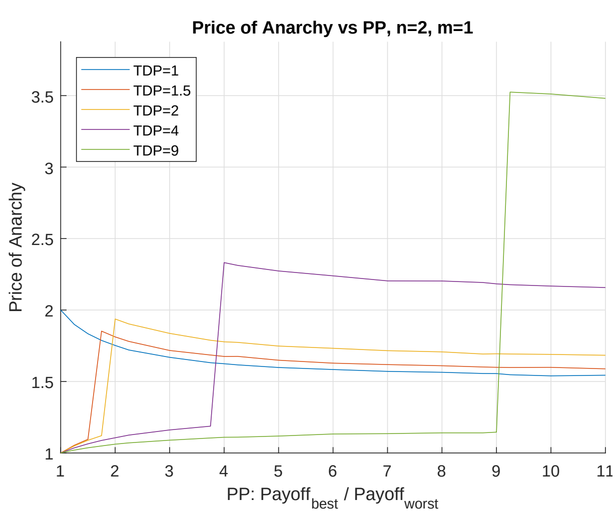

Figure 3: Price Of Anarchy vs. Payoff parameter

It is also possible to see that a higher value for the PoA is reached if the payoff on the preferred channel is much higher than the other (higher *PP*).
Then, we calculated the behavior of the PoA as a function of the *PP* parameter (Figure 3). The curves in Figure 3 remain almost equal to 1 for low values of the *PP* parameter, then they increase vertically and finally, for higher values of *PP* they remain constant.
This behavior is again motivated by the fact that for very low values of the *PP* parameter, the selfish BNE is not a solution of the game. When the selfish BNE becomes a solution for this game, the price of anarchy grows and then stabilizes to a fixed value.
A higher value for the PoA is reached when the *TDP* parameter is higher, but in this cases the selfish BNE arises only for higher values of the *PP* parameter. This can be explained with the fact that if the uncertainty in the game is low, there needs to be a big advantage for the players in terms of payoff to encourage them to play selfishly.
We noticed that there is a threshold value for the selfish BNE to become a solution for the game, this threshold is reached when the *PP* parameter value is almost equal to the *TDP* one:

$$\mathrm{PP}_{\mathrm{threshold}} \sim \mathrm{TDP}$$

It is possible to notiche this threshold behavior in the 3D plot showing the PoA behavior as a funcition both of the *TDP* parameter and of the *PP* parameter (Figure 4).

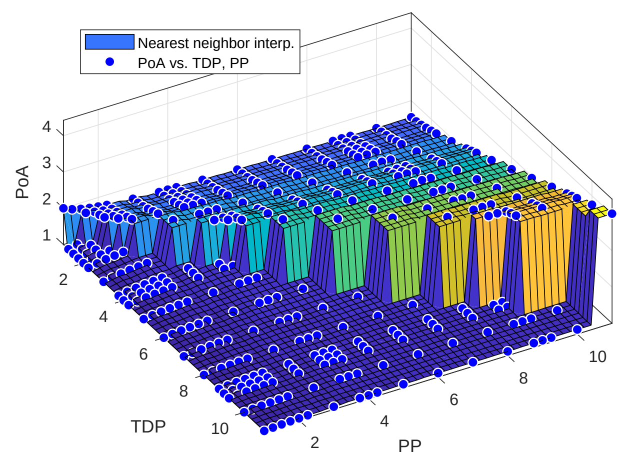

Figure 4: Price Of Anarchy vs. Payoff parameter and Type distribution parameter

In the next part of this section we expose the results of the simulations run with:

- a variable number of channels: $n \in \{4, 6\}$
- a variable number of channels choosen by each player: $m \in \{2, 3\}$
- a fixed number of kinds of channel (and, therefore, of players types): $k = 2$.

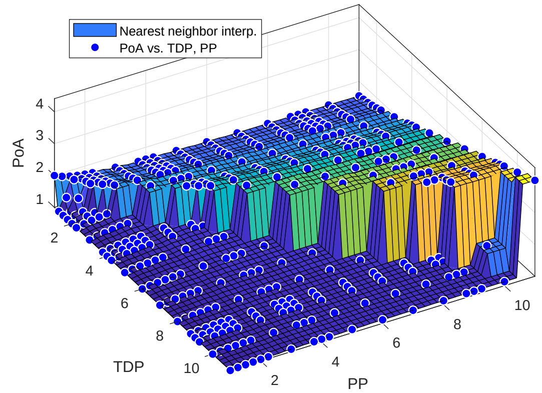

Figure 5: Price Of Anarchy vs. TDP and PP with $n = 4, m = 2$

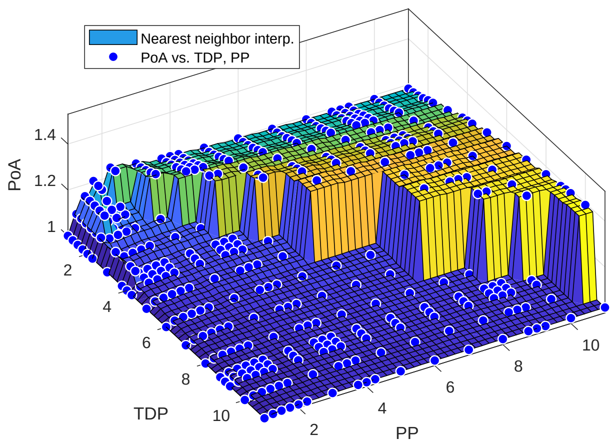

Figure 6: Price Of Anarchy vs. TDP and PP with $n = 6, m = 2$

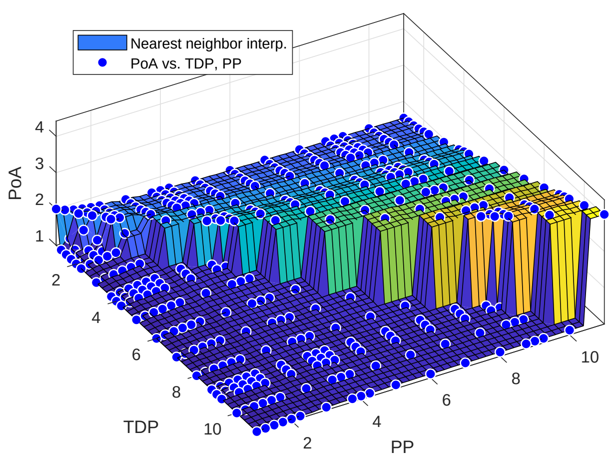

Figure 7: Price Of Anarchy vs. TDP and PP with $n = 6, m = 3$

More precisely, we tested the following combinations of parameters:

- $n = 4, m = 2$: (Figure 5)
- $n = 6, m = 2$: (Figure 6)
- $n = 6, m = 3$: (Figure 7)

The simulations showed that the behavior that we noticed for the case with two channel are maintained when the number of channel is increased. For the two cases in Figure 5 and 7 the outcomes for the PoA values are almost equal to the two channel case explained above. For the case with $n = 6$ channels in total and $m = 2$ channels to choose among for every player (Figure 6), the behaviour of the curve is similar, but the values reached for the PoA are lower (the maximum value reached is less than 2). This can be explained with the fact that since more channels are available (2 channels remain free), it is more difficult that the two players transmissions collide. This is also visible in the 2D graph for the *TDP* parameter (Figure 8). In this graph in fact, the first part of the curves is no longer linear, but it is now a sub-linear growth.

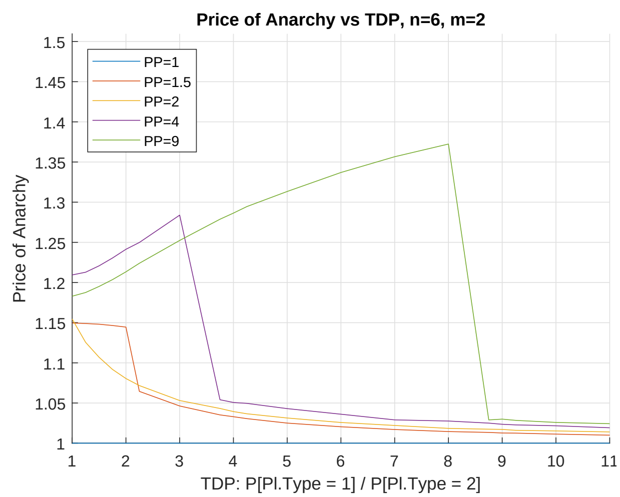

Figure 8: Price Of Anarchy vs. TDP with $n = 6, m = 2$

V. Conclusions

We created a game to describe in a simple way a wireless communication over multiple channels. We analysed this Bayesian game in order to draw some conclusions on what are the cases in which the players are encouraged to play selfishly. The results of this simulations show that players are encouraged to play in a selfish manner when there is enough uncertainty in the game. In particular, a selfish BNE arises in the game when two elements are present in the game:

1. There is enough uncertainty about the other players preferences
2. The ratio between the payoff on the preferred channel and the payoff on the not preferred channel is high enough

From our simulations, we concluded that this general principle is valid with any number of channels considered.

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