4 A 6 DOF VEHICLE RIDE & HANDLING MODEL

Full vehicle motion assuming small roll & pitch angles on a flat road

The popular 'bicycle' handling model only goes so far in describing real vehicle dynamics. It's advantages are that it is easy to formulate mathematically (forces and moments in two dimensions are easily visualised and applied). Also it can provide a platform for a combined slip, load dependent tyre model, and simplified load transfer calculations can be included so that vertical loads are approximately correct. However it doesn't consider suspension effects such as roll, pitch and heave (ride), nor does it cope with anything other than a flat, smooth road.

5.1 Consider cornering. The forces generated by the tyres have by far the greatest influence on vehicle dynamics. If you can approximate load transfer using the bicycle model, why bother with a proper model for roll ?

The bicycle model can be extended to include roll, while retaining a lot of the 'forces and moments in two dimensions' thinking, by using a roll centre which acts as a force centre and also a motion centre (eg using a maths model based on fig 1). But, this only brings roll to the party, with pitch and heave still left out. It also means we have to deal with 3d force and moment calcs, and hence some vector algebra.

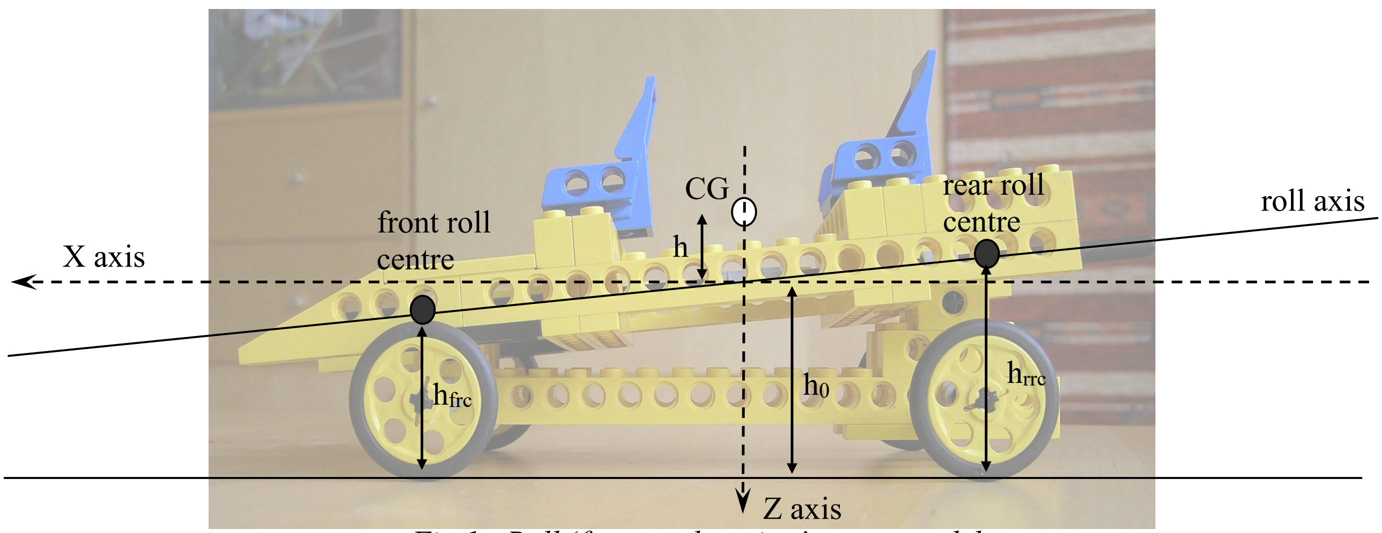


Fig 1 : Roll 'force and motion' centre model

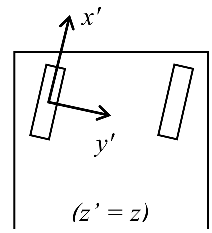
If we 'bite the bullet', and live with the additional complexity of using vector algebra, we can move to a full six degrees of freedom model, which will simulate full ride and handling dynamics using surprisingly few equations. Also the suspension can be modeled using force centre principles (roll centres for roll, and trailing arms to include anti effects) so we can use the 'suspension force compensation' method introduced at the end of Chapter 3.

To keep things reasonably simple however, we will constrain this model to operate on a 'flat' road, and for motion where vehicle pitch and roll angles are 'small'. By 'small' I mean small enough for the approximations $\sin\theta = \theta$ and $\cos\theta = 1$ to be suitable, and this is true up to around 10º, which is easily enough for most realistic pitch and roll. By 'flat' I mean that there are no significant camber angles or 'hills' on the road. It is still simple to extend the model to include road profiles of the type seen in the ride section from the vehicle dynamics and simulation course however.

Axis systems and vector notation

The vector algebra might be a bit off-putting at first, but it is very efficient, and is not terribly complicated once you learn to follow the rules. First consider the axis systems, and the principal vectors which describe the motion (fig 2).

Three axis systems exist :
(x,y,z) : vehicle local axis system (or 'local frame')
(x',y',z) : tyre axis system
(X,Y,Z) : environment (world / track) axis system (or 'global frame')



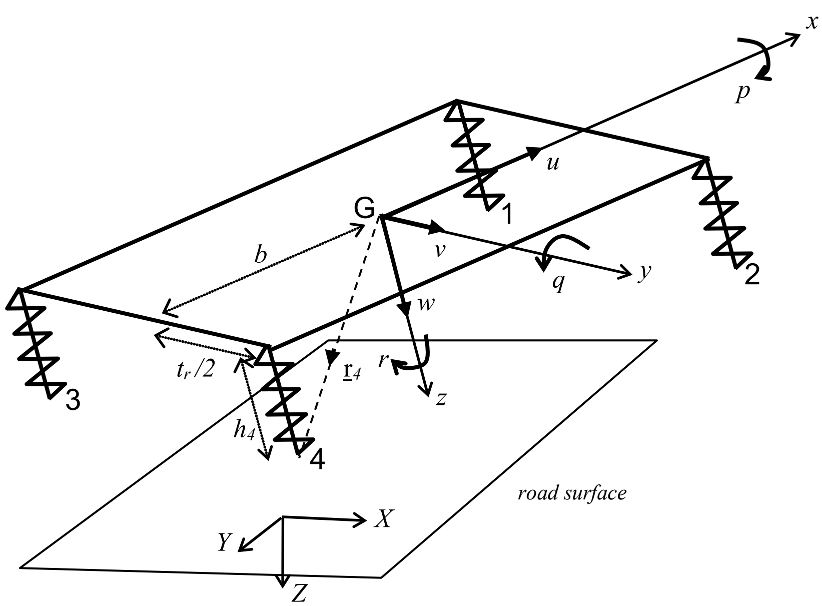


Figure 2 : Axis systems, state and other vectors

The vehicle velocity components are the 'core' vehicle states – described in two vectors; translation, $\underline{v}$, and rotation, $\underline{\omega}$:

$$\underline{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \qquad \underline{\omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

Notes :

1) As for the bicycle model, vehicle motion is described relative to the local axis system
2) The axis systems follow the right-hand rule, and all variables are *signed* (and processed in SI units). So, if the vehicle were reversing straight backwards at a speed of 10m/s, the translational velocity vector would be $\underline{v} = \begin{pmatrix} -10 \\ 0 \\ 0 \end{pmatrix}$. When turning to the left at 10 degrees per second (assuming steady-state – so no pitch or roll velocity), its rotational vector would be $\underline{\omega} = \begin{pmatrix} 0 \\ 0 \\ -10\pi/180 \end{pmatrix}$
3) Tyre contact patches are described by points, and located assuming that they lie a distance h in the z (local) direction from each corner, with the corners numbered as above. Suspension compliance exists in the local z direction, so h will vary dynamically.
4) The position of tyre contact patch 4 is given with respect to the centre of gravity, G : $\underline{r}_4 = \begin{pmatrix} -b \\ t_r/2 \\ h_4 \end{pmatrix}$. Note that confusion can arise in the sign of variables; b is a distance here, not a signed variable. Another way of writing this would be $\underline{r}_4 = \begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix}$, where, eg, $x_4 = -1.1$, $y_4 = 0.75$, $z_4 = 0.3$.

Use of vectors for vehicle dynamics

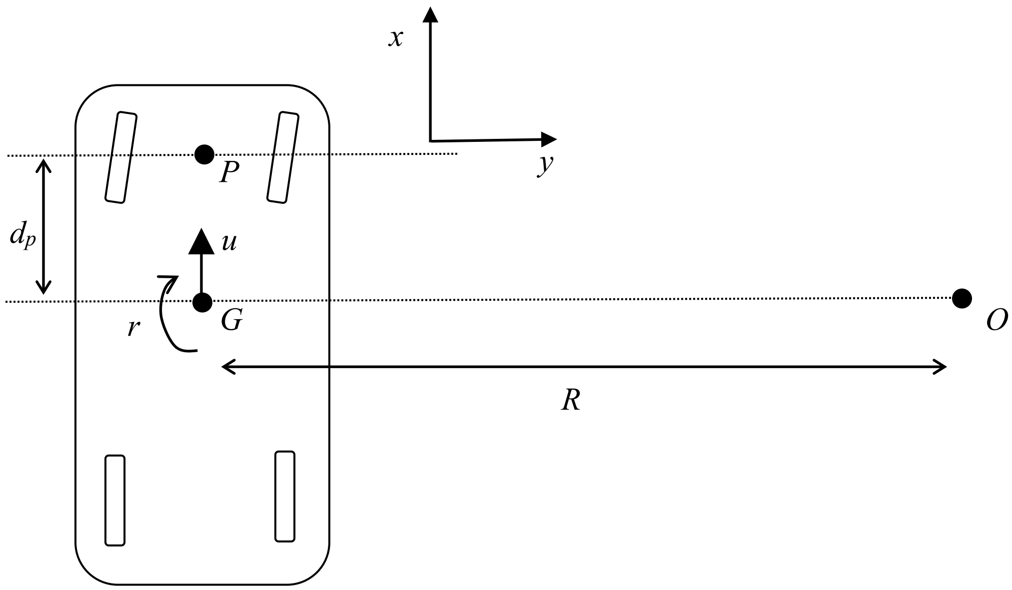
Hopefully you're comfortable with the concept of rotation of a body causing velocities and accelerations at a point on the body away from the centre of rotation ($v = R\omega$, $a = R\omega^2$ for constant ω). For the bicycle model, we only considered yaw of the body, so 'ω' is the amount of yaw rotation. With a vector approach, we use the cross product to find velocity (or accel / other 'rate of change') of a point, which might be caused by a combination of rotations (eg yaw and roll together).

Reminder of cross product :

$$\underline{r} \times \underline{p} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} bf - ce \\ -(af - cd) \\ ae - bd \end{pmatrix}$$

Vector theory : Effect of rotation of the axis system, and vector summation

If the vehicle (and hence its axis system) rotates, then any vector describing position or motion within that axis system experiences a change. To understand this, consider the position vector $\underline{d}_p$, as the vehicle rotates about point G, with constant angular velocity r (ignore the u vector for now). The position vector doesn't change it's own magnitude or direction within the axis system, yet clearly the point P now has a velocity in the vehicle y direction.



Written generally, the rule for rate of change of a vector is :

$$\boxed{\frac{d}{dt}\left(\underline{V}\right) = \underline{\dot{V}} + \underline{\omega} \times \underline{V}} \qquad \text{......(1)}$$

Where $\underline{V}$ is *any* vector, and ω is the vector describing the axis rotation (*this is a very powerful and useful equation*). Note the two components : $\underline{\dot{V}}$ describes the rate of change of the vector (rate of change of it's length), and $\underline{\omega} \times \underline{V}$ describes its change of direction. Typically we don't know the $\underline{\dot{V}}$ part, so we rearrange the equation to give $\underline{\dot{V}}$ as the subject (recall the state-space models, where the model is written, $\dot{x}$ = f(x,u)).

Confirm this works with the $\underline{d}_p$ vector :

$$\frac{d}{dt}\left(\underline{d}_p\right) = \underline{\dot{d}}_p + \underline{\omega} \times \underline{d}_p = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix} \times \begin{pmatrix} d_p \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ rd_p \\ 0 \end{pmatrix}$$

Note how the vectors $\underline{\omega}$ and $\underline{d}_p$ are formed; how $\underline{\omega}$ would be drawn as a vector into the page (right handed rule 'sweeps' the rotation about the index finger, which points along the vector), and how the notation uses $\underline{d}_p$ the vector, and d_p the magnitude of the vector.

Also, the result is as expected, following the expected rule '$v = R\omega$', and giving the resulting velocity in the y direction.

Now consider a combination of translation and rotation. Consider the point G has the velocity vector u, and yaw rotation r simultaneously (the car is cornering to the right). If the car is stable in the steady-state corner, the magnitude of u and r will be related ('$v = R\omega$', so $u = rR$). NB: this doesn't have to be the case – the car could be in a spin.

Rotation is different from translation. Rotation causes different points on the body to have different velocities and accels, whereas translation is more obvious; if the CG has velocity u, then clearly all the other points on the body *also* have velocity u (because it's a rigid body) PLUS any velocity due to rotation.

So the total velocity of a point away from G on the car is

$$\boxed{\underline{v}_p = \underline{v}_G + \frac{d}{dt}\left(\underline{d}_p\right)} \qquad \text{.... (1a)}$$

Find the velocity at point P, and the acceleration at point G :

$$\underline{v}_p = \underline{u} + \frac{d}{dt}\left(\underline{d}_p\right) = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ rd_p \\ 0 \end{pmatrix} = \begin{pmatrix} u \\ rd_p \\ 0 \end{pmatrix}$$

$$\underline{a}_G = \frac{d}{dt}\left(\underline{u}\right) = \underline{\dot{u}} + \underline{\omega} \times \underline{u} = \begin{pmatrix} \dot{u} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix} \times \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \dot{u} \\ ru \\ 0 \end{pmatrix}$$

NB :

(i) here we're assuming the total velocity vector at G is $[u,0,0]'$, so zero velocity in y and z directions. The same equation form applies if these aren't zero.

(ii) The total acceleration at G is made up of any forward acceleration / deceleration ($\dot{u}$), plus the centripetal acceleration towards O (positive y direction) ru, which if $u = rR$ becomes Rr^2, which is the expected '$a = R\omega^2$' term

5.2 Again, if u=rR, the body would be acting like part of a disc, spinning about the centre of the turn. Check that the magnitude of $\underline{v}_p$ *above is equal to 'rω' for the (slightly larger) radius that P has from O.*

5.3 A vehicle is travelling on a flat road in a right hand turn at 20m/s, and has a body side slip angle of 1^o (in the same sense as shown on p20). (i) Write down the total velocity vector of the CG. (ii) What would the velocity vector be if the vehicle were in the same, steady condition in a left turn? (iii) Considering the left turn case, if the yaw rate magnitude is 0.2^c/s and roll and pitch velocities are zero, write down the total body rotation velocity vector, and use vector algebra to calculate the total CG acceleration vector. (iv) Does this agree with a simple hand calculation for acceleration? (v) If the vehicle has CG to rear axle distance 1.2m and rear track 1.4m, use vector algebra to find the total velocity vector at the centre of the rear axle. Hence what is the slip angle here ? (vi) Calculate the slip angle at one of the rear tyre contact patches in the same way. (vii) Comment on the difference between your answers to (v) and (vi).

The Equations of motion : Deriving a state-space model

The vehicle states are those variables which dictate the forces on the body, and hence 'drive' its motion. The full list of states we need for our simple 6dof model is :

$$\underline{x}_b = \begin{pmatrix} \underline{v} \\ \underline{\omega} \\ \underline{\theta} \\ z \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \\ \psi \\ z \end{pmatrix} \qquad \underline{x}_s = \begin{pmatrix} \underline{\omega}_w \\ \underline{F}_{ylag} \\ \underline{F}_{xlag} \end{pmatrix}$$

Where the primary states $\underline{x}_b$ are directly related to the body, and $\underline{x}_s$ are other necessary states, ($\underline{\omega}_w$ are the wheel spin speeds – needed for longitudinal force generation, and the others are tyre force lag states).

Tyre forces, which drive the motion, are governed by relative velocities, so $\underline{v}$ and $\underline{\omega}$ are most relevant as the focus for deriving the model. The angles ($\underline{\theta}$) and body CG height above the road, z are required to find the geometry of the motion relative to the road.

For all these states we seek the equations which explain their rates of change, so we can form (recall from the VDS course) :

$$\dot{\underline{x}}_p = f\left(\underline{x}_p, \underline{u}\right)$$

Where $\underline{\boldsymbol{u}}$ in this equation refers generally, to 'inputs' (not forward speed !).

For gross body motion, we do this by forming the translation and angular *momentum* vectors :

$$\underline{D} = m\underline{v} \qquad \underline{H} = I_G\underline{\omega}$$

Where *m* is the (scalar) vehicle mass, and I_G is the matrix of moments of inertia :

$$I_G = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$$

Each component in this matrix is the sum of mass times distance in each subscript, for each component mass of the vehicle. So for example a component in I_{xx} is its mass times its distance from *G* in the *x* direction squared, whereas an I_{xy} component uses mass times distance from G in the *y* direction times distance from *G* in *x*. As a result, 'off diagonal' elements will cancel each other out if the components are symmetrical about one of the axes, whereas the 'core' components on the leading diagonal provide the 'mass times radius squared' information which is usually meant by moment of inertia.

It's important that we allow for I_G as a matrix, since the presence of non-zero off-diagonal terms (unsymmetric weight) causes momentum in directions other than those in $\underline{\omega}$. For example :

$$\underline{\omega} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \qquad I = \begin{pmatrix} 200 & 0 & -30 \\ 0 & 1500 & 0 \\ -30 & 0 & 1500 \end{pmatrix}, \qquad \underline{H} = I\underline{\omega} = \begin{pmatrix} 400 \\ 0 \\ -60 \end{pmatrix}$$

The effect, dynamically, is that rotation in one direction *causes* momentum in another, which causes rotation to start in that direction. In non-symmetrical objects, this leads to tumbling (transfers between direction of rotation) when you see free rotation in space (eg rotation in mid-air).

5.4 You can confirm this by spinning objects and throwing them in the air. If they are symmetrically weighted, the spin direction remains consistent. If not, the object will tumble, or the spin direction relative to the object will realign.

Now, Newton's first law is written in terms of momentum as :

$$\sum \underline{F} = \frac{d}{dt}\left(\underline{D}_G\right), \qquad \sum \underline{M} = \frac{d}{dt}\left(\underline{H}_G\right)$$

And using our 'magic' vector rate of change equation (1) we can say :

$$\sum \underline{F} = \frac{d}{dt}\left(m\underline{v}_G\right) = m\dot{\underline{v}}_G + \underline{\omega}\times\left(m\underline{v}_G\right) \qquad \ldots\ldots(2)$$

5.5 Do a 'sanity check' on this for a car negotiating a steady-state left hand corner. What direction must the tyre forces be in, and what is the acceleration vector (assume lateral and vertical velocities are negligible).

Also,

$$\sum \underline{M} = \frac{d}{dt}\left(\underline{H}_G\right) = I_G\dot{\underline{\omega}} + \underline{\omega}\times\left(I_G\underline{\omega}\right) \qquad \ldots\ldots(3)$$

NB : We write $\underline{\omega}$ rather than $\underline{\omega}_G$ (why ?), yet we do need to specify the 'G' in I_G. What if the axis system for this equation was not aligned with the vehicle ?

Given the general model free body diagram of figure 3, and assuming no aerodynamic forces (for now), the forces and moments can be determined as :

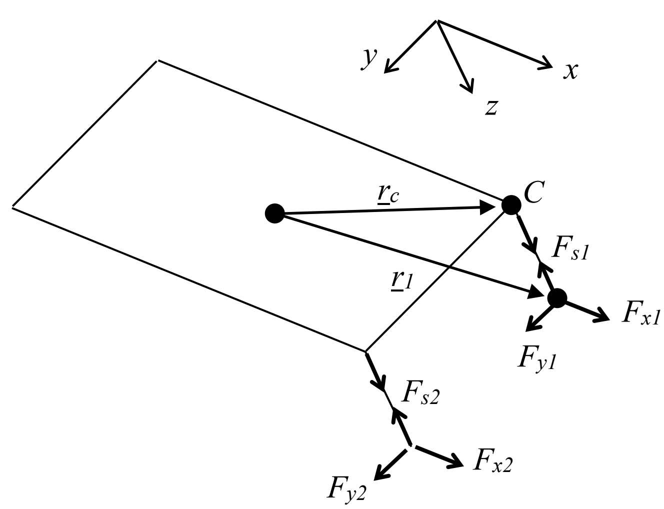


Figure 3 : Vehicle body free-body diagram

$$\sum \underline{M} = \sum_{i=1}^{4}\left(\underline{r}_i \times \underline{F}_i\right), \qquad \sum F = \sum_{i=1}^{4} \underline{F}_i$$

where $\underline{F}_i = \begin{pmatrix} F_{xi} \\ F_{yi} \\ F_{si} \end{pmatrix}, \quad (F_{si} = F_{zi})$

You'll notice there is no weight force in figure 3. This is omitted because we need only calculate how variables change dynamically from their rest state (we took the same approach with the quarter car ride model introduced in Vehicle Dynamics and Simulation).

Also note that given the 'springy table leg' interpretation , F_x and F_y apply at the contact patch, but strictly, F_s applies on to the body at the corner (C). As the direction of F_{si} is the same as for F_{zi}, $\underline{F}$ is not altered by defining it in this way, and since $\underline{r}_1 = \underline{r}_C + \begin{pmatrix} 0 \\ 0 \\ h_G \end{pmatrix}$, $\underline{M}$ is also identical.

5.6 You can check this, using $\underline{r}_1 \times \begin{pmatrix} F_{xi} \\ F_{yi} \\ 0 \end{pmatrix} + \underline{r}_C \times \begin{pmatrix} 0 \\ 0 \\ F_{si} \end{pmatrix} = \underline{r}_1 \times \begin{pmatrix} F_{xi} \\ F_{yi} \\ F_{si} \end{pmatrix}$

Equations (2) and (3) can now be rearranged to give us the derivatives we need :

$$\dot{\underline{v}}_G = \frac{1}{m}\left\{\sum_{i=1}^{4} \underline{F}_i - \underline{\omega} \times \left(m\underline{v}_G\right)\right\}$$

$$\dot{\underline{\omega}} = I_G^{-1}\left\{\sum_{i=1}^{4} \left(\underline{r}_i \times \underline{F}_i\right) - \underline{\omega} \times \left(I_G \underline{\omega}\right)\right\}$$

And the full state derivative set is completed by considering the kinematic relationships that exist for the last four states :

$$\dot{\underline{x}}_b = \begin{pmatrix} \dot{\underline{v}}_G \\ \dot{\underline{\omega}} \\ \underline{\omega} \\ w \end{pmatrix}$$

Finding tyre and suspension forces

The analysis above effectively amounts to a scheme for finding accelerations from the tyre forces $\underline{F}_i$; ie it shows how tyre forces dictate motion. The accelerations can be integrated to give velocity and position information, and it is the velocity information which in turn dictates tyre forces.

Horizontal motion and forces

Although we will still work with 3D vector algebra, we'll consider the vertical motion separately from the vehicle *xy* plane motion. We do this because the tyres determine the x-y forces (Chapter 1 notes), but the suspension and road determine the z forces (Chapter 3 notes).

The figure below condenses the process of finding $\underline{F}_i$ from $\underline{v}$ and $\underline{\omega}$ into five stages :

(i) Find point velocities at the contact patches in the vehicle axis frame
(ii) What are these velocities when viewed in ('rotated into') the tyre axis frame
(iii) What is the lateral and longitudinal slip angle in each tyre
(iv) Find the tyre force in its own axis frame, from these slip angles
(v) How does this translate back into $\underline{F}_i$ (ie aligned in the vehicle frame).

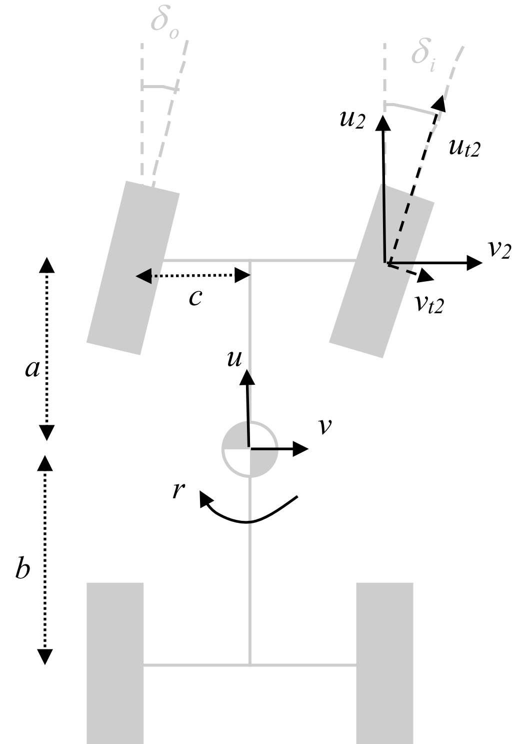
The method here finds slip angles 'correctly' by '*rotating'* the contact patch velocities through δ. 'rotation' is the commonly used term here, because the matrix transformation that is performed uses a rotation matrix. Note that we aren't actually rotating the vectors to just align them with the tyre however; we're rotating the axis system the vector is described in. (Draw the total vector given by u_2+v_2 on the diagram, and you'll see this is exactly the same vector as $u_{t2}+v_{t2}$.)

5.7 Show that, for small steer angles, $\underline{v}=\begin{pmatrix}u\\v\\0\end{pmatrix}$, $\underline{\omega}=\begin{pmatrix}0\\0\\r\end{pmatrix}$, $\underline{r}=\begin{pmatrix}a\\c\\h\end{pmatrix}$, α_2 *still approximates the bicycle model expectation,* $\alpha=\dfrac{-v-ar}{u}+\delta$. *(NB, it's not* exactly *the same !)*

5.8 A front wheel drive car of mass 1600kg has properties $C_{\alpha f}=C_{\alpha r}=50$kN/rad and $C_x=70$kN/rad for each single tyre, and geometry $a=1.1$m, $b=1.2$m, $c=0.8$m and $r_w=0.3$m. In a steady state turn, $\underline{v}=[4\ \ 0.84\ \ 0]^T$, $\underline{\omega}=[0\ \ 0\ \ 0.75]^T$, $\underline{\omega}_w=[16.33\ \ 12.68\ \ 15.32\ \ 11.32]^T$ and both steered wheels are at $\delta=25^o$.

(i) Assuming a linear tyre model, calculate the tyre force vector for each tyre. (ii) Explain each of the 8 components in the tyre forces. Why are the front lateral tyre forces opposite sign ? (HINT : Look back at the first diagram in Section 2 of these notes, and consider what R, R_f and R_r will apply here) (iii) CG acceleration can be given by the sum of forces divided by mass. Calculate the car's CG acceleration vector and explain why the longitudinal acceleration is negative. (The Section 2 diagram is relevant again.) (iii) Was it reasonable to assume a linear tyre model ?

**NB* : Keep accurate records of intermediate values in your calculations, since rounding errors can be significant here.*



$$\underline{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \qquad \underline{\omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

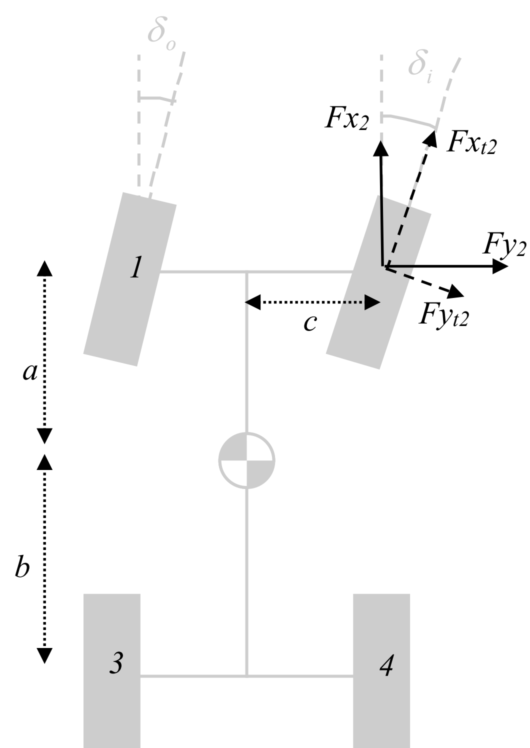
$$\underline{v}_2 = \begin{pmatrix} u_2 \\ v_2 \\ w_2 \end{pmatrix} = \underline{v} + \underline{\omega} \times \underline{r}_2$$

$$\underline{v}_{t2} = \begin{pmatrix} u_{t2} \\ v_{t2} \\ w_{t2} \end{pmatrix} = \begin{pmatrix} u_2 \cos\delta_2 + v_2 \sin\delta_2 \\ v_2 \cos\delta_2 - u_2 \sin\delta_2 \\ w_{t2} \end{pmatrix}$$

$$\alpha_2 = \frac{-v_{t2}}{u_{t2}}, \quad S_2 = \frac{r_w \omega_{w2} - u_{t2}}{u_{t2}}$$

$$Fx_{t2}, Fy_{t2} = f_{TYRE}\left(\alpha_2, S_2, \underline{F}z_{t2}\right)$$

$$\underline{F}_2 = \begin{pmatrix} Fx_2 \\ Fy_2 \\ Fz_2 \end{pmatrix} = \begin{pmatrix} Fx_{t2} \cos\delta_2 - Fy_{t2} \sin\delta_2 \\ Fy_{t2} \cos\delta_2 + Fx_{t2} \sin\delta_2 \\ ? \end{pmatrix}$$

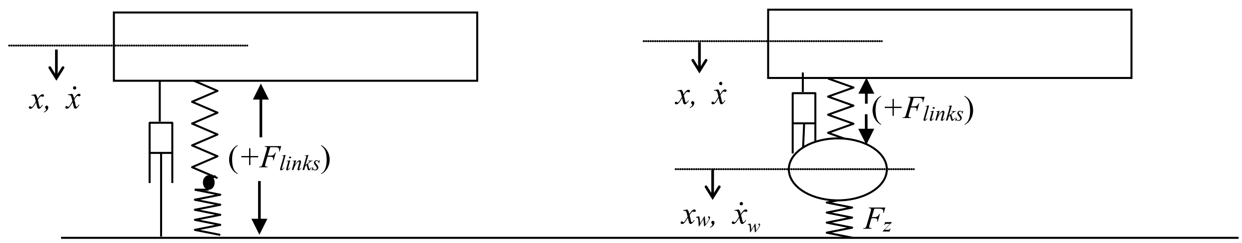


<u>Vertical motion and forces</u>

Two alternatives exist for the vertical suspension model, depending on whether the vertical degree of freedom of the wheel is required or not. (A more realistic ride model, with the 10-12Hz wheel hop mode included clearly needs case b.)

case (a) : single mass

case (b) : sprung and unsprung masses



In both of these cases, F_{links} models the effect of the suspension links, as discussed at the end of Chapter 3, and the total force in the suspension and on the tyre also requires knowledge of the vertical deflection and velocity, $x,\ \dot{x}$ at each corner. It isn't very complicated to include the unsprung mass motion, case (b), and of course this is necessary for any serious ride simulation work, but here we will model the simplest case, (a), where vertical force upwards on the body is equal in magnitude to the force down on the tyre contact patch.

The total corner spring force will be from the series combination of the tyre and suspension springs,

$$\frac{1}{K_{seff}} = \frac{1}{K_s} + \frac{1}{K_t}$$

and

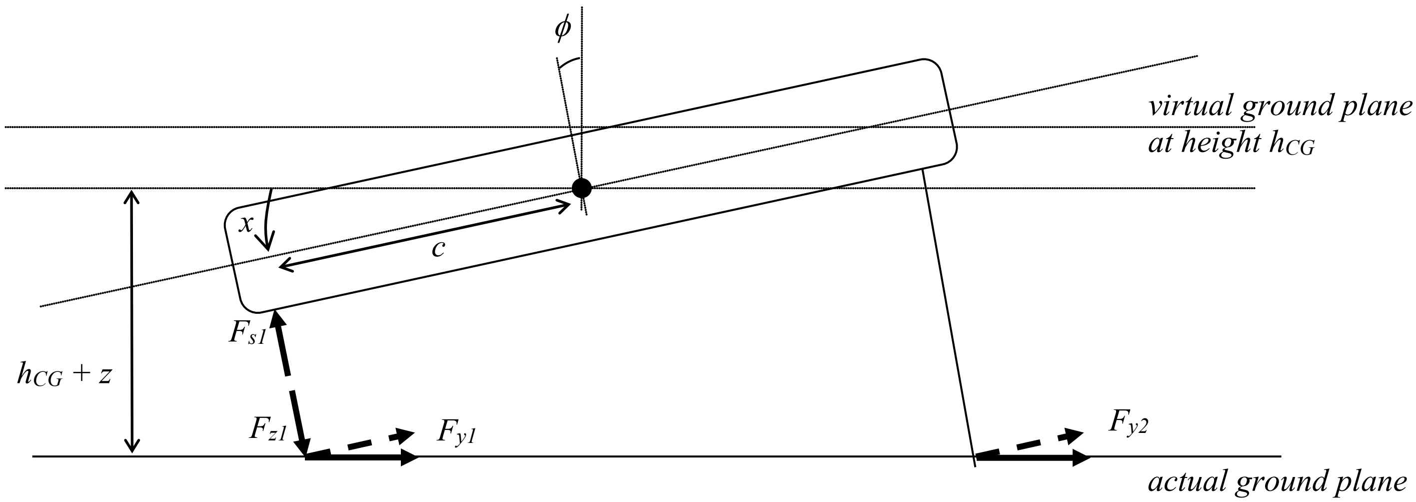
$$F_s = -F_z = K_{seff}x + B\dot{x} + F_{links}$$

In the 2d example shown below, x is approximated as the arc length caused by rotation ϕ, and $\dot{x}$ is the vertical corner velocity ('v = rω') :

$$x = c\phi \qquad \dot{x} = c\dot{\phi}$$

Since ϕ is 'small', all the distances and forces can be viewed as the same vertically on the page as they would be in the (tilted) z direction of the vehicle, since $\cos\phi \approx 1$

Note how the tyre force calculations we make are actually applied in the dashed direction, when in reality the forces are formed in the plane of the road surface (solid arrows). (More complex / complete models which deal with road bank angles and / or large vehicle rotation angles require considerable extra complexity in the vector calculations to include this effect, while still separating suspension and tyre forces).



The vector notation gives us a very simple solution for x and $\dot{x}$, taking into account roll ϕ, pitch θ and heave z, at corner i :

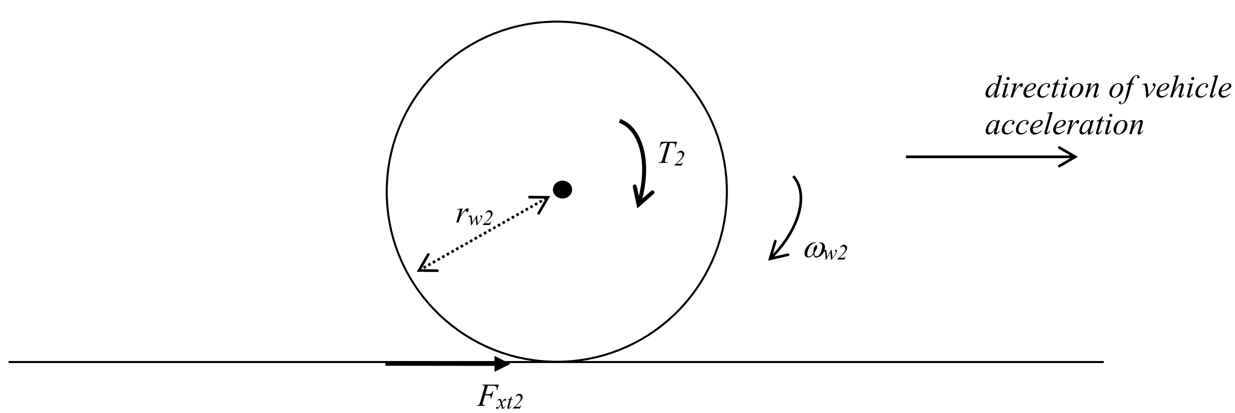
$$\underline{x}_i = \underline{\theta} \times \underline{r}_i \qquad \dot{\underline{x}}_i = \underline{\omega} \times \underline{r}_i$$
$$x_i = z + \underline{x}_i(3) \qquad \dot{x}_i = w + \dot{\underline{x}}_i(3)$$

where z and w are the vertical deflection and velocity of the CG, respectively, from the body state set on p48.

Wheel spin and tyre lag states

Wheel spin states

In order to find longitudinal slip and hence longitudinal tyre force (on p53), we used the variable for wheel angular velocity ω_w, which is one of the 'other necessary' states listed on p48. The equation of motion for the spin of the wheel can be derived by considering the wheel's free body diagram (shown in 'drive' mode, for corner 2) :



The torque T drives the whole vehicle (strictly shared between at least two driven wheels, but you get the idea), and is therefore very large. This torque is reacted at the contact patch by the drive force F_x. If we think of T as an internal force in the whole vehicle, F_x actually pushes the whole vehicle along the road, so it too is very large. The *difference* between these two large values is what accelerates the *wheel inertia* itself :

$$\boxed{T - F_{xt} = I_w \dot{\omega}_w}$$

This is very easily arranged to give $\dot{\omega}_w$ as a function of the input engine or braking torque T, and the tyre longitudinal force. Note that I_w is small (about 1kgm^2 for a wheel & tyre assembly), but when T actually represents the torque from the engine (appropriately geared), I_w should be replaced by the *total* effective inertia of the engine (flywheel), gearbox and all upstream transmission components. For simplicity we will use a nominal I_w, and model brake and drive torques simply using a change of sign. Clearly better drivetrain models (including velocity and load dependent engine torque models) are desirable additions, depending on the simulations to be carried out; in first gear the effective drivetrain inertia can be equivalent to as much as 20% of the vehicle mass.

Tyre lag states

Tyre lags exist, but are very small; typically the tyre force change caused by a change in applied slip develops within 1/3 of a rotation of the wheel. We might not bother including additional states for this in a vehicle model, but for the existence of an 'algebraic loop' whereby one variable in the model instantaneously depends on itself. The loop here is :

i) F_x and F_y depend instantaneously on F_z (see the tyre model)
ii) through the suspension links, F_x and F_y instantaneously affect F_s
iii) $F_s = -F_z$ (in our model)

This loop drastically slows numerical integration solvers like Simulink, since at every time step, it is necessary to repeatedly loop through calculations (i) – (iii) above until accurate convergence is achieved. To break the loop we have a number of options; we could include the unsprung mass vertical mode, or a link bushing compression model, or a suitable tyre delay.

A simple 1st order tyre lag solves the problem, at the expense of introducing 8 additional lag states (though note that both of the alternative solutions also involve introducing new states). The relevant lag equation is :

$$\dot{x}_{lag} = \frac{1}{\tau}\left(x - x_{lag}\right)$$

or in transfer function form, from input state x to output lagged state x_{lag}, it is

$$H(s) = \frac{1}{s\tau + 1}$$

where τ is the time constant, representing the time it takes to achieve approx 63% of the lagged step change. (The output lagged state exponentially approaches the input state.)

5.9 I use $\tau = 0.025$ in the 6dof model. What nominal vehicle forward speed does this relate to, if we assume $r_w = 0.3$, and 1/3 of a revolution of the driven wheels achieves 63% of a step change in drive torque ?

NB : It really is *not* worth modeling τ to vary with forward speed unless you're looking at very low vehicle speeds, in which case you almost certainly need a more complex tyre lag model in any case.