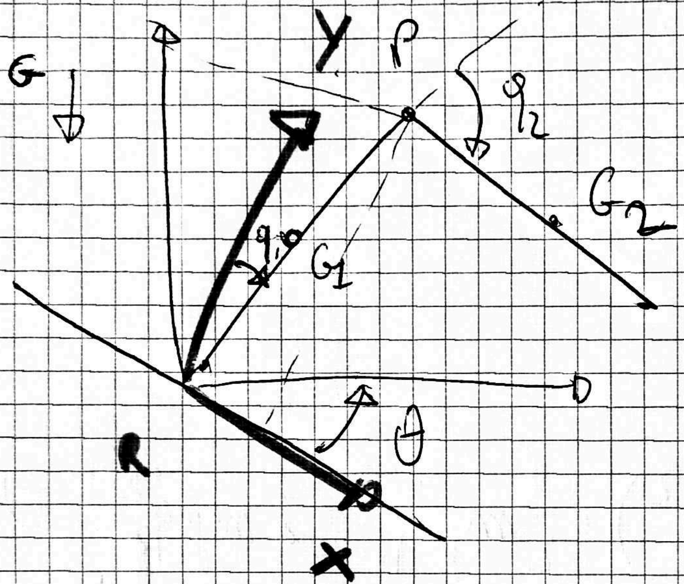
$$\boxed{\tau = \tau_q - B^T A^{-T} \tau_x}$$

# HUMRO LAB

\$ how

$l \quad m \quad s \quad J \quad g$

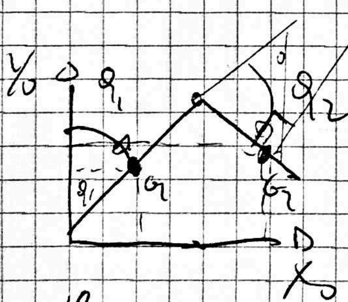



$$[A, H] = \text{function\_dyn}(q_1, q_2, \dot q_1, \dot q_2, \ldots$$

$$A \begin{bmatrix} \ddot q_1 \\ \ddot q_2 \end{bmatrix} + H = B\tau$$

+ counterclockwise

**PART 1** (1) → that $q_1$ is negative! — clockwise




$$G_1 = \begin{bmatrix} X \\ Y \end{bmatrix} = l \cdot \begin{bmatrix} \sin q_1 \\ c q_1 \end{bmatrix} = (l-s) \begin{bmatrix} \sin(-q_1) \\ \cos(-q_1) \end{bmatrix}$$

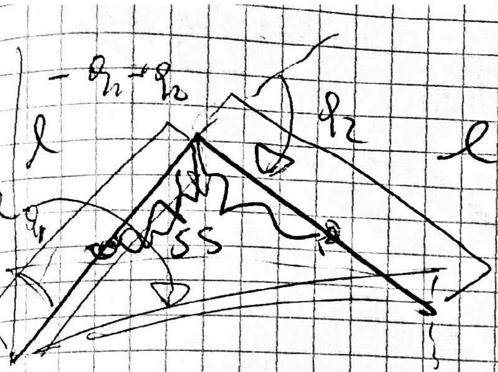
$$G_2 = \begin{bmatrix} X \\ Y \end{bmatrix} \cancel{= l \begin{bmatrix} s q_1 \\ c q_1 \end{bmatrix}}$$

$l$


$\Delta = l - s$

by ANGOLI ASSOCIATI

$$= (l-s) \begin{bmatrix} -s q_1 \\ c q_1 \end{bmatrix}$$




$$\overline{OP} = \begin{bmatrix} -l\, s q_1 \\ l\, c q_1 \end{bmatrix} + S \begin{bmatrix} S(q_1 + q_2) \\ c(q_1 + q_2) \end{bmatrix}$$

BY ASSOCIATED ANGLES

$\overline{PG_2}$

$$\overline{OG_2} = l\begin{bmatrix} -sq_1 \\ cq_1 \end{bmatrix} + S\begin{bmatrix} -s(q_1+q_2) \\ c(q_1+q_2) \end{bmatrix}$$

② Velocities, derivate w.r.t. time

$$\dot{\overline{OG_1}} = (l-S)\begin{bmatrix} -sq_1\,\dot q_1 \\ -s q_1 \dot q \end{bmatrix} = (l-S)\dot q_1 \begin{bmatrix} c q_1 \\ -sq_1 \end{bmatrix} = (S-l)\dot q_1 \begin{bmatrix} cq_1 \\ sq_1 \end{bmatrix}$$

$$-(l-S)\dot q_1 \begin{bmatrix} cq_1 \\ sq_1 \end{bmatrix}$$

$$\dot{\overline{OG_2}} = -l\,\dot q_1 \begin{bmatrix} cq_1 \\ sq_1 \end{bmatrix} + S\begin{bmatrix} -c(q_1+q_2) \\ -s(q_1+q_2) \end{bmatrix}(\dot q_1 + \dot q_2) =$$

$$= -\left[ l\dot q_1 \begin{bmatrix} cq_1 \\ sq_1 \end{bmatrix} + S\begin{bmatrix} c(q_1+q_2) \\ s(q_1+q_2) \end{bmatrix}(\dot q_1+\dot q_2) \right]$$

③

$\omega_1 = \dot q_1 \quad (\dot q_1 + \dot q_2) = \omega_2$

⑤ $T_1 = \frac{1}{2} m_1 \cdot (S-l)^2 \,[cq_1 \; sq_1]\begin{bmatrix} cq_1 \\ sq_1 \end{bmatrix} \dot q_1^2 =$

$$= \frac{1}{2} m_1 (S-l)^2 \cancel{\left[c^2 q_1 + s^2 q_1\right]}\, \dot q_1^2 = \underbrace{\frac{1}{2} m_1 (S-l)^2}\, \dot q_1^2$$

$+ \frac{1}{2} I_{zz}\, \dot q_1^2$

$l-S$

~~$(l-S)$~~

$\boxed{T_2}$ $\frac{1}{2} m_2 l^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 S^2 (\dot{\varphi}_1 + \dot{\varphi}_2)^2 + m_2 l S \, \dot{\varphi}_1 (\dot{\varphi}_1 + \dot{\varphi}_2) c\varphi_2$

$+ 2 I_{zz} (\dot{\varphi}_1 + \dot{\varphi}_2)^2$

$-l\dot{\varphi}_1 s\varphi_1$

# 5 Inertia matrix

$T_{TOT} = T_1 + T_2 = \frac{1}{2} [\dot{\varphi}_1 \ \dot{\varphi}_2] \, A \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix}$

don't forget the $2 m l S c\varphi_2$

$$A = \begin{bmatrix} m_1 (l-S)^2 + m_2 S^2 + 2 I_{zz} + m l S c\varphi_2 & m_1 S^2 + I \\ m S^2 + I + m l S c\varphi_2 & m_2 S^2 + I \end{bmatrix}$$

$\frac{1}{2} [\dot{\varphi}_1 \ \dot{\varphi}_2] \, A \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix}$

# 6 Compute potential energies

$U = -m ({}^0 g^T) \, \overline{OG_i}$

$U_{G_1} = -m [0 \ -g \ 0 \ 0] \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $m$

or, much simpler, $-m\begin{bmatrix}0 & -g & 0\end{bmatrix}\begin{bmatrix}(l-s)(-sq_1)\\(l-s)\,cq_1\\0\end{bmatrix} =$

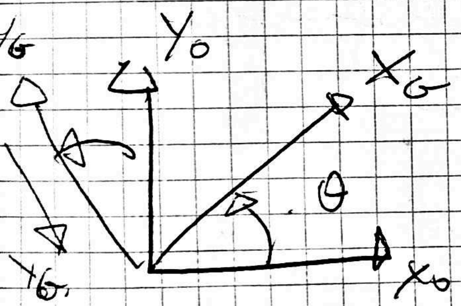
$+m(l-s)\,g\,q_1 = U_1$

$U_2 = -m\begin{bmatrix}0 & -g & 0\end{bmatrix}\begin{bmatrix}- - - -\\ l\,cq_1 + S\,c(q_1+q_2)\\0\end{bmatrix}$

$= +m\,g\left(l\,cq_1 + S\,c(q_1+q_2)\right) = U_2$

CHANGE OF FRAME

$\Rightarrow$ need to put $g$ in the proper frame




change of frame

I need $Y_\theta$ in $X_0, Y_0$

$\boxed{Y_\theta = -s\theta\,\overline{X_0} + c\theta\,\overline{Y_0}}$

$\Rightarrow -g\overline{Y_\theta} = gs\theta\,\overline{X_0} - c\theta\,\overline{Y_0}$

in frame $\theta$

$U_1 = -m\begin{bmatrix}+s\theta g & -c\theta g & 0\end{bmatrix}\begin{bmatrix}(l-s)(-sq_1)\\(l-s)\,cq_1\\0\end{bmatrix}$

$U_1 = +m\left[gs\theta\,(l-s)(-sq_1)\right.$

$\left.- g(l-s)\,c\theta\,cq_1\right]$

$-s\alpha\,s\beta + c\alpha\,c\beta = -\left[s\alpha\,s\beta + c\alpha\,c\beta\right] = c(\theta - \cdots$

$U_1 = \cancel{q} - mg(l-s)\,c(\theta - q_1)$

$U_2 = ?$

$$-m\begin{bmatrix} gs\theta & -gc\theta & 0 \end{bmatrix}\begin{bmatrix} -ls q_1 & -S s(q_1+q_2) \\ +lc q_1 & +S c(q_1+q_2) \end{bmatrix}$$

$$= -mg\left[-s\theta\left[ls q_1 + S s(q_1+q_2)\right] - \cancel{g}\, c\theta\left[lc q_1 + S c(q_2+q_1)\right]\right]$$

$$= -mg\left[-ls\theta s q_1 - \cancel{g} lc\theta c q_1 - Ss\theta s(q_1+q_2) - S c\theta c(q_1+q_2)\right]$$

~~[illegible]~~ MENOS POR TRIGONOM

by using some trigonometric function.

$$+mg\left[-l\,c(\theta - q_1) - S\,c(\theta - q_1 - q_2)\right] =$$

$$mgl\,c(\theta - q_1) + mgS\,c(\theta - q_1 - q_2) = U_2$$

$U_{TOT} = U_1 + U_2$

7) $Q_i = \frac{\partial U}{\partial q_i} = \begin{bmatrix} \frac{\partial U}{\partial q_1} \\ \frac{\partial U}{\partial q_2} \end{bmatrix}$

$$\frac{\partial U}{\partial q_1} = -mg(l-S)\,s(\theta - q_1) - mgl\,s(\theta - q_1)$$

$\cancel{\ell m a}$ $- mg S \,s(\theta - q_1 - q_2)$

$$\frac{\partial U}{\partial q_2} = -mg S \,s(\theta - q_1 - q_2)$$

8) $H = \underbrace{B \dot{q}\dot{q}}_{} + C \dot{q}$

$\hookrightarrow \begin{bmatrix} \dot{q}_1 \dot{q}_2 & \cancel{\dot{q}} \\ \dot{q}_2 \dot{q}_1 & \end{bmatrix}^2 \qquad \dot{q}^2 = [\dot{q}_1 \quad \dot{q}_2]^T$

$$B = \frac{\partial A}{\partial q_k} + \frac{\partial A}{\partial q_j} - \frac{\partial A_{jk}}{\partial q_i}$$

$$C = \frac{\partial A}{\partial q_j} - \frac{1}{2}\frac{A}{\partial q_i}$$

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Our case.

$\dot{q}^2 = \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \end{bmatrix} \qquad B = [2 \times 1] \qquad C = [2 \times 2]$

$\dot{q}\dot{q} = \dot{q}_1 \dot{q}_2$

$B = [redacted] = \left[ \cancel{ \begin{matrix} i=1 \; j=1 \; k=2 \quad = B_{1,2} \\ \frac{\partial A_{11}}{\partial q_2} + \frac{\partial A_{12}}{\partial q_1} - \frac{\partial A_{12}}{\partial q_1} \end{matrix} } \right.$

$\cancel{\ell m}$

APPLY B STRICTLY

$B = \begin{bmatrix} b_{12} \end{bmatrix}$ $\quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$i=1 \quad J=1 \quad K=2$

$$\cancel{B_{1,2} = \left[ \underbrace{\frac{\partial A_{11}}{\partial q_2}}_{=0} + \underbrace{\frac{\partial A_{12}}{\partial q_1}}_{=0} - \underbrace{\frac{\partial A_{12}}{\partial q_1}}_{=0} \right] = 0}$$

$i=2 \quad J=1 \quad K=2$

$$\cancel{B_{2,2} = \left[ \underbrace{\frac{\partial A_{21}}{\partial q_2}}_{=-s\, m l S_{sq_2}} + \underbrace{\frac{\partial A_{22}}{\partial q_1}}_{=0} - \underbrace{\frac{\partial A_{12}}{\partial q_2}}_{=+ m l S_{sq_2}} \right] = - m l S_{sq_{12}}}$$

$i=1 \quad J=1 \quad K=2$ → first elent of A

$$B_{1,2} = \left[ \frac{\partial A_1}{\partial q_2} + \underbrace{\frac{\partial A_2}{\partial q_1}}_{=0} - \underbrace{\frac{\partial A_2}{\partial q_1}}_{=0} \right] = -2 m l S_{cq_2}$$

$i=2 \quad J=1 \quad K=2$

$$B_{2,2} = \left[ \frac{\partial A_2}{\partial q_2} + \frac{\partial A_4}{\partial q_1} - \frac{\partial A_2}{\partial q_2} \right] = 0$$

$$B = \begin{bmatrix} -2 m l S_{cq_2} \\ 0 \end{bmatrix}$$

$i = 1 \quad j = 1$

$$C_{11} = \left[\frac{\partial A_{11}}{\partial q_1} - \frac{1}{2}\frac{A_{11}}{\partial q_1}\right] = 0$$

$i = 2 \quad j = 1$

$$C_{21} = \left[\underbrace{\frac{\partial A_{21}}{\partial q_1}}_{=0} - \frac{1}{2}\frac{\partial A_{11}}{\partial q_2}\right] = +m S l s q_2$$

$i = 1 \quad j = 2$

$$C_{12} = \left[\underbrace{\frac{\partial A_{12}}{\partial q_2}}_{-m l S s q_2} - \frac{1}{2}\underbrace{\frac{\partial A_{22}}{\partial q_1}}_{=0}\right] = -m l S s q_2$$

$i = 2 \quad j = 2$

$C_{22}$

$$C_{22} = \left[\underbrace{\frac{\partial A_{22}}{\partial q_2}}_{=0} - \frac{1}{2}\underbrace{\frac{\partial A_{22}}{\partial q_2}}_{=0}\right] = 0$$

$$C = \begin{bmatrix} 0 & -m l S s q_2 \\ m S l s q_2 & 0 \end{bmatrix}$$

$$H = (q, \dot{q}) \quad B \dot{q}_1 \dot{q}_2 + C \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2 \end{bmatrix}$$