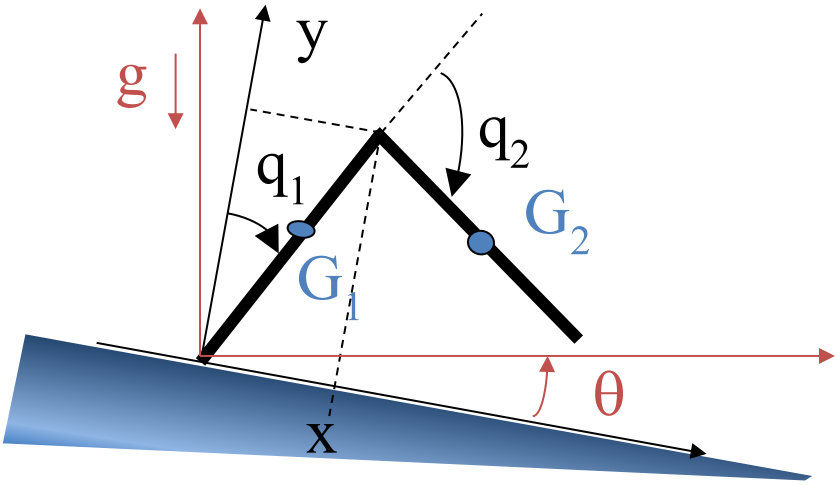
Humanoid and walking robots –lab n° 2: Passive walking of a compass robot



For each leg :
l : length
m : mass
I : inertia
S : distance between G and the hip

We consider a compass robot (2 links) walking along a slope.

The black frame is the reference frame R_0, it is attached to the ground (with slope).

For numerical application, we will consider 2 cases:

- Case 1: l=0.8m; m=2 kg; I=0.1 kg·m²; s=0.5m; θ =3*pi/180 rd;
- Case 2: l=0.8m; m=2kg; I=0.08 kg·m²; s=0.45m; θ=3*pi/180 rd;

1) Simulation of the passive gait

The gait studied is passive no torque are applied. The robot is placed on a slope (θ=3*pi/180 rd).

The gait is composed of phase of support on leg tip 1, impact where the leg 2 touches the ground and the leg 1 takes off, support on leg tip 2, impact where the leg 1 touches the ground and the leg 2 takes off …

In the first TP, we have prepared modelling for the single support on leg 1 and impact where the leg 2 touches the ground and the leg 1 takes off. To avoid deriving new model, we introduce a relabeling of the number of the leg (and change of q_i).

A. Simulation of one single support phase

The simulation will be done with matlab using the function 0de45. From the dynamic model, the evolution of the state of the robot is deduced; the state is the vector of position and velocity (here denoted as z, the function denoted SS_passif defines the derivative of the states as function of the state: function zdot = SS_passif(t, z)). The integration is not done until a given time but until the fact that the free leg tip touches the ground.

Example:

```
options = odeset('Events', @PEvents);

z0=[qi;qpi];

[t, z, te, ze] = ode45(@SS_passif, [0:0.02:10], z0, options);
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The initial state for the integration is z0.

The outputs of the function ode45 are the vector of time t, and the corresponding value of state z. Since the simulation is stop of an event, the corresponding value of time and state are given in te and ze.

In the example the simulation can be done from t=0 to 10s with a time step of 20ms; but the simulation will be stop before 10s when the output of the function Events will be 0.

[T,Y,TE,YE] = ODE45(ODEFUN,TSPAN,Y0,OPTIONS...) with the 'Events' property in OPTIONS set to a function PEvents, solves as above while also finding where functions of (T,Y), called event functions, are zero. For each function you specify whether the integration is to terminate at a zero and whether the direction of the zero crossing matters. These are the three vectors returned by EVENTS: [VALUE,ISTERMINAL,DIRECTION] = PEvents (T,Y). For the I-th event function: VALUE(I) is the value of the function, ISTERMINAL(I)=1 if the integration is to terminate at a zero of this event function and 0 otherwise. DIRECTION(I)=0 if all zeros are to be computed (the default), +1 if only zeros where the event function is increasing, and -1 if only zeros where the event function is decreasing.

Here ISTERMINAL(1)=1, DIRECTION(1)=0 or -1. VALUE(1) is the height of the free leg tip when the position of the free leg tip is greater than 0.1 (in order to avoid that the step stop when the top leg tips coincide in (0,0)).

A.1) Write the files PEvents and SS_passif

Test the simulation with the following initial condition: l=0.8m; m=2 kg; I=0.1 kg·m²; s=0.5m; θ=3*pi/180 rd; q_1=0.1860rd , q_2= 2.7696rd , $\dot{q}_1$= -1.4281rd/s, $\dot{q}_2$= 0.3377rd/s.

A.2) Prepare an animation of the passive walking

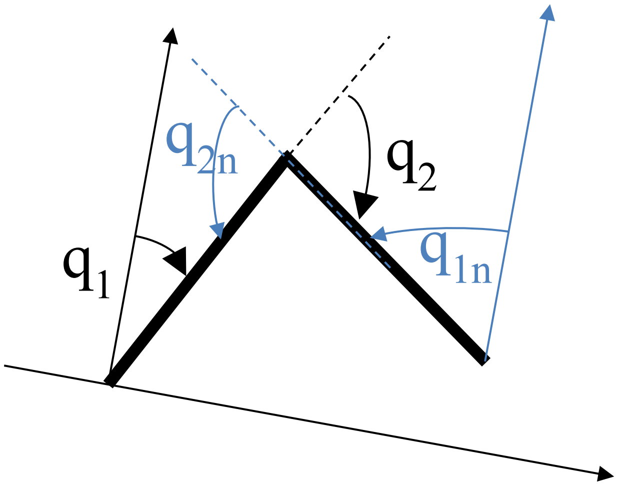
B. Simulation of several half steps

B.1) Write the relabeling equations

q_1 define the orientation of the leg in support and q_2 for the leg in transfer. This must be the case for any leg in support. Before impact the leg one is in support, after impact it is the leg 2. We define a new set of coordinates q_{1n} and q_{2n} that will be the use for the support on leg 2. The figure below allows to defined q_{1n} and q_{2n} as function of q_1 and q_2 to obtained the relabelling equations.

The relabelling equation must be derive to obtain the relabelling relation for the velocity.

THe support leg changes hence the legs change too!



B.2) Starting from a configuration of DS and initial velocity for q1 and q2,

1) Simulate one single support until impact,
2) Calculate the state before impact without implicit constraint,
3) Calculate the joint velocity after impact,
4) Change the label of the joint in order to be ready for a new single support on leg 1
5) Goto step 1 to simulate a next half step

2) Periodic motion

The objective is to obtain a cyclic passive motion on the slope. The variables that must be defined to achieve this objective are the initial configuration in double support and the initial velocity.

A. The Poincaré return map.

A cyclic motion is a fixed point of the Poincaré return map.
The Poincaré section is defined just before impact (this choice is arbitrary).

A.1) What is the condition of the configuration of the compass to be in double support? Write a relation between q1 and q2.

A.2) What are the state variables X for the Poincaré return map?

A.3) Write the Poincaré return map $X_{k+1}=P(X_k)$.

This function includes

1) Starting from the state variable X, definition of the complete state of the robot for a parameterization without implicit constraint
2) Impact model
3) Relabeling of the joint
4) Simulation of one single support on leg 1
5) Extraction of the state variable X

B. The periodic passive motion on a slope

A cyclic motion is a fixed point of the Poincaré return map.

B.1) Define a fixed point by optimization technique, a fixed point is such that X*=P(X*) and simulate one step starting from X*. Show the joint evolution in a phase plan.

The matlab function fminsearch can be used.
options = OPTIMSET ('display','iter','MaxFunEvals',1000,'TolX', 1.0e-010,'TolFun', 1.0e-006);
X = fsolve('test_periodic',X0,options)
with f=test_ periodic (X)
Starting from initial optimization variables X0, it will find the state X that minimize f. f is P(X*)- X*.

Remarks: 2 angles that differ from 2π are equal.
For case 1, a cyclic motion exists close to q_1=-0.1860 rd, $\dot{q}_1$= -2.0504 rd/s, $\dot{q}_2$= -0.0428 rd/s
For case 2, a cyclic motion exists close to q_1=-0.1933 rd, $\dot{q}_1$= -2.0262 rd/s, $\dot{q}_2$= -0.1253 rd/s

3) Stability analysis

A periodic motion is stable, if starting close to the periodic motion, the motion will converge to the periodic one. A periodic motion is unstable, if starting close to the periodic motion, the motion will moves away from the periodic one.

The eigenvalues of the Jacobian of the Poincaré map is a tool to check the stability of a fixed point.

3-1) Write a program that calculate the Jacobian of the Poincaré return map. (The error on velocity can be 0.5e-3 and on position 0.5e-4). $J_i = \dfrac{P(X^* + \delta x_i e_i) - P(X^* - \delta x_i e_i)}{2\delta x_i}$

3-2) Evaluate the Jacobian for the two previous cases, calculate the eigenvalues and conclude on stability.

3-3) For the 2 cases, starting for the fixed point, simulate 25 to 30 steps of walking, record the states in the Poincaré section, draw the joint evolution in the phase plane and conclude.