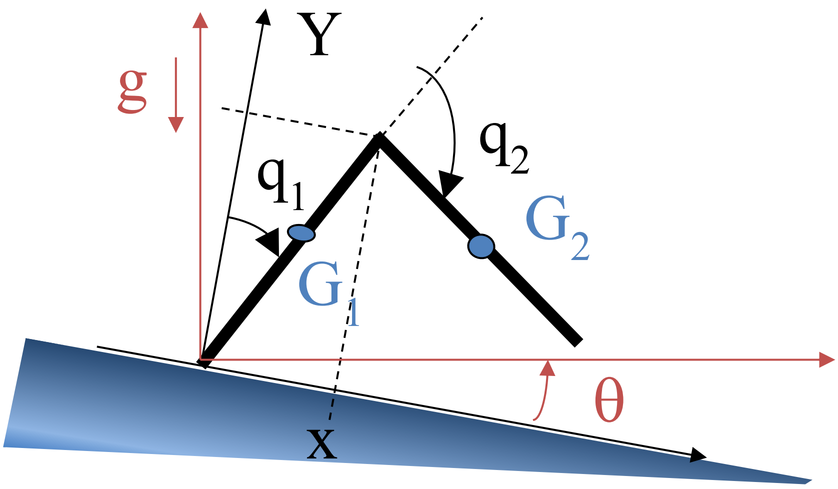
Humanoid and walking robots –lab n° 1: Dynamic modeling of a compass

Work in autonomy

The report and matlab files must be send for the November 25

To acces to matlab use the procedure given in https://box.ec-nantes.fr/index.php/s/s6mGPay8DgYL4Yr#pdfviewer



For each leg :
l : length
m : mass
I : inertia
S : distance between G and the hip

We consider a compass robot (2 links) walking along a slope.

The black frame is the reference frame R_0, it is attached to the ground (with slope).

The objective is to define three Matlab functions that will be used in the next lab.

1°) The dynamic model of the biped in single support

$$[A, H] = function\_dyn(q_1, q_2, \dot{q}_1, \dot{q}_2, \theta)$$

The matrices defined allow to write the dynamic model under the form : $A\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + H = B\Gamma$

The matrix **B** will depend on the actuation and is not defined in this file.

This model will be defined by the Lagrange approach.

1) Based on an implicit contact between the ground and leg 1, the position of G1 and G2 are defined as function of q1 and q2
2) The velocity of the mass centre G1 and G2 are calculated
3) The angular velocity of each leg is defined
4) The kinetic energy of each link is calculated $E_{C_i} = \frac{1}{2}\left(mV_{G_i}{}^T V_{G_i} + \omega_i{}^T I \omega_i\right)$

5) The inertia matrix is deduced ($E_C = E_{C_1} + E_{C_2} = \frac{1}{2}\begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix}\mathbf{A}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$)

6) Calculate the potential energy; note that the gravity vector in the reference frame depends on θ. $U_i = -m\left({}^0g^T\right){}^0\overrightarrow{OG_i}$

7) Deduce the gravity effect $Q_i = \frac{\partial U}{\partial q_i}$

8) The vector H is $B\dot{q}\dot{q} + C\dot{q}^2$

with $\dot{q}\dot{q} = \left[\dot{q}_1\dot{q}_2 \ldots \dot{q}_1\dot{q}_n\ \dot{q}_2\dot{q}_3 \ldots \dot{q}_{n-1}\dot{q}_n\right]^T$ and $\dot{q}^2 = \left[\dot{q}_1^2 \ldots \dot{q}_n^2\right]^T$

$$B_{i,jk} = \frac{\partial A_{ij}}{\partial q_k} + \frac{\partial A_{ik}}{\partial q_j} - \frac{\partial A_{jk}}{\partial q_i},$$

Here $i = 1,\ldots,n,\ j = 1,\ldots,n-1$ and $k = j+1,\ldots,n$

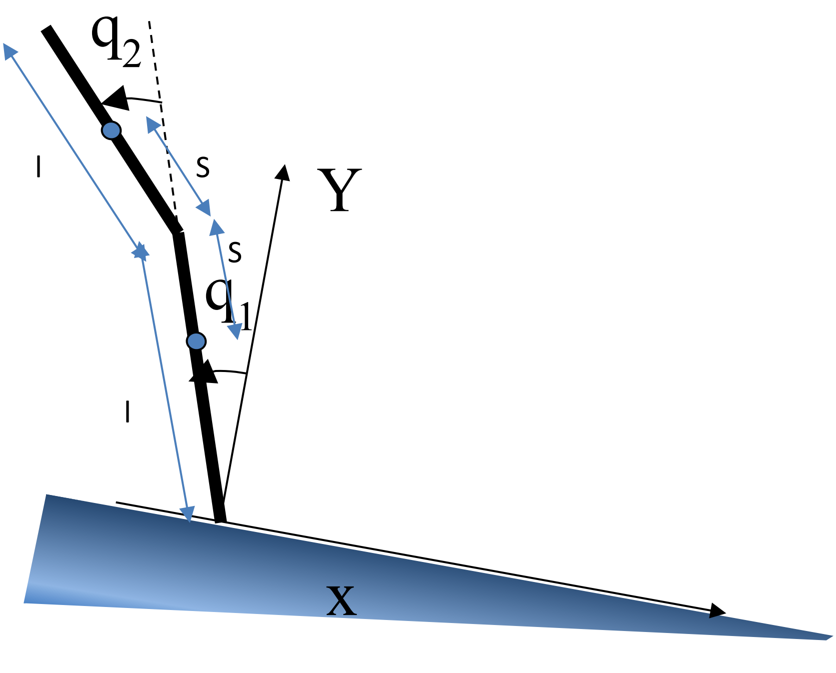
$$C_{ij} = \frac{\partial A_{ij}}{\partial q_j} - \frac{1}{2}\frac{\partial A_{jj}}{\partial q_i}$$

Here $i = 1,\ldots,n,\ j = 1,\ldots,n$

In our case, since there are only two joints, $\dot{q}\dot{q}$ is a scalar variable $\dot{q}_1\dot{q}_2$, $\dot{q}^2$ should be a vector $\dot{q}^2 = \begin{bmatrix}\dot{q}_1^2 \\ \dot{q}_2^2\end{bmatrix}$. Thus $B = B(2\times 1)$; $C = C(2\times 2)$;

<u>Advices :</u> Write the position and velocity of the mass centre in the black frame to calculate the energy (any frame can be used).

To avoid error in the calculation of the position of 1 point, it is better to draw the robot in a configuration such that all angle are positive and small. The frame (X,Y) should be direct. As a consequence in the above picture angular variables are positive in the counter-clock wise. For this robot, a configuration with a small positive value of q_1, q_2 is :



2°) Reaction force

In order to check the condition of contact in support it is useful to calculate the reaction force during support on leg 1. The contact is on a point thus only the reaction force exist without momentum.

$$[F] = function\_reactionforce(q_1, q_2,, \dot{q}_1, \dot{q}_1, \ddot{q}_1, \ddot{q}_2)$$

1) Using results of 1-1, calculate the position of the mass center x_G of the robot as function of q_1 and q_2
2) Derive this expression to have the velocity of the mass center and its acceleration $\ddot{x}_G$
3) Deduce the reaction force : $F = M\ddot{x}_G - M\,\vec{g}$ where M is the total mass of the robot *i.e. M=2m*.

3°) The impact model

The leg tip 1 was the stance foot and the swing tip 2 touches the ground.

$$[A_1, J_{R2}] = function\_impact(q_1, q_2)$$

It is recalled that this matrix can be used to defined the state of the robot after impact and the impulsive forces with the following equation

$$\begin{bmatrix} A_1 & -J_{R2}^T \\ J_{R2} & 0_{2x2} \end{bmatrix} \begin{bmatrix} \dot{x}^+ \\ \dot{y}^+ \\ \dot{q}_1^+ \\ \dot{q}_2^+ \\ I_{R_{2x}} \\ I_{R_{2y}} \end{bmatrix} = \begin{bmatrix} A_1 \\ 0_{2x4} \end{bmatrix} \begin{bmatrix} \dot{x}^- \\ \dot{y}^- \\ \dot{q}_1^- \\ \dot{q}_2^- \end{bmatrix}$$

1) The matrix A_1 is defined as in the first question 1) but without implicit constraint (5 first steps : with $E_c = E_{c1} + E_{c2} = \frac{1}{2}[\dot{x} \quad \dot{y} \quad \dot{q}_1 \quad \dot{q}_2] A_1 \begin{bmatrix} \dot{x} \\ \dot{x} \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$, the angle q1 and q2 are the same as in the first part but since the contact with the ground is not assumed, the position of the hip has to be known. The coordinates of the hip in the black frame are x and y)
2) Considering un impact such that the leg 1 was in support, the leg 2 arrives in impact, the leg 1 takes off and leg 2 stay on the ground without sliding.

Write the contact conditions after impact (the extremity of the leg 2 is fixed) as function of x, y, q1, q2.

Derive these contact conditions, to have condition on velocity written as $J_{R2}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{x}\\ \dot{y}\end{bmatrix} = 0$ where J_{R2} is a (2x4) matrix.

How to check your calculations

Let us consider the parameters:

l =0.8 m; m=2 Kg; I=0.1 Kg.m^2; S=0.5 m; g=9.8 Kg s^{-2} , θ=2*π/180

with

$[q_1,q_2]$=[1,-0.5]; $[\dot{q}_1,\dot{q}_2]$ = [0.1,0.2]; $[\ddot{q}_1,\ddot{q}_2]$ = [0.2,0.3];

You can check:

1°) The dynamic model

A= [3.5641 1.3021
1.3021 0.6000];

H= [0.0307,
-0.0038]

1°) Ground reaction:

R=[-1.9843
38.5153]

3°) Impact:

A_1=[4.0000 0 -0.3373 -0.8776
0 4.0000 0.3620 -0.4794
-0.3373 0.3620 1.2000 0.6000
-0.8776 -0.4794 0.6000 0.6000];

J_{R2}= [1.0000 0 -0.7021 -0.7021;
0 1.0000 -0.3835 -0.3835]