

# Localization lab and homework

## System description

### The robot and its sensors

The robot used to collect the data is a Lego NXT robot equipped with a sensor developed at ECN. The figure below shows the robot and the sensor, located at the front of the robot.

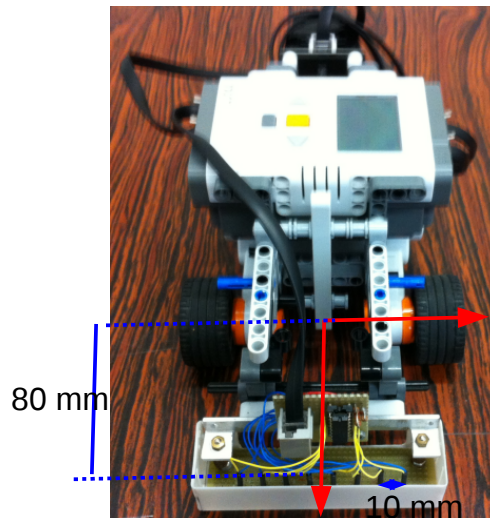


Figure 1. The NXT robot and its sensor.

The sensor is made of eight Reed sensors. Reed sensors are binary magnetic field detectors. The sensor is made of 8 such binary elements, visible on the above photograph. It is connected to the I2C bus of the robot and returns an eight bit word reflecting the state of the eight Reed sensors.

In our case, the Reed sensors are organized on a line, with a spacing of 1 cm. The line of sensors is located perpendicular to axis  $X_m$  of the robot, at a known position along  $X_m$ .

The magnets are the beacons of the localization system. They are arranged as a square array of magnets, with a spacing of 55 mm. They are not visible on the photograph because they are below the table on which the robot moves, right under the surface.

The robot also has two wheel encoders, with a resolution of 360 dots per revolution.

### System equations

The localization system uses an EKF (Extended Kalman Filter).

The discrete evolution equation is the equations of odometry, as presented in the lectures. The only difference is that the localization algorithm uses a slightly different form of the Kalman filter, with respect to the one presented in the lectures. See the file “LocalizationBook.pdf”, paragraph 5.2 “Kalman filter with a noisy input”.

When the sensor detects a magnet, the position of the magnet along the sensor direction is given by the number of the Reed sensor/sensors which is/are activated. This gives the  $y$  coordinate of the magnet in the robot frame, since the sensor is perpendicular to  $X_m$ . But the position of the line of sensors along  $X_m$  being known, we also simultaneously measure the  $x$  component of the coordinates of the magnet in the robot frame. Only this measurement is always equal to the known position of the sensor along  $X_m$ .

Students sometimes have a little difficulty understanding that. But imagine the robot has several lines of Reed sensors. Then reading the sensors would indeed give the two coordinates of the magnet, the line and row index of the magnet in the (low resolution) binary image formed by the array of Reed sensors. Our case is similar, but we have a “linear camera”, a single line of “magnetic detection pixels”.

So in our Kalman filter, the measurement equation will be the two components of the position of the magnet in the robot frame. The only thing we need in order to complete the equations is to express the measurement vector as a function of the state. We will do it for a certain magnet at position

${}^0P_{mag} = \begin{bmatrix} {}^0x_{mag} \\ {}^0y_{mag} \\ 1 \end{bmatrix}$  in the absolute reference frame, in homogeneous coordinates.

Let  $X = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$  be the posture of the robot and  ${}^0T_m = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}$

The position of the magnet in the robot frame is obtained by left-multiplying the position in the absolute frame by the inverse of  ${}^0T_m$ .

We obtain:

$${}^mP_{mag} = \begin{bmatrix} \cos\theta({}^0x_{mag} - x) + \sin\theta({}^0y_{mag} - y) \\ \cos\theta({}^0y_{mag} - y) - \sin\theta({}^0x_{mag} - x) \\ 1 \end{bmatrix} = \begin{bmatrix} Y \\ 1 \end{bmatrix} = \begin{bmatrix} g_{mag}(X) \\ 1 \end{bmatrix}$$

As indicated above, the observation equation of course depends on the particular magnet considered. Notice that, as indicated in the lectures, anything in the observation equation that is not an element of the state vector is a known constant.

To understand the code of the program, you also need to understand that the 2D vector  $Y$  is formed of a 1st component which is a known constant (the position of the sensor along  $X_m$ ) and a second (variable) component which depends on the Reed sensor which reacted to the magnetic field.