Sección 1.5.7

2.a.) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$$\nabla(\phi\psi) = \frac{\partial(\phi\psi)}{\partial x}\hat{\imath} + \frac{\partial(\phi\psi)}{\partial y}\hat{\jmath} + \frac{\partial(\phi\psi)}{\partial z}\hat{k}$$

$$= \psi\frac{\partial\phi}{\partial x}\hat{\imath} + \psi\frac{\partial\phi}{\partial y}\hat{\jmath} + \psi\frac{\partial\phi}{\partial z}\hat{k} + \phi\frac{\partial\psi}{\partial x}\hat{\imath} + \phi\frac{\partial\psi}{\partial y}\hat{\jmath} + \phi\frac{\partial\psi}{\partial z}\hat{k}$$

$$= \psi\nabla\phi + \phi\nabla\psi = \phi\nabla\psi + \psi\nabla\phi$$

2.d.) $\nabla\cdot(\nabla\times a)$ y $\nabla\times(\nabla\cdot a)$

$$\nabla\cdot(\nabla\times a) = \frac{\partial}{\partial x}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$= \frac{\partial^2 a_3}{\partial x\partial y} - \frac{\partial^2 a_2}{\partial x\partial z} + \frac{\partial^2 a_1}{\partial y\partial z} - \frac{\partial^2 a_3}{\partial y\partial x} + \frac{\partial^2 a_2}{\partial z\partial x} - \frac{\partial^2 a_1}{\partial z\partial y}$$

$$= \frac{\partial^2 a_3}{\partial x\partial y} - \frac{\partial^2 a_1}{\partial z\partial y} + \frac{\partial^2 a_2}{\partial z\partial x} - \frac{\partial^2 a_3}{\partial x\partial y} + \frac{\partial^2 a_1}{\partial y\partial z} - \frac{\partial^2 a_2}{\partial y\partial x}$$

$$\nabla\cdot(\nabla\times a) = 0$$

$\nabla\times(\nabla\cdot a) = ?$ → No tiene sentido, porque la divergencia de a es un campo escalar, y el rotacional se aplica sobre campos vectoriales.

2.f.) $\nabla\times(\nabla\times a) = \nabla(\nabla\cdot a) - \nabla^2 a$

$$\nabla\times(\nabla\times a) = \left[\frac{\partial}{\partial y}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right)\right]\hat{\imath} + \left[\frac{\partial}{\partial z}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)\right]\hat{\jmath}$$

$$+ \left[\frac{\partial}{\partial x}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right)\right]\hat{k}$$

$$= \left[\frac{\partial^2 a_2}{\partial y\partial x} - \frac{\partial^2 a_1}{\partial y^2} - \frac{\partial^2 a_1}{\partial z^2} + \frac{\partial^2 a_3}{\partial z\partial x}\right]\hat{\imath} + \left[\frac{\partial^2 a_3}{\partial z\partial y} - \frac{\partial^2 a_2}{\partial z^2} - \frac{\partial^2 a_2}{\partial x^2} + \frac{\partial^2 a_1}{\partial x\partial y}\right]\hat{\jmath} + \left[\frac{\partial^2 a_1}{\partial x\partial z} - \frac{\partial^2 a_3}{\partial x^2}\right.$$

$$\left. - \frac{\partial^2 a_3}{\partial y^2} + \frac{\partial^2 a_2}{\partial y\partial z}\right]\hat{k}$$

$$= \left[\frac{\partial^2 a_2}{\partial y\partial x} + \frac{\partial^2 a_3}{\partial z\partial x}\right]\hat{\imath} - \left[\frac{\partial^2 a_1}{\partial y^2} + \frac{\partial^2 a_1}{\partial z^2}\right]\hat{\imath} + \left[\frac{\partial^2 a_3}{\partial z\partial y} + \frac{\partial^2 a_1}{\partial x\partial y}\right]\hat{\jmath} - \left[\frac{\partial^2 a_2}{\partial z^2} + \frac{\partial^2 a_2}{\partial x^2}\right]\hat{\jmath} + \left[\frac{\partial^2 a_1}{\partial x\partial z} + \frac{\partial^2 a_2}{\partial y\partial z}\right]\hat{k}$$

$$- \left[\frac{\partial^2 a_3}{\partial x^2} + \frac{\partial^2 a_3}{\partial y^2}\right]\hat{k}$$

$$= \left[\frac{\partial^2 a_2}{\partial y\partial x} + \frac{\partial^2 a_3}{\partial z\partial x} + \frac{\partial^2 a_1}{\partial x^2}\right]\hat{\imath} + \left[\frac{\partial^2 a_1}{\partial x\partial y} + \frac{\partial^2 a_3}{\partial z\partial y} + \frac{\partial^2 a_2}{\partial x^2}\right]\hat{\jmath} + \left[\frac{\partial^2 a_1}{\partial x\partial z} + \frac{\partial^2 a_2}{\partial y\partial z} + \frac{\partial^2 a_3}{\partial z^2}\right]\hat{k} - \left(\frac{\partial^2 a_1}{\partial x^2}\hat{\imath} + \frac{\partial^2 a_2}{\partial y^2}\hat{\jmath} + \frac{\partial^2 a_3}{\partial z^2}\hat{k}\right)$$

$$= \nabla(\nabla\cdot a) - \nabla^2 a$$