Sección 3.1.4

6.)

6.a.) $e^i = \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)}$

$\langle e^i | e_i \rangle = 1 \Rightarrow \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} \cdot e_i = \dfrac{e_i \cdot (e_j \times e_k)}{e_i \cdot (e_j \times e_k)} = 1$ ✓

$\langle e^i | e_j \rangle = 0 \Rightarrow \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} \cdot e_j = \dfrac{e_j \cdot (e_j \times e_k)}{e_i \cdot (e_j \times e_k)} = \dfrac{0}{e_i \cdot (e_j \times e_k)} = 0$ ✓✓

$\langle e^i | e_k \rangle = 0 \Rightarrow \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} \cdot e_k = \dfrac{e_k \cdot (e_j \times e_k)}{e_i \cdot (e_j \times e_k)} = \dfrac{0}{e_i \cdot (e_j \times e_k)} = 0$ ✓✓

Esto se cumple para los diferentes combinaciones de i, j, k.

6.b.) $V = e_1 \cdot (e_2 \times e_3)$, $\bar{V} = e^1 \cdot (e^2 \times e^3)$

$V\bar{V} = e_1 \cdot (e_2 \times e_3) \cdot e^1 \cdot (e^2 \times e^3)$

$\bar{V} = e^1 \cdot (e^2 \times e^3) = \dfrac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} \cdot \left[\dfrac{e_3 \times e_1}{e_2 \cdot (e_3 \times e_1)} \times \dfrac{e_1 \times e_2}{e_3 \cdot (e_1 \times e_2)} \right]$

$\bar{V} = \dfrac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3) \cdot e_2 \cdot (e_3 \times e_1) \cdot e_3 \cdot (e_1 \times e_2)} \left[e_1 \cdot ((e_3 \times e_1) \cdot e_2) - e_2 (\overset{0}{\cancel{(e_3 \times e_1) \cdot e_1}}) \right]$

$\bar{V} = \dfrac{(e_2 \times e_3) \cdot e_1 ((e_3 \times e_1) \cdot e_2)}{e_1 \cdot (e_2 \times e_3) \cdot e_2 \cdot (e_3 \times e_1) \cdot e_3 \cdot (e_1 \times e_2)} = \dfrac{\cancel{e_2 \cdot (e_3 \times e_1)} \cdot \cancel{e_1 \cdot (e_2 \times e_3)}}{\cancel{e_1 \cdot (e_2 \times e_3)} \cdot \cancel{e_2 \cdot (e_3 \times e_1)} \cdot e_3 \cdot (e_1 \times e_2)} = \dfrac{1}{e_3 \cdot (e_1 \times e_2)}$

$\bar{V} = \dfrac{1}{e_3 \cdot (e_1 \times e_2)} = \dfrac{1}{e_1 \cdot (e_2 \times e_3)} = \dfrac{1}{V}$

$\Rightarrow V\bar{V} = V \cdot \dfrac{1}{V} = 1$

6.c.) ¿Qué vector satisface $a \cdot e^i = 1$? Demuestre que es único

$a \cdot e^i = a \cdot \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} = 1 \Rightarrow a \cdot (e_j \times e_k) = e_i \cdot (e_j \times e_k) \Rightarrow \boxed{a = e_i}$

Ahora, demostrando que a es único, tomamos $a \cdot e^i = 1 = b \cdot e^i$

$a \cdot e^i = b \cdot e^i \Rightarrow a \cdot \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} = b \cdot \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} \Rightarrow a \cdot (e_j \times e_k) = b \cdot (e_j \times e_k)$

$\Rightarrow a = b$

Sección 3.3.5

7.)

7.a) Si tenemos los vectores $A = \hat{i} + 2\hat{j} + 3\hat{k}$, $B = 2\hat{i} + \hat{j} + 3\hat{k}$, exprése los en términos de O'

$$M_1 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \text{Primera rotación}$$

$$M_2 = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \rightarrow \text{Segunda rotación}$$

Entonces ; $A = 1\hat{i} + 2\hat{j} + 3\hat{k}$

$$M_1 \times A = \begin{pmatrix} \frac{\sqrt{3}-2}{2} \\ \frac{2\sqrt{3}+1}{2} \\ 3 \end{pmatrix} = A^* \longrightarrow M_2 \times A^* = \begin{pmatrix} \frac{\sqrt{3}-2}{2} \\ \frac{-\sqrt{3}+2}{4} \\ \frac{2\sqrt{3}+1}{2} \end{pmatrix} = A'$$

$B = 2\hat{i} + \hat{j} + 3\hat{k}$

$$M_1 \times B = \begin{pmatrix} \frac{2\sqrt{3}-1}{2} \\ \frac{-2\sqrt{3}+1}{4} \\ \frac{\sqrt{3}+2}{2} \end{pmatrix} = B^* \longrightarrow M_2 \times B^* = \begin{pmatrix} \frac{2\sqrt{3}-1}{2} \\ \frac{-2\sqrt{3}+1}{4} \\ \frac{\sqrt{3}+2}{2} \end{pmatrix} = B'$$

7.b.) Tensor de esfuerzos

$$P_j = \begin{pmatrix} P_1 & 0 & P_4 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{pmatrix}$$ ¿Cuál será su expresión en O'?

Será $P' = M_2 \times P^*$, donde $P^* = M_1 \times P$

$$M_1 \times P = \begin{pmatrix} \frac{\sqrt{3} \cdot P_1}{2} & -\frac{P_2}{2} & \frac{\sqrt{3} \cdot P_4}{2} \\ \frac{P_1}{2} & \frac{\sqrt{3} \cdot P_2}{2} & \frac{P_4}{2} \\ 0 & 0 & P_3 \end{pmatrix} = P^*$$

$$M_2 \times P^* = \begin{pmatrix} \frac{\sqrt{3}\, P_1}{2} & -\frac{P_2}{2} & \frac{\sqrt{3}\, P_4}{2} \\ -\frac{\sqrt{3}\, P_1}{4} & \frac{P_2}{4} & -\frac{\sqrt{3}\, P_4}{4} \\ \frac{P_1}{2} & \frac{\sqrt{3}\, P_2}{2} & \frac{P_4}{2} \end{pmatrix} = P'$$

8.) $q^1 = x + y$, $q^2 = x - y$, $q^3 = 2z$

8.a.) Compruebe que (q^1, q^2, q^3) son ortogonales

$q^1 \cdot q^2 = (x+y) \cdot (x-y) = (|\hat{\imath}\rangle + |\hat{\jmath}\rangle) \cdot (|\hat{\imath}\rangle - |\hat{\jmath}\rangle) = 1 - 0 + 0 - 1 = 0$

$q^1 \cdot q^3 = (x+y) \cdot (2z) = (|\hat{\imath}\rangle + |\hat{\jmath}\rangle) \cdot (2|\hat{k}\rangle) = 2(0+0) = 0$

$q^2 \cdot q^3 = (|\hat{\imath}\rangle - |\hat{\jmath}\rangle) \cdot (2|\hat{k}\rangle) = 0$

8.b.)

$(x+y, x-y, 2z) = x(1,1,0) + y(1,-1,0) + z(0,0,2)$

$e_1 = (1,1,0)$, $e_2 = (1,-1,0)$, $e_3 = (0,0,2)$ son la base

8.c.) Tensor métrico

$$g_{ij} = \begin{pmatrix} e_1 \cdot e_1 & e_1 \cdot e_2 & e_1 \cdot e_3 \\ e_2 \cdot e_1 & e_2 \cdot e_2 & e_2 \cdot e_3 \\ e_3 \cdot e_1 & e_2 \cdot e_3 & e_3 \cdot e_3 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Elemento de volumen

$$dV = dq_1\, dq_2\, dq_3 = d(x+y)\, d(x-y)\, d(2z) = (dx+dy)(dx-dy)(2dz) = (dx^2-dy^2)\, 2dz$$

$$dV = 2(dx^2 - dy^2)\, dz$$

8.d.)

$$A = 2\hat{j} = e_1 - e_2$$

$$B = \hat{i} + 2\hat{j} = \frac{3}{2} e_1 - \frac{1}{2} e_2$$

$$C = \hat{i} + 7\hat{j} + 3\hat{k} = 4e_1 - 3e_2 + \tfrac{3}{2} e_3$$

8.e.)

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 1 & 2 & 0 \end{vmatrix} \quad 0\hat{i} + 0\hat{j} - 2\hat{k} = -e_3$$