Sección 1.5.7

2.a.) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$$\nabla(\phi\psi) = \frac{\partial(\phi\psi)}{\partial x}\hat{\imath} + \frac{\partial(\phi\psi)}{\partial y}\hat{\jmath} + \frac{\partial(\phi\psi)}{\partial z}\hat{k}$$

$$= \psi\frac{\partial\phi}{\partial x}\hat{\imath} + \psi\frac{\partial\phi}{\partial y}\hat{\jmath} + \psi\frac{\partial\phi}{\partial z}\hat{k} + \phi\frac{\partial\psi}{\partial x}\hat{\imath} + \phi\frac{\partial\psi}{\partial y}\hat{\jmath} + \phi\frac{\partial\psi}{\partial z}\hat{k}$$

$$= \psi\nabla\phi + \phi\nabla\psi = \phi\nabla\psi + \psi\nabla\phi$$

2.d.) $\nabla\cdot(\nabla\times a)$ y $\nabla\times(\nabla\cdot a)$

$$\nabla\cdot(\nabla\times a) = \frac{\partial}{\partial x}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$= \frac{\partial^2 a_3}{\partial x\partial y} - \frac{\partial^2 a_2}{\partial x\partial z} + \frac{\partial^2 a_1}{\partial y\partial z} - \frac{\partial^2 a_3}{\partial y\partial x} + \frac{\partial^2 a_2}{\partial z\partial x} - \frac{\partial^2 a_1}{\partial z\partial y}$$

$$= \frac{\partial^2 a_1}{\partial y\partial z} - \frac{\partial^2 a_1}{\partial z\partial y} + \frac{\partial^2 a_2}{\partial z\partial x} - \frac{\partial^2 a_2}{\partial x\partial z} + \frac{\partial^2 a_3}{\partial x\partial y} - \frac{\partial^2 a_3}{\partial y\partial x}$$

$$\nabla\cdot(\nabla\times a) = 0$$

$\nabla\times(\nabla\cdot a) = \,?$ $\rightarrow$ No tiene sentido, porque la divergencia de a es un campo escalar, y el rotacional se aplica sobre campos vectoriales.

2.f.) $\nabla\times(\nabla\times a) = \nabla(\nabla\cdot a) - \nabla^2 a$

$$\nabla\times(\nabla\times a) = \left[\frac{\partial}{\partial y}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right)\right]\hat{\imath} + \left[\frac{\partial}{\partial z}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)\right]\hat{\jmath}$$

$$+ \left[\frac{\partial}{\partial x}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right)\right]\hat{k}$$

$$= \left[\frac{\partial^2 a_2}{\partial y\partial x} - \frac{\partial^2 a_1}{\partial y^2} - \frac{\partial^2 a_1}{\partial z^2} + \frac{\partial^2 a_3}{\partial z\partial x}\right]\hat{\imath} + \left[\frac{\partial^2 a_3}{\partial z\partial y} - \frac{\partial^2 a_2}{\partial z^2} - \frac{\partial^2 a_2}{\partial x^2} + \frac{\partial^2 a_1}{\partial x\partial y}\right]\hat{\jmath} + \left[\frac{\partial^2 a_1}{\partial x\partial z} - \frac{\partial^2 a_3}{\partial x^2} - \frac{\partial^2 a_3}{\partial y^2} + \frac{\partial^2 a_2}{\partial y\partial z}\right]\hat{k}$$

$$= \left[\frac{\partial^2 a_2}{\partial y\partial x} + \frac{\partial^2 a_3}{\partial z\partial x}\right]\hat{\imath} - \left[\frac{\partial^2 a_1}{\partial y^2} + \frac{\partial^2 a_1}{\partial z^2}\right]\hat{\imath} + \left[\frac{\partial^2 a_3}{\partial z\partial y} + \frac{\partial^2 a_1}{\partial x\partial y}\right]\hat{\jmath} - \left[\frac{\partial^2 a_2}{\partial z^2} + \frac{\partial^2 a_2}{\partial x^2}\right]\hat{\jmath} + \left[\frac{\partial^2 a_1}{\partial x\partial z} + \frac{\partial^2 a_2}{\partial y\partial z}\right]\hat{k}$$

$$- \left[\frac{\partial^2 a_3}{\partial x^2} + \frac{\partial^2 a_3}{\partial y^2}\right]\hat{k}$$

$$= \left[\frac{\partial^2 a_2}{\partial x\partial y} + \frac{\partial^2 a_3}{\partial z\partial x} + \frac{\partial^2 a_1}{\partial x^2}\right]\hat{\imath} + \left[\frac{\partial^2 a_1}{\partial y\partial x} + \frac{\partial^2 a_2}{\partial y^2} + \frac{\partial^2 a_3}{\partial y\partial z}\right]\hat{\jmath} + \left[\frac{\partial^2 a_1}{\partial z\partial x} + \frac{\partial^2 a_2}{\partial z\partial y} + \frac{\partial^2 a_3}{\partial z^2}\right]\hat{k} - \left(\frac{\partial^2 a_1}{\partial x^2}\hat{\imath} + \frac{\partial^2 a_2}{\partial y^2}\hat{\jmath} + \frac{\partial^2 a_3}{\partial z^2}\hat{k}\right)$$

$$= \nabla(\nabla\cdot a) - \nabla^2 a$$

Sección 1.6.6

$z = re^{i\theta}$

2) Demuestre

a) $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2(\alpha)$

$\cos(2\alpha+\alpha) = \cos(2\alpha)\cos(\alpha) - \text{sen}(2\alpha)\,\text{sen}(\alpha) = \cos(\alpha+\alpha)\cos(\alpha) - \text{sen}(\alpha+\alpha)\,\text{sen}(\alpha)$

$= (\cos^2(\alpha) - \text{sen}^2(\alpha))\cos(\alpha) - (2\,\text{sen}(\alpha)\cos(\alpha))\,\text{sen}(\alpha)$

$= \cos^3(\alpha) - \text{sen}^2(\alpha)\cos(\alpha) - 2\,\text{sen}^2\alpha\cos\alpha$

$= \cos^3\alpha - 3\,\text{sen}^2\alpha\cos\alpha$ //

b.) $\text{sen}(3\alpha) = 3\cos^2(\alpha)\,\text{sen}(\alpha) - \text{sen}^3(\alpha)$

$\text{sen}(2\alpha+\alpha) = \text{sen}(2\alpha)\cos(\alpha) + \text{sen}(\alpha)\cos(2\alpha) = \text{sen}(\alpha+\alpha)\cos(\alpha) + \text{sen}(\alpha)\cos(\alpha+\alpha)$

$= (2\,\text{sen}(\alpha)\cos(\alpha))\cos\alpha + \text{sen}(\alpha)(\cos^2(\alpha) - \text{sen}^2(\alpha))$

$= 2\,\text{sen}(\alpha)\cos^2(\alpha) + \text{sen}\,\alpha\cos^2\alpha - \text{sen}^3\alpha$

$= 3\,\text{sen}\,\alpha\cos^2\alpha - \text{sen}^3\alpha$ //

5.) Encuentre las raíces de

a.) $(2i)^{1/2}$ $\qquad i = \sqrt{-1} \qquad i^2 = -1 \qquad i^3 = -i \qquad i^4 = 1 \qquad i^5 = i$

$\sqrt{2i} = \sqrt{2(i^5)} = i^3\sqrt{2} = -i\sqrt{2}$

b.) $(1-\sqrt{3}\,i)^{1/2} = (i^4 - \sqrt{3}\,i)^{1/2} = [i(i^3 - \sqrt{3})]^{1/2} = -i(-i-\sqrt{3})^{1/2} = -i[(-1)(i+\sqrt{3})]^{1/2}$

$= (i+\sqrt{3})^{1/2}$

c.) $(-1)^{1/3} = -1$

d.) $(8)^{1/6} = (2^3)^{1/6} = 2^{1/2} = \sqrt{2}$

e.) $(-8-8\sqrt{3}\,i)^{1/4} = [-8(1+\sqrt{3}\,i)]^{1/4} = (-8)^{1/4}(1+\sqrt{3}\,i)^{1/4} = i^3(8)^{1/4}(1+\sqrt{3}\,i)^{1/4}$

$= -i(8+8\sqrt{3}\,i)^{1/4}$