

Paul Erdős

Robert Osburn

University College Dublin

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- ▶ He travelled from country to country, attending conferences, lectures and visiting fellow mathematics.

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- ▶ We have that 2 and $105 = 3 \cdot 5 \cdot 7$ are relatively prime, but 6 and 105 are not.

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- ▶ “The man who loved only numbers”, Paul Hoffman.