$$\left(S_{i}^{\chi_{\ell}}\right)_{p_{i}}^{1-p_{i}} \quad \left(S_{i+1}^{\chi_{\ell}}\right)_{b_{i}:=\perp}$$