MATH 222 (Lectures 1,2, and 4) Fall 2015
Practice Midterm 2.2

Name: ________________________________ Student ID#: __________________________

Circle your TA's name from the following list.

Carolyn Abbott	Tejas Bhojraj	Zachary Carter	Mohamed Abou Dbai	Ed Dewey
Jale Dinler	Di Fang	Bingyang Hu	Canberk Irimagzi	Chris Janjigian
Tao Ju	Ahmet Kabakulak	Dima Kuzmenko	Ethan McCarthy	Tung Nguyen
Jaeun Park	Adrian Tovar Lopez	Polly Yu		

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6	Problem 7
Score							

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- On Problems 1, 2, and 3, only the answer will be graded.
- On Problems 4, 5, 6, and 7 you must show your work and we will grade the work and your justification, and not just the final answer.
- Problem 3 is worth 10 points. All other problems worth 15 points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1 - x^2}$. Note that we have provided some formulas on the next page to help with this.

Formulas

- $T_\infty e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$
- $T_\infty \sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$
- $T_\infty \cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$
- $T_\infty \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$
- $T_\infty \frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k$
- $T_\infty (1+x)^b = \sum_{k=0}^{\infty} \binom{b}{k} x^k$ where $\binom{b}{k} = \frac{b(b-1)(b-2)\cdots(b-k+1)}{k!}$

1. For each statement below, CIRCLE the correct answer. You do not need to show your work.

(a)		(b)		(c)		(d)	(e)
True	False	True	False	True	False	I II	I II
						III	III

True or false:

(a) $(x\cos(x) - x)$ is $o(x^5)$.

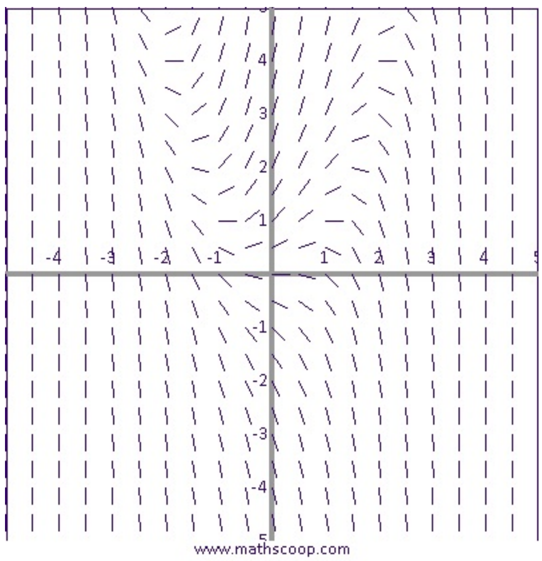
(b) If $f(x)$ is a degree 5 polynomial then $T_{15}f(x) = f(x)$.

(c) $R_4 \sin x = \sin x - (x - \frac{x^3}{3!})$

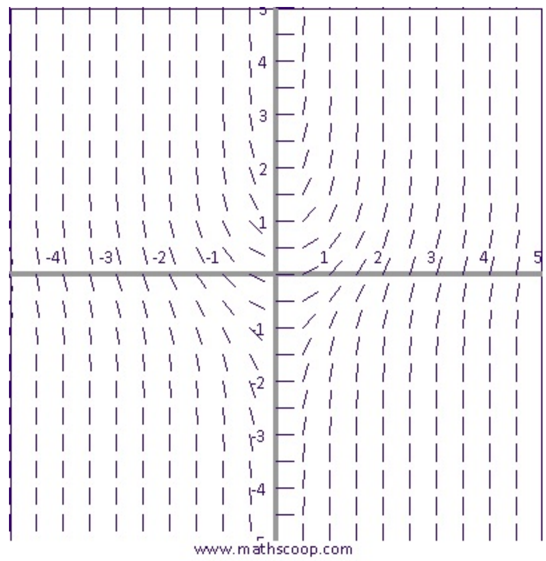
Below are three direction fields. The equations for *two* of those fields are given below. Match the equation to the appropriate direction field and record your answer on the previous page.

(d) $\frac{dy}{dx} = x^2 - y$

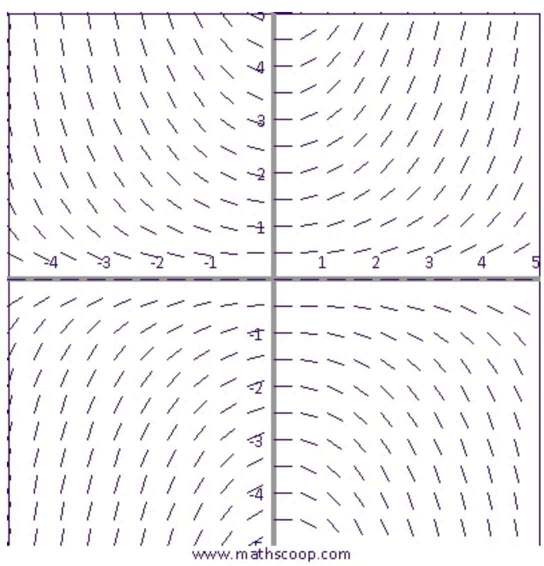
(e) $\frac{dy}{dx} = xy$



I.



II.



III.

2. (a) Use Euler's method with step size $h = 0.1$ to estimate $y(0.1)$ where $y(x)$ satisfies

$$\frac{dy}{dx} = x + y \text{ and } y(0) = 1.$$

Answer: ______________________________

(b) Find $T_3\left(x^3 + \frac{1}{1+2x^2}\right)$.

Answer: ______________________________

(c) Find $T_2^1(x^3)$.

Answer: ______________________________

3. In the problem below: 1. Clearly define variables (including units!); 2. Set up the appropriate differential equation; and 3. Write down the appropriate initial condition. DO NOT SOLVE THE DIFFERENTIAL EQUATION.

   Ten thousand dollars is deposited in a bank account on January 1, 1990 with a nominal annual interest rate of 5% compounded continuously. No further deposits are made. Money is withdrawn continuously at a rate of \$4000 per year. We are interested in a function that models the amount of money left in the account.

- **Variables (2pts):**

- **Differential equation (6pts)**

- **Initial condition (2pts):**

4. Find a solution to each initial value problem.

(a)

$$\frac{dy}{dx} = 4x^3(y + e^{x^4}) \text{ and } y(0) = 1.$$

Solution Satisfying Initial Condition: $y =$ ______________________________

(b)

$$\frac{1}{\sqrt{1 - y^2}} \frac{dy}{dx} = \cos x \text{ and } y(0) = 0.$$

Solution Satisfying Initial Condition: $y =$ ______________________________

5. Let t stand for time in minutes from 12:00pm and let $B(t)$ denote the number of bacteria in a petri dish at time t. Assume that B satisfies $\frac{dB}{dt} = 50 \cdot B \cdot (1 - B)$. Also assume that at 12:00pm there were 2 bacteria in the dish. Compute $B(t)$.

6. Let $f(x) = \sin(2x)$. Find n such that $|f(x) - T_n f(x)| \leq \frac{1}{100}$ for x in the range $-\frac{1}{2} \leq x \leq \frac{1}{2}$.
It may be helpful to know that $2! = 2, \quad 3! = 6, \quad 4! = 24, \quad 5! = 120$ *and* $6! = 720$.

7. Let $f(x)$ be a function satisfying the differential equation

$$f^{''}(x) + 2e^{2x^2} - f(x) = 0$$

and also satisfying the initial conditions $f(0) = 0$ and $f'(0) = -1$. Compute $T_4 f(x)$.