Worksheet 11

Spring 2016

MATH 222, Week 11: Sequences!

Name: ________________________________

You aren't necessarily expected to finish the entire worksheet in discussion. There are a lot of problems to supplement your homework and general problem bank for studying.

Problem 1. Find

$$\lim_{n\to\infty} \frac{n^2+n+1}{3n^2-n-2}$$

Problem 2. Find an example of a sequence a_n which is bounded but not convergent.

Problem 3. Let $a_n = (-1)^n$ for $n = 1, 2, 3, \ldots$.

(a) Does a_n converge? i.e. does $\lim_{n\to\infty} a_n = L$ for some real number L? If it exists, what is it?

(b) Let $f(x) = x^2$. Does $f(a_n)$ converge? If so, what is $\lim_{n\to\infty} f(a_n)$?

(c) Try to state in words what (a) and (b) illustrate.

Problem 4. Let's try to think of the last problem in the opposite direction. Let $a_n = \frac{1}{n}$ for $n = 1, 2, 3, \ldots$.

(a) Does a_n converge? If so, what is $\lim_{n\to\infty} a_n$?

(b) Define a function on the interval from 0 to 1 by

$$f(x) = \begin{cases} (-1)^{\frac{1}{x}} & x \in (0, 1] \\ 0 & x = 0 \end{cases}$$

Does $f(a_n)$ converge? If so, what is $\lim_{n\to\infty} f(a_n)$?

(c) Does $f(\lim_{n\to\infty} a_n)$ exist, and if so what is it?

(d) Try to state in words what (a) and (b) illustrate.

Problem 5. We've essentially been playing with examples that lead to an interesting question that I want you to try to answer. Under what circumstances is it true that if $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} f(a_n) = f(\lim_{n\to\infty} a_n) = f(L)$?