WES Worksheet 1.2

Fall 2018

MATH 222, Week 1: More IBP

Name: _____

Problem 1. Consider the integral $I_n = \int e^{nx} \cos(x)\, dx$. Integrate by parts and solve for I_n.

Problem 2. (a) If $\theta = \arctan(x)$, what is $\cos(\theta)$? Hint: draw a right angle triangle.

(b) Show that the derivative of $y = \arctan(x)$ is $\frac{dy}{dx} = \frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$. (Hint: Implicit differentiation)

(c) Integrate $\int x \arctan(x)\, dx$.

Problem 3. For $m, n = 1, 2, 3, \ldots$, consider the integral $I_{m,n} = \int x^m (\ln x)^n \, dx$.

(a) Integrate $I_{m,n}$ by parts once to find a reduction formula in terms of $I_{m,(n-1)}$.

(b) Now consider the definite integral $A_{m,n} = \int_0^1 x^m (\ln x)^n \, dx$.
Write down the reduction formula for $A_{m,n}$ in terms of $A_{m,(n-1)}$. You may use as a fact that $\lim_{x \to 0^+} (\ln x)^n x^m = 0$.

(c) Using part (b), write down a formula for $A_{m,n}$ in terms of $A_{m,0}$.

(d) Evaluate $A_{m,0} = \int_0^1 x^m \, dx$. What is $A_{m,n}$?