$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\sin 2\theta \cos 2\theta$

$[\sin\theta\cos\theta]^2$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

WES Worksheet 1.1/2.1

Fall 2018

MATH 222, Week 1: Review

Name: ______________________________

$\frac{u}{1+u^2}du \rightsquigarrow \frac{c}{s}ds$

1 More differentiation and integration review

$s = 1+u^2$

ds

Problem 1. Find the following:

(a) $\int 2^x dx$ $= 2^x/\ln(2) + C$

(b) $\frac{d}{dx}(e^{2x}\cos(x))$ $= 2e^{2x}\cos(x) - e^{2x}\sin(x)$

(c) $\frac{d}{dx}\int_0^{x^2}\sin^4(t)dt$ $\sin^4(x^2)\cdot 2x$

(d) $\int \frac{2x+2}{x^2+2x+1}dx$ (There are at least two ways to approach this) $= \int \frac{2(x+1)}{(x+1)^2}dx$

$= \int \frac{2}{(x+1)}dx = 2\ln(x+1)+C$

2 Serious Trig Identity Problem

Problem 2. Compute $\int \sin^2(\theta)\cos^2(\theta)\, d\theta$. There are at least two ways to approach this.

ws1

$\frac{1}{4}\sin(4\theta)^2$

3 Integration by Parts (IBP)

Problem 3. Evaluate the following integrals:

(a) $\int xe^x dx$ $= e^x(x-1)+C$

(b) $\int x^2\cos(x)dx$ $= 2x\cos(x) + (x^2-2)\sin(x)$

(c) $\int (x+1)\sin(x)dx$ $= -\cos x - x\cos x + \sin x$

(d) $\int x^5 e^{2x}dx$ $= \frac{1}{8}e^{2x}(-15+30x-30x^2+20x^3-10x^4+4x^5)$

$\int fg' = fg - \int f'g$

$\frac{d}{dx}(fg) = f'g + g'f$

integrate this

4 Some Theory

Problem 4. Discuss how integration by parts could be considered the "backwards product rule." (This is similar to the way that substitution could be considered the "backwards chain rule"). How is this apt? Where does it fall apart? Use this perspective to derive the integration by parts formula.

Problem 5. This about how integration by parts works to come up with a way to decide what should be "u" and what should be "dv". In particular, consider the types of functions you know and in what order they should be chosen. Generally, what happens to the function we choose as "u" and what happens to the function we choose as "dv" and how does this dictate your choices? u becomes easier dv becomes no worse...

5 Common IBP "trick"

Using IBP, we are able to integrate functions like $f(x) = \ln(x)$. The secret is to choose 1 as your "dv". Carry out this integration technique in the following problems.

Problem 6. Figure out the following integrals $u = \tan^{-1}(x)$ $du = \frac{1}{1+x^2}$

(a) $\int \arctan(x)dx = x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2)$

(b) $\int \arcsin(x)dx = \sqrt{1-x^2} + x\sin^{-1}(x)$. $u = \sin^{-1}(x)$ $du = \frac{1}{\sqrt{1-x^2}}$

(c) Did the trick let you integrate these or was something further required?

6 Finished Everything Else?

Another common way to use integration by parts is for the following problems. The idea is sometimes called "doubling back," (why does this make sense?). Try to solve these integrals and see if you can figure out the method. We will get more practice with this on Monday.

Problem 7. Evaluate the following integrals

(a) $\int e^x \cos(x)dx$

(b) $\int \sin(2x)\cos(5x)dx = \frac{1}{6}\cos(3x) - \frac{1}{14}\cos(7x)$