MATH 222 (002 and 004) Fall 2013
Practice Final.

Name: ________________________________

Circle your TA's name from the following list.

Ed Dewey	Stephen Neal	Dae Han Kang	Sharath Prased	Alisha Zachariah
Chris Janjigian	Animesh Anand	Reese Johnston	Jeremy Schwend	Alex Troesch

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6
Score						
	Problem 7	Problem 8	Problem 9	Problem 10	Problem 11	Problem 12
Score						

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- You must show all of your work, except on Problem 1.
- All problems graded out of 10.
- No calculators, books, or notes (except for those notes on your 3×5 notecard.)

Note: Everything on this page will appear on the actual exam as well.

Formulas

- $\cos(\arcsin x) = \sqrt{1 - x^2}$
- $\sec(\arctan x) = \sqrt{1 + x^2}$.
- $\tan(\operatorname{arcsec} x) = \sqrt{x^2 - 1}$.
- $\csc(\arcsin x) = \frac{1}{x}$
- $\cot(\arcsin x) = \frac{\sqrt{1-x^2}}{x}$

Bound for remainder term

If f is a $n+1$ differentiable function on an interval containing $x = 0$ and if we have a constant M_n such that

$$\left| f^{(n+1)}(t) \right| \leq \text{ for all } t \text{ between } 0 \text{ and } x$$

then

$$|R_n f(x)| \leq \frac{M_n |x|^{n+1}}{(n+1)!}$$

1. For each statement below, CIRCLE true or false. You do not need to show your work.

(a)		(b)		(c)		(d)		(e)	
True	False	True	False	True	False	True	False	True	False

(a) $\int_3^\infty \frac{1}{e^{3x}} dx \geq \int_3^\infty \frac{1}{e^{x^2}} dx$.

(b) $(x^2 + x^3)^2 = o(x^3)$.

(c) $\sum_{n=1}^\infty \frac{1}{n^4+5}$ is a finite number.

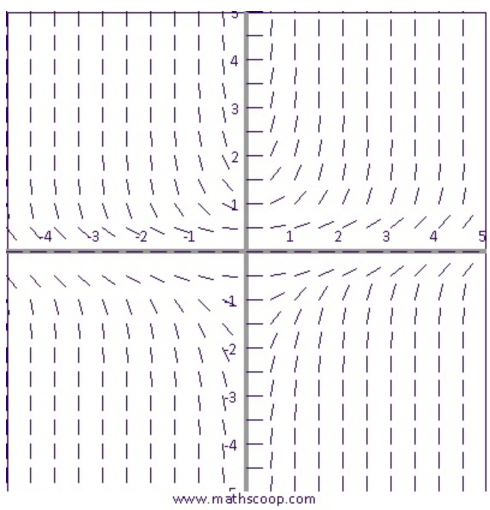
(d) Let $\mathbf{a}$ and $\mathbf{b}$ be any space vectors and let t be any number. Then $\mathbf{a} \cdot (\mathbf{b} + t\mathbf{a}) = \mathbf{a} \cdot \mathbf{b} + ||t\mathbf{a}||^2$.

(e) $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 0 \end{pmatrix}$.

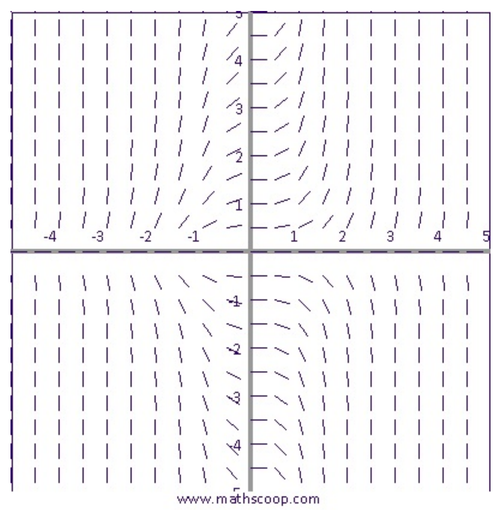
2. Below are four direction fields and two equations. Match the equation to the appropriate direction field. (5 points each).

   (a) $\frac{dy}{dx} = \sin x + \cos y$. Answer: ________

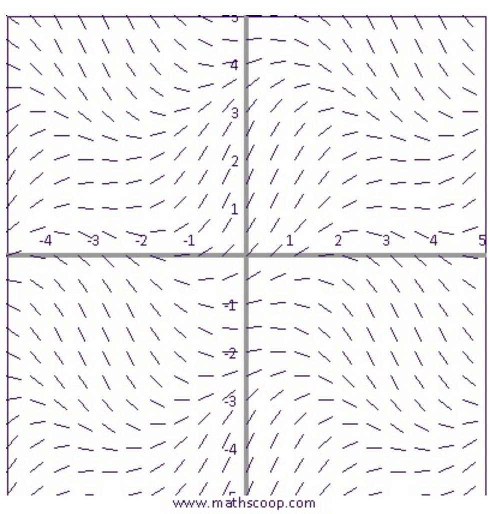
   (b) $\frac{dy}{dx} = xy^2$. Answer: ________



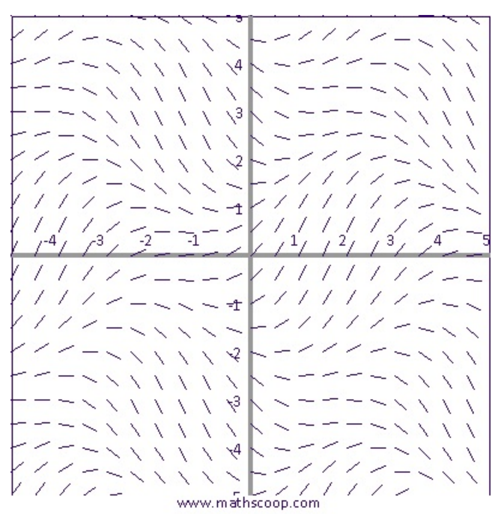
I.



II.



III.



IV.

3. Below you will find a number of mathematical expressions. Circle those which are *nonsense*. For instance, writing $\begin{pmatrix} 1 \\ 0 \end{pmatrix}^5$ is nonsense since we cannot raise a vector to the fifth power. Let

$$\mathbf{a} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -2 \\ 11 \\ 6 \end{pmatrix}, \text{ and } \mathbf{d} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

(a) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

(b) $\mathbf{a} \cdot \mathbf{b} \cdot \mathbf{c}$

(c) $3\mathbf{a} + 5\mathbf{b} - \mathbf{d}$

(d) $\mathbf{a} \times \mathbf{a}$

(e) $\frac{(\mathbf{a} \cdot \mathbf{b})^2}{||\mathbf{a}||}\mathbf{d}$.

4. Compute $\int \frac{5x-1}{(x+5)(x^2+1)} dx$.

5. Compute $\int \cos^3(5\theta + 1)dx$.

6. Compute $\int_0^\infty x^2 e^{-x} dx$. (*Note: The original copy of the practice exam had* xe^{-x} *instead of* x^2e^{-x}. *The problems are similar, but I like this one more.*)

7. The squirrel population in Madison has a continuous birth rate of 8% and a natural continuous death rate of 3%. In addition, each year 300 squirrels are eaten by foxes and 100 squirrels are run over by cars. There were $10,000$ squirrels in Madison on January 1, 2010. We are interested in explicitly finding a function S that models the **squirrel population in Madison at a given time**. Use the following space to work out your answer, and record the various parts of the problem at the bottom of the page.

- **Variables:**

- **Differential equation and initial condition for S:**

- **Solution for P satisfying initial conditions:**

8. Compute a solution to the initial value problem

$$\frac{dy}{dx} = \frac{x + xy^2}{2y} \quad \text{and} \quad y(0) = \sqrt{e^2 - 1}$$

9. Find the Taylor polynomial of degree 14 at $x = 0$ (i.e. find T_{14}) of the function $f(x) = \frac{10x^4}{(1-x^5)^2}$. Remember to use notation correctly!

10. Does $\sum_{n=1}^{\infty} \frac{e^{-n}+n+\sqrt{n}}{e^{n}+n^{-3}+\sqrt[3]{n}}$ converge? You must justify your answer.

11. Imagine that you have a function $f(x)$ that satisfies

$$|f^{(n+1)}(x)| \leq (n+1)$$

for all n. Show that the Taylor series $T_\infty f(x)$ converges to $f(x)$ for any value of x. You should make use of the "bound for the remainder term" on the second page of this exam.

12. Let $\mathcal{P}$ be the plane spanned by the points $(5, 0, 0)$, $(4, 2, 0)$ and $(0, 10, 6)$. Use the space below to compute the following:

(a) (4 points). The normal vector $\mathbf{n}$ to $\mathcal{P}$.

(b) (3 points). An equation (standard or parametric is fine) for $\mathcal{P}$.

(c) (3 points). Does the points $A = (2, 6, 11)$ line in $\mathcal{P}$?