Quiz 4

Spring 2016

MATH 222-004

Name: ________________________________

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Compute $\displaystyle\int_0^1 \frac{1}{x+\sqrt{x}}\,dx$.

Solution 1.

We first notice this is an improper integral so we set up the limit

$$\lim_{a\to 0}\int_a^1 \frac{1}{x+\sqrt{x}}\,dx$$

Now we figure out the antiderivative. To do this we let $u=\sqrt{x}$ and thus $2udu = dx$. Substituting

$$\lim_{a\to 0}\int_*^* \frac{2u}{u^2+u}\,du = \lim_{a\to 0} 2\ln|u+1|\Big|_*^* = \lim_{a\to 0} 2\ln|\sqrt{x}+1|\Big|_a^1 = \lim_{a\to 0} 2\ln|\sqrt{1}+1| - 2\ln|\sqrt{a}+1| = 2\ln(2)$$

$\square$

Problem 2. Determine if there is a constant a such that for $x > a$ the following inequality is true. If such an a exists, state the minimal such a.

$$\frac{x}{\sqrt{x^3-1}} > x^{-1/2}$$

Solution 2.

To see if such an a exists we want to reduce this statement to a more understandable equivalent statement. To do this we can assume that $x > 1$. In this case everything is positive so we can multiply both sides of the inequality by $\sqrt{x}\sqrt{x^3-1}$. So our original inequality when $x > 1$ is equivalent to

$$x\sqrt{x} > \sqrt{x^3-1}$$

This is true if and only if it remains true for the square of both sides because once again everything is positive. So we have another equivalent statement

$$x^3 > x^3 - 1$$

This is clearly true. So we have found an a that works, mainly $a = 1$. As this is equivalent to our original statement this means that when $x > 1$ our original statement is true. $\square$