Worksheet 10

Spring 2016

MATH 222, Week 10: Taylor Series and Little-oh Notation

Name: ______________________________

You aren't necessarily expected to finish the entire worksheet in discussion. There are a lot of problems to supplement your homework and general problem bank for studying.

Problem 1. Solve the following initial value problem exactly, then compute its degree two Taylor polynomial around zero and use this to compute an estimate for $y(.3)$. Then use Euler's method with step size $\Delta x = .1$ to estimate $y(.3)$.

$$\frac{dy}{dx} = -2xy$$

$y(0) = 1$. Just as a sanity check, the true value of $y(.3)$ is about .914.

Problem 2. Hasdrubal has designed a rocket. While proving mathematically that it won't explode, he used the approximation $e^{1/3} \approx 1 + \frac{1}{3} + \frac{1}{3^2 2!} + \frac{1}{3^3(3!)}$ If this approximation is off by more than $\frac{2}{4!}\left(\frac{1}{3}\right)^4$, the rocket might blow up. Convince Hasdrubal that it won't.

Problem 3. Find a bound for $R^0_n \sin(3x)$ and use this to show that $T^0_n \sin(3x) \to \sin(3x)$ for all x as $n \to \infty$.

Problem 4. Find a bound on $R_n^0 e^{2x}$ and use this to show that for every x, $T_n^0 e^{2x} \to e^{2x}$ as $n \to \infty$.

Problem 5. First, define what it means if we write $h(x) = o(x^n)$. Then finish the following rules concerning little-oh notation:

(a) $x^n \cdot o(x^m) =$

(b) $o(x^n) \cdot o(x^m) =$

(c) When does $x^m = o(x^n)$?

(d) When does $o(x^n) + o(x^m) = o(x^n)$

(e) Given any constant C, $o(Cx^n) =$

Problem 6. Is it true that $\sin(x) - x + \frac{x^3}{3} = o(x^4)$?

Problem 7. Show that $e^x - \sqrt{1+2x} = o(x)$.