Worksheet 9

Spring 2016

MATH 222, Week 9: Taylor Series!

Name: ______________________________

You aren't necessarily expected to finish the entire worksheet in discussion. There are a lot of problems to supplement your homework and general problem bank for studying.

Problem 1. Write the following series in summation notation:

(a) $1 + x + x^2 + x^3 + x^4 + \cdots$

(b) $1 + x^2 + x^4 + x^6 + \cdots$

(c) $1 + \frac{x^2}{2} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$ (This is an extremely important Taylor Series! Which one?)

Problem 2. Compute the second order Taylor polynomial of $\sin(x^2)$ around 0 and use this to approximate $\sin(1/4)$. Note that the actual value is $\sin(1/4) \approx 0.247404$

Problem 3. Compute the degree two Taylor polynomial of the function $f(x) = e^{\tan(x)}$ around 0. Use this to estimate $e^{\tan(.1)}$. Note that the actual value is $e^{\tan(.1)} \approx 1.10554$.

Problem 4. Find the second order Taylor polynomial around 0 for $f(x) = \displaystyle\int_0^x e^{-t^2} dt$ and use this to estimate $f(.1)$. This allows us to approximate this integral for different bounds! To show you how useful this is, try to think about taking the antiderivative of e^{-t^2}.

Problem 5. Solve the following initial value problem exactly, then compute its degree two Taylor polynomial around zero and use this to compute an estimate for $y(.3)$. Then use Euler's method with step size $\Delta x = .1$ to estimate $y(.3)$.

$$\frac{dy}{dx} = -2xy$$

$y(0) = 1$. Just as a sanity check, the true value of $y(.3)$ is about .914.

Problem 6. Hasdrubal has designed a rocket. While proving mathematically that it won't explode, he used the approximation $e^{1/3} \approx 1 + \frac{1}{3} + \frac{1}{3^2 2!} + \frac{1}{3^3(3!)}$ If this approximation is off by more than $\frac{2}{4!}\left(\frac{1}{3}\right)^4$, the rocket might blow up. Convince Hasdrubal that it won't.

Problem 7. Find a bound for $R_n^0 \sin(3x)$ and use this to show that $T_n^0 \sin(3x) \to \sin(3x)$ for all x as $n \to \infty$.