Worksheet 4

Fall 2018

MATH 222, Week 4: Trig Sub with Bounds, Partial Fractions and

Name: ________________________________

Trig Sub with Bounds

Problem 1. The point of the following questions is to get comfortable with changing bounds in a definite integral when making trig sub.

(a) Say we have a definite integral and we make a trig sub $x = 2\sin(\theta)$. We want to evaluate our integral from $x = 1$ to $x = 2$ what would the new bounds be in terms of θ after the trig sub?

(b) If $x = a\sin(\theta)$ and I want to evaluate my integral from $x = -a/2$ to $x = 0$, what would my new bounds be in terms of θ?

(c) Same question as above but what if we make the trig sub $x = 3\tan(\theta)$ and we want to evaluate our integral from $x = -3$ to $x = 0$?

(d) Same question as above but what if we make the trig sub $x = 2\sec(\theta)$ and we want to evaluate our integral from $x = 2\sqrt{2}$ to $x = 4$?

Problem 2. Evaluate the following integrals. For each, make a trig sub, change the bounds and evaluate, do not convert back to x. Be mindful of which bounds you choose for θ.

(a) $\int_2^4 \frac{x^2-4}{x}\,dx$

(b) $\int_1^2 \frac{4-x^2}{x^2}\,dx$

(c) $\int_2^4 \frac{1}{\sqrt{x^2+2}}\,dx$

(d) $\int_0^1 t\sqrt{1-t^2}\,dt$

(e) $\int_0^1 e^{4x}\sqrt{1-e^{2x}}\,dx$.

(f) For problems (d) and (e) you can consult your previous worksheet and check if your answer agrees with the answer you would get if you converted back to x and evaluated at the given bounds.

Partial Fractions

Problem 3. Fill in the appropriate numerators and denominators in the following partial fraction decomposition. (The first is an example. We will assume the degree of $f(x)$ is smaller than the degree of the denominator.)

(a) $$\frac{f(x)}{(x-1)(x-2)} = \left[\quad \frac{A}{x-1} \quad + \quad \frac{B}{x-2} \quad \right]$$

(b) $$\frac{f(x)}{(x-1)(x-2)(x-\pi)} = \left[\quad \frac{}{} \quad + \quad \frac{}{} \quad + \quad \frac{}{} \quad \right]$$

(c) $$\frac{f(x)}{(x+1)^3} = \left[\quad \frac{}{} \quad + \quad \frac{}{} \quad + \quad \frac{}{} \quad \right]$$

(d) $$\frac{f(x)}{(x-1)(x^2+1)} = \left[\quad \frac{}{} \quad + \quad \frac{}{} \quad \right]$$

Problem 4. Evaluate the following integrals

(a) $\int \frac{1}{x^2-4}\,dx$

(b) $\int \frac{1}{2+e^{2t}}\,dt$

(c) $\int \frac{x^3}{x^2+2}\,dx$

(d) $\int \frac{x^2+2x}{x^3-8x^2-7x}\,dx$

(e) $\int \frac{dx}{(x+1)^3(x-2)^2}$

(f) $\int \frac{dx}{(x^2+1)(x-2)}$

(g) $\int \frac{dx}{(x^2-4)(x^2+1)^2}$

(h) (**Challenge**) $\int \frac{2x^3+3x+1}{(x^2-1)^2(x^2+1)^2(x-1)}\,dx$

Problem 5. (a) Compute $\int_2^4 \frac{1}{x^2}\,dx$

(b) Compute $\int_2^4 \frac{1}{x(x-h)}\,dx$ where h is any positive real number.

(c) What happens as $h \to 0$ in the integral for part (b)? How is this related to part (a)?

Problem 6. Compute $\int \frac{1}{x^2-a^2}\,dx$ where a is any real number.

Strategy for Integration

Problem 7. For each of the following integrals, indicate which integration technique(s) should/could be used to evaluate the integral. There could be more than one answer, or multiple techniques combined. You do not need to evaluate the integrals.

(a) $\int \frac{\tan^3(x)}{\cos^3(x)}\,dx$

(b) $\int \frac{x^5+1}{x^3-3x^2-10x}\,dx$

(c) $\int x^8 \sin(x)\,dx$

(d) $\int \sec^3(x)\,dx$

(e) $\int \frac{x}{\sqrt{4-x^2}}\,dx$

(f) $\int \ln(x)\,dx$

(g) $\int \cos^2(x)\sin^3(x)\,dx$

(h) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

Improper Integrals, if we get there...

Problem 8. Evaluate the following integrals

(a) $\int_1^\infty \frac{1}{x^2}\,dx$

(b) $\int_1^\infty \frac{\ln(x)}{x^3}\,dx$

(c) $\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

(d) $\int_3^{10} \frac{1}{(x-9)^{1/3}}\,dx$

(e) $\int_0^4 \frac{1}{x^2+x-6}\,dx$

(f) $\int_0^1 \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

For the last three problems, what would happen if you forgot about the asymptote?