

Worksheet 9

Fall 2016

MATH 221, Week 9

Name: _____

1 Intermediate Value Theorem

State the intermediate value theorem and use it to prove the following functions have a zero in the given interval:

(a) $x^4 - 5x^3 + 3x^2 - 1$ on the interval $4 \leq x \leq 5$

(b) $2^x - 3^x + (x + 1)^2$ on the interval $2 \leq x \leq 3$ (2^x and 3^x are continuous)

2 Max and Min!

Find all the critical points of the following function on the interval $-3 \leq x \leq 4$ and determine which are global or local max/min or neither:

$$f(x) = \begin{cases} -x & x \leq 0 \\ x & 0 < x \leq 1 \\ 1 & 1 < x \leq 2 \\ x - 1 & 2 < x \leq 3 \\ -x + 5 & 3 < x \end{cases}$$

(Hint: It will be a very good idea to graph this).

3 Graph Sketching

For the following functions do all of the following (you'll probably need to use another sheet of paper):

- (a) Find all zeroes of the function
- (b) Find the first and second derivatives
- (c) Classify where it is increasing and decreasing
- (d) Classify where it is concave up and concave down
- (e) Compute all asymptotes
- (f) Put all of this together and sketch the graph!

(a) $f(x) = x^4 - 4x^3$

(b) $f(x) = \frac{1}{x^2-9}$

(c) $f(x) = x - 3x^{1/2}$

(d) $f(x) = \frac{4x}{x^2+1}$

(e) $f(x) = x\sqrt{4-x^2}$