MATH 222 (Lectures 1,2, and 4) Fall 2015
Midterm 2a

Name: ______________________________ Student ID#: ________________________

Circle your TA's name from the following list.

Carolyn Abbott	Tejas Bhojraj	Zachary Carter	Mohamed Abou Dbai	Ed Dewey
Jale Dinler	Di Fang	Bingyang Hu	Canberk Irimagzi	Chris Janjigian
Tao Ju	Ahmet Kabakulak	Dima Kuzmenko	Ethan McCarthy	Tung Nguyen
Jaeun Park	Adrian Tovar Lopez	Polly Yu		

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6	Problem 7
Score							

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- On Problems 1, 2, and 3, only the answer will be graded.
- On Problems 4, 5, 6, and 7 you must show your work and we will grade the work and your justification, and not just the final answer.
- Problem 3 worth 10 points. All other problems worth 15 points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1 - x^2}$. Note that we have provided some formulas on the next page to help with this.

Formulas

- $T_\infty e^x = \sum_{k=0}^\infty \frac{x^k}{k!}$
- $T_\infty \sin x = \sum_{k=0}^\infty (-1)^k \frac{x^{2k+1}}{(2k+1)!}$
- $T_\infty \cos x = \sum_{k=0}^\infty (-1)^k \frac{x^{2k}}{(2k)!}$
- $T_\infty \frac{1}{1-x} = \sum_{k=0}^\infty x^k$
- $T_\infty \frac{1}{1+x} = \sum_{k=0}^\infty (-1)^k x^k$
- $T_\infty (1+x)^b = \sum_{k=0}^\infty \binom{b}{k} x^k$ where $\binom{b}{k} = \frac{b(b-1)(b-2)\cdots(b-k+1)}{k!}$

1. For each statement below, CIRCLE the correct answer. The direction field matching appears on the next page. You do not need to show your work.

(a)	(b)	(c)	(d)	(e)
True False	True False	True False	I II III	I II III

True or false:

(a) $\left(x^2 + x\right)^3$ is $o(x^3)$.

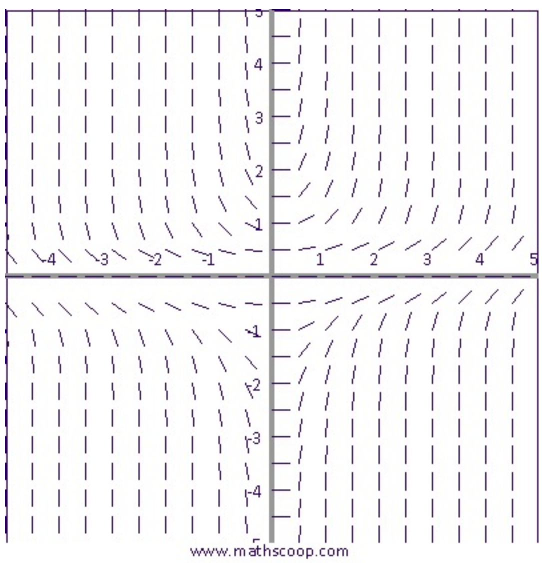
(b) $\binom{1/3}{2} = -\frac{1}{9}$.

(c) $R_3 \cos x = \cos x - (1 + \frac{x^2}{2!})$

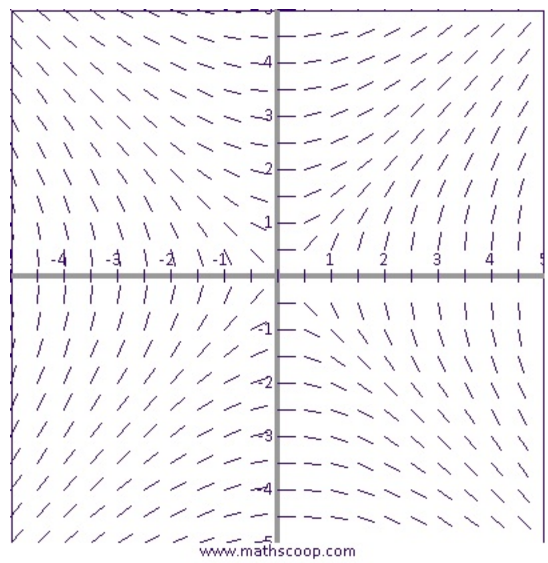
Below are three direction fields. The equations for *two* of those fields are given below. Match the equation to the appropriate direction field and record your answer on the previous page.

(d) $\frac{dy}{dx} = \frac{x}{y}$

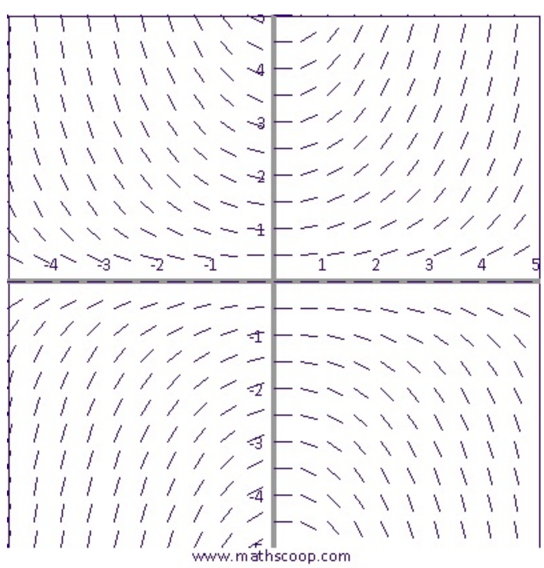
(e) $\frac{dy}{dx} = xy$



I.



II.



III.

2. (a) Use Euler's method with step size $h = 0.1$ to estimate $y(1.1)$ for the initial value problem

$$\frac{dy}{dx} = x + xy + y \text{ and } y(1) = 1.$$

Answer: ______________________________

(b) Find $T_1^2 x^3$.

Answer: ______________________________

(c) Find $T_2 \cos(\sin x)$.

Answer: ______________________________

3. In the problem below:

   - Clearly define variables (including units!).
   - Set up the appropriate differential equation
   - Write down the appropriate initial condition.

   DO NOT SOLVE THE DIFFERENTIAL EQUATION!!

   We start with a full 10,000 gallon vat containing a solution of 3% acid. There is a pipe bringing in a solution of 5% acid at a rate of 10 gallons per minute, and another pipe removes the solution at a rate of 15 gallons per minute. We are interested in finding a function that describes the **total amount of acid in the vat at time** t.

4. Find a solution to each initial value problem.

(a)

$$\frac{dy}{dx} + y \tan x = \cos x \text{ and } y(0) = 2.$$

Hint: you may want to recall that $\int \tan x \; dx = -\ln|\cos x| + C$.

Solution Satisfying Initial Condition: $y =$ ____________________

(b)

$$\frac{dy}{dx} = 6x^5 + y^2 6x^5 \text{ and } y(0) = 0.$$

Solution Satisfying Initial Condition: $y =$ ____________________

5. Let t stand for time in minutes from 12:00pm and let $N(t)$ denote the number of gallons of salt in a vat at time t. Assume that N satisfies $\frac{dN}{dt} = 50 \cdot (1 - N^2)$. Also assume that at 12:00pm there were 3 gallons of salt in the vat. Compute $N(t)$.

6. Let $f(x) = e^{-x}$. Find n such that $|f(x) - T_n f(x)| \le \frac{1}{100}$ for x in the range $-\frac{1}{2} \le x \le \frac{1}{2}$. It may be helpful to know that $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, and $\sqrt{e} \le 2$.

7. Let $f(x)$ be a function satisfying the differential equation

$$xf''(x) + x\cos(x^2) + f(x) = 0$$

and also satisfying the initial conditions $f(0) = 0$ and $f'(0) = 11$. Compute $T_4 f(x)$.

Note: it is essential that you use notation correctly in your answer, as part of what we are testing is whether you understand what the notation means.

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