Worksheet 8

Fall 2016

MATH 221

Name: _____

1

Find $\frac{dy}{dx}$ for the following implicit functions.

- $x^2 - y^2 = x$
- $x^2 + xy - y^2 = 4$
- $y^5 + 3x^2y^2 + 5x^2 = 12$

2

Use implicit differentiation to find

$$\frac{d}{dx}(\arctan(x))$$

[Hint: $y = \arctan x \Rightarrow \tan y = x$]

3

Air is being pumped into a spherical balloon at the rate of 7 cubic centimeters per second. What is the rate of change of the radius at the instant the volume equals 36π?

4

A conical paper cup 3 inches across at the top and 4 inches deep is full of water. The cup springs a leak at the bottom and loses water at a rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water level is exactly 3 inches deep? [Hint:$V = \frac{\pi r^2 h}{3}$. And use similar triangles. Once.]

5

A kite 100 feet above the ground is being blown away from the person holding its string in a direction parallel to the ground at the rate of 10 feet per second. At what rate must the string be let out when the length of the string is 200 feet?

6

An square alien blob monster (OK it's 3-D but its height is fixed)lands in the middle of a field and starts growing in area at a constant rate of 40 square feet per second, consuming everything it touches.

How fast do you need to be running to avoid consumption if you are right on the edge of the monster when its area is 2500 square feet?

7

A pitcher throws a pitch at 80 feet per second, directly towards home plate. How fast is the ball moving away from the first baseman when it crosses home plate? (You need a little knowledge of baseball to do this problem, but not much. Ask around if you don't know it!).

8

Sketch a rough graph of

$$f(x) = \frac{1}{x^2 - 9}$$

That is, figure out where f is positive and negative, increasing and decreasing, its concavity if we have covered this yet, and its asymptotics (behavior at ∞ and around any points where it blows up). Once you have this information, you can sketch the graph!