Quiz 7

Fall 2016

MATH 221

Name: ________________________________

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Find all the critical points of the following function:

$$f(x) = \frac{x+6}{x^2-4}$$

Solution 1.

Notice that $f(x)$ is not defined at $x = \pm 2$, it has vertical asymptotes here so these points cannot be critical points. This is an important distinction. A point can only be critical if the function is defined there. We will see $f'(x) = 0$ as ± 2 as well, but these are not critical points for this reason.

To find the critical points we take the derivative and set it equal to zero or find where it doesn't exist:

$$f'(x) = -\frac{x^2+12x+4}{(x^2-4)^2}$$

We see $f'(x)$ is not defined at $x = \pm 2$ but by the above these are not critical points, they are vertical asymptotes! The only other option is $f'(x) = 0$. This will occur when the numerator equals zero. So we need to solve $x^2 + 12x + 4 = 0$. This requires the quadratic formula (you want to know this!).

$$x = \frac{-12 \pm \sqrt{144-16}}{2}$$

So these are our only critical points.

$\square$

Problem 2. Compute the following limit. Show all your work (write lim at each step, equals signs etc.):

$$\lim_{\theta \to 0} \frac{\tan(x)}{x}$$

Solution 2.

This is like asking what's $\lim_{x \to 0} 3$ because the variable in the function has nothing to do with the variable we are taking the limit of, so it just stays the same. So

$$\lim_{\theta \to 0} \frac{\tan(x)}{x} = \frac{\tan(x)}{x}$$

This is an important distinction!! Now if we took the derivative

$$\lim_{x\to 0} \frac{\tan(x)}{x}$$

You should notice $\tan(x) = \sin(x)/\cos(x)$ and so

$$\lim_{x\to 0} \frac{\tan(x)}{x} = \lim_{x\to 0} \frac{\sin(x)}{x} \cdot \frac{1}{\cos(x)} = 1$$

$\square$