Worksheet 2

Spring 2016

MATH 222, Week 2: I.6,I.8,I.10

Name: SOLUTIONS

Problem 1. Compute $\int x \ln(x)\, dx$.

Solution 1.

We integrate by parts. Let $u = \ln(x)$ and $v' = x$. Then $u' = 1/xdx$ and $v = x^2/2$. We then know:

$$\int x \ln(x) dx = uv - \int vu' = \ln(x)(x^2/2) - \int \frac{x}{2}\, dx = \frac{x^2 \ln(x)}{2} - x^2/4 + C$$

$\square$

Problem 2. (a) Compute $\int_0^\pi \cos(x)dx$ and $\int_0^\pi x^2 \cos(x)dx$

(b) Show that:

$$\int x^n \cos(x)\, dx = x^n \sin(x) + nx^{n-1} \cos(x) - n(n-1) \int x^{n-2} \cos(x)\, dx$$

(Hint: The steps are very similar to what you did in part (a) for $\int_0^\pi x^2 \cos(x)\, dx$).

(c) Use the identity you just proved and part (a) to compute $\int_0^\pi x^4 \cos(x)\, dx$.

Solution 2.

(a) $\int_0^\pi \cos(x) = 0$ and $\int_0^\pi x^2 \cos(x)dx = -2\pi$.

(b) Apply integration by parts twice with $u = x^n$ $v' = cos(x)$ in the first step and $u = x^{n-1}$ and $v' = sin(x)$ in the second step.

(c) $\int_0^4 x^2 \cos(x)dx = -4\pi(\pi^2 - 6)$

$\square$

Problem 3. Compute $\int x^7 \sin(2x^4)\, dx$.

Solution 3.

This is tricky and should serve as a reminder that the split for u and v' won't always be obvious. Intuitively you want to set $v' = \sin(2x^4)$, but this is nearly impossible to antidifferentiate. The issue is that when you take the derivative of $-\cos(2x^4)$ you pick up an extra $8x^3$ thanks to the chain rule. You can fix this though! Choose $v' = x^3 \sin(2x^4)$ and $u = x^4$. You'll only need one round of integration by parts here (not seven)! $u' = 4x^3$ and $v = -\cos(2x^4)/8$, so we know:

$$\int x^7 \sin(2x^4)\, dx = uv - \int vu' dx = -x^4 \cos(2x^4)/8 + \int x^3 \cos(2x^4)/2\, dx$$

You can compute the integral by u-sub or just straight up guess and check to find:

$$\int x^7 \sin(2x^4)\ dx = \frac{1}{16}(\sin(2x^4) - 2x^4\cos(2x^4)) + C$$

$\square$

Problem 4. Compute $\int \frac{1}{x^2-4}\ dx$

Solution 4.

Using partial fractions we can write $\frac{1}{x^2-4} = \frac{1/4}{x-2} - \frac{1/4}{x+2}$ we can then integrate:

$$\int \frac{x^3}{x^2+2}\ dx = \frac{1}{4}(\ln(2-x) - \ln(2+x)) + C$$

$\square$

Problem 5. Compute $\int \frac{x^3}{x^2+2}\ dx$

Solution 5.

Use partial fractions again to write $\frac{x^3}{x^2+2} = \frac{-2x}{x^2+2} + x$. We can then integrate:

$$\int \frac{x^3}{x^2+2}\ dx = \frac{x^2}{2} - \ln(x^2+2) + C$$

$\square$

Problem 6. Compute $\int \frac{1}{2+e^{2t}}\ dt$

Solution 6.

We use partial fractions again, writing $\frac{1}{2+e^{2t}} = 1/2 - \frac{1/2e^{2t}}{2+e^{2t}}$ we can then integrate:

$$\int \frac{1}{2+e^{2t}}\ dt = \frac{t}{2} - \frac{1}{4}\ln(e^{2t}+2) + C$$

$\square$