Worksheet 1

Spring 2016

MATH 222, Week 1: I.1,I.3,I.5

Name: Bobby Laudane (~~A~~ Answer Key)

Problem 1. Use the identity $sin^2(\theta) + \cos^2(\theta) = 1$ to show that $\tan^2(\theta) + 1 = \sec^2(\theta)$.

$$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \tan^2\theta + 1 = \sec^2\theta$$

Problem 2. (a) Circle the correct answer:

$2\sin(\theta)\cos(\theta) =$	$\sin(2\theta)$	$\cos(2\theta)$
$\cos^2(\theta) - \sin^2(\theta) =$	$\sin(2\theta)$	$\cos(2\theta)$

(b) Using the previous part and other trig identities, prove the following half angle formulas:

(a) $cos^2(\theta) = \frac{1}{2}(cos(2\theta) + 1)$. There's a very similar identity for $\sin^2(\theta)$ that could be useful later on.

$\cos^2\theta + \sin^2\theta = 1$ but $-\sin^2\theta = \cos 2\theta - \cos^2\theta$

$$\Rightarrow \cos^2\theta = 1 + \cos 2\theta - \cos^2\theta$$

$$\Rightarrow \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

(b) $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

~~Divide~~ Multiply by $\frac{\cos^2\theta}{\cos^2\theta}$:

$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

Problem 3. True of False. In either case, briefly explain why.

(a) $\frac{d}{dx}(\ln(x^2)) = \frac{2}{x^2}$ False $= \frac{2}{x}$

(b) $\frac{d}{dz}\int_0^z \frac{dy}{4-y^2} = \frac{1}{4-z^2}$ True Fund. Thm. Calc.

(c) $\sqrt{x^4+36} = x^2+6$ False

(d) $\int e^x dx = e^x$ False $= e^x + C$ indefinite integral

(e) $\int \ln(x) dx = \frac{1}{x} + C$ False see prob 4

Problem 4. Compute $\int \ln(x)\ dx$ (Slight hint for part (e) of the last problem).

$u = \ln(x)$ $\quad dv = dx$ $\quad = x\ln(x) - \int dx$

$du = \frac{dx}{x}$ $\quad v = x$

$$\boxed{= x\ln(x) - x + C}$$

Problem 5. Compute $\int \arcsin(3x)\ dx$.

$u = \sin^{-1}(3x)$ $\quad dv = dx$ $\quad = x\sin^{-1}(3x) - \int \frac{3x}{\sqrt{1-9x^2}}\ dx$

$du = \frac{3}{\sqrt{1-9x^2}}\ dx$ $\quad v = x$

Use u-sub:

$$\boxed{= x\sin^{-1}(3x) + \frac{1}{3}\sqrt{1-9x^2}}$$

Problem 6. Let a be any fixed real constant. Compute $\frac{d}{dx}\int_{x^3}^a \ln(t)\ dt$. (Hint: Fundamental Theorem of Calc).

$$= \boxed{-\ln(x^3)\cdot 3x^2}$$

Problem 7. Compute $\int \sin^2(\theta)\cos^2(\theta)\ d\theta$. There are at least two ways to approach this.

$$= \int (\sin\theta\cos\theta)^2 d\theta = \int \left(\tfrac{1}{2}\sin(2\theta)\right)^2 d\theta$$

$$= \frac{1}{4}\int \sin^2 2\theta\ d\theta = \frac{1}{4}\int \frac{1}{2}\left(1 - \cos(4\theta)\right) d\theta$$

$$= \frac{1}{8}\left[\theta - \sin(4\theta)\cdot\frac{1}{4}\right] + C = \boxed{\frac{1}{8}\theta - \frac{1}{32}\sin(4\theta)}$$