Worksheet 3

Fall 2016

MATH 221, Week 3

Name: _____

1 Absolute Value Refresher

Find all the solutions to the following equations:

$$|2x - 1| = 5$$

$$|x^2 - 4x - 5| < 7$$

$$|2x - 1| = |4x + 9|$$

(Hint: For the third problem in particular, think about what the absolute value sign does and how you could think about the function in different pieces. It may help to graph these two lines to get a geometric intuition.)

2 Rigorous Limits

Use the $\epsilon - \delta$ definition to prove the following limits:

(a) $\lim_{x\to 3} x = 3$

(b) $\lim_{x\to 1} 2x - 4 = 6$

(c) $\lim_{x\to 2} x^2 = 4$

(d) $\lim_{x\to 2} x^2 - 4 = 0$

(e) $\lim_{x\to 1} \frac{2-x}{4-x} = \frac{1}{3}$

(f) $\lim_{x\to 3} \sqrt{x+6} = 9$

3 Different Limits

Compute the following limits (please show work):

(a) $\lim_{x \nearrow 1} \frac{x^2+2x-3}{x^2-1}$

(b) $\lim_{x \to 1} \frac{x^2+2x-3}{x^2-1}$

(c) $\lim_{x \to \infty} \frac{1}{x}$

(d) $\lim_{x \to \infty} \frac{x^2+3}{x^2+4}$

(e) $\lim_{x \to -\infty} \frac{x^3+2x^2-2}{x-2x^2}$

(f) $\lim_{x \to \infty} \frac{16x^{29}}{2534x^{12} \times 33245x^7+14x^3}$

4 To Think About

Use the limit you computed in 3(c) to show that $\lim_{x \to \infty} x$ does not exist. (Hint: Suppose $\lim_{x \to \infty} x = L$ does exist. Then apply the limit properties to $\lim_{x \to \infty} x \cdot \frac{1}{x}$).

True or false (if true, provide an example; if false, provide a reason why)

(a) If $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} g(x)$ does not exist, then $\lim_{x \to a} f(x) + g(x)$ could still exist.

(b) If $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} g(x)$ does not exist, then $\lim_{x \to a} f(x)g(x)$ could still exist.

(c) If $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} g(x)$ does not exist, then $\lim_{x \to a} f(x)/g(x)$ could still exist.

(d) If $\lim_{x \to a} f(x)$ does not exist and $\lim_{x \to a} g(x)$ does exist, then $\lim_{x \to a} f(x)/g(x)$ could still exist.