Practice Final a

MATH 222 (Lectures 1,2, and 4) Fall 2015.

Name: ______________________________ Student ID#: ________________________

Circle your TAs name:

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	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
Score					
	Problem 6	Problem 7	Problem 8	Problem 9	Problem 10
Score					

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- Please read the instructions on every problem carefully.
- On Problems 1–4 only the answer will be graded.
- On Problems 5–10 you must show your work and we will grade the work and your justification, and not just the final answer.
- Each problem is worth ten points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1-x^2}$. Note that we have provided some formulas on the next page to help with this.

Formulas

You may freely quote any algebraic or trigonometric identity, as well as well as any of the following formulas or minor variants of those formulas.

Integrals

- $\cos(\arcsin x) = \sqrt{1-x^2}$
- $\sec(\arctan x) = \sqrt{1+x^2}$.
- $\tan(\operatorname{arcsec} x) = \sqrt{x^2-1}$.
- $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & \text{when } n \neq -1 \\ \ln|x| + C & \text{when } n = -1 \end{cases}$
- $\int e^x dx = e^x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \tan x dx = -\ln|\cos x| + C$
- $\int \cot x dx = \ln|\sin x| + C$
- $\int \sec x dx = \ln|\sec x + \tan x| + C$.
- $\int \csc x dx = -\ln|\csc x + \cot x| + C$.
- $\int \frac{1}{1+x^2} dx = \arctan(x) + C$.

Taylor series

- $T_\infty e^x = \sum_{k=0}^\infty \frac{x^k}{k!}$
- $T_\infty \sin x = \sum_{k=0}^\infty (-1)^k \frac{x^{2k+1}}{(2k+1)!}$
- $T_\infty \cos x = \sum_{k=0}^\infty (-1)^k \frac{x^{2k}}{(2k)!}$
- $T_\infty \frac{1}{1-x} = \sum_{k=0}^\infty x^k$
- $T_\infty \frac{1}{1+x} = \sum_{k=0}^\infty (-1)^k x^k$
- $T_\infty (1+x)^b = \sum_{k=0}^\infty \binom{b}{k} x^k$ where $\binom{b}{k} = \frac{b(b-1)(b-2)\cdots(b-k+1)}{k!}$

Other

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$.

1. For each statement below, CIRCLE true or false. You do not need to show your work.

(a)		(b)		(c)		(d)		(e)	
True	False	True	False	True	False	True	False	True	False

(a) The integral $\int_1^\infty \frac{1}{\sqrt{x-3}} dx$ is finite.

(b) $\int_2^\infty \frac{1}{x^3+1} dx \leq \int_2^\infty \frac{1}{x^2+5} dx$.

(c) The series $\sum_{k=0}^\infty \frac{k^2+e^{-k}}{2^k+\sqrt{k}}$ converges.

(d) The series $\sum_{k=0}^\infty \frac{k!}{k^k}$ is finite.

(e) $\sin(x^2) - x^2$ is $o(x^4)$.

2. On this page only the answer will be graded.

   (a) Compute $\int \frac{1}{x^2(x+1)} dx$.

   (b) Compute $\int x^{2013} \ln(x) dx$.

3. On this page only the answer will be graded.

(a) Compute $T_4(\frac{e^{x^4}}{\sqrt{1+x^3}})$

(b) For which values of b does the Taylor series for $\frac{1}{3+2x^2}$ converge at $x = b$?

4. On this page, only the answer will be graded.

   (a) Let P be the plane through the points $(1,1,1),(3,2,0)$ and $(0,4,0)$. Compute the normal vector to P.

   (b) Let $\vec{a}=(1,2)$. Compute $\vec{v}^{\perp}$ with respect to $\vec{a}$ where $\vec{v}=(3,4)$.

   (c) At what point (x,y) does the line $\ell=(1+2t,3+4t)$ intersect the line $y=3x+2$?

5. On this page, you must show your work to receive full credit.

   (a) Let $f(x) = x^3 \sin(x^2)$. Compute $f^{(409)}(0)$.

   (b) Let t denote time in years since January 1, 2010 and let $S(t)$ denote the number of squirrels in Madison at time t. This squirrel population has a continuous birth rate of 8% and a natural continuous death rate of 2%. In addition, each year 250 squirrels are eaten by foxes and 150 squirrels are run over by cars. Write down a different equation for $S(t)$. DO NOT SOLVE THE DIFFERENTIAL EQUATION.

6. On this page, you must show your work to receive full credit.

(a)

$$\frac{dy}{dx} - \frac{2x}{1+x^2}y = 1 + x^2 \qquad \textit{and} \qquad y(0) = 7.$$

(b)

$$\frac{dy}{dx} = \frac{e^x}{2y+1} \text{ and } y(0) = 1$$

7. On this page, you must show your work to receive full credit.

$$\text{Compute } \int \sqrt{2x - x^2}\,dx.$$

For your final answer, you should simplify any expression that combines a trigonometric and inverse trigonometric function (e.g. $\cos(\arcsin x) = \sqrt{1 - x^2}$).

8. On this page, you must show your work and justify your answer to receive full credit. Let $a > 0$ be a real number, and let $I_n = \int x^n e^{ax} dx$. Derive a reduction formula for I_n.

9. On this page, you must show your work and justify your answer to receive full credit. Let $f(x) = e^{-2x} + x^5$.

$$\text{Find } B \text{ so that } |R_2 f(x)| \leq B \text{ for all } -2 \leq x \leq 2.$$

If using the Error Bound Formula, you must clearly indicate your chosen value for M and explain why this choice of M is valid for the desired range.

10. On this page, you must show your work and justify your answer to receive full credit. You must clearly state any convergence test that you use and explicitly verify each hypothesis for that test.

For which values of b does the series $\sum_{k=1}^{\infty} \frac{e^{kb}+\sqrt{k}}{k!}$ converge?

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