Quiz 4 Solutions

Fall 2018

MATH 222-001

Name: ______________________________

Problem 1. (5 Points) Evaluate $\int \frac{x^2+1}{x^2-3x+2}$.

Solution 1.

We have to notice that this isn't a proper rational function so we have to do long division and divide x^2+1 by x^2-3x+2. We then find:

$$\frac{x^2+1}{x^2-3x+2} = 1 + \frac{3x-1}{(x-1)(x-2)}$$

Now we can do partial fractions:

$$\frac{3x-1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

Multiplying to get a common denominator and equating the numerators gives us $A(x-2)+B(x-1) = 3x-1$. If we let $x=2$ we find that $B=5$ and letting $x=1$ we find that $A=-2$. So we now have:

$$\int 1 + \frac{-2}{x-1} + \frac{5}{x-2} dx$$

Now we can integrate:

$$\int \frac{x^2+1}{x^2-3x+2}\, dx = x - 2\ln|x-1| + 5\ln|x-2| + C$$

$\square$

Problem 2. (5 Points) Compute $\int_3^\infty \frac{dx}{x^2-4}$. You may use the fact that $\frac{1}{x^2-4} = \frac{1/4}{x-2} - \frac{1/4}{x+2}$.

Solution 2.

If we use the given fact, before we can integrate we must set up the limit

$$\lim_{t\to\infty}\int_3^t \frac{1/4}{x-2} - \frac{1/4}{x+2} = \lim_{t\to\infty} 1/4\ln|x-2| - 1/4\ln|x+2|\Big|_3^t = \lim_{t\to\infty} 1/4\ln|t-2| - 1/4\ln|t+2| - (1/4\ln|3-2| - 1/4\ln|3+2|)$$

If we tried to take the first limit, we would get something indeterminate because there are two terms going to ∞. $\infty - \infty$ is nonsensical, it could be anything and so it is wrong to say $\infty - \infty = 0$. If you see multiple infinities appearing, it means you have to simplify your expression further. In this case we must combine logs

$$\lim_{t\to\infty} 1/4\ln\left|\frac{t-2}{t+2}\right| - (1/4\ln|3-2| - 1/4\ln|3+2|)$$

Now when we take the limit, it is also wrong to say $\frac{\infty}{\infty} = 1$ because this is not true. You have to consider the function as a whole. In particular if you factor out a t to find

$$\lim_{t\to\infty} 1/4\ln\left|\frac{t-2}{t+2}\right| = \lim_{t\to\infty} 1/4\ln\left|\frac{t}{t}\cdot\frac{1-2/t}{1+2/t}\right| = \ln|1| = 0.$$

So in this case, the limit is 0. Now that we have evaluated the limit, this tells our answer is

$$-1/4\ln(5).$$

. $\square$