Worksheet 6

Problem 1:

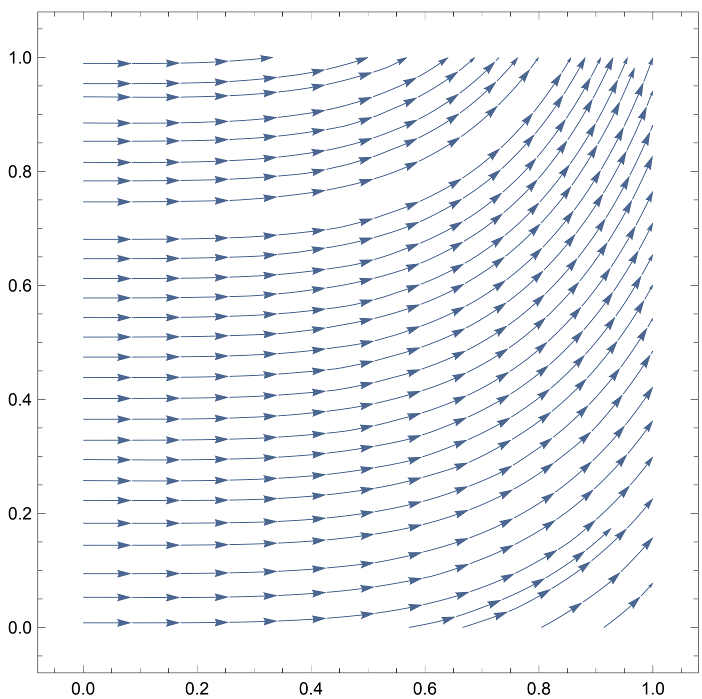
Find a solution to the initial value problem:
$\frac{dy}{dx} = e^y x^3$, $y(0) = 0$

We're first going to get a feel for what the funciton should look like. What we're given tells us in what direction and how much our function changes in the y direction depending on how much time has transpired (think of the x value as time). In other words, we have a tangent vector at each point. If we plot these tangent vectors they should roughly sketch out the family of curves that satisfy the differential equation. The first picture here will connect the vectors for us, the second will just plot the tangent vectors at each point.

In[26]:=
```
g[x_, y_] := E^y * x^3
```

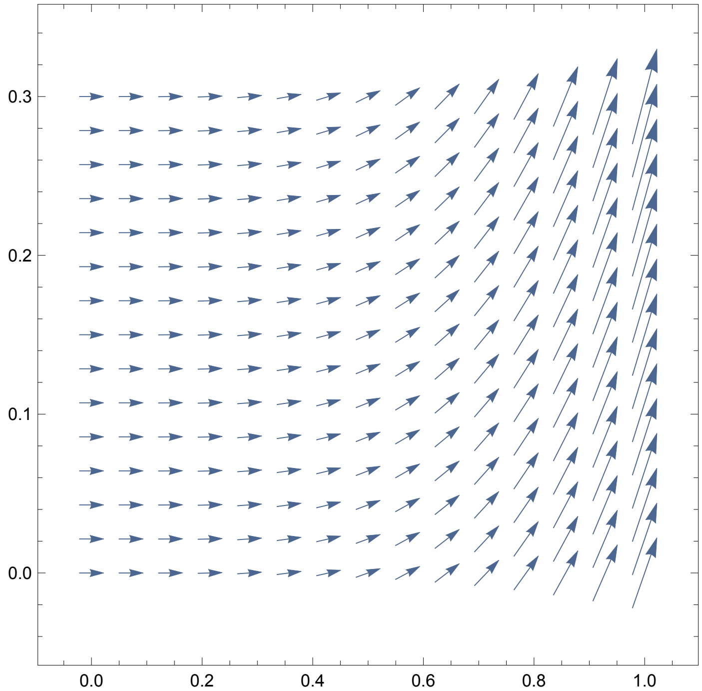
In[32]:=
```
StreamPlot[{1, g[x, y]}, {x, 0, 1}, {y, 0, 1}]
```

Out[32]=



In[33]:=
```
VectorPlot[{1, g[x, y]}, {x, 0, 1}, {y, 0, .3}]
```

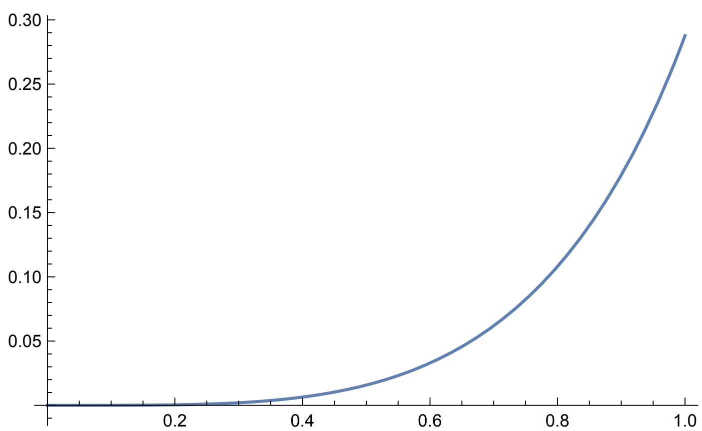
Out[33]=



Now, when we solve the differential equation (as seen on the solutions to the worksheet) we get $\mathrm{Ln}\left(-\frac{(x^{\wedge}4)}{4}+1\right)$. If we plot this it should match up with the vector stream we saw above that began at the point (0,0). And we see it does. My hope is that this gives a little intuition behind what we're doing with differential equations.

In[39]:=
```
Plot[-Log[-x^4/4+1], {x, 0, 1}]
```

Out[39]=

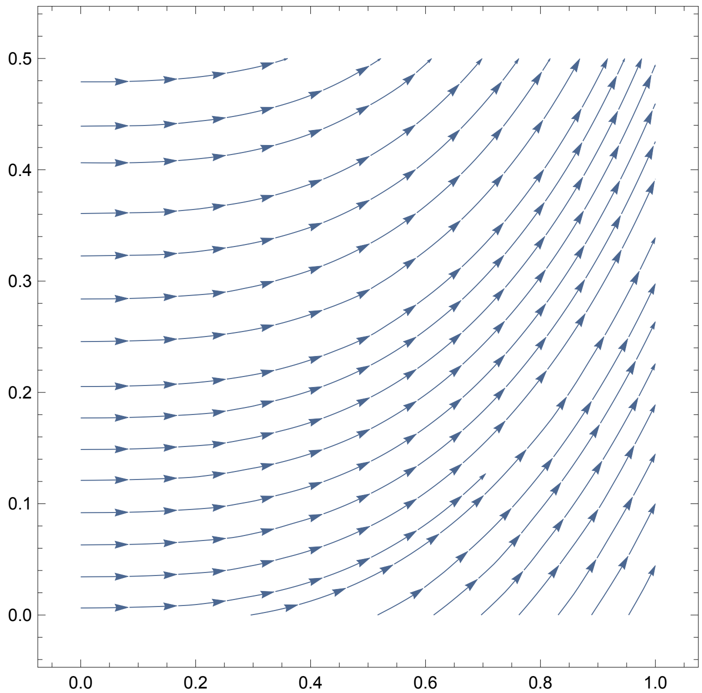


Problem 3:

In[4]:=
```
f[x_, y_] := x^2 + y^2 * x^2
```

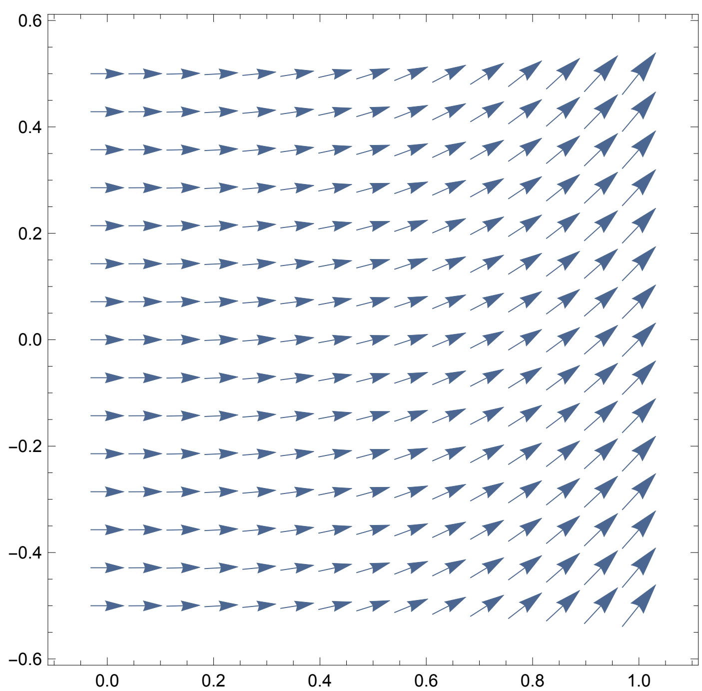
In[25]:=
```
StreamPlot[{1, f[x, y]}, {x, 0, 1}, {y, 0, .5}]
```

Out[25]=



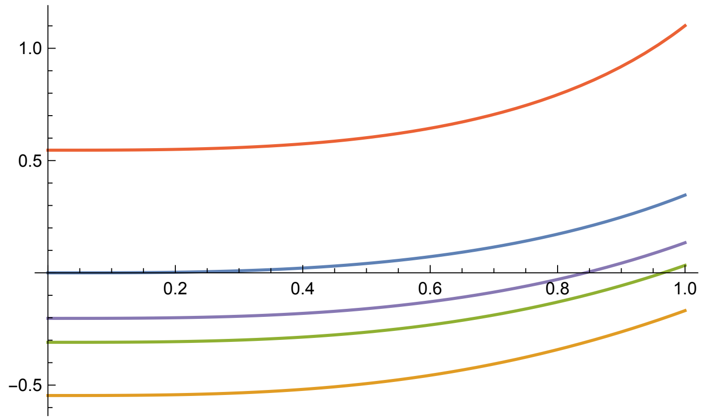
In[43]:=
```
VectorPlot[{1, f[x, y]}, {x, 0, 1}, {y, -.5, .5}]
```

Out[43]=



In[44]:=
```
Plot[{Tan[x^3/3], Tan[x^3/3 - .5],
  Tan[x^3/3 - .3], Tan[x^3/3 + .5], Tan[x^3/3 - .2]}, {x, 0, 1}]
```

Out[44]=



When we solve the differential equation we get a family of curves that give us most of the solutions to the differential equation depending on the different initial values we choose. I've plotted a couple that align with different initial values above. You can see how they correspond to the vector plots above. This stuff is really cool and has incredible applications. If you're interested feel free to look into it a little more or ask any of the TAs!