Worksheet 4

Fall 2016

MATH 221

Name: _____

1

Find and justify the following limits

(a) $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5}$

(b) $\lim_{x\to 2} \frac{\sqrt[3]{x+6}-2}{x-2}$

(c) $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

(d) $\lim_{x\to\infty} \frac{\sin(x)}{x}$

(e) $\lim_{x\to\infty} \frac{x^2+bx+c}{x^3+2}$

(f) $\lim_{x\to-\infty} \frac{-3x^2+2x+7}{x^2+1}$

(g) $\lim_{x\to-\infty} \frac{x^3+6}{-x+3}$

(h) $\lim_{x\to\infty} \frac{1}{x^2+1}$

(i)

$$\lim_{x\to\infty} \sqrt{x^2+9x+1}-\sqrt{x^2+x+2}$$

(j)

$$\lim_{x\to\infty} \sqrt{x^2+4x+1}-\sqrt{2x^2+6x}$$

2

Find numbers a and b so that the following function is continuous

$$f(x) = \begin{cases} \sin(ax)+b & : x \le 0 \\ bx^2+a & : 0 < x < 1 \\ 2 & : x \ge 1 \end{cases}$$

3

Find the limit

$$\lim_{x \to a} \frac{\sqrt[3]{x} - \sqrt[3]{a}}{x - a}$$

4 To think about

- Let $A(x)$ be the area of a square with side x, and let $L(x)$ be the perimeter of the square. Show that $A'(x) = \frac{1}{2}L(x)$. That is, show the instantaneous rate of change of the area of a square admits this relationship with the perimeter, as you change the side length. Can you geometrically reason why this is true?

- Consider the following problem–determine the rate of change in total area under a smooth, positive function. How would you describe the instantaneous rate of change of this area? What could you relate it to on a graph of the function?