Name: _____

Solve the following problems. Some of these are based on past/practice midterm questions. The problems are purposefully mixed, as on an exam, so you must also recognize which method you should be using.

1.   (a) True or false? $\int \dfrac{1}{1+x^2}\,dx = \ln|1+x^2| + C.$

(b) True or false? $\dfrac{1}{x^2} \leq \dfrac{1 + \sin x \cos x}{x^2}$ for $x \geq 1$.

(c) True or false? $\dfrac{1}{x^2} \leq \dfrac{1 + \sin x \cos x}{x^2}$ for $0 \leq x \leq \frac{\pi}{2}$.

(d) True or false? $x^x \geq 1$ for $0 < x < 1$.

(e) If $x = \sqrt{2}\sec\theta$, what is $\tan\theta$ equal to in terms of x?

(f) Given the reduction formula

$$\int \cos^n x\,dx = \frac{1}{n}\sin x \cos^{n-1} x + \frac{n-1}{n}\int \cos^{n-2} x\,dx,$$

write down a reduction formula for $\displaystyle\int_0^{\frac{\pi}{4}} \cos^n x\,dx.$

2. Compute $\int x \ln(x)dx$.

3. Compute $\int \frac{1}{y\sqrt{1-y^2}}dy$.

4. Compute $\int \frac{dx}{\sqrt{1-e^{2x}}}$.

5. Compute $\int_0^1 \ln(2t+1)dt$.

6. Compute $\int (\cos(x) + \sin(x))^2\, dx$.

7. Determine whether
$$\int_3^\infty \frac{4-x}{2x^2 + 2x - 4}\, dx$$
coverges or diverges. If it converges, what does it converge to?

8. Show that $\int_1^\infty \frac{dx}{x^2-4}$ is not a finite number. What answer do you get if you forget that the integrand has an asymptote at 2 and fail to split the integral up there?

9. Compute $\int \frac{p+2}{p^2 - 1}dp$

10. Compute $\int e^x \sin(x)dx$.

11. Compute $\int \sec^2(x)\tan(x)dx$.

12. Find a solution to the initial value problem
$$\frac{dy}{dx} = y\sqrt{y^2-1}\cos(x)$$
$$y(0) = 1$$

13. Determine whether
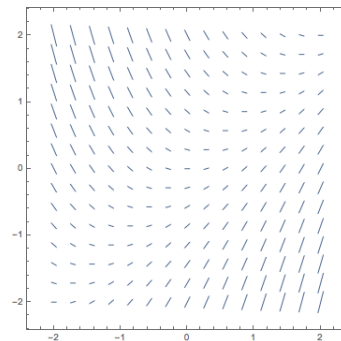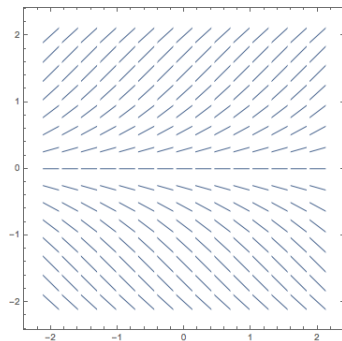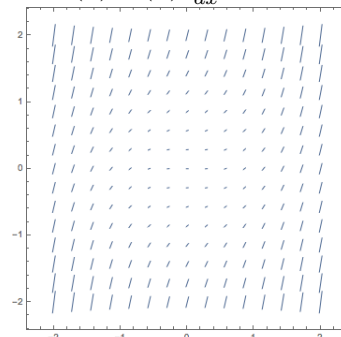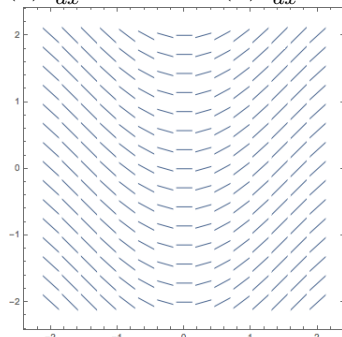$$\int_1^\infty \frac{1 - e^{-x}}{x}\, dx$$
converges or diverges.

14. Compute $\int \dfrac{x^3}{x^2 + 2} dx$.

15. Compute $\int \cos^4(x) dx$.

16. Identify which of the following differential equations are associated to each of the following direction fields:

(a) $\frac{dy}{dx} = x - y$ (b) $\frac{dy}{dx} = \sin(x)$ (c) $\frac{dy}{dx} = \sin(y)$ (d) $\frac{dy}{dx} = x^2 + y^2$
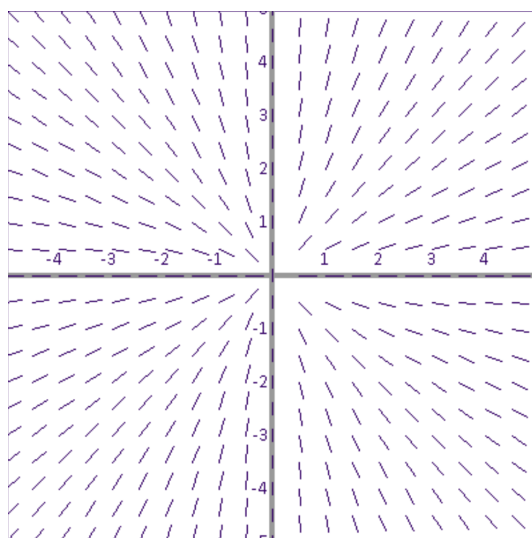

17. Circle the differential equation that corresponds to the slope field shown below.

$$y' = y/x \qquad\qquad y' = \sin(x) \qquad\qquad y' = x + y \qquad\qquad y' = -x/y$$
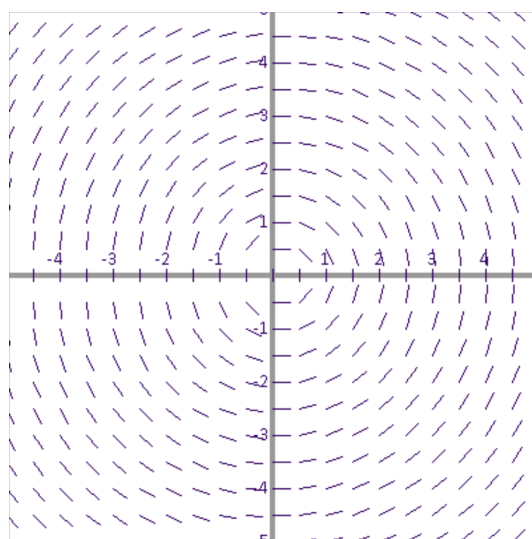
18. Circle the differential equation that corresponds to the slope field shown below.

$$y' = y/x \qquad\qquad y' = \sin(x) \qquad\qquad y' = x + y \qquad\qquad y' = -x/y$$

19. Consider a continuous function $f(x)$:

$$f(x) = \begin{cases} 2x^{3/2} & \text{if } 0 \le x \le 4 \\ 4x & \text{if } 4 \le x < \infty \end{cases}$$

For what p values is the integral $\displaystyle\int_0^\infty x^p f(x)\,dx$ convergent? For what p values is the integral divergent?