Quiz 2

Fall 2016

MATH 221

Name: _____

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Let $f(x) = 5x - 6$.

(a) Find $L = \lim_{x\to 1} f(x)$ without proving it.

$L = -1$

(b) Find a number $\delta > 0$ such that for all x with $0 < |x - 1| < \delta$ we have $|f(x) - L| < 1$.

We want to find $\delta > 0$ such that $|5x - 6 - (-1)| < 1$. That is when $|5x - 5| < 1$. Notice that

$$|5x - 5| = 5|x - 1|$$

So

$$|5x - 5| < 1 \iff 5|x - 1| < 1 \iff |x - 1| < 1/5$$

So we can take $\delta = 1/5$.

Problem 2. Let

$$f(x) = \begin{cases} 2x & x \geq 0 \\ x^2 - 2 & x < 0 \end{cases}$$

Find $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0^-} f(x)$. Does $\lim_{x\to 0} f(x)$ exist, why or why not?

The left limit $x \to 0^-$ will be given by $\lim_{x\to 0} x^2 - 2 = -2$ as this is how our function behaves for negative x values. Similarly $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 2x = 0$. The left and right hand limits are not equal, so $\lim_{x\to 0} f(x)$ does not exist!