Worksheet 11

Fall 2016

MATH 221, Week 11

Name: ________________________________

1 Optimization!

Solve the following optimization problems on a separate sheet of paper:

(a) You are making a square-bottomed box with no top and want to maximize the total volume that it can hold while using no more than 600 square inches of material. What's the biggest box you can make?

(b) Now you are making a square-bottomed box with no top and want to minimize the total material used while making a box that can hold 2000 cubic inches. What's the most efficient box you can make?

(c) A zoo has decided, for some reason, to place the enclosure for sheep between the lion and tiger exhibits. Assume that all three enclosures are rectangles of the same size and shape, lined up in a row. In order to make sure that the large cats don't break into the sheep pen and eat them, thereby traumatizing countless children, the fences between the cats and the sheep must be taller and stronger. (We don't really care if they can get out and eat visitors). Assuming that normal fencing costs \$50 per meter, the reinforcing fencing costs \$200 per meter, and we have a total budget of $\$500,000$, how big can we make the enclosures?

(d) I have 3 meters of wire. I take some of it and bend it into a circle, and I bend the rest of it into a square. What is the possible range of values for the sum of the areas of the two figures I've made? How should I allocate wire to maximize and minimize the total area? What does this say about circles and squares?

2 Limits!

Why might you have some issues applying L'Hopital's rule to the following limit:

$$\lim_{x\to 0} \frac{\frac{1}{2}x + x^2 \sin(\frac{\pi}{x})}{x}$$

Evaluate the following limits using any methods you know:

(a) $\lim_{x\to 0} \frac{\sin(3x^2)}{x^2}$

(b) $\lim_{x\to 1} \frac{5t^4-4t^2-1}{10-t-9t^3}$

(c) $\lim_{x\to 0} \frac{e^x}{x^n}$ where n is some positive integer. What does this say about e^x?

(d) $\lim_{x\to\infty} \frac{4x^2-5x}{1-3x^2}$

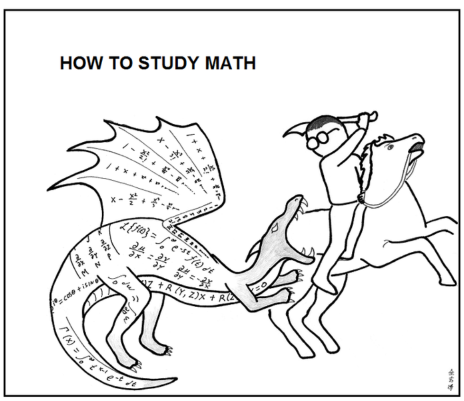
3 To Think About

You are with some friends fishing (or doing something else) in one of Wisconsin's rivers. The river is swollen from recent snow melts. You are 100 meters upstream of a deadly waterfall. Your idiot friend Tim decides to have a little fun and push you in the river. Unknown to him, there is a rip current that immediately drags you to out to the middle. You are now 10 meters off the bank, and traveling downstream towards the waterfall. This worksheet helps you answer the question of what to do. And whether you're going to live or die. Thanks a lot Tim. At least Tim has shown some practical applications of what we've been learning!

Solve this on a separate sheet of paper using the following steps:

- Draw a picture encapsulating the situation. Label the directions of the various velocities involved, including your own swimming velocity (although the magnitudes and directions are as of yet unknown to us).
- Now the fun begins. Let's say the river is moving at 5 m/s, and you can swim at 2 m/s. Write down an equation for how far you travel in the river before you land on shore in terms of the direction in which you swim (assuming you can only swim in straight lines, and you don't get tired, but for now this is not a big deal). Hint: introduce a suitable angle. I think one in particular is the most natural.
- Optimize this equation for minimal distance traveled down the river. Notice you can't travel negative distance here, because you can't swim faster than the river is moving.
- In the previous iteration of the problem you could tell immediately whether you would live or die if you took the optimal route (why?). Now we're going to change the parameters around to make it less clear. We'll turn up the river and turn down your fitness. That is, let's say the river is rushing at 10 m/s, and you can only swim at 1 m/s. Can you survive the situation now? If so, how? In which direction do you swim?
- Now let's see how these change in general. Let's say the river is rushing at A m/s and you can swim at B m/s, $A, B > 0$. Can you figure out when you can survive and when you can't?
- What happens if you can swim against the current (that is, $B > A$)?

Good luck on your exam! I'm thinking massive dragons all over the exams...



Don't just read it; fight it!

— Paul R. Halmos