

Worksheet 9

Fall 2016

MATH 221, Week 9

Name: _____

1 Intermediate Value Theorem

State the intermediate value theorem and use it to prove the following functions have a zero in the given interval:

(a) $x^4 - 5x^3 + 3x^2 - 1$ on the interval $4 \leq x \leq 5$

(b) $2^x - 3^x + (x + 1)^2$ on the interval $2 \leq x \leq 3$ (2^x and 3^x are continuous)

Solution 1.

If you don't know the IVP this would be a good time to look it up again :).

For both of these parts notice that at one endpoint the function is negative, and at the other the function is positive. These are both continuous functions so the IVP implies that they must achieve every value in between the negative and positive value. So in particular they both equal zero at some point.

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2 Max and Min!

Find all the critical points of the following function on the interval $-3 \leq x \leq 4$ and determine which are global or local max/min or neither:

$$f(x) = \begin{cases} -x & x \leq 0 \\ x & 0 < x \leq 1 \\ 1 & 1 < x \leq 2 \\ x - 1 & 2 < x \leq 3 \\ -x + 5 & 3 < x \end{cases}$$

(Hint: It will be a very good idea to graph this).

Solution 2.

You could graph this or differentiate it piece by piece, but if you take the second option you have to be careful. In either case you should find that $x = 0$ is a local min, $x = -3$ is a global max, $x = 1$ is a local max, the interval $1 < x < 2$ are all local max and mins. The point $x = 2$ is a local min, $x = 3$ is a global max and $x = 4$ is a local min. The important part here is classifying the line!

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3 Graph Sketching

For the following functions do all of the following (you'll probably need to use another sheet of paper):

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|--|--|
| (a) Find all zeroes of the function | (d) Classify where it is concave up and concave down |
| (b) Find the first and second derivatives | (e) Compute all asymptotes |
| (c) Classify where it is increasing and decreasing | (f) Put all of this together and sketch the graph! |

(a) $f(x) = x^4 - 4x^3$

(b) $f(x) = \frac{1}{x^2-9}$

(c) $f(x) = x - 3x^{1/2}$

(d) $f(x) = \frac{4x}{x^2+1}$

(e) $f(x) = x\sqrt{4-x^2}$

Solution 3.

Check your answers with your graphing calculator or any online graphing software!

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