Worksheet 7

Fall 2016

MATH 221, Week 7

Name: ______________________________

1 A few more derivatives!

Differentiate the following using the definition of the derivative:

(a) $f(x) = \sqrt{x+3}$

(b) $f(x) = 4 - \sqrt{x+3}$

(c) $f(x) = \sqrt[3]{x}$ (this should look familiar)

(d) $f(x) = x^{2/3}$

Solution 1.

You can check your solutions using the derivative rules you know now. The point of this was to recall the definition of the derivative. $\square$

2 Solving for an Unknown

Solve for y' in the following equations:

(a) $x(x-y)^3 + 2x(x-y^2)(1-y') = 2x - 3y'$
(b) $1 + y'\cos(y) = y + x\sin(y^2)y'$

Solution 2.

(a) $y' = \frac{2x-2x^2-x^4+3x^3y+2xy^2-3x^2y^2+xy^3}{3-2x^2+2xy^2}$

(b) $y' = \frac{1-y}{\cos(y) - x\sin(y^2)}$

$\square$

3 Chain Rule!

Differentiate the following functions:

(a) $f(x) = (x+1)^2$

(b) $f(x) = \cos(\sin(x))$

(c) $f(x) = \tan(2x^2 - 4)$

(d) $f(x) = ((2x^2)^5 + 4)^3$

(e) $f(x) = \tan^3(\sqrt{\cot(7x)})$

(f) $f(x) = h(g(k(x)))$ Where h, g, k are all functions.

Solution 3.

(a) $f'(x) = 2(x+1)$

(b) $f'(x) = -\sin(\sin(x))\cos(x)$

(c) $f'(x) = \sec^2(2x^2 - 4)(4x)$

(d) $f'(x) = 3((2x^2)^5 + 4)^2 \cdot (5(2x^2)^4) \cdot 4x$

(e) $f'(x) = 3\tan^2(\sqrt{\cot(7x)}) \cdot \sec^2(\sqrt{\cot(7x)}) \cdot \frac{1}{2}(\cot(7x))^{-1/2} \cdot -\csc^2(7x) \cdot 7$

(f) $f'(x) = h'(g(k(x))) \cdot g'(k(x)) \cdot k'(x)$.

$\square$

4 Continuity/Differentiability of Functions

Find a family of numbers a and b so that the following function is continuous. So this answer will be like an equation that a and b have to satisfy.

$$f(x) = \begin{cases} ax + b & x \leq 0 \\ bx^2 + a & x > 0 \end{cases}$$

Are there any numbers a and b so that this function is continuous AND differentiable? If so, what are they? If not, why not?

Solution 4.

For the function to be continuous the left and right hand limits must exist and be equal to the functions value at every point. Away from 0 as both pieces are polynomials the piecewise function will be continuous. So we just have to check that the limits are equal at $x = 0$. So we need

$$f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

That is

$$a(0) + b = b(0) + a$$

So this function will be continuous as long as $b = a$. If we also want it to be differentiable we have to differentiate the pieces and do the same analysis. I'll leave this to you, if you have questions feel free to email me. You should get

$$a = 0$$

So if we want this function to be continuous and differentiable $a = b = 0$, so $f(x) = 0$. You may have noticed this was an option from the start, but it's important to know how to work through a problem like this.

$\square$

5 To think about a little more (from last week)

Draw some curves that have maxima and minima. Draw their tangent lines at those points. Look at the geometry there. What is true at these points? Look at your answers to parts 1 and 3. What are some potential maxima and minima of these functions. Can you find any? How could you use this in an applied setting?