Worksheet 3

Fall 2018

MATH 222, Week 3: Trig Sub and Partial Fractions

Name: _____

Integrals of Products of Sine and Cosine

Problem 1. Use trig identities and what we learned in class to calculate the following integrals.

(a) $\int \sin^2(x)\cos^3(x)\,dx$

(b) $\int \sin^3(x)\cos^3(x)\,dx$

(c) $\int \sin^3(x)\cos^{2018}(x)\,dx$.

There's more to life than sine and cosine!

Problem 2. Calculate these integrals using trig identities:

(a) $\int \sec(x)\,dx$

(b) $\int \tan(x)\,dx$

(c) $\int \tan^5(x)\sec^{2017}(x)\,dx$.

(d) $\int \sec^6(x)\tan^{2018}(x)\,dx$

(e) (**Challenge**) $\int \sec^3(x)\,dx$

(f) (**Challenge**) $\int \frac{\sec^3(x)}{\tan(x)}\,dx$

(g) (**Challenge**) $\int \sec(x)\tan^2(x)\,dx$ (Hint: Solving parts (a) and (e) can help)

Trig Substitution

Problem 3. Choose the best trig substitution (when all else fails) in each case:

(a) $\int \sqrt{a^2-x^2}\,dx$: $x = a\sin\theta$ **or** $x = a\sec\theta$ **or** $x = a\tan\theta$

(b) $\int \sqrt{x^2-a^2}\,dx$: $x = a\sin\theta$ **or** $x = a\sec\theta$ **or** $x = a\tan\theta$

(c) $\int \sqrt{a^2+x^2}\,dx$: $x = a\sin\theta$ **or** $x = a\sec\theta$ **or** $x = a\tan\theta$

Problem 4. Use trig substitution to eliminate the root in $\sqrt{1-4x-2x^2}$. (Hint: Complete the square)

Problem 5. If we have $x = \sin(\theta)$, then what is $\cos(\theta)$ and $\tan(\theta)$ in terms of x? What if $x = \cos(\theta)$, what is $\sin(\theta)$ and $\tan(\theta)$ in terms of x? This will be useful when doing trig sub. (Hint: Use Triangles!)

Problem 6. Compute the following integrals using trig sub (use Problems 3,4 and 5 as your guide):

(a) $\int t\sqrt{1-t^2}\,dt$

(b) $\int \frac{1}{\sqrt{2x-x^2}}\,dx$

(c) $\int t^3(3t^2-4)^{3/2}\,dt$

(d) $\int \frac{ds}{(2+s^2)^{3/2}}\,ds$

(e) $\int \frac{e^t}{\sqrt{4-e^{2t}}}\,dt$

(f) $\int \frac{x^3}{\sqrt{4-x^2}}\,dx$

(g) $\int e^{4x}\sqrt{1-e^{2x}}\,dx$.

(h) (**Semi-Challenge**) $\int \frac{1}{(1+x^2)\sqrt{\arctan(x)^2-1}}\,dx$

(i) (**Challenge**) $\int \sqrt{\frac{x}{1-x^3}}\,dx$. Hint: First make a usual kind of substitution.

(j) (**Semi-Challenge**) $\int \frac{x}{\sqrt{2x^2-4x-7}}\,dx$.

1 Partial Fractions

Problem 7. Fill in the appropriate numerators and denominators in the following partial fraction decomposition. (The first is an example.)

(a) $\frac{f(x)}{(x-1)(x-2)} = \left[\quad \frac{A}{x-1} \quad + \quad \frac{B}{x-2} \quad \right]$

(b) $\frac{f(x)}{(x-1)(x-2)(x-\pi)} = \left[\quad \frac{\qquad}{\qquad} \quad + \quad \frac{\qquad}{\qquad} \quad + \quad \frac{\qquad}{\qquad} \quad \right]$

(c) $\frac{f(x)}{(x+1)^3} = \left[\quad \frac{\qquad}{\qquad} \quad + \quad \frac{\qquad}{\qquad} \quad + \quad \frac{\qquad}{\qquad} \quad \right]$

(d) $\frac{f(x)}{(x-1)(x^2+1)} = \left[\quad \frac{\qquad}{\qquad} \quad + \quad \frac{\qquad}{\qquad} \quad \right]$

Problem 8. Compute $\int \frac{dx}{(x^2-4)(x^2+1)^2}$

Problem 9. (a) Compute $\int_2^4 \frac{1}{x^2}\,dx$

(b) Compute $\int_2^4 \frac{1}{x(x-h)}\,dx$ where h is any positive number.

(c) What happens as $h \to 0$ in the integral for part (b)? How is this related to part (a)?

Problem 10. Compute $\int \frac{1}{x^2-a^2}\,dx$