Quiz 6

Spring 2016

MATH 222-004

Name: ______________________________

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Solve the following initial value problem. Please show every step for full credit, i.e. do not find the integrating factor and use a memorized equation.

$$\frac{dy}{dx} = -xy + x^3$$

With initial value $y(0) = 0$

Solution 1.

This was problem 12 on page 63 in the course notes. We write this in a recognizable form

$$\frac{dy}{dx} + xy = x^3$$

Our integrating factor is then $m(x) = e^{x^2/2}$. Multiplying through we have

$$\frac{d}{dx}(e^{x^2/2}y) = e^{x^2/2}x^3$$

Now we integrate both sides

$$e^{x^2/2}y = \int e^{x^2/2}x^3\;dx$$

Now comes the tricky part of this problem, finding that integral. We'll use integration by parts with $u = x^2$ and $v' = xe^{x^2/2}$. Then $u' = 2x$ and $v = e^{x^2/2}$, so

$$\int e^{x^2/2}x^3\;dx = x^2e^{x^2/2} - 2\int xe^{x^2/2}\;dx$$

Now we could use u-sub or just recognize this integral and so we have

$$\int e^{x^2/2}x^3\;dx = x^2e^{x^2/2} - 2\int xe^{x^2/2}\;dx = x^2e^{x^2/2} - 2e^{x^2/2} + C$$

Hence

$$e^{x^2/2}y = x^2e^{x^2/2} - 2e^{x^2/2} + C$$

Dividing through by $e^{x^2/2}$

$$y = x^2 - 2 + Ce^{-x^2/2}$$

To find C we use $y(0) = 0$ so

$$0 = -2 + C$$

Hence $C = 2$ and we get the final solution is

$$y = x^2 - 2 + 2e^{-x^2/2}$$

If you got to the point where you found the integrating factor but struggled to take the integral you received most of the credit, but I wanted to do a little refresher or integration by parts! $\square$

Problem 2. Write, but **DO NOT SOLVE** a differential equation that describes the rabbit population at time t in the following scenario:

Rabbits in Madison have a birth rate of 6% per year and a death rate (from old age) of 2% per year. Each year 1200 rabbits get run over and 700 rabbits move in from Sun Prairie.

Solution 2.

Breaking this into pieces if we let $y(t)$ be the number or rabbits after t years we have

$$\frac{dy}{dt} = .06y - .02y - 1200 + 700 = .04y - 500$$

As we have a constant rate fatalities and move ins and a proportional growth of .06 due to breeding and death of .02 due to natural causes. $\square$