

# Worksheet 12

Spring 2016

MATH 222, Week 12: Series! (and vectors maybe...)

**Name:** \_\_\_\_\_

You aren't necessarily expected to finish the entire worksheet in discussion. There are a lot of problems to supplement your homework and general problem bank for studying.

**Problem 1.** Let  $a_n = \frac{1}{n^2-n}$  and  $S_N = \sum_{n=2}^N a_n$ .

(a) Use one of your convergence tests to conclude that this series converges.

(b) Now we'll find what it converges to. Use partial fractions to rewrite  $a_n$

(c) Use part(a) to write out  $S_2, S_3, S_4$  explicitly and notice how terms cancel. Generalize this to find a formula for  $S_N$ .

(d) Compute  $\sum_{n=2}^{\infty} a_n$  i.e.  $\lim_{N \rightarrow \infty} S_N$ .

**Problem 2.** If  $x > 2$ , use the geometric series formula to find  $\sum_{n=0}^{\infty} \frac{2^{n+1}}{x^n}$

**Problem 3.** Using convergence tests determine the convergence or divergence of the following series:

(a)  $\sum_{n=1}^{\infty} n e^{-n^2}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4+1}$

(c)  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(d)  $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

(e)  $\sum_{n=1}^{\infty} \sin(n)$

(f)  $\sum_{n=1}^{\infty} \frac{5^k}{3^k+4^k}$

**Problem 4.** Let  $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ . Compute

(a)  $||\vec{a}||$

(b)  $2\vec{a}$

(c)  $||2\vec{a}||^2$

(d)  $\vec{a} + \vec{b}$