Quiz 9

Spring 2016

MATH 222-004

Name: ________________________________

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Find $T_5\{e^{1+t}\}$ using any method you want [**5 points**]. As a reminder $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

Solution 1.

We realize that

$$T_5\{e^{1+t}\} = eT_5\{e^t\} = e + ex + \frac{ex^2}{2} + \frac{ex^3}{3!} + \frac{ex^4}{4!} + \frac{ex^5}{5!}$$

$\square$

Problem 2. Let f be the real valued function defined below

$$f(x) = \begin{cases} 0 & x \neq 0 \\ 1 & x = 0 \end{cases}$$

If we take the sequence $a_n = \frac{2^n}{n!}$. What is $\lim_{n\to\infty} a_n$? [**2 points**]. Is it true that $\lim_{n\to\infty} f(a_n) = f(\lim_{n\to\infty} a_n)$? [**3 points**] You don't need to use the rigorous definition of a limit to justify your answer, but you do need to discuss how you arrived at your decision.

Solution 2.

Recall that factorials grow far faster than exponentials so $\lim_{n\to\infty} a_n = 0$. However $\frac{2^n}{n!} > 0$ for all n even though it goes to zero. So $f(a_n) = 0$ for all n. This implies

$$\lim_{n\to\infty} f(a_n) = \lim_{n\to\infty} 0 = 0$$

However $f(\lim_{n\to\infty} a_n) = f(0) = 1$. So $\lim_{n\to\infty} f(a_n) \neq f(\lim_{n\to\infty} a_n)$. This can occur because f is not continuous. $\square$