

Worksheet 5

Fall 2016

MATH 221, Week 5

Name: _____

1 Some Continuity

Consider the following functions:

$$f(x) = \begin{cases} 1 & x > 0 \\ -1 & x \leq 0 \end{cases}$$

and $g(x) = x^2$. Which of the following are continuous and why? What does this illustrate about compositions of functions?

(a) $g(x)$:

(c) $g \circ f(x)$:

(b) $f(x)$:

(d) $f \circ g(x)$:

2 Trig Limits!

Evaluate the following limits assuming $\lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{x \rightarrow 0} \frac{1 - \cos(\theta)}{\theta^2} = 1/2$. Double angle identities will be helpful...

(a) $\lim_{x \rightarrow \infty} \frac{\cos(x)}{x^2 + 1}$

(c) $\lim_{\theta \rightarrow 0} \sin(\theta) \cot(2\theta)$

(b) $\lim_{x \rightarrow 0} \frac{\tan(2x)}{\tan(\pi x)}$

(d) $\lim_{x \rightarrow 0} \frac{x + \tan(x)}{\sin(2x)}$

3 Derivatives!

Differentiate the following using the definition of the derivative:

(a) $f(x) = 1/x$ for $x \neq 0$

(b) $f(x) = \sqrt{x+3}$

(c) $f(x) = 4 - \sqrt{x+3}$

(d) $f(x) = \sqrt[3]{x}$ (this should look familiar)

(e) $f(x) = x^{2/3}$