MATH 222, Week 1: Review

## 1 Trig Identity Review

**Problem 1.** Use the identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  to show that  $\tan^2(\theta) + 1 = \sec^2(\theta)$ .

**Problem 2.** (a) Circle the correct answer:

$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
  $\cos(2\theta)$ 

$$\cos^2(\theta) - \sin^2(\theta) = \sin(2\theta) \qquad \cos(2\theta)$$

- (b) Using part (a) and  $\sin^2(\theta) + \cos^2(\theta) = 1$ , prove the following half angle formulas:
  - (a)  $\cos^2(\theta) = \frac{1}{2}(\cos(2\theta) + 1)$ . There's a very similar identity for  $\sin^2(\theta)$  that could be useful later on.

(b) 
$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

## 2 Integration and Fundamental Theorem of Calculus Review

**Problem 3.** For each of the following, state whether the object is a function or a number. If it is a function, state what variable it is a function of.

(a)  $\int_e^x e^{\sec^2(\ln(t))} dt$ 

(d)  $\int_0^{\pi t} \arccos(x) dx$ 

(b)  $\int \arcsin(x) dx$ 

(e)  $\int_{x}^{x^2} \cos^3(t) \sin^2(t) dt$ 

(c)  $\int_1^3 \arctan(s) ds$ 

(f)  $\int_t^t f(x) dx$ 

**Problem 4.** Compute  $\int_0^x (\int_0^t \cos(s)ds)dt$ .

**Problem 5.** Define  $f(x) = \int_x^{x^2} e^{t^3} dt$ . Compute f'(x). Hint: Split the integral into  $\int_x^1$  and  $\int_1^{x^2}$  and use the Fundamental Theorem of Calculus.

**Problem 6.** Let a be any fixed real constant. Compute  $\frac{d}{dx} \int_{x^3}^a \ln(t) dt$ . (Hint: Fundamental Theorem of Calculus).

## 3 Challenge Problem

**Problem 7.** Compute  $\int \sin^2(\theta) \cos^2(\theta) d\theta$ . There are at least two ways to approach this.