MATH 222 (Lectures 1,2, and 4) Fall 2015
Practice Midterm 2.1

Name: _____ Student ID#: _____

Circle your TA's name from the following list.

Carolyn Abbott	Tejas Bhojraj	Zachary Carter	Mohamed Abou Dbai	Ed Dewey
Jale Dinler	Di Fang	Bingyang Hu	Canberk Irimagzi	Chris Janjigian
Tao Ju	Ahmet Kabakulak	Dima Kuzmenko	Ethan McCarthy	Tung Nguyen
Jaeun Park	Adrian Tovar Lopez	Polly Yu		

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6	Problem 7
Score							

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- On Problems 1, 2, and 3 only the answer will be graded.
- On Problems 4, 5, 6, and 7 you must show your work and we will grade the work and your justification, and not just the final answer. Limited partial credit will be available.
- Problem 3 is worth 10points. All other problems worth 15 points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1-x^2}$. Note that we have provided some formulas on the next page to help with this.

Formulas

- $T_\infty e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$
- $T_\infty \sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$
- $T_\infty \cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$
- $T_\infty \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$
- $T_\infty \frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k$
- $T_\infty (1+x)^b = \sum_{k=0}^{\infty} \binom{b}{k} x^k \quad \text{where} \quad \binom{b}{k} = \frac{b(b-1)(b-2)\cdots(b-k+1)}{k!}$

1. On this page are three True/False statements. On the following page you will be asked to match direction fields to their defining equations. CIRCLE the correct answers below.

(a)		(b)		(c)		(d)			(e)		
True	False	True	False	True	False	I	II	III	I	II	III

True or false:

(a) The function $(x^2 + x^3)^2$ is $o(x^3)$.

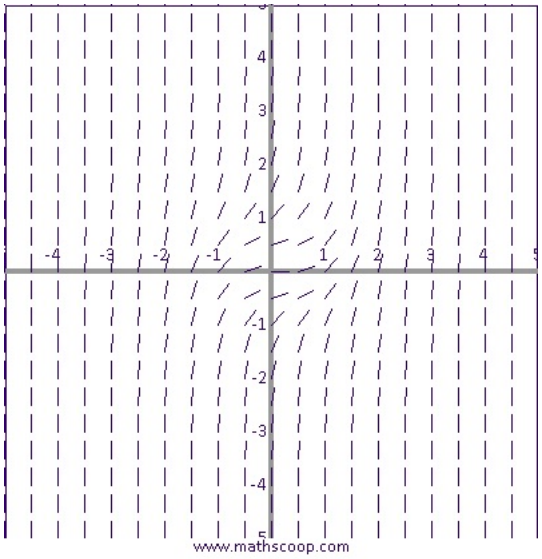
(b) $R_4 e^x = e^x - (1 + x + x^2 + x^3 + x^4)$

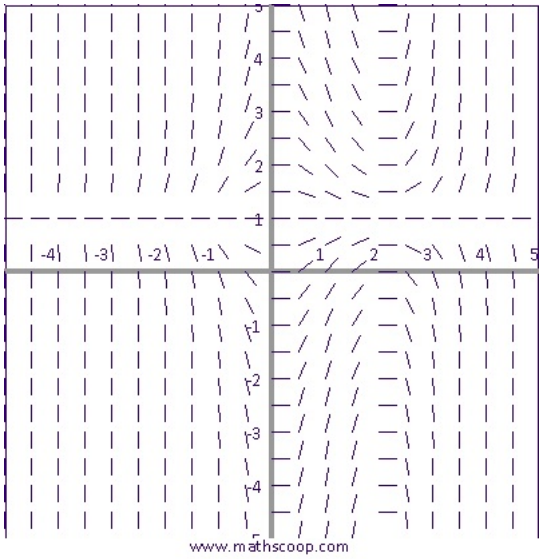
(c) Let $f(x)$ and $g(x)$ be functions whose Taylor series exist. Then for any n we have $T_n(f(x)g(x)) = (T_n f(x))(T_n g(x))$.

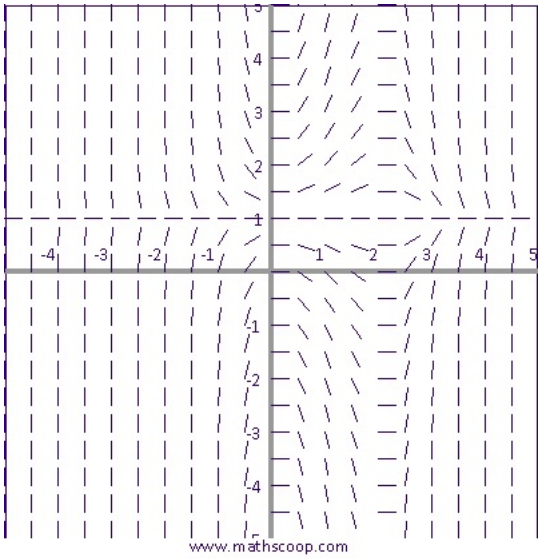
Below are three direction fields. The equations for *two* of those fields are given below. Match the equation to the appropriate direction field and record your answer on the previous page.

(d) $\frac{dy}{dx} = x(x-2)(1-y)$

(e) $\frac{dy}{dx} = x^2 + y^2$

I. 

II. 

III. 

2. (a) Use Euler's method with step size $h = 0.1$ to approximate $y(1.1)$ where $y(x)$ is the solution of

$$\frac{dy}{dx} = -2y + 3x \text{ and } y(1.0) = 0.$$

Answer: _____

(b) Find $T_1^2(\arctan x)$. *You may find it helpful to recall that* $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$. *You do not need to simplify any values of* arctan *in your answer.*

Answer: _____

(c) Find T_2 of $\sqrt{1+2x}$ (your final answer must not contain any binomial expression $\binom{a}{b}$).

Answer: _____

3. In the problem below: 1. Clearly define variables (including units!); 2. Set up the appropriate differential equation; and 3. Write down the appropriate initial condition. DO NOT SOLVE THE DIFFERENTIAL EQUATION.

   (b) Start with a full 50 quart vat containing a 8% (by volume) solution of vinegar, at 10:00am. A solution of 5% vinegar flows in at a rate of 2 quarts per minute. The solution is kept thoroughly mixed and drawn off at a rate of 3 quarts per minute. We are interested in a function describing the **total amount of vinegar in the vat at a given time**.

- **Variables (2pts):**

- **Differential equation (5pts)**

- **Initial condition (3pts):**

4. Find a solution to each initial value problem.

(a)

$$\frac{dy}{dx} - e^x y^2 = e^x \text{ and } y(0) = 0.$$

Solution Satisfying Initial Condition: $y =$ _____

(b)

$$(1 + x^2)\frac{dy}{dx} + 2xy = 3(1 + x^2) \text{ and } y(1) = \frac{5}{2}.$$

Solution Satisfying Initial Condition: $y =$ _____

5. We have a vat containing a mixture of acid and water. Let:

   - t stand for time in minutes from 12:00pm
   - $A(t)$ denote the total amount of acid in the vat at time t
   - $V(t)$ denote the total volume of liquid in the vat at time t.

   Assume that

   $$\frac{dA}{dt} = 3 - \frac{2A}{20 - t} \qquad \text{and} \qquad A(0) = 0 \qquad \text{and} \qquad V(t) = 1000 - 50t.$$

   What is the *concentration* (%) of acid in the vat after 10 minutes?

6. Let $f(x) = e^{5-2x}$. Find a number B such that $|f(x) - T_4 f(x)| \leq B$ for all x in the range $-1 \leq x \leq 1$. You must justify your answer.

7. Let $f(x)$ be a function satisfying the differential equation

$$f''(x) + \sin(2x^2) - f(x) = 0$$

and also satisfying the initial conditions $f(0) = 2$ and $f'(0) = 0$. Compute $T_4 f(x)$.

Note: it is essential that you use notation correctly in your answer, as part of what we are testing is whether you understand what the notation means.