Worksheet 12

Fall 2016

MATH 221

Name: _____

Some more optimizations

(a) We want to build a window with a rectangular base and semicircular top. We have 12 meters of framing materials; what should the dimensions be to let in the most light?

(b) Suppose you can row at 10 miles an hour and run at 8 miles an hour. Your friend Tim is back and wants to race you to the house of your other friend Mike. This house is across a 1-mile-wide lake, and 2 miles inland, 1 mile to the left of you as you look across the lake. Where should you land on shore to beat the hell out of Tim?

(c) Determine the point(s) on $y = x^2 + 1$ that are closest to the point $(0, 2)$

Some integrations with Riemann sums We are gonna try and integrate x^2.

(a) Our first goal is to do this on $[0, 1]$. Draw a graph of $y = x^2$ on $[0, 1]$, and split up the interval $[0, 1]$ into subintervals of length $\frac{1}{n}$.

(b) First, choose the right endpoint of each of these subintervals. Now write down a Riemann sum (the kind of sums we've been doing in class) with these endpoints. Remember, you're adding up areas of rectangles.

(c) Now, using the identity

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

write down a formula for the area you are computing in step (2).

(d) Take that limit as $n \to \infty$.

(e) What happens if you choose the left endpoint instead?

(f) Now, what happens if we are doing this integral from 0 to A, where A is just some fixed positive constant? Redo the steps above with 1 replaced by A. What do you get?

Fundamental Theorem of Calculus work

(g) We touched on the fundamental theorem of calculus at the end of lecture last Monday. This says, for

$$F(x) = \int_a^x f(t)dt$$

we have

$$\frac{d}{dx}F(x) = f(x)$$

Can you use this to integrate x^2 from 0 to 1? How about x^3, or x^n for $n > 0$?

(h) The function $F(x)$ defined above is the area under the graph of f (let's assume f is continuous and positive or whatever). Draw a picture that represents this situation. Can you tell me why I should believe that $F'(x) = f(x)$?

(i) Integrate

$$\int_0^{2\pi} \sin(x)dx$$

(j) Integrate

$$\int_0^{2\pi} \cos(nx)dx$$

where $n \neq 0$ is a constant.

(k) This problem may make more sense later, but integrate

$$\int_0^{\pi/4} 2x \sec^2(x^2)dx$$