Worksheet 2 Fall 2018

MATH 222, Week 2: Integration by Parts and Trigonometric Integrals

Name: _____

1 Integration by Parts without Trig Identities

Problem 1. Compute the following

(a) $\int \frac{1}{x} \ln(x) dx$

(b) $\int x \ln(x)\ dx$

(c) $\int \ln(x) dx$

What integration techniques did you use to solve each of these integrals and what lead you to choose them?

Problem 2. (a) Compute $\int_0^\pi \cos(x) dx$ and $\int_0^\pi x^2 \cos(x) dx$

(b) Show that:
$$\int x^n \cos(x)\ dx = x^n \sin(x) + nx^{n-1}\cos(x) - n(n-1)\int x^{n-2}\cos(x)\ dx$$

(Hint: The steps are very similar to what you did in part (a) for $\int_0^\pi x^2 \cos(x)\ dx$).

(c) Use the identity you just proved and part (a) to compute $\int_0^\pi x^4 \cos(x)\ dx$.

2 Integration by Parts with Trig Identities

Problem 3. There are some extra Integration by Parts problems that require some of the trig identities from the last worksheet. We worked through this problem in lecture, but it is worth slowly repeating yourselves:

(a) Compute $\int \cos^2(x)dx$.

(b) Compute $\int \cos^3(x)dx$.

(c) Compute $\int \cos^4(x)dx$.

(d) How do the previous three problems compare? Did you use the same idea for each or was there some difference? If so, how would you separate them? (Hint: How would you compute $\int \cos^5(x)dx$?)

(e) (**Challenge**) Try to find a recursive formula as in Problem 2 for $\int \cos^n(x)dx$ using IBP. That is let $J_n = \int \cos^n(x)dx$ express J_n in terms of smaller J_k (This is what we did in lecture, try to do it just using the previous parts, not your notes).

Problem 4. We will do the same problem as above, but for $\sin(x)$ this time:

(a) Compute $\int \sin^2(x)dx$

(b) Compute $\int \sin^3(x)dx$

(c) Compute $\int \sin^4(x)dx$

(d) How do the previous three problems compare? Did you use the same idea for each or was there some difference? If so, how would you separate them?

(e) How did problem 4 compare to problem 3? What differences and similarities were there? How would you try to explain some of the differences?

(f) (**Challenge**) Try to find a recursive formula as in Problem 2 and Problem 3 for $\int \sin^n(x)dx$ using IBP.

Problem 5. Compute $\int \arctan(x)\ dx$.

3 Challenge Problem

Problem 6. Find $\int e^x \cos(x)dx$.