Practice Final b
MATH 222 (Lectures 1,2, and 4) Fall 2015.

Name: ________________________________ Student ID#: __________________________

Circle your TAs name:

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	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
Score					
	Problem 6	Problem 7	Problem 8	Problem 9	Problem 10
Score					

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- Please read the instructions on every problem carefully.
- On Problems 1–4 only the answer will be graded.
- On Problems 5–10 you must show your work and we will grade the work and your justification, and not just the final answer.
- Each problem is worth ten points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1-x^2}$. Note that we have provided some formulas on the next page to help with this.

Formulas

You may freely quote any algebraic or trigonometric identity, as well as well as any of the following formulas or minor variants of those formulas.

Integrals

- $\cos(\arcsin x) = \sqrt{1-x^2}$
- $\sec(\arctan x) = \sqrt{1+x^2}$.
- $\tan(\operatorname{arcsec} x) = \sqrt{x^2-1}$.
- $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & \text{when } n \neq -1 \\ \ln|x| + C & \text{when } n = -1 \end{cases}$
- $\int e^x dx = e^x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \tan x dx = -\ln|\cos x| + C$
- $\int \cot x dx = \ln|\sin x| + C$
- $\int \sec x dx = \ln|\sec x + \tan x| + C$.
- $\int \csc x dx = -\ln|\csc x + \cot x| + C$.
- $\int \frac{1}{1+x^2} dx = \arctan(x) + C$.

Taylor series

- $T_\infty e^x = \sum_{k=0}^\infty \frac{x^k}{k!}$
- $T_\infty \sin x = \sum_{k=0}^\infty (-1)^k \frac{x^{2k+1}}{(2k+1)!}$
- $T_\infty \cos x = \sum_{k=0}^\infty (-1)^k \frac{x^{2k}}{(2k)!}$
- $T_\infty \frac{1}{1-x} = \sum_{k=0}^\infty x^k$
- $T_\infty \frac{1}{1+x} = \sum_{k=0}^\infty (-1)^k x^k$
- $T_\infty (1+x)^b = \sum_{k=0}^\infty \binom{b}{k} x^k$ where $\binom{b}{k} = \frac{b(b-1)(b-2)\cdots(b-k+1)}{k!}$

Other

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$.

1. For each statement below, CIRCLE true or false. You do not need to show your work.

(a)		(b)		(c)		(d)		(e)	
True	False	True	False	True	False	True	False	True	False

(a) The series $\sum_{k=1}^{\infty} \frac{\cos^k(1/k)}{2^{2k+3}}$ converges.

(b) The series $\sum_{k=1} (1 + \frac{1}{k})^k$ converges.

(c) The integral $\int_0^1 \frac{1}{\sqrt{x}} + \frac{1}{(x-2)^2}$ converges.

(d) The integral $\int_1^{\infty} \frac{1}{x \ln(x)} dx$ converges.

(e) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ then $\vec{b} = \vec{c}$.

2. On this page only the answer will be graded.

   (a) Compute $\int \frac{1}{\sqrt{4-(x+2)^2}} dx$.

   (b) Compute $\int \frac{x^2}{x(x-1)} dx$.

3. On this page only the answer will be graded.

   (a) Compute $T_3(\sin x \cos x)$.

   (b) For which values of x does $\sum_{k=0}^{\infty} \frac{2^{2k+3}}{x^k}$ converge?

4. On this page, only the answer will be graded.

(a) Let ℓ and m be the lines $\ell : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1+3s \\ 2+4s \end{pmatrix}$ and $m : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2+t \\ 2t \end{pmatrix}$. Compute the angle between ℓ and m. You do not need to simplify any arccos value.

(b) Compute $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 6 \\ -5 \end{pmatrix}$.

(c) Compute the distance from the point $(1, 1, 0)$ to the plane $x + 2y + 3z = 0$.

5. On this page, you must show your work to receive full credit.

(a) Compute $T_2(\frac{1}{\cos x})$.

(b) A tank begins with 100 litres of salt water in it at 10:00am. Fresh water is pumped in at a rate of twenty litres per minute and the mixed water is pumped out at a rate of ten litres per minute. Let t stand for time in minutes from 10:00am and let $A(t)$ for the amount of salt in the tank, in kilograms, at time t. Write down a differential equation for $A(t)$. DO NOT SOLVE THE DIFFERENTIAL EQUATION.

6. On this page, you must show your work to receive full credit.

(a)

$$\frac{dy}{dx} = \frac{x + xy^2}{2y} \qquad \text{and} \qquad y(0) = e.$$

(b)

$$x\frac{dy}{dx} = y + x^2 e^x \qquad \text{and} \qquad y(1) = 10 + e.$$

Note: The original version of this problem had a typo. It has now been corrected.

7. On this page, you must show your work to receive full credit.

$$\text{Compute } \int x \arctan(x) + \frac{2}{1-x^2} dx.$$

8. On this page, you must show your work and justify your answer to receive full credit.

Compute $\displaystyle\int_1^\infty \frac{8}{x^3 - 4x} dx$ or explain why the integral does not exist. You may freely use the formula $\frac{8}{x^3-4x} = -\frac{2}{x} + \frac{1}{x+2} + \frac{1}{x-2}$.

9. On this page, you must show your work and justify your answer to receive full credit.

   We have a 300 gallon tank that begins full of milk containing 2% butterfat. Let t stand for time in minutes from 10:00am and let $A(t)$ stand for the total amount of butterfat in gallons in the vat. Starting at 10:00am, $A(t)$ satisfies the differential equation:

$$\frac{dA}{dt} = .1 - \frac{15A}{100 - 5t}.$$

   Compute A(t).

10. On this page, you must show your work and justify your answer to receive full credit. Justifying your answer must include: clearly stating any convergence test that you use and explicitly verifying each hypothesis for that test.

$$\text{Does } \sum_{k=0}^{\infty} \frac{e^{-k} + k + \sqrt{k}}{e^{k} + k^3 + \sqrt[3]{k}} \text{ converge?}$$

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