

Worksheet 1

Fall 2016

MATH 221, Week 1

Name: Guide

1

Factor the following:

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$$

$$x^n - y^n = (x - y) \sum_{i=0}^{n-1} x^{n-1-i} y^i$$

(Hint: if you plug in a number a to a polynomial in x , $p(x)$, and get 0, you can factor $x - a$ out of the polynomial. For example $p(x) = x^3 - 2x + 1$, $a = 1$. $p(1) = 1 - 2 + 1 = 0$, so we know we can factor $p(x) = (x - 1)q(x)$, and we just need to figure out what $q(x)$ is. In this case it's $x^3 - 2x + 1 = (x - 1)(x^2 + x - 1)$)

2 Approximating Derivatives

- (a) Find an equation for the line passing through $(576, 432)$ and having slope 190392.

Using point slope form:

$$(y - 432) = 190392(x - 576)$$

- (b) Find the secant line to the graph $f(x) = x^3$ determined by the points $(1, 1)$ and (x_0, x_0^3) . It may be useful to use the previous part when computing the slope. (What should y be?)

We know the slope is rise over run, given by

$$m = \frac{x_0^3 - 1}{x_0 - 1}$$

Now we want to factor the top to cancel the denominator. We can factor it as $(x_0 - 1)(x_0^2 + x_0 + 1)$ taking $y = 1$ in problem 1. Using this we can cancel the denominator and we find

$$m = x_0^2 + x_0 + 1$$

Using point slope form, the secant line is then

$$(y - 1) = (x_0^2 + x_0 + 1)(x - 1)$$

- (c) Based on the previous part, what would you guess the derivative of x^3 is at the point $x = 1$? Explain your answer.

We would expect it to be what the slopes of the secant lines converge to as we get really close to $x_0 = 1$. Hence we expect the slope to be $\lim_{x_0 \rightarrow 1} x_0^2 + x_0 + 1 = 3$. We will see soon that this is true.

3 Average Speed

Ariella drove 40 miles to see her cousin. The trip took her 2 hours. Then, Ariella left her cousin's house and drove another 30 miles to the store. It took Ariella 3 hours to arrive at the store. What was Ariella's average speed for the trip?

Average speed is distance divided by time so the answer will be

$$\frac{40 + 30}{2 + 3} = \frac{70}{5}$$

4 To think about

Suppose you are traveling on a highway with a speed limit of 60 mph. You travel down this highway for two hours and go 150 miles. A police officer somehow happens upon this information. Could he write you a ticket for speeding? Why or why not?

This is hinting at the Mean Value Theorem. Which essentially tells you that your instantaneous speed equaled your average speed at some point along your trip. For the purpose of this problem it means that since your average speed was 75mph that at some point you must have been traveling at that speed and hence you could get a ticket.