Quiz 3

Spring 2016

MATH 222-004

Name: ______________________________

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Use rational substitution to find $\int \sqrt{x^2-4}\,dx$. For sake of time, you may leave your answer in terms of t, but you should know how to convert back to x.

Solution 1.

Remember that $U(t) = \frac{1}{2}(t + \frac{1}{t})$ and $V(t) = \frac{1}{2}(t - \frac{1}{t})$, so that $U^2(t) - V^2(t) = 1$. Also recall that $t = U + V$ and $1/t = U - V$. These are important identities. To solve this integral we let $x = 2U(t)$. Then $dx = 2U'(t)dt$. We then substitute:

$$\int \sqrt{x^2-4}\,dx = \int \sqrt{4(U(t)^2-1)}\,2U'(t)dt = \int 4\,V(t)U'(t)dt = \int (t-1/t)(1+1/t^2)\,dt$$

If we expand $(t-1/t)(1-1/t^2) = t - 2/t + 1/t^3$. Now we should be in comfortable territory:

$$\int t - 2/t + 1/t^3\,dt = \frac{t^2}{2} - 2\ln|t| - \frac{1}{2t^2} + C$$

You may leave your answer in this form, but I would recommend converting back to x on your own. You should ultimately get $\frac{x}{2}\sqrt{x^2-4} - 2\ln|x/2 + \sqrt{(x/2)^2-1}| + C$ □

Problem 2. Compute $\int \frac{1}{4x^2-8x-5}\,dx$. Recall that $\int \sec(\theta)\,d\theta = \ln|\sec(\theta)+\tan(\theta)| + C$.

Solution 2.

We first have to complete the square. Notice that $4x^2 - 8x - 5 = 4(x-1)^2 - 9$. Substituting this into our integral we have:

$$\int \frac{1}{4x^2-8x-5}\,dx = \int \frac{1}{4(x-1)^2-9}\,dx$$

Now we can go two ways from here. Partial fractions or trig sub. I actually think trig sub might be better. Let $x - 1 = \frac{3}{2}\sin(\theta)$. Then $dx = \frac{3}{2}\cos(\theta)$. Substituting we have:

$$\int \frac{1}{4(x-1)^2-9}\,dx = \int \frac{(3/2)\cos(\theta)}{-9\cos^2(\theta)}\,d\theta = -\frac{1}{6}\int \sec(\theta)\,d\theta$$

This is where the hint comes in. We now have:

$$-\frac{1}{6}\int \sec(\theta)\,d\theta = -\frac{1}{6}\ln|\sec(\theta)+\tan(\theta)| + C$$

Using our triangle we know that $\sin(\theta) = \frac{2x-2}{3}$. This implies that $\sec(\theta) = \frac{3}{2x-2}$ and $\tan(\theta) = \frac{2x-2}{\sqrt{9-(2x-2)^2}}$. So our final answer is:

$$\int \frac{1}{4x^2 - 8x - 5}\, dx = -\frac{1}{6} \ln \left| \frac{2x-2}{3} + \frac{2x-2}{\sqrt{9-(2x-2)^2}} \right| + C$$

□