Quiz 4

Fall 2016

MATH 221

Name: _____

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Compute the following limit:

$$\lim_{\theta\to 0} \frac{2}{\theta\tan(\theta)} - \frac{2}{\theta\sin(\theta)}$$

We rewrite tangent and combine the fractions over the common denominator:

$$\lim_{\theta\to 0} \frac{2}{\theta\tan(\theta)} - \frac{2}{\theta\sin(\theta)} = \lim_{\theta\to 0} \frac{2\cos(\theta)-2}{\theta\sin(\theta)}$$

Now we split this up and multiply by θ/θ

$$\lim_{\theta\to 0} \frac{2\cos(\theta)-2}{\theta\sin(\theta)} = \lim_{\theta\to 0} \frac{2(\cos(\theta)-1)}{\theta^2}\cdot\frac{\theta}{\sin(\theta)}$$

Now we know how to take all of these limits if we slightly tweak how it's written

$$\lim_{\theta\to 0}(-2)\frac{(1-\cos(\theta))}{\theta^2}\cdot\frac{\theta}{\sin(\theta)} = (-2)(1/2)(1) = -1$$

Problem 2. Compute the following limit:

$$\lim_{x\to\infty} \sqrt{x^2+x} - x$$

We multiply by the conjugate,

$$\lim_{x\to\infty} \sqrt{x^2+x} - x\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

Now if we factor an x out of both of the terms in the denominator

$$\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \lim_{x\to\infty}\frac{x}{x(\sqrt{1+1/x}+1)} = \lim_{x\to\infty}\frac{1}{\sqrt{1+1/x}+1} = 1/2$$