

MATH 222 (Lectures 1,2, and 4) Fall 2015
Practice Midterm 1.1

Name: _____

Circle your TA's name from the following list.

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	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6	Problem 7
Score							

Instructions

- Write neatly on this exam. If you need extra paper, let us know.
- On Problems 1, 2, and 3, only the answer will be graded.
- On Problems 4, 5, 6, and 7 you must show your work and we will grade the work and your justification, and not just the final answer.
- Each problem worth either 14 or 15 points.
- No calculators, books, or notes (except for those notes on your 3 inch by 5 inch notecard.)
- Please simplify any formula involving a trigonometric function and an inverse trigonometric function. For example, please write $\cos(\arcsin x) = \sqrt{1 - x^2}$. Note that we have provided some formulas on the next page to help with this.

Formulas

You may freely quote any algebraic or trigonometric identity, as well as any of the following formulas or minor variants of those formulas.

- $\cos(\arcsin x) = \sqrt{1 - x^2}$
- $\sec(\arctan x) = \sqrt{1 + x^2}$.
- $\tan(\operatorname{arcsec} x) = \sqrt{x^2 - 1}$.
- $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & \text{when } n \neq -1 \\ \ln |x| + C & \text{when } n = -1 \end{cases}$
- $\int e^x dx = e^x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \tan x dx = -\ln |\cos x| + C$
- $\int \cot x dx = \ln |\sin x| + C$
- $\int \sec x dx = \ln |\sec x + \tan x| + C$.
- $\int \csc x dx = -\ln |\csc x + \cot x| + C$.
- $\int \frac{1}{1+x^2} dx = \arctan(x) + C$.

1. For each statement below, CIRCLE true or false. You do not need to show your work.

(a)		(b)		(c)		(d)		(e)	
True	False	True	False	True	False	True	False	True	False

(a) $\int_3^\infty \frac{x-\sqrt{x}}{3x^3+11} dx$ is a finite number.

(b) $\int_3^\infty \frac{1}{2x^2} dx \geq \int_3^\infty \frac{1}{x^2+3x} dx$.

(c) $\int \cos^2(5\theta + 1) \sin(5\theta + 1) d\theta = -\frac{1}{15} \cos^3(5\theta + 1) + C$.

(d) $\int_3^{10} \frac{1}{\sqrt{x-3}} dx$ is a finite number.

(e) Let $I_n = \int x^n e^x dx$ then a reduction formula for these integrals is given by:

$$I_n = x^n e^x + (n-1)I_{n-1}.$$

2. On this page, only the answer will be graded.

(a) Compute $\int \frac{7}{(t-1)(2t+5)} dt$.

Answer: _____

(b) Compute $\int x \cos(4x) dx$.

Answer: _____

(c) Compute $\int \frac{x}{1+(x+3)^2} dx$.

Answer: _____

3. On this page, only the answer will be graded.

- (a) Find a such that $\frac{1}{x^2+x-5} < \frac{1}{x^2+2} < \frac{1}{x^2+1-x}$ for all $x > a$.

Answer: _____

- (b) Compute $\int \sin^2(3x+1)dx$.

Answer: _____

- (c) Compute $\int_0^\pi \sin^7 x dx$. (You may use $A_n = \frac{n-1}{n}A_{n-2}$ where $A_n = \int_0^\pi \sin^n x dx$.)

Answer: _____

4. Compute $\int \frac{1}{(1+x^2)^2} dx$.

5. For $n = 0, 1, \dots$ let $I_n = \int \sec^n x dx$. Use integration by parts to derive a reduction formula for I_n .

6. Compute $\int \frac{e^{-x} dx}{1 + e^{2x}}$.

7. Compute $\int_1^\infty \frac{1}{x(x^2 + 1)} dx$. (You may freely use the formula $\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$.)