Quiz 2

Spring 2016

MATH 222-004

Name: _____

Problem 1. Find $\int \frac{x^2+1}{x^2-3x+2}\,dx$

Solution 1.

We have to notice that this isn't a proper rational function so we have to do long division and divide x^2+1 by x^2-3x+2. We then find:

$$\frac{x^2+1}{x^2-3x+2} = 1 + \frac{3x-1}{(x-1)(x-2)}$$

Now we can do partial fractions:

$$\frac{3x-1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

Multiplying to get a common denominator and equating the numerators gives us $A(x-2)+B(x-1) = 3x-1$. If we let $x=2$ we find that $B=5$ and letting $x=1$ we find that $A=-2$. So we now have:

$$\int 1 + \frac{-2}{x-1} + \frac{5}{x-2} dx$$

Now we can integrate:

$$\int \frac{x^2+1}{x^2-3x+2}\,dx = x - 2\ln|x-1| + 5\ln|x-2| + C$$

sdf □

Problem 2. Compute $\int \frac{1}{\sqrt{9-x^2}}\,dx$

Solution 2.

This is in the classic form $\frac{1}{\sqrt{a^2-x^2}}$ so we make the substitution $x = 3\sin(\theta)$ and so $dx = 3\cos(\theta)d\theta$. Substituting we find:

$$\int \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{3\sin(\theta)}{3\sin(\theta)} d\theta = \int 1 d\theta = \theta + C$$

Putting this back in terms of x we find $\int \frac{1}{\sqrt{9-x^2}}\,dx = \arcsin(x/3) + C$ □