Quiz 5

Spring 2016

MATH 222-004

Name: ______________________________

For full credit please explain all of your answers. **No calculators** are allowed.

Problem 1. Solve the following initial value problem [**4.5 points**] and indicate which, if any, solutions were lost while separating variables [**.5 points**].

$$\frac{dy}{dx} = xy$$

With initial value $y(2) = 1$.

Solution 1.

We notice immediately this is separable, so we divide through by y. However at this step we tacitly assume $y \neq 0$, so we quickly check that this is actually a solution to our differential equation that we are discarding. Hence we will lose the solution $y = 0$ as we continue. Integrating

$$\int \frac{1}{y} dy = \int x \, dx$$

We find $\ln(y) = x^2/2 + C$. We'll solve for C here using our initial value $y(2) = 1$. So we must have $\ln(1) = 2 + C$. Hence $C = -2$. So the solution to our initial value problem is $y = e^{x^2/2-2}$. □

Problem 2. Find the general solution to the differential equation.

$$\frac{dy}{dx} + 1 + y^2 = 0$$

Solution 2.

This is also separable with a little bit of work. If we subtract $1 + y^2$ over to the other side and divide through we have

$$\int \frac{1}{1+y^2} dy = \int -1 dx$$

We recognize this as the derivative of $\arctan(y)$,

$$\arctan(y) = -x + C$$

Thus the general solution to the differential equation is

$$y = \tan(C - x) = -\tan(x - C)$$

Notice that we didn't discard any solutions as we divided by $1 + y^2$ which is strictly positive. □