Worksheet 6

Spring 2016

MATH 222, Week 6: 3.1, 3.2, 3.3, 3.5

Name: ______________________________

You aren't necessarily expected to finish the entire worksheet in discussion. There are a lot of problems to supplement your homework and general problem bank for studying.

Problem 1. Find a solution to the initial value problem:

$$\frac{dy}{dx} = e^y x^3$$

With initial value $y(0) = 0$.

Solution 1.

This is separable so we can write $e^{-y}dy = x^3dx$ and integrate both sides to find $-e^{-y} = x^4/4 + C$. However we need $y(0) = 0$ so $-e^0 = 0 + C$, thus $C = -1$. So the solution is $y = -\ln(-x^4/4 + 1)$. □

Problem 2. Find a solution to the initial value problem:

$$\frac{dy}{dx} = y\sqrt{y^2 - 1}\cos(x)$$

With initial value $y(0) = 1$.

Solution 2.

Once again this is separable so we write

$$\frac{dy}{y\sqrt{y^2-1}} = \cos(x)dx$$

Integrating both sides

$$\int \frac{dy}{y\sqrt{y^2-1}} = \int \cos(x)dx$$

To solve the first integral we use trig sub. Let $y = \sec(\theta)$ then $dy = \sec(\theta)\tan(\theta)$ substituting:

$$\int \frac{\sec(\theta)\tan(\theta)}{\sec(\theta)\sqrt{\sec(\theta)^2 - 1}}d\theta = \int d\theta = \theta = \sec^{-1}(y)$$

We know $\int \cos(x)dx = \sin(x)$. So this tells us that $\sec^{-1}(y) = \sin(x) + C$. So $y = \sec(\sin(x) + C)$. We need $y(0) = 1$, so $1 = \sec(0 + C)$, so we can take $C = 0$. Thus a solution to the initial value problem is $y = \sec(\sin(x))$. To check that this makes sense:

$$\frac{dy}{dx} = \sec(\sin(x))\tan(\sin(x))\cos(x) = \sec(\sin(x))\sqrt{\sec(\sin(x))^2 - 1}\cos(x) = y\sqrt{y^2-1}\cos(x)$$

$\square$

Problem 3. Find the general solution to the differential equation

$$\frac{dy}{dx} = x^2 + y^2x^2$$

Solution 3.

This is also separable, but not as obviously. We can factor our an x^2 first

$$\frac{dy}{dx} = x^2(1 + y^2)$$

Then we can separate and integrate

$$\int \frac{dy}{1 + y^2} = \int x^2 dx$$

We recognize the integral on the left as $\tan^{-1}(y)$ and we know the integral on the right is $x^3/3$. So

$$\tan^{-1}(y) = x^3/3 + C$$

This tells us that $y = \tan(x^3/3 + C)$ is the general solution to the differential equation. $\square$

Problem 4. Find the general solution to the differential equation (for $x \neq 0$):

$$x\frac{dy}{dx} = -y + x$$

Solution 4.

See Worksheet 7 Solutions. $\square$

Problem 5. Find the general solution to the differential equation

$$\frac{1}{2x}\frac{dy}{dx} = y + e^{x^2}$$

Solution 5.

See Worksheet 7 Solutions. $\square$

Problem 6. Find a solution to the initial value problem

$$\cos(x)\frac{dy}{dx} = 1 - \sin(x)y$$

With initial value $y(0) = 1$.

Solution 6.

See Worksheet 7 Solutions □