1.0 Languages, Expressions, Automata

Alphabet: a finite set, typically a set of symbols.
Language: a particular subset of the strings that can be made from the alphabet.

ex: *an alphabet of digits* = {–,0,1,2,3,4,5,6,7,8,9}
a language of integers = {0,1,2,...,101,102,103,...,-1,-2,etc.}
Note that strings such as 2-20 *would not be included in this language.*

Regular Expression:

A pattern that generates (only) the strings of a desired language. It is made up of letters of the language's alphabet, as well as of the following special characters:

( )	used for grouping
$*$	repetition
•	concatenation (usually omitted)
+	denotes a choice ("or").
λ	a special symbol denoting the null string

Precedence from highest to lowest: ( ) $*$ • +

formal (recursive) definition:

If **A** is an alphabet, and **a** $\in$ **A** , then **a** is a regular expression.
λ is a regular expression.
If **r** and **s** are regular expressions, then the following are also regular expressions: **r*** , **r • s = rs** , **r + s** , *and* **(r)**

examples: (assume that A = {*a, b*})

a • b • a (or just **aba**)	matched only by the string *aba*
ab + ba	matched by exactly two strings: *ab* and *ba*
b*	matched by { λ, b, bb, bbb, ...}
b(a + ba*)*a (b + λ)	matched by bbaaab, and many others

Some convenient extensions to regular expression notation:

aa = a^2, bbbb = b^4, etc.
a^+ = a•a* = { any string of a's of positive length, i.e. excludes λ }
ex: $(ab)^2$ = abab $\neq$ a^2 b^2 , so don't try to use "algebra".
ex: $(a+b)^2$ = (a+b)(a+b) = *aa* or *ab* or *ba* or *bb.*
ex: (a+b)* any string made up of a's and b's.

<u>Examples</u> of regular expressions over **{a, b}** :

- all strings that begin with **a** and end with **b**

  $a\ (a + b)^*\ b$

- all non empty strings of even length

  $(aa + ab + ba + bb)^+$

- all strings with at least one **a**

  $(a + b)^*\ a\ (a + b)^*$

- all strings with at least two **a**'s

  $(a + b)^*\ a\ (a + b)^*\ a\ (a + b)^*$

- all strings of one or more **b**'s with an optional single leading **a**

  $(a + \lambda)\ b^+$

- the language { **ab**, **ba**, **abaa**, **bbb** }

  $ab + ba + abaa + bbb$ or
  $ab\ (\lambda + aa) + b\ (a + bb)$ or
  $(a + bb)\ b + (b + aba)\ a$ or?

Tips:

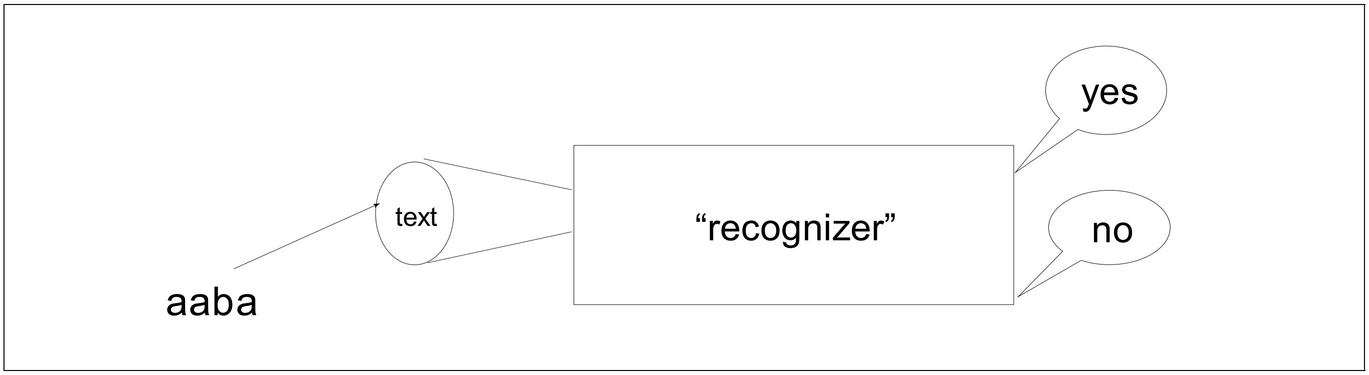
Check the simplest cases
Check for "sins of omission" (forgot some strings)
Check for "sins of commission" (included some unwanted strings)

<u>More examples</u>

Find a regular expression for the following sets of strings on { a, b }:

- All strings with at least two **b**'s.
  $(a + b)^*\ b\ (a + b)^*\ b\ (a + b)^*$
- All strings with exactly two **b**'s.
  $a^*\ b\ a^*\ b\ a^*$
- All strings with at least one **a** and at least one **b**.
  $(a + b)^*\ (ab + ba)\ (a + b)^*$
- All strings which end in a double letter (two **a**'s or two **b**'s).
  $(a + b)^*\ (aa + bb)$
- All strings of even length (includes 0 length).
  $(aa + bb + ab + ba)^*$

Finite Automata: a particular, simplified model of a computing machine, that is a "language recognizer":



A finite automaton (FSA) has five pieces:

1. S = a finite number of states,
2. A = the alphabet,
3. S_i = the **start** state,
4. Y = one or more final or "accept" states, and
5. F = a transition function (mapping) between states, F: S x A ➜ S.

The transition function F is usually presented in one of two ways:

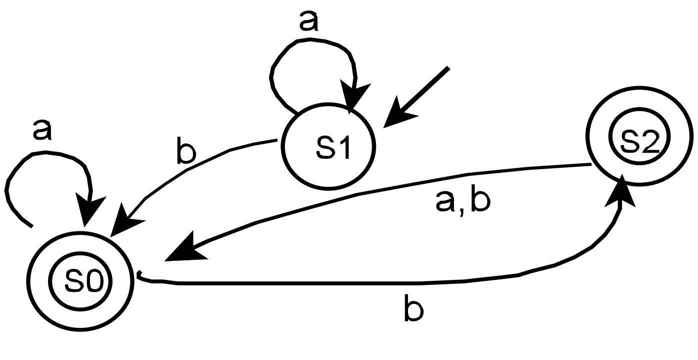
- as a table (called a transition table), or
- as a graph (called a transition diagram).

Transition Table (example):
A= { *a, b* }, S = { s_0 , s_1 , s_2 }, S_i = s_1, Y = { s_0 , s_2 }

current input	F	a	b
current state	s_0	s_0	s_2
	s_1	s_1	s_0
	s_2	s_0	s_0

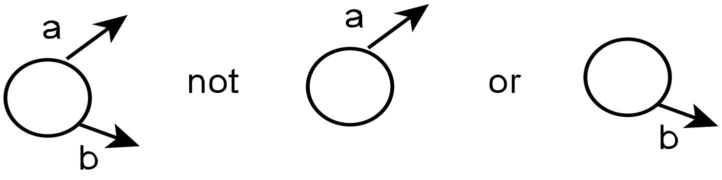
gives the next state ➶

Transition Diagram (example):

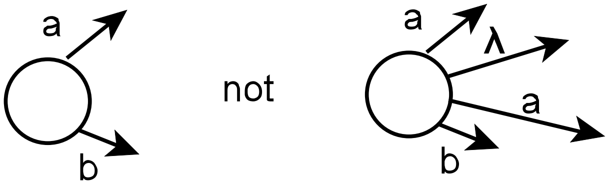


Note that this FSA is:

- *Complete*
  (no undefined
  transitions)



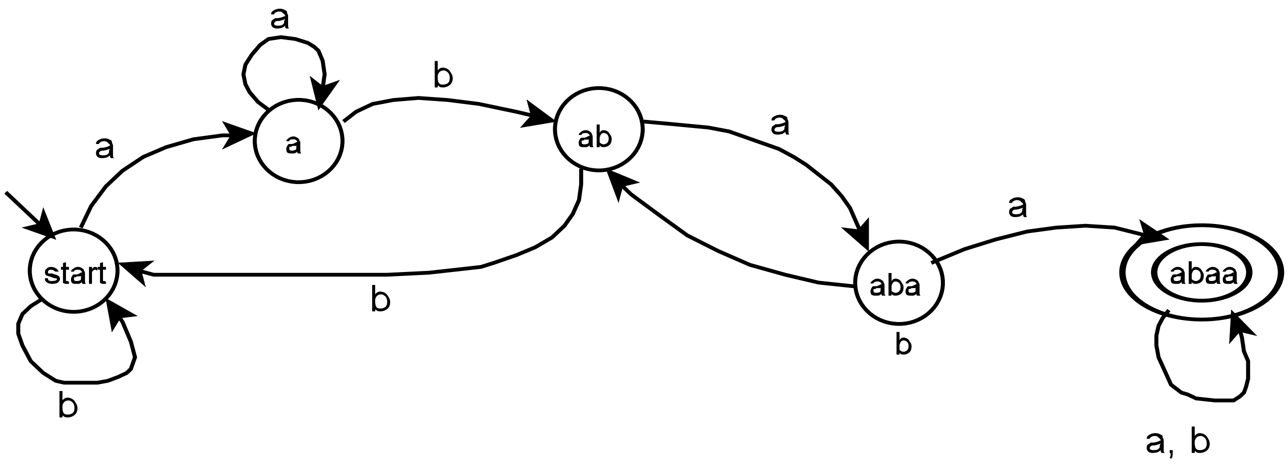
- *Deterministic*
  (no choices)



"Skeleton Method" - a useful solution technique in limited cases:

- The "skeleton" is a sequence of states assuming legal input.
- Construct the skeleton, presume that no additional states will be needed.
- The FSA must be **complete and deterministic:** for A= { a,b }, every state has exactly two arcs leaving it, one labeled "a" and one labeled "b".

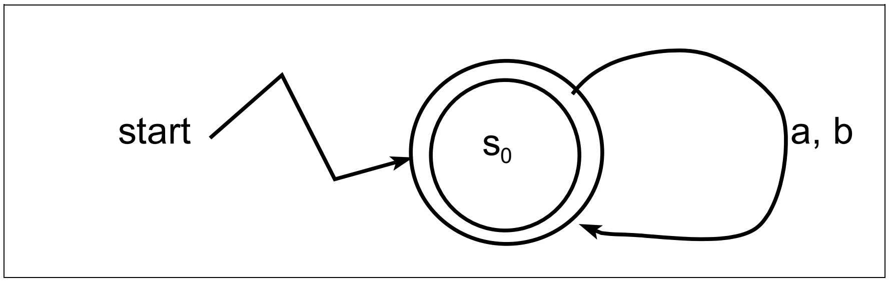
example (skeleton): All strings containing abaa



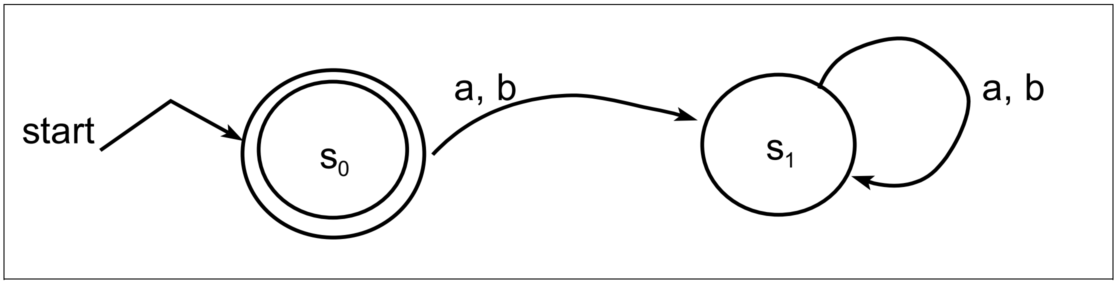
Examples

Assume A= { a, b }. Construct the following automata which:

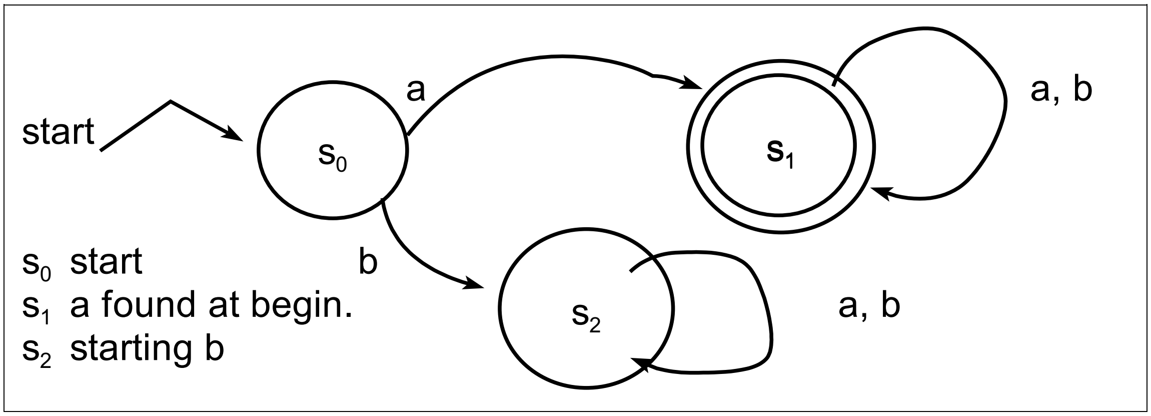
1. Accepts strings of the form **(a+b)***



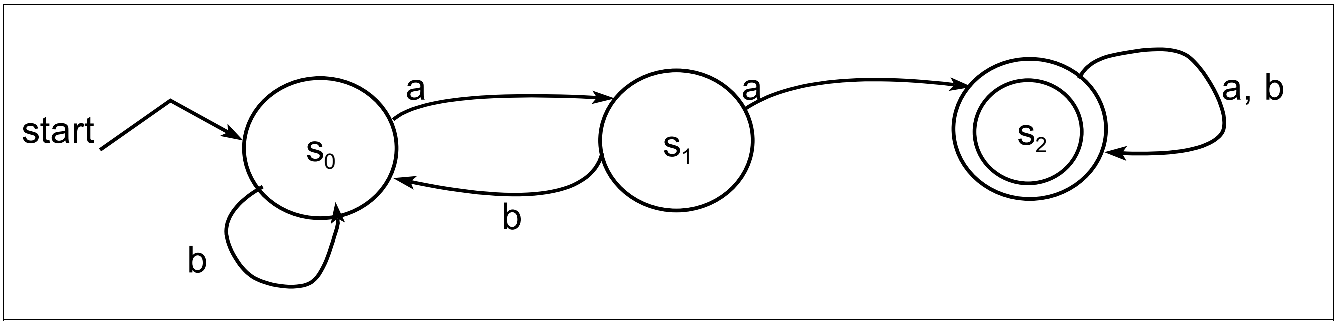
2. Accepts **λ** only.



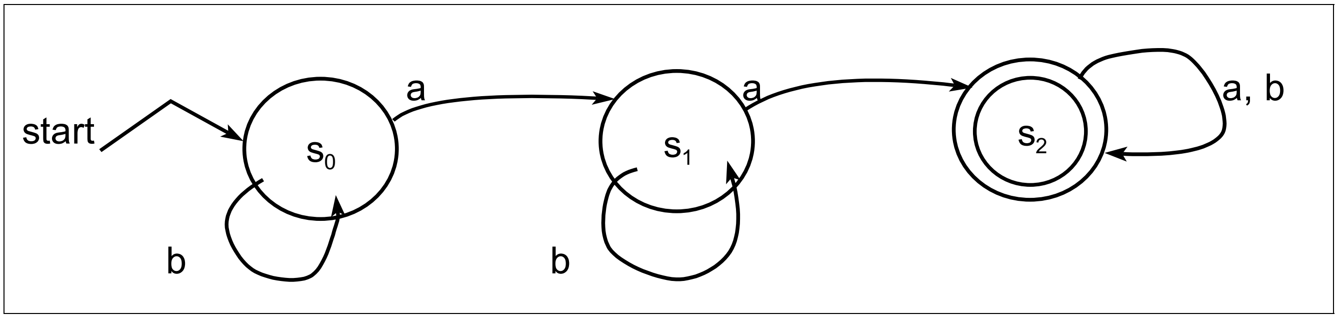
3. Accepts strings which begin with **a**



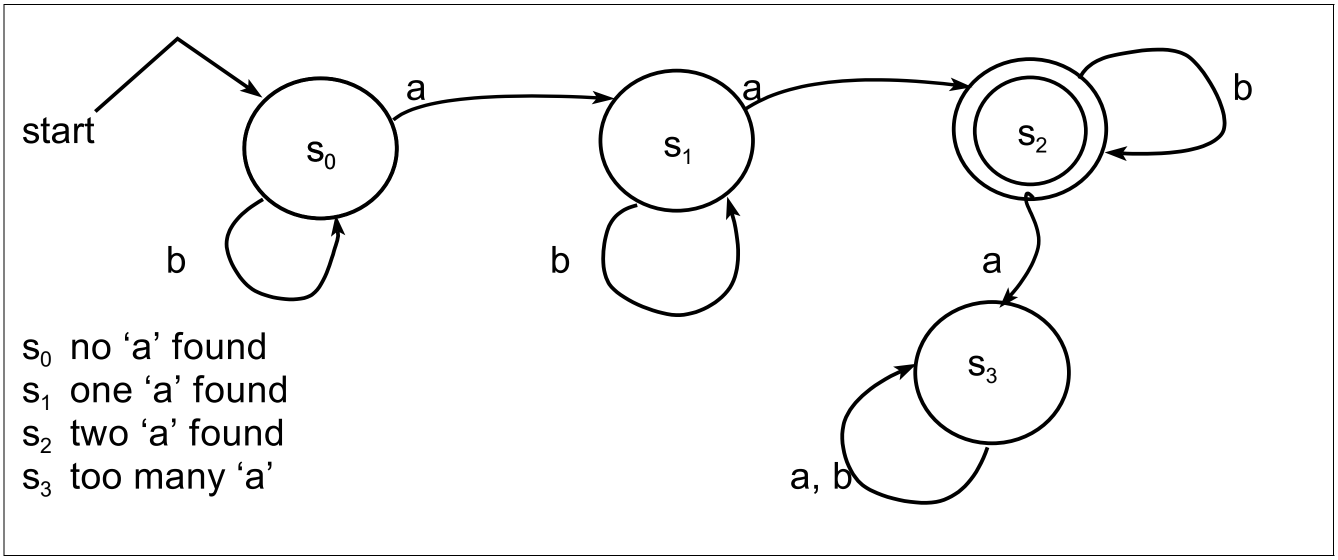
4. Accepts strings containing '**aa**' (skeleton method)



5. All words containing at least two a's



4. All words containing exactly two a's



Equivalence of Regular Expressions and Finite-State Automata

1. For every regular expression “R”, defining a language “L”, there is a FSA “M” recognizing exactly L.

2. For every FSA “M”, recognizing a language “L”, there is a regular expression “R” matching all the strings of L and no others.
   (we will prove this later)

Question: is there a FSA that can recognize { λ, ab, aabb, aaabbb, . . . } ??
Answer: No, because we need to “remember” how many a’s have been seen to verify that there are as many b’s. Since an FSA can only have a finite number of states there cannot be enough states to count the a’s.

We need a more powerful kind of recognizer... that is, a *grammar*.