Chapter 28 Lecture

physics

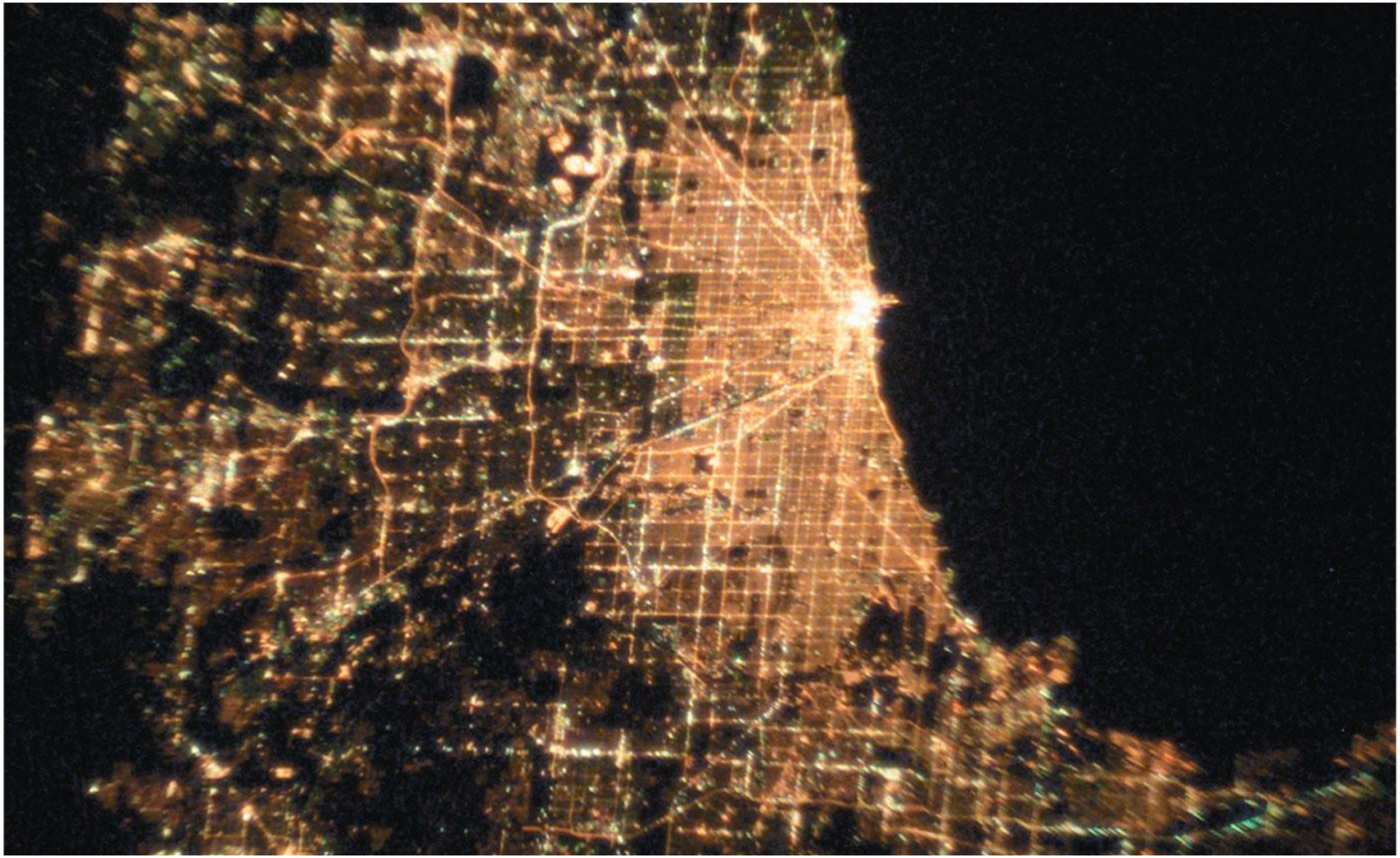
FOR SCIENTISTS AND ENGINEERS

a strategic approach

THIRD EDITION

randall d.
knight

Chapter 28 The Electric Potential



Chapter Goal: To calculate and use the electric potential and electric potential energy.

Electric Energy

Energy allows things to happen. You want your lights to light, your computer to compute, and your stereo to keep your neighbors awake. All these require energy—*electric* energy.

This is the first of two chapters that explore electric energy and its connection to electric forces and fields.

Lightning is a dramatic example of the transformation of electric energy into light, sound, and thermal energy.

Chapter 28 Preview

You'll learn to calculate the electric potential energy of charged particles and to solve problems using conservation of mechanical energy.

There's a close connection between electric potential energy and gravitational potential energy because both forces obey inverse-square laws.

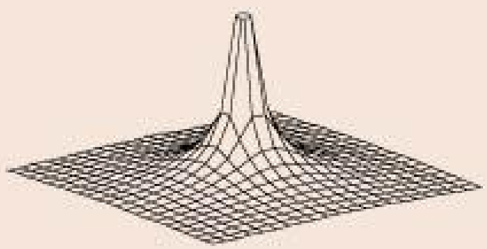
The Electric Potential

Just as source charges create an electric field, they also create an **electric potential.** A charge moving in an electric potential has an electric potential energy.

The unit of electric potential is the **volt,** perhaps the most well known of all electrical units. A voltmeter reads the *potential difference* between two points.

Calculating Electric Potential

You'll learn how to calculate the electric potential for several important charge distributions.



Elevation graph

Equipotential surfaces

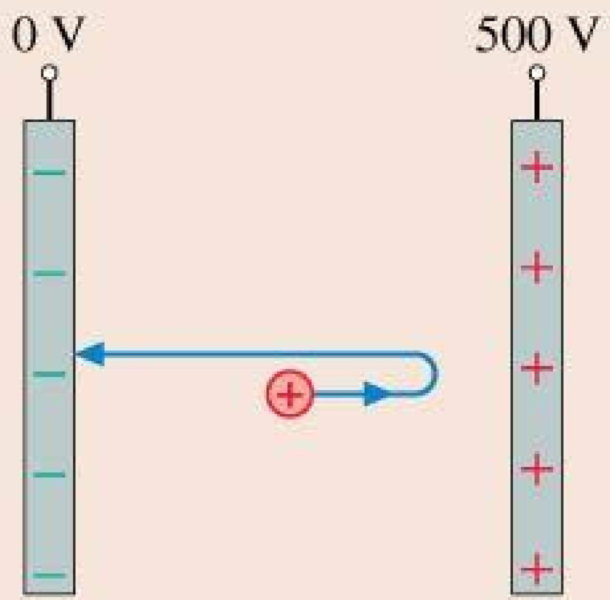
You'll also learn to use several different representations of the electric potential.

Chapter 28 Preview

Using Electric Potential

Charged particles *accelerate* as they move through a potential difference.

You'll learn to use the electric potential and a conservation of energy problem-solving strategy to solve problems about the motion of charged particles.



Sources of Electric Potential

In practice, electric potential is created by separating positive and negative charges—an idea we'll explore more thoroughly in Chapter 29.

A battery is the most common source of electric potential. As you'll learn, its *voltage* is the potential difference between separated charges—the plus and minus terminals.

Chapter 28 Reading Quiz

Reading Question 28.1

The electric potential energy of a system of two point charges is proportional to

A. The distance between the two charges.

B. The square of the distance between the two charges.

C. The inverse of the distance between the two charges.

D. The inverse of the square of the distance between the two charges.

Reading Question 28.1

The electric potential energy of a system of two point charges is proportional to

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B. The square of the distance between the two charges.

C. **The inverse of the distance between the two charges.**

D. The inverse of the square of the distance between the two charges.

Reading Question 28.2

What are the units of *potential difference*?

A. Amperes.
B. Potentiometers.
C. Farads.
D. Volts.
E. Henrys.

Reading Question 28.2

What are the units of *potential difference*?

A. Amperes.

B. Potentiometers.

C. Farads.

D. **Volts.**

E. Henrys.

Reading Question 28.3

New units of the electric field were introduced in this chapter. They are:

A. V/C.
B. N/C.
C. V/m.
D. J/m^2.
E. Ω/m.

Reading Question 28.3

New units of the electric field were introduced in this chapter. They are:

A. V/C.

B. N/C.

C. **V/m.**

D. J/m^2.

E. Ω/m.

Reading Question 28.4

Which of the following statements about equipotential surfaces is true?

A. Tangent lines to equipotential surfaces are always parallel to the electric field vectors.
B. Equipotential surfaces are surfaces that have the same value of potential energy at every point.
C. Equipotential surfaces are surfaces that have the same value of potential at every point.
D. Equipotential surfaces are always parallel planes.
E. Equipotential surfaces are real physical surfaces that exist in space.

Reading Question 28.4

Which of the following statements about equipotential surfaces is true?

A. Tangent lines to equipotential surfaces are always parallel to the electric field vectors.

B. Equipotential surfaces are surfaces that have the same value of potential energy at every point.

C. **Equipotential surfaces are surfaces that have the same value of potential at every point.**

D. Equipotential surfaces are always parallel planes.

E. Equipotential surfaces are real physical surfaces that exist in space.

Reading Question 28.5

The electric potential inside a capacitor

A. Is constant.

B. Increases linearly from the negative to the positive plate.

C. Decreases linearly from the negative to the positive plate.

D. Decreases inversely with distance from the negative plate.

E. Decreases inversely with the square of the distance from the negative plate.

Reading Question 28.5

The electric potential inside a capacitor

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Chapter 28 Content, Examples, and QuickCheck Questions

Energy

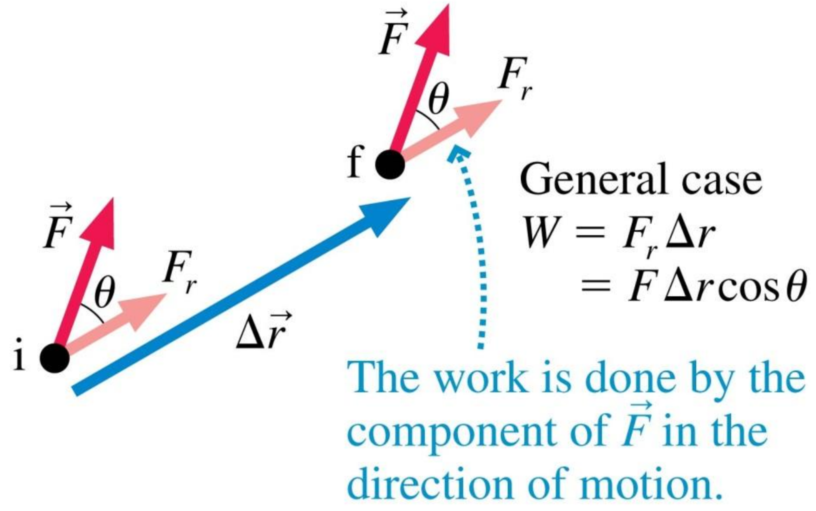
- The kinetic energy of a system, K, is the sum of the kinetic energies $K_i = 1/2m_iv_i^2$ of all the particles in the system.
- The potential energy of a system, U, is the *interaction energy* of the system.
- The change in potential energy, ΔU, is -1 times the work done by the interaction forces:

$$\Delta U = U_{\mathrm{f}} - U_{\mathrm{i}} = -W_{\text{interaction forces}}$$

- If all of the forces involved are *conservative forces* (such as gravity or the electric force) then the total energy $K + U$ is *conserved*; it does not change with time.

Work Done by a Constant Force

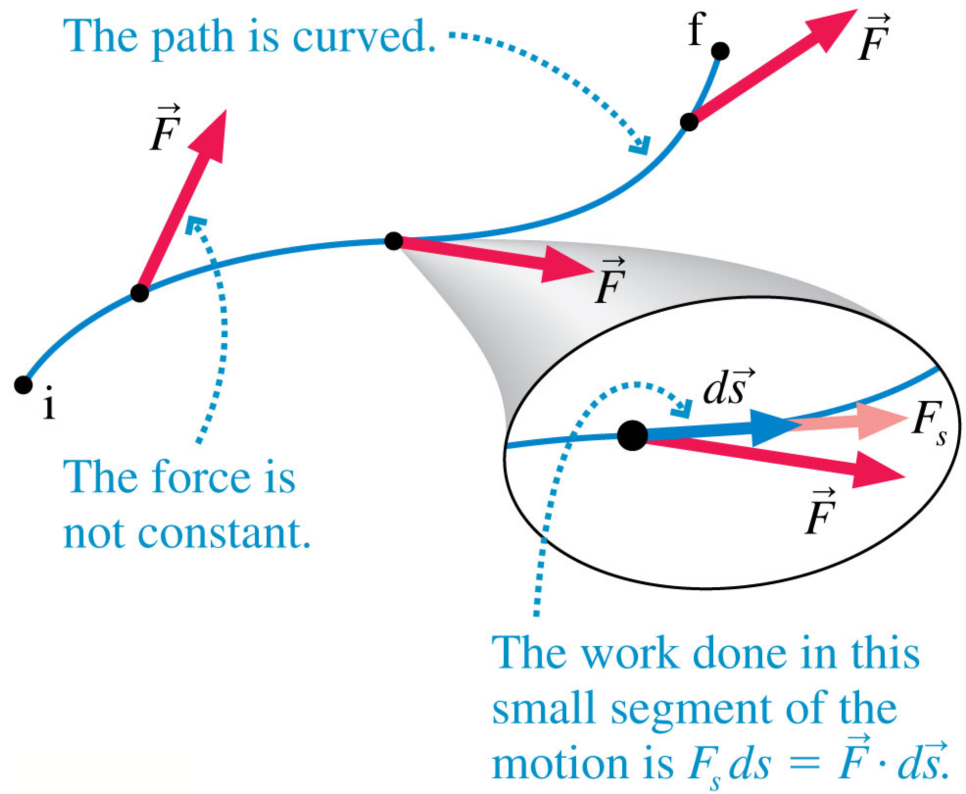
- Recall that the work done by a constant force depends on the angle θ between the force F and the displacement Δr.



- If $\theta = 0°$, then $W = F\Delta r$.
- If $\theta = 90°$, then $W = 0$.
- If $\theta = 180°$, then $W = -F\Delta r$.

Work

If the force is *not* constant or the displacement is *not* along a linear path, we can calculate the work by dividing the path into many small segments.



$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_i^f \vec{F}\cdot d\vec{s}$$

Gravitational Potential Energy

- Every conservative force is associated with a potential energy.
- In the case of gravity, the work done is:

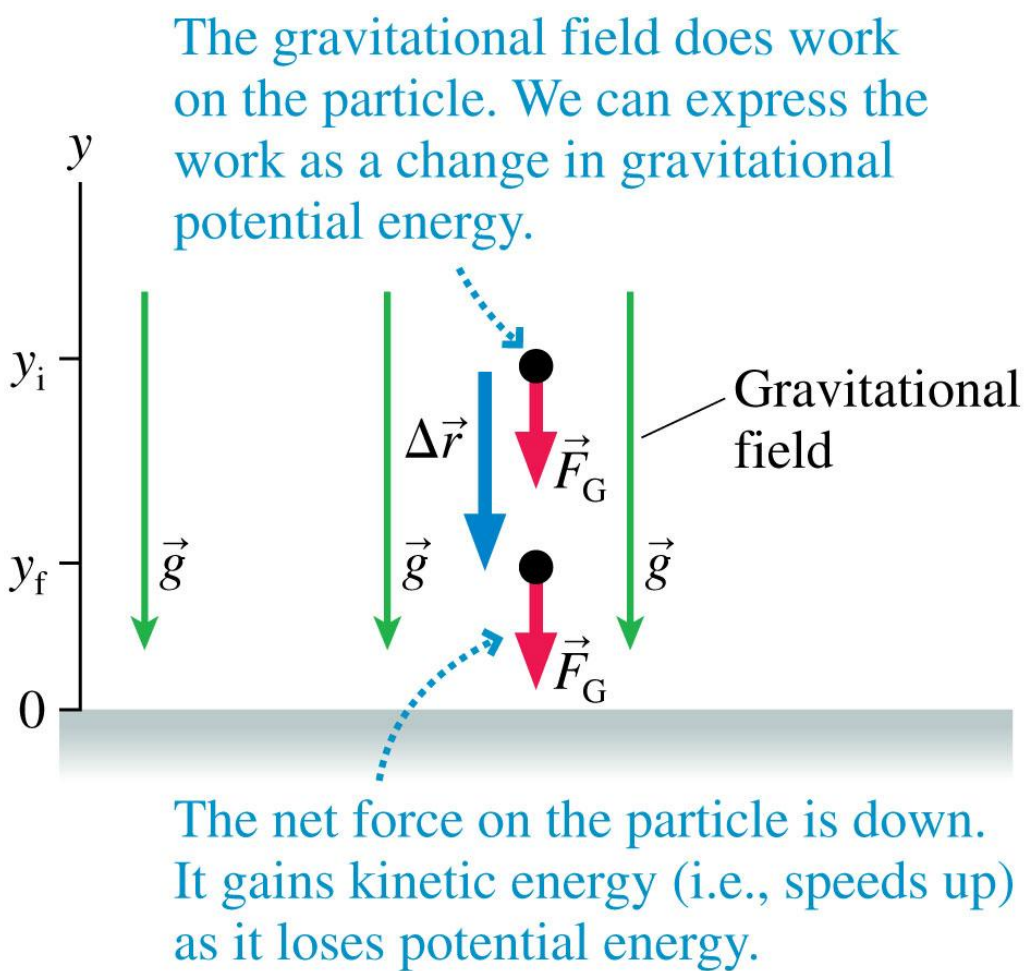
$$W_{\text{grav}} = mgy_{\text{i}} - mgy_{\text{f}}$$

- The change in gravitational potential energy is:

$$\Delta U_{\text{grav}} = -W_{\text{grav}}$$

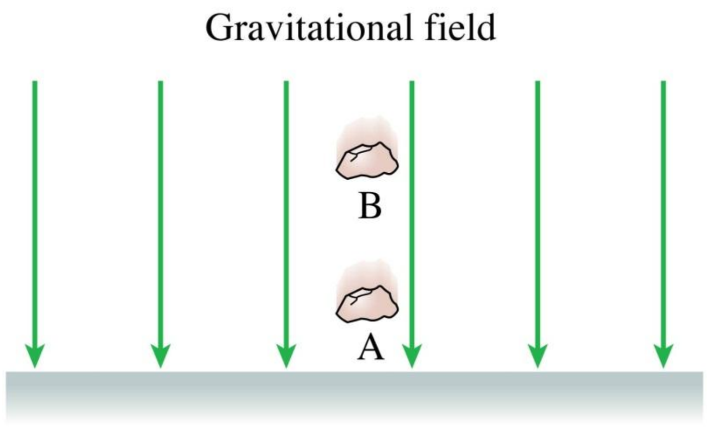
where

$$U_{grav} = U_0 + mgy$$



QuickCheck 28.1

Two rocks have equal mass. Which has more gravitational potential energy?



A. Rock A.

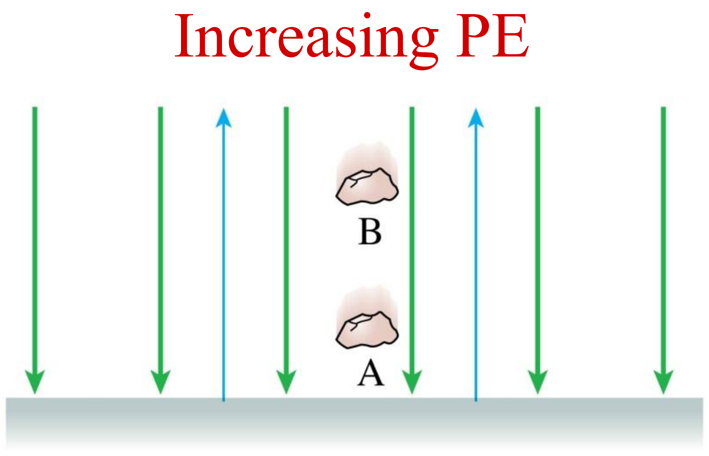
B. Rock B.

C. They have the same potential energy.

D. Both have zero potential energy.

QuickCheck 28.1

Two rocks have equal mass. Which has more gravitational potential energy?



A. Rock A.

B. **Rock B.**

C. They have the same potential energy.

D. Both have zero potential energy.

Electric Potential Energy in a Uniform Field

- A positive charge q inside a capacitor speeds up as it "falls" toward the negative plate.
- There is a constant force $F = qE$ in the direction of the displacement.
- The work done is:

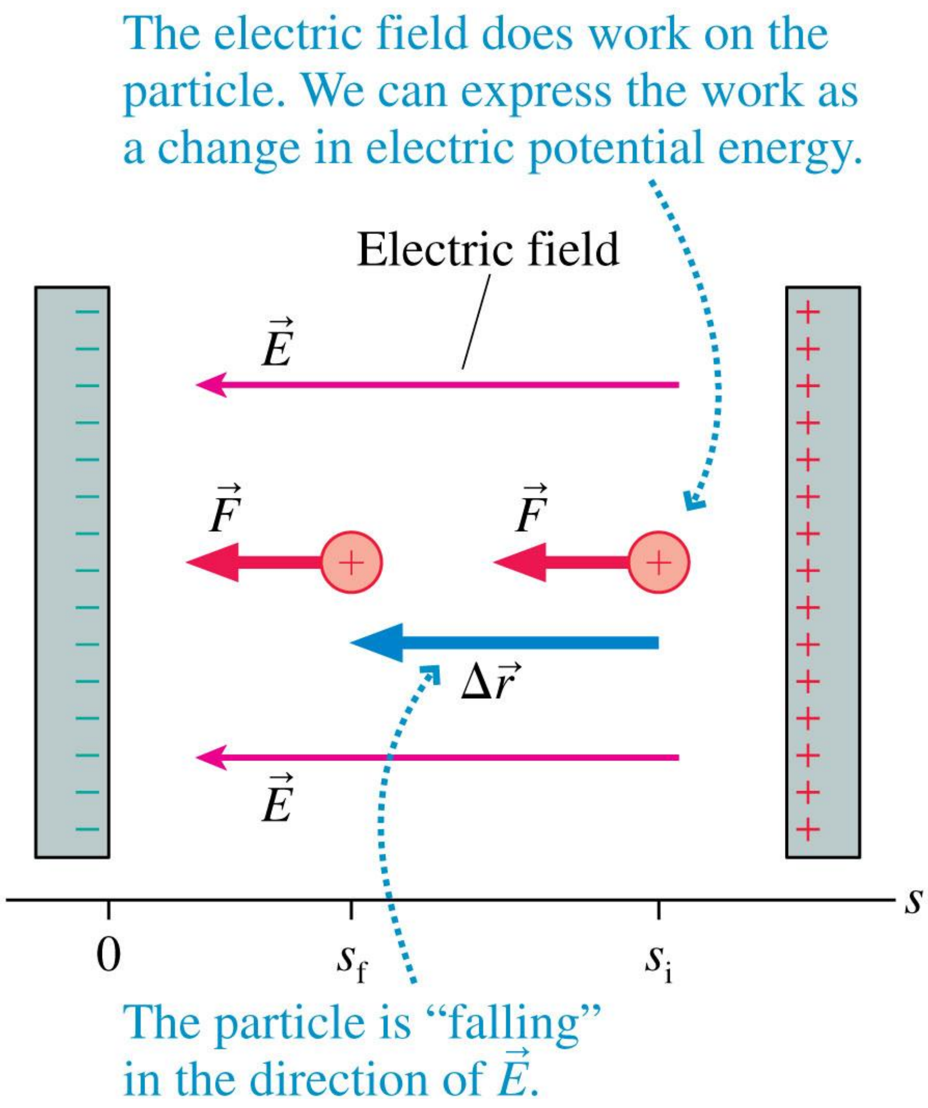
$$W_{\text{elec}} = qEs_{\text{i}} - qEs_{\text{f}}$$

- The change in **electric potential energy** is:

$$\Delta U_{\text{elec}} = -W_{\text{elec}}$$

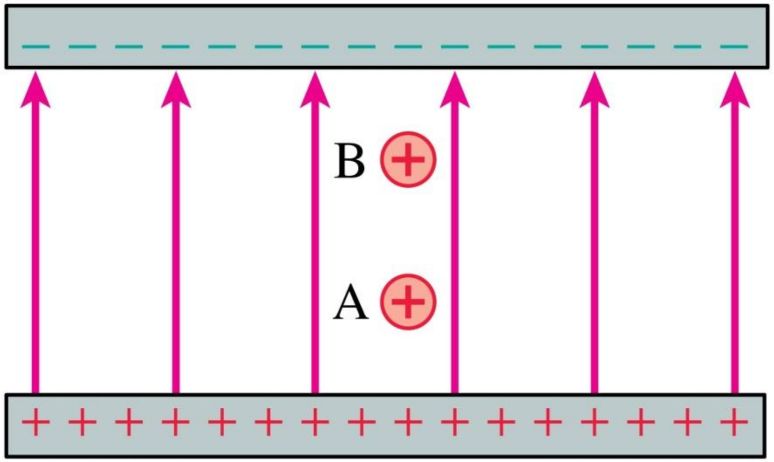
where

$$U_{\text{elec}} = U_0 + qEs$$



QuickCheck 28.2

Two positive charges are equal. Which has more electric potential energy?



A. Charge A.

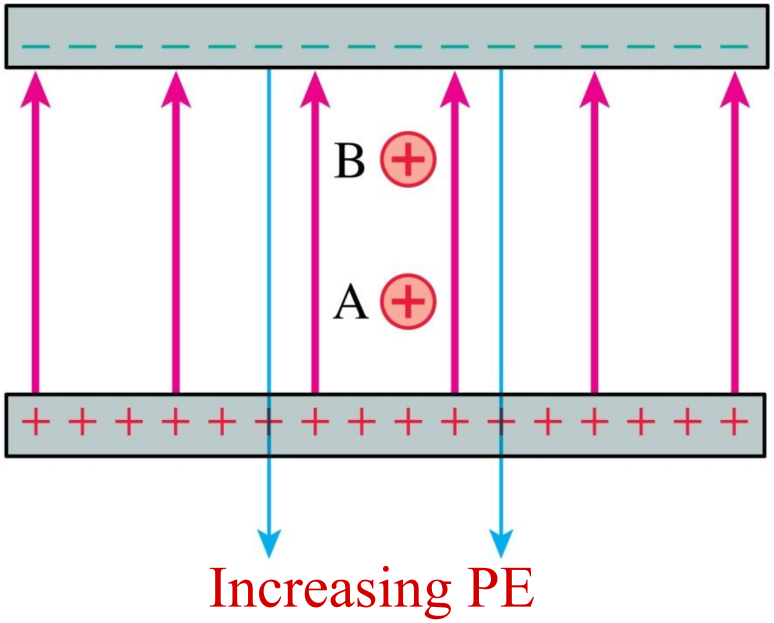
B. Charge B.

C. They have the same potential energy.

D. Both have zero potential energy.

QuickCheck 28.2

Two positive charges are equal. Which has more electric potential energy?



A. **Charge A.**

B. Charge B.

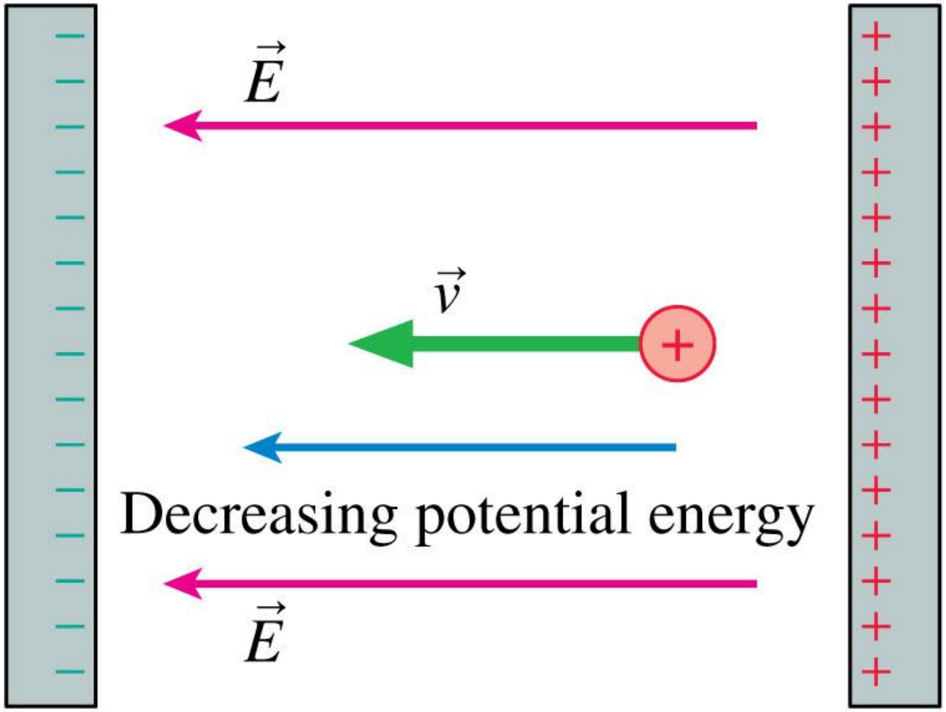
C. They have the same potential energy.

D. Both have zero potential energy.

Electric Potential Energy in a Uniform Field

$$U_{\text{elec}} = U_0 + qEs$$

A positively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.

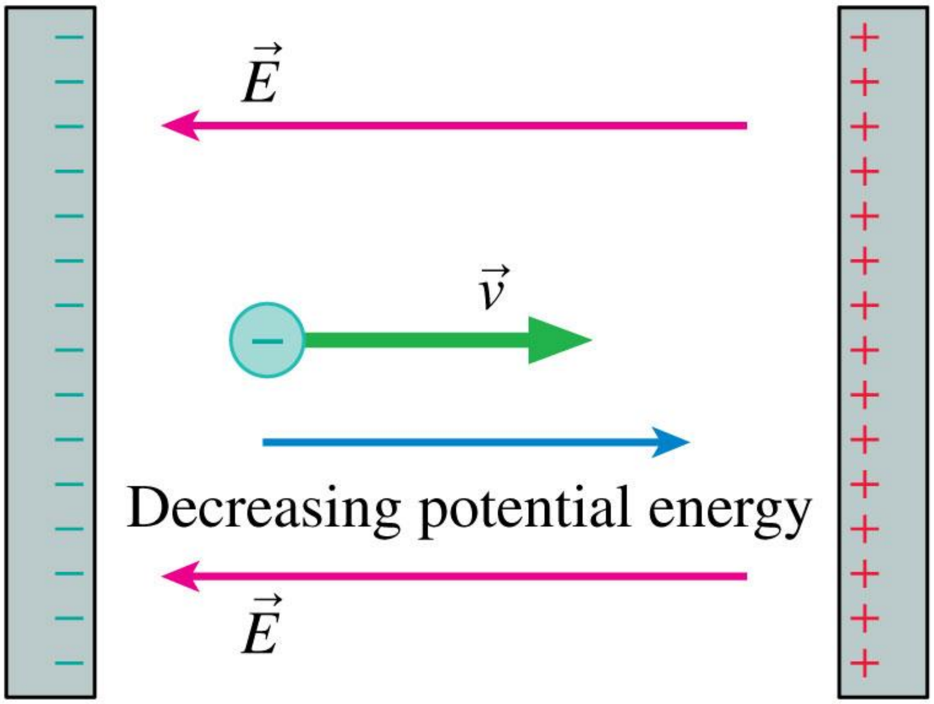


The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

Electric Potential Energy in a Uniform Field

$$U_{\text{elec}} = U_0 + qEs$$

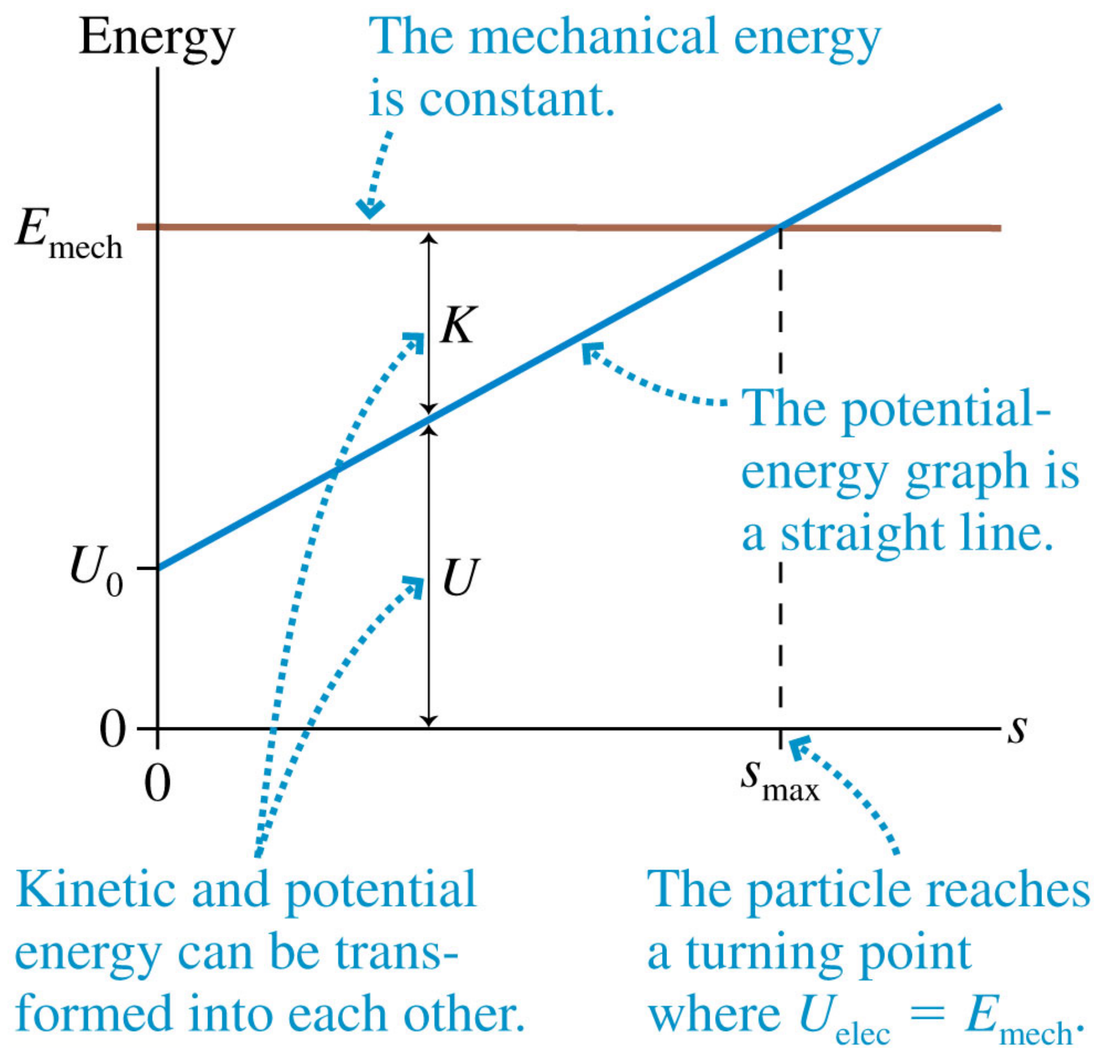
A negatively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.



The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

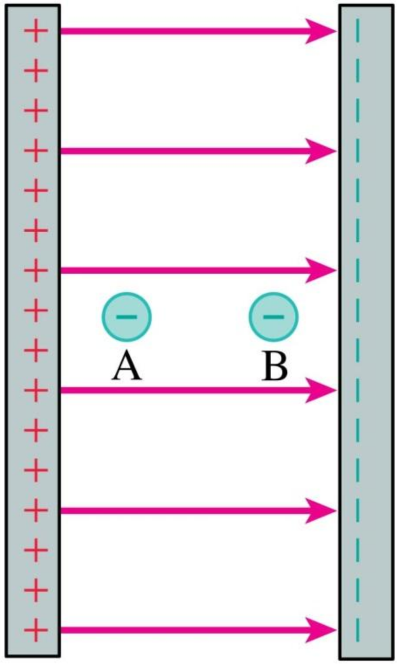
Electric Potential Energy in a Uniform Field

- The figure shows the **energy diagram** for a positively charged particle in a uniform electric field.
- The potential energy increases linearly with distance, but the total mechanical energy E_{mech} is fixed.



QuickCheck 28.3

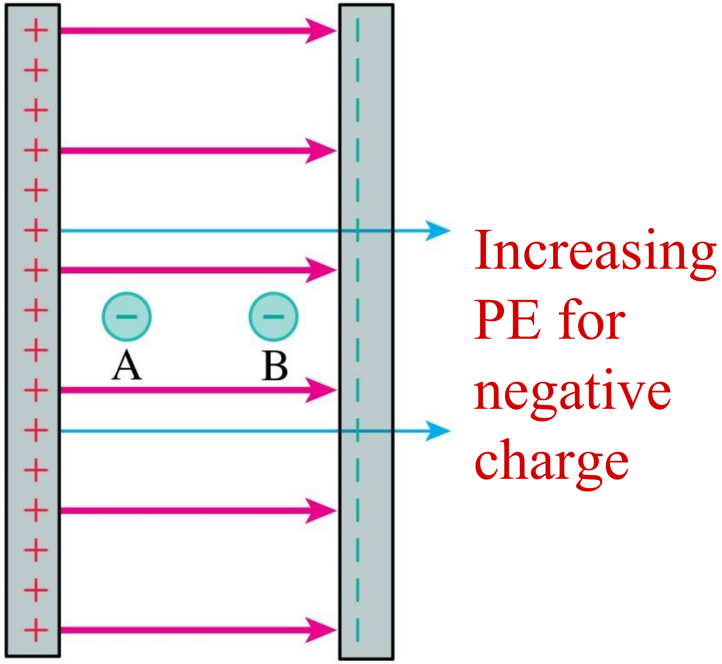
Two negative charges are equal. Which has more electric potential energy?



A. Charge A.
B. Charge B.
C. They have the same potential energy.
D. Both have zero potential energy.

QuickCheck 28.3

Two negative charges are equal. Which has more electric potential energy?



A. Charge A.

B. Charge B.

C. They have the same potential energy.

D. Both have zero potential energy.

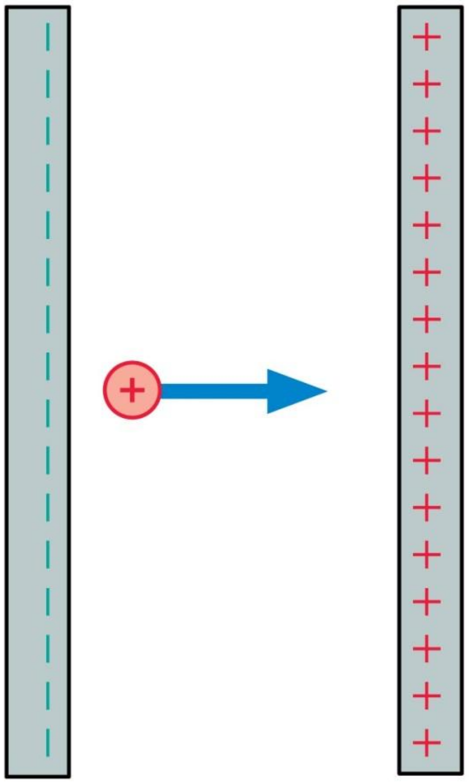
QuickCheck 28.4

A positive charge moves as shown. Its kinetic energy

A. Increases.

B. Remains constant.

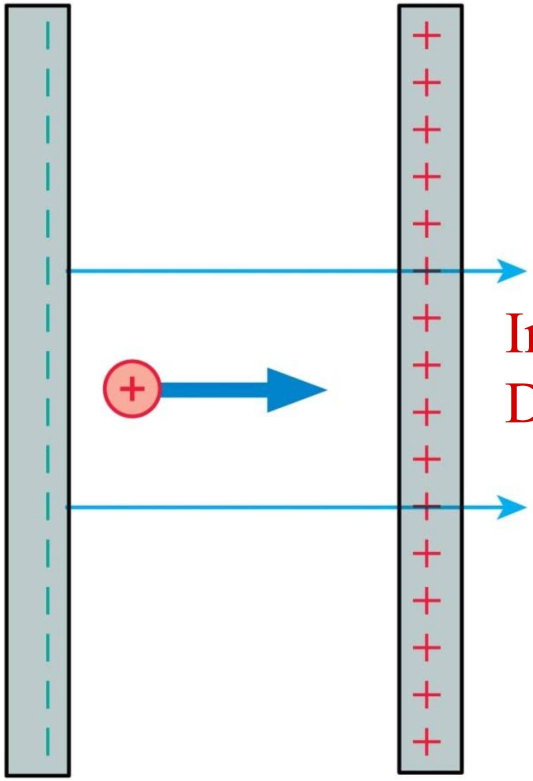
C. Decreases.



QuickCheck 28.4

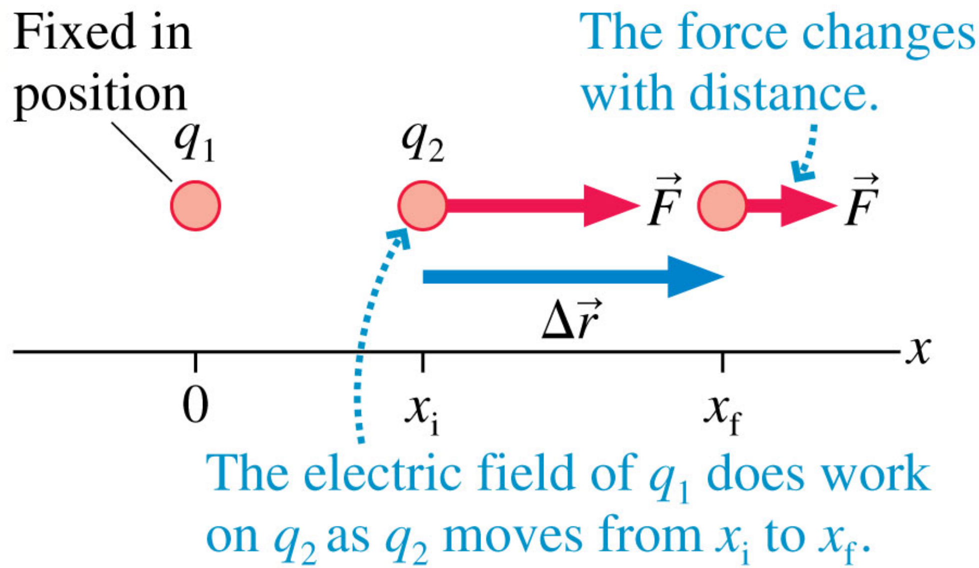
A positive charge moves as shown. Its kinetic energy

A. Increases.
B. Remains constant.
✔C. **Decreases.**



The Potential Energy of Two Point Charges

- Consider two like charges q_1 and q_2.
- The electric field of q_1 *pushes* q_2 as it moves from x_i to x_f.
- The work done is:



$$W_{\text{elec}} = \int_{x_i}^{x_f} F_{1\text{ on }2}\, dx = \int_{x_i}^{x_f} \frac{Kq_1q_2}{x^2}\, dx = Kq_1q_2 \frac{-1}{x}\bigg|_{x_i}^{x_f} = -\frac{Kq_1q_2}{x_f} + \frac{Kq_1q_2}{x_i}$$

- The change in electric potential energy of the system is $\Delta U_{\text{elec}} = -W_{\text{elec}}$ if:

$$U_{\text{elec}} = \frac{Kq_1q_2}{x}$$

The Potential Energy of Two Point Charges

Consider two point charges, q_1 and q_2, separated by a distance r. The electric potential energy is

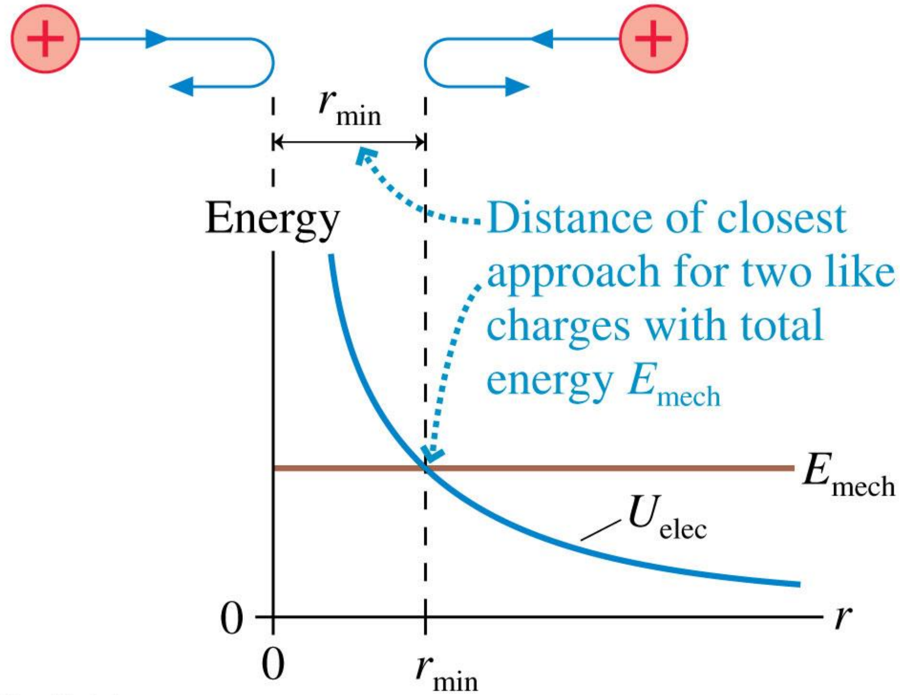
$$U_{\text{elec}} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

This is explicitly the energy of *the system*, not the energy of just q_1 or q_2.

Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

The Potential Energy of Two Point Charges

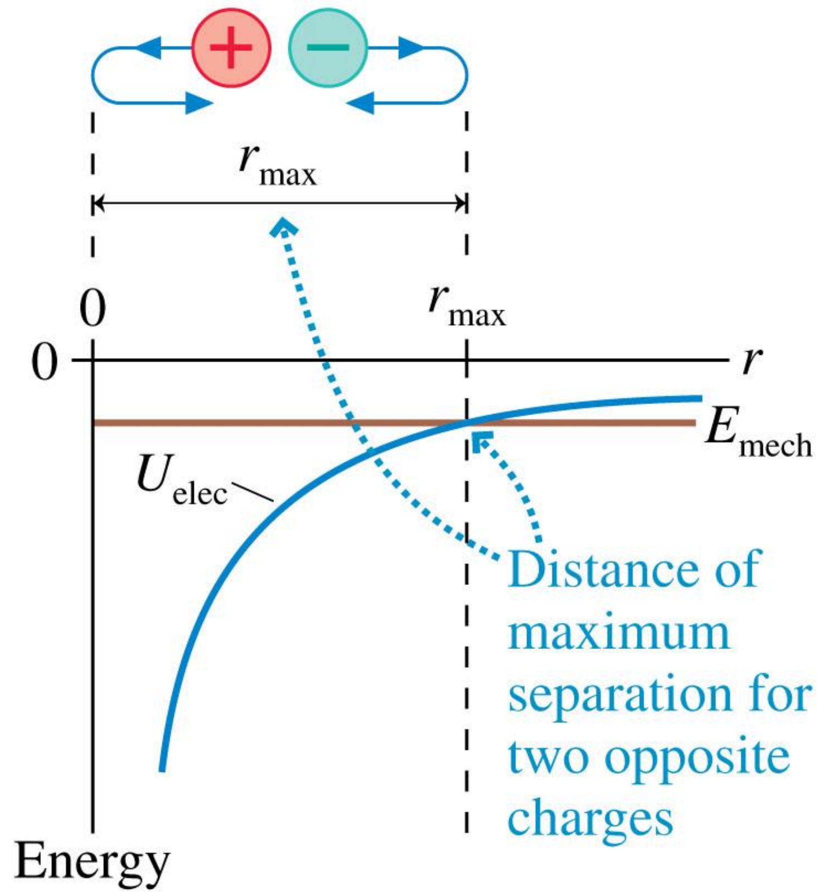
- Two like charges approach each other.
- Their total energy is E_{mech} • 0.
- They gradually slow down until the distance separating them is r_{min}.
- This is the *distance of closest approach.*



$$U_{\text{elec}} = \frac{Kq_1q_2}{r}$$

The Potential Energy of Two Point Charges

- Two opposite charges are shot apart from one another with equal and opposite momenta.
- Their total energy is E_{mech} • 0.
- They gradually slow down until the distance separating them is r_{max}.
- This is their *maximum separation.*



$$U_{elec} = \frac{Kq_1q_2}{r}$$

QuickCheck 28.5

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do:

A. A positive potential energy becomes more positive.

B. A positive potential energy becomes less positive.

C. A negative potential energy becomes more negative.

D. A negative potential energy becomes less negative.

E. A positive potential energy becomes a negative potential energy.

QuickCheck 28.5

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do:

A. A positive potential energy becomes more positive.

B. A positive potential energy becomes less positive.

C. **A negative potential energy becomes more negative.**

D. A negative potential energy becomes less negative.

E. A positive potential energy becomes a negative potential energy.

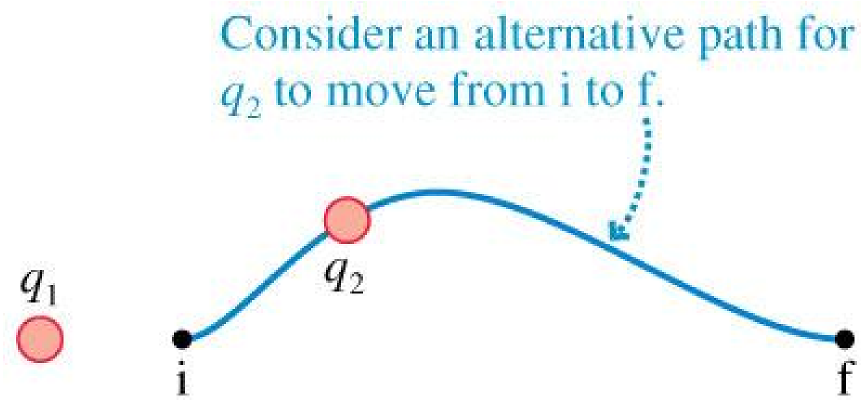
$$U_{\text{elec}} = \frac{Kq_1q_2}{r}$$

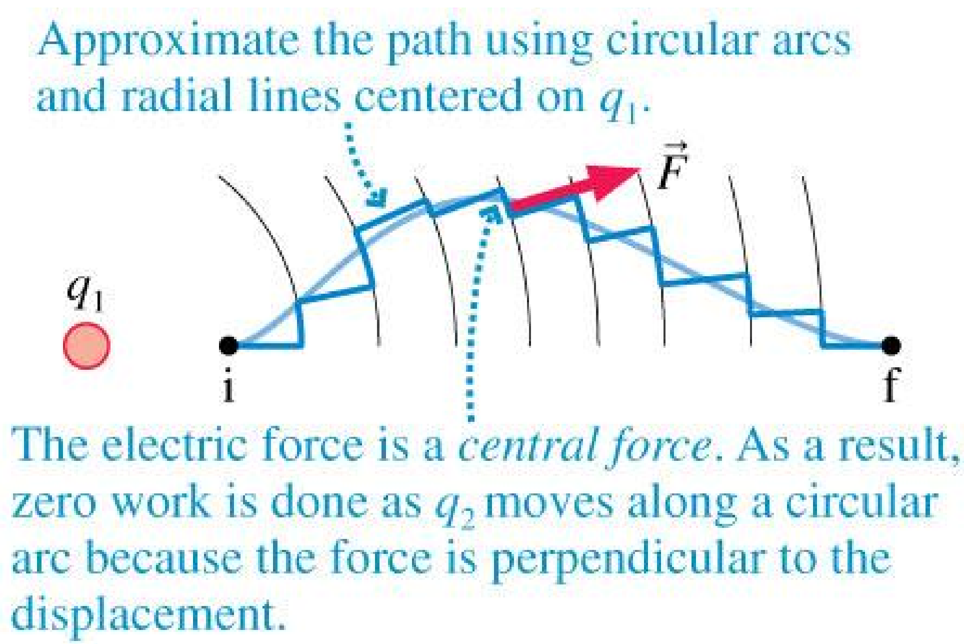
Opposite signs, so U is Negative.

U increases in magnitude as r decreases.

The Electric Force Is a Conservative Force

- Any path away from q_1 can be approximated using circular arcs and radial lines.
- All the work is done along the radial line segments, which is equivalent to a straight line from i to f.
- Therefore the work done by the electric force depends only on initial and final position, not the path followed.





Example 28.2 Approaching a Charged Sphere

EXAMPLE 28.2 Approaching a charged sphere

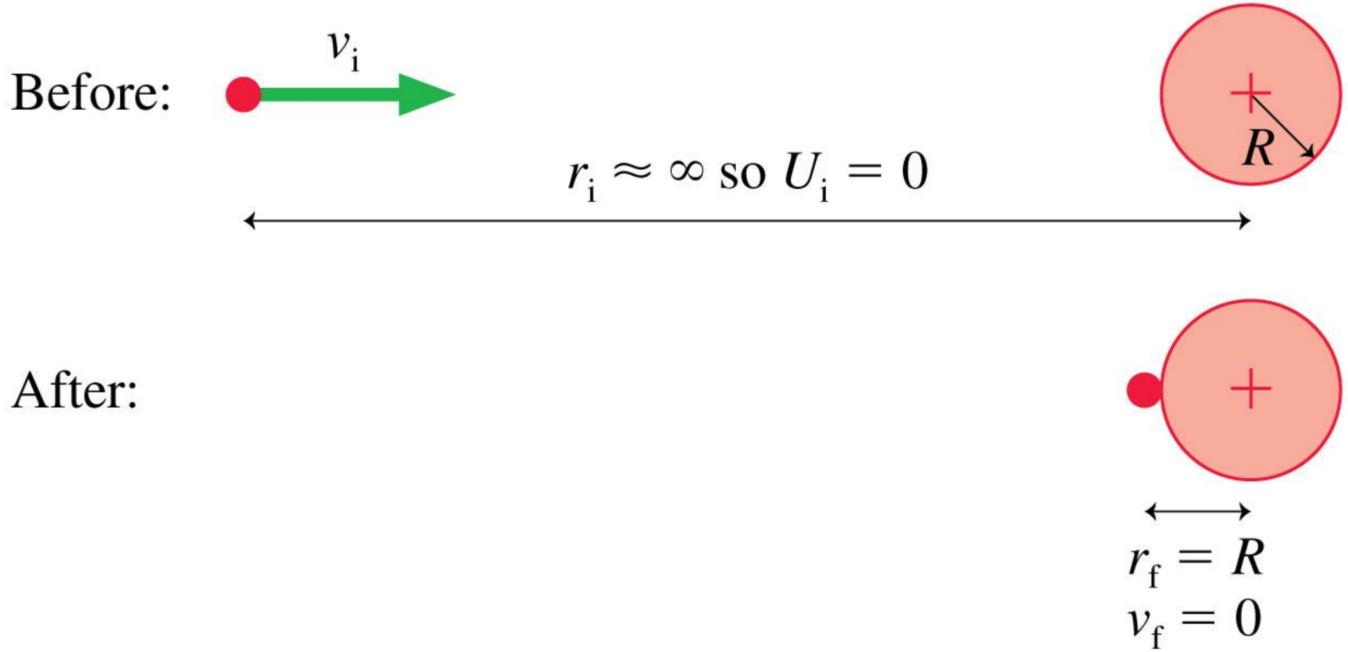
A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to $+100$ nC. What initial speed must the proton have to just reach the surface of the glass?

MODEL Energy is conserved. The glass sphere can be treated as a charged particle, so the potential energy is that of two point charges. The proton starts "far away," which we interpret as sufficiently far to make $U_{\mathrm{i}} \approx 0$.

Example 28.2 Approaching a Charged Sphere

EXAMPLE 28.2 Approaching a charged sphere

VISUALIZE The figure below shows the before-and-after pictorial representation. To "just reach" the glass sphere means that the proton comes to rest, $v_f = 0$, as it reaches $r_f = 0.50$ mm, the *radius* of the sphere.



Example 28.2 Approaching a Charged Sphere

EXAMPLE 28.2 Approaching a charged sphere

SOLVE Conservation of energy is

$$0 + \frac{{}_{\mathrm{p}}\quad \mathrm{sphere}}{r} = \frac{1}{2}mv_{\mathrm{i}}^2 + 0$$

The proton charge is $_{\mathrm{p}}$ With this, we can solve for the proton's initial speed:

$$v_{\mathrm{i}} = \sqrt{\frac{2Keq_{\mathrm{sphere}}}{mr}} = 1.86 \times 10^7 \text{ m/s}$$

Example 28.3 Escape Velocity

EXAMPLE 28.3 Escape velocity

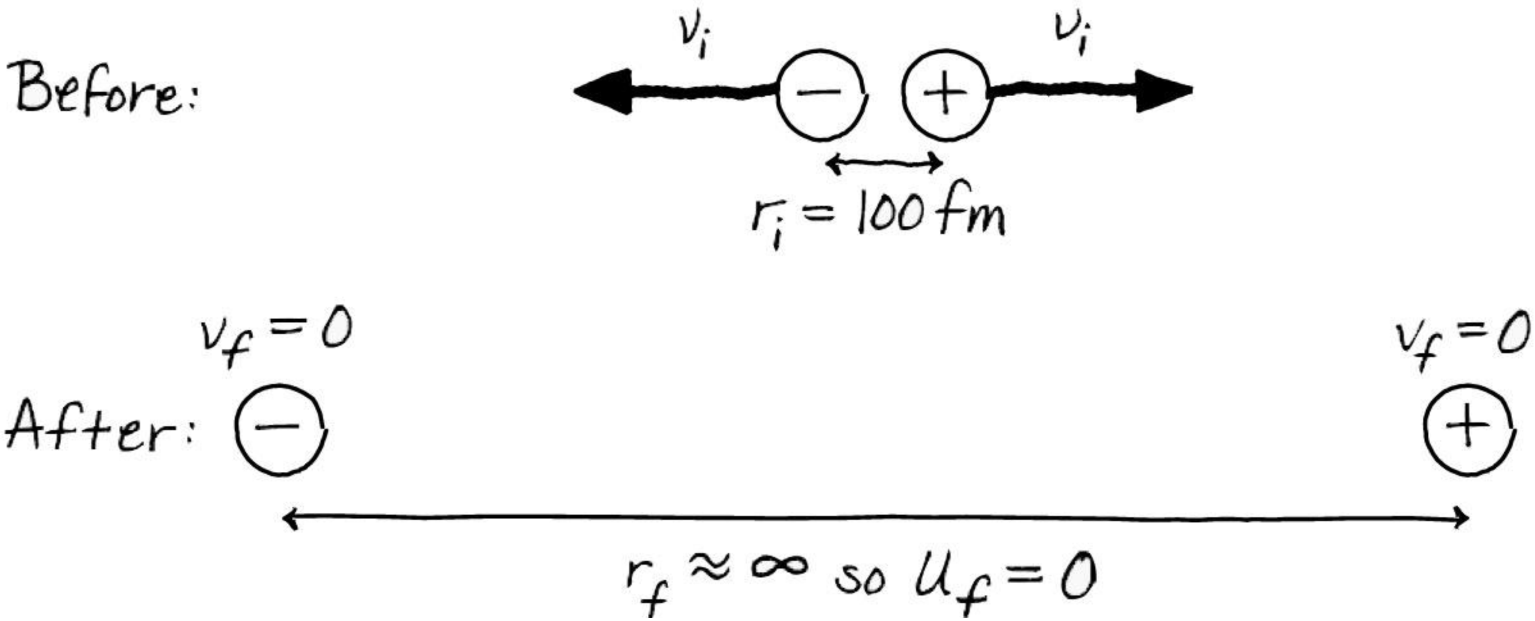
An interaction between two elementary particles causes an electron and a positron (a positive electron) to be shot out back to back with equal speeds. What minimum speed must each have when they are 100 fm apart in order to escape each other?

MODEL Energy is conserved. The particles end "far apart," which we interpret as sufficiently far to make $U_f \approx 0$.

Example 28.3 Escape Velocity

EXAMPLE 28.3 Escape velocity

VISUALIZE The figure below shows the before-and-after pictorial representation. The minimum speed to escape is the speed that allows the particles to reach $r_f = \infty$ with $v_f = 0$.



Example 28.3 Escape Velocity

EXAMPLE 28.3 **Escape velocity**

SOLVE Here it is essential to interpret U_{elec} as the potential energy of the electron + positron system. Similarly, K is the *total* kinetic energy of the system. The electron and the positron, with equal masses and equal speeds, have equal kinetic energies. Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + 0 + 0 = \frac{1}{2}mv_i^2 + \frac{1}{2}mv_i^2 + \frac{Kq_e q_p}{r_i} = mv_i^2 - \frac{Ke^2}{r_i}$$

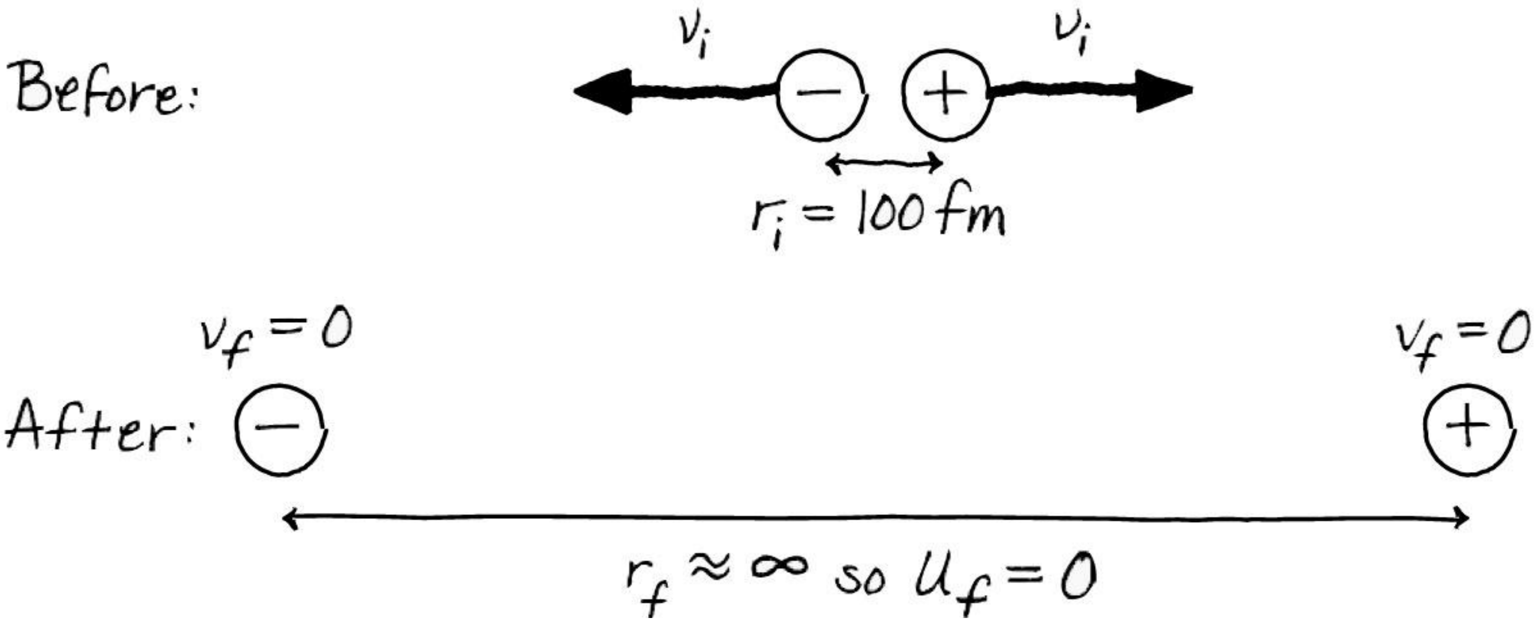
Using $r_i = 100 \text{ fm} = 1.0 \times 10^{-13}$ m, we can calculate the minimum initial speed to be

$$v_i = \sqrt{\frac{Ke^2}{mr_i}} = 5.0 \times 10^7 \text{ m/s}$$

Example 28.3 Escape Velocity

EXAMPLE 28.3 **Escape velocity**

ASSESS v_i is a little more than 10% the speed of light, just about the limit of what a "classical" calculation can predict. We would need to use the theory of relativity if v_i were any larger.



The Potential Energy of Multiple Point Charges

Consider more than two point charges, the potential energy is the sum of the potential energies due to all pairs of charges:

$$U_{\text{elec}} = \sum_{i<j} \frac{K q_i q_j}{r_{ij}}$$

where r_{ij} is the distance between q_i and q_j.
The summation contains the $i \bullet j$ restriction to ensure that each pair of charges is counted only once.

Example 28.4 Launching an Electron

EXAMPLE 28.4 **Launching an electron**

Three electrons are spaced 1.0 mm apart along a vertical line. The outer two electrons are fixed in position.

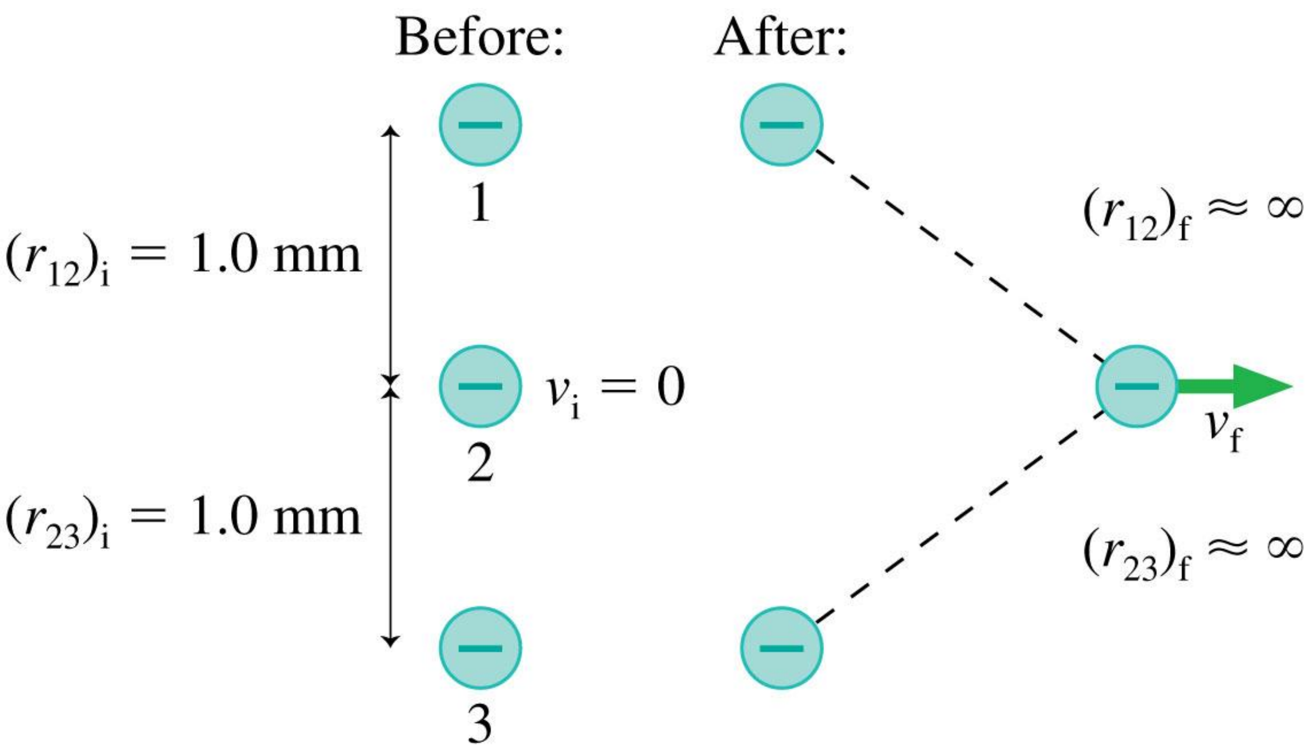
a. Is the center electron at a point of stable or unstable equilibrium?
b. If the center electron is displaced horizontally by a small distance, what will its speed be when it is very far away?

MODEL Energy is conserved. The outer two electrons don't move, so we don't need to include the potential energy of their interaction.

Example 28.4 Launching an Electron

EXAMPLE 28.4 Launching an electron

VISUALIZE The figure below shows the situation.



Example 28.4 Launching an Electron

EXAMPLE 28.4 **Launching an electron**

SOLVE a. The center electron is in equilibrium *exactly* in the center because the two electric forces on it balance. But if it moves a little to the right or left, no matter how little, then the horizontal components of the forces from both outer electrons will push the center electron farther away. This is an unstable equilibrium for horizontal displacements, like being on the top of a hill.

Example 28.4 Launching an Electron

EXAMPLE 28.4 Launching an electron

b. A small displacement will cause the electron to move away. If the displacement is only infinitesimal, the initial conditions are $(r_{12})_i = (r_{23})_i = 1.0$ mm and $v_i = 0$. "Far away" is interpreted as $r_f \to \infty$, where $U_f \approx 0$. There are now *two* terms in the potential energy, so conservation of energy $K_f + U_f = K_i + U_i$ gives

$$\frac{1}{2}mv_f^2 + 0 + 0 = 0 + \left[\frac{Kq_1q_2}{(r_{12})_i} + \frac{Kq_2q_3}{(r_{23})_i}\right]$$

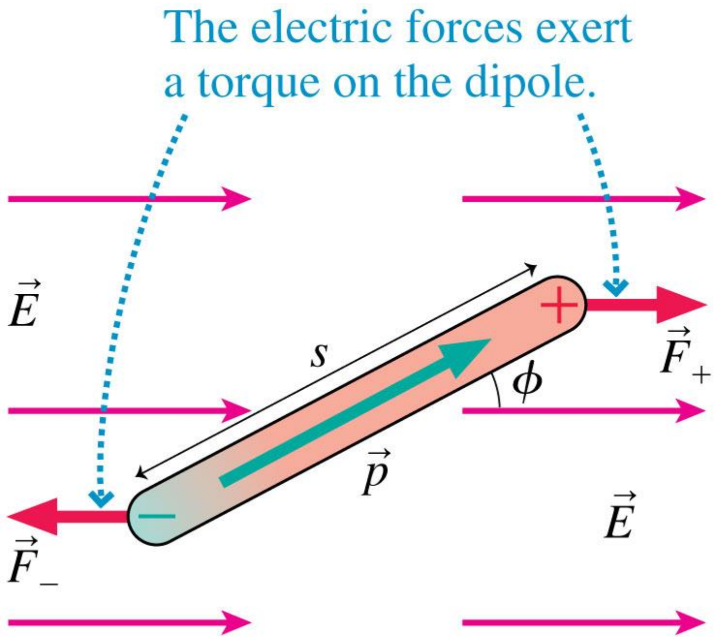
$$= \left[\frac{Ke^2}{(r_{12})_i} + \frac{Ke^2}{(r_{23})_i}\right]$$

This is easily solved to give

$$v_f = \sqrt{\frac{2}{m}\left[\frac{Ke^2}{(r_{12})_i} + \frac{Ke^2}{(r_{23})_i}\right]} = 1000 \text{ m/s}$$

The Potential Energy of a Dipole

- Consider a dipole in a uniform electric field.
- The forces F_+ and F_- exert a torque on the dipole.
- The work done is:



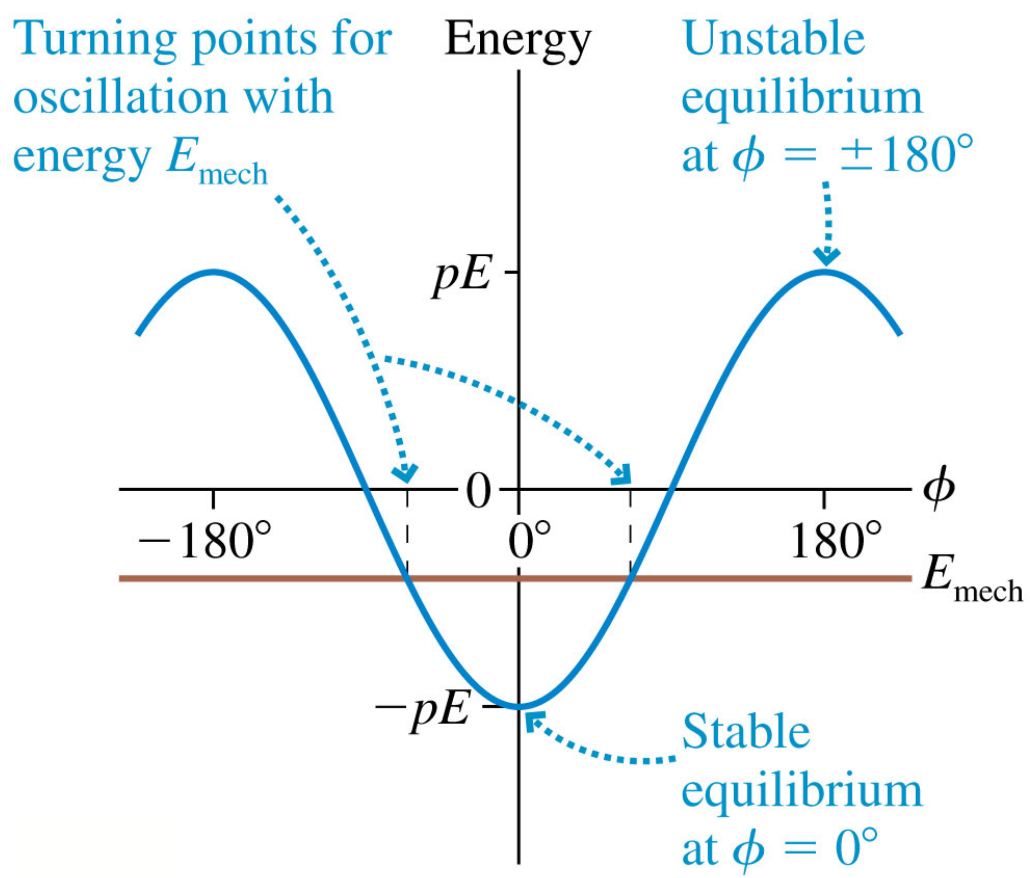
$$W_{\text{elec}} = -pE\int_{\phi_i}^{\phi_f} \sin\phi \, d\phi = pE\cos\phi_f - pE\cos\phi_i$$

- The change in electric potential energy of the system is $\Delta U_{\text{elec}} = -W_{\text{elec}}$ if:

$$\boxed{U_{\text{dipole}} = -pE\cos\phi = -\vec{p}\cdot\vec{E}}$$

The Potential Energy of a Dipole

- The potential energy of a dipole is $\varphi = 0°$ minimum at where the dipole is aligned with the electric field.
- A frictionless dipole with mechanical energy E_{mech} will oscillate back and forth between turning points on either side of $\varphi = 0°$.



$$U_{\text{dipole}} = -pE\cos\phi = -\vec{p} \cdot \vec{E}$$

Example 28.5 Rotating a Molecule

EXAMPLE 28.5 Rotating a molecule

The water molecule is a permanent electric dipole with dipole moment 6.2×10^{-30} C m. A water molecule is aligned in an electric field with field strength 1.0×10^{7} N/C. How much energy is needed to rotate the molecule 90°?

MODEL The molecule is at the point of minimum energy. It won't spontaneously rotate 90°. However, an external force that supplies energy, such as a collision with another molecule, can cause the water molecule to rotate.

Example 28.5 Rotating a Molecule

EXAMPLE 28.5 **Rotating a molecule**

SOLVE The molecule starts at $\phi_i = 0°$ and ends at $\phi_f = 90°$. The increase in potential energy is

$$\Delta U_{dipole} = U_f - U_i = -pE\cos 90° - (-pE\cos 0°)$$
$$= pE = 6.2 \times 10^{-23}\ \text{J}$$

This is the energy needed to rotate the molecule 90°.

ASSESS ΔU_{dipole} is significantly less than $k_B T$ at room temperature. Thus collisions with other molecules can easily supply the energy to rotate the water molecules and keep them from staying aligned with the electric field.

The Electric Potential

- We define the electric potential V (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

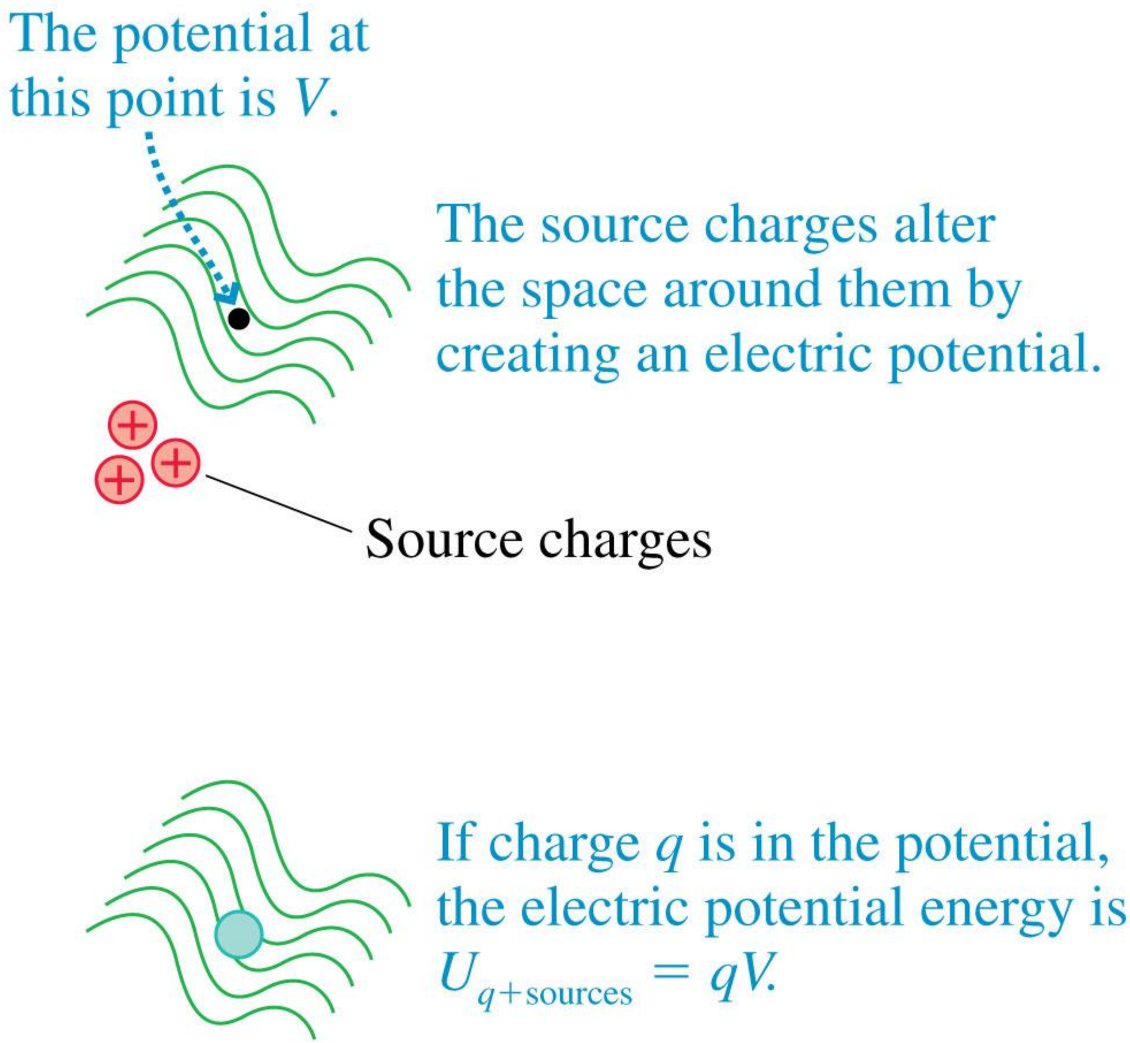
- The unit of electric potential is the joule per coulomb, which is called the volt V:

$$1 \text{ volt} = 1 \text{ V} \equiv 1 \text{ J/C}$$

This battery is a source of *electric potential*. The electric potential difference between the + and − sides is 1.5 V.

The Electric Potential

- Test charge q is used as a probe to determine the electric potential, but the value of V is *independent of* q.
- **The electric potential, like the electric field, is a property of the source charges.**

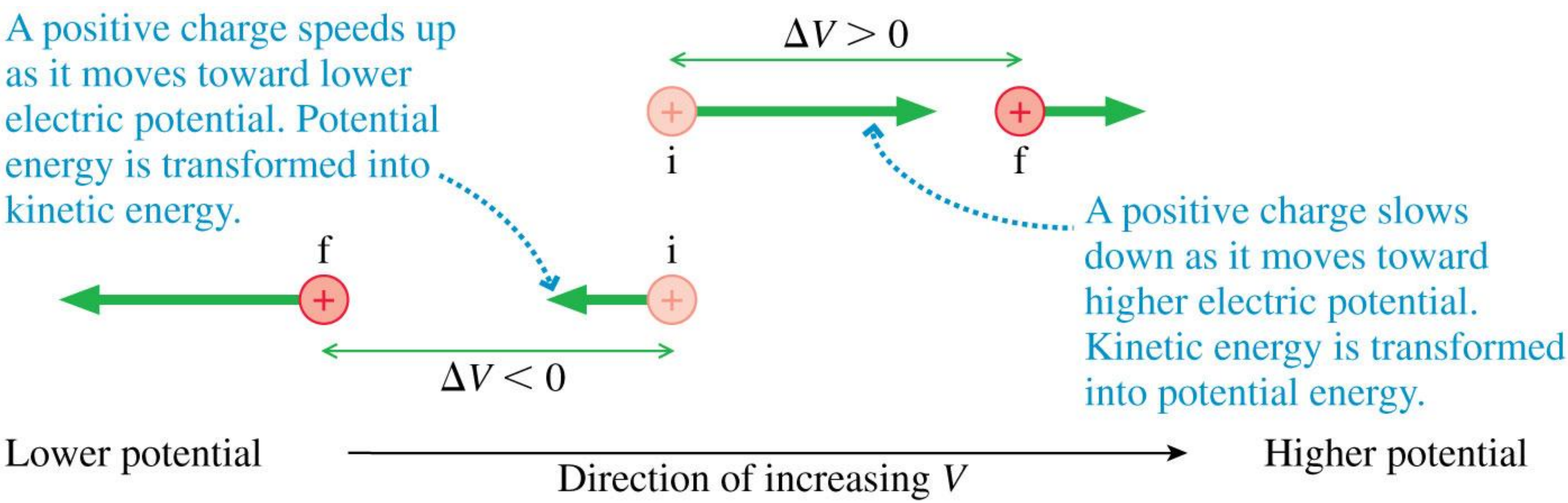


Using the Electric Potential

As a charged particle moves through a changing electric potential, energy is conserved:

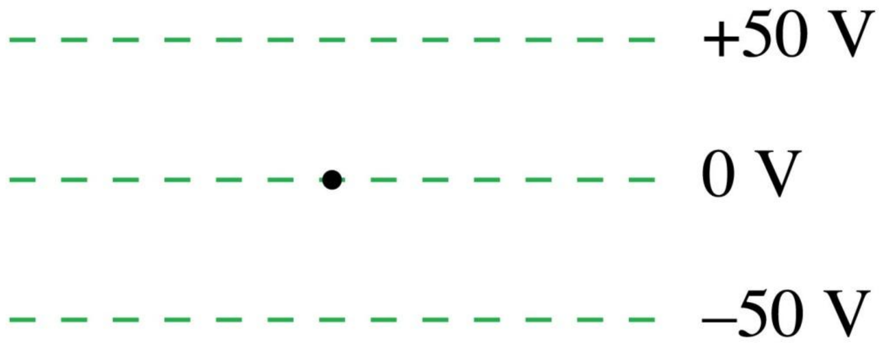
$$K_f + qV_f = K_i + qV_i$$

	Electric potential	
	Increasing ($\Delta V > 0$)	Decreasing ($\Delta V < 0$)
+ charge	Slows down	Speeds up
− charge	Speeds up	Slows down



QuickCheck 28.6

A proton is released from rest at the dot. Afterward, the proton



A. Remains at the dot.

B. Moves upward with steady speed.

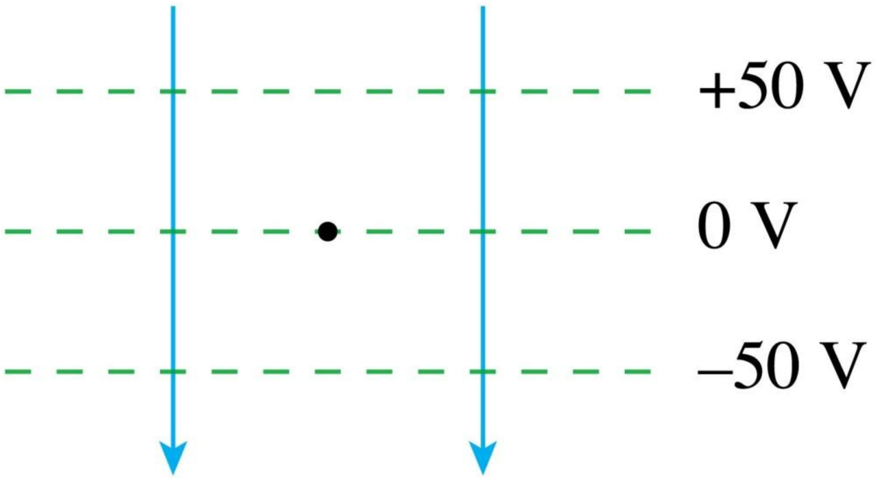
C. Moves upward with an increasing speed.

D. Moves downward with a steady speed.

E. Moves downward with an increasing speed.

QuickCheck 28.6

A proton is released from rest at the dot. Afterward, the proton



Decreasing PE
Increasing KE

A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. **Moves downward with an increasing speed.**

QuickCheck 28.7

If a positive charge is released from rest, it moves in the direction of

A. A stronger electric field.

B. A weaker electric field.

C. Higher electric potential.

D. Lower electric potential.

E. Both B and D.

QuickCheck 28.7

If a positive charge is released from rest, it moves in the direction of

A. A stronger electric field.

B. A weaker electric field.

C. Higher electric potential.

D. **Lower electric potential.**

E. Both B and D.

Problem-Solving Strategy: Conservation of Energy in Charge Interactions

PROBLEM-SOLVING STRATEGY 28.1 **Conservation of energy in charge interactions**

MODEL Check whether there are any dissipative forces that would keep the mechanical energy from being conserved.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

Problem-Solving Strategy: Conservation of Energy in Charge Interactions

PROBLEM-SOLVING STRATEGY 28.1 **Conservation of energy in charge interactions**

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + qV_f = K_i + qV_i$$

- Is the electric potential given in the problem statement? If not, you'll need to use a known potential, such as that of a point charge, or calculate the potential using the procedure given later, in Problem-Solving Strategy 28.2.
- K_i and K_f are the sums of the kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 22

Example 28.6 Moving Through a Potential Difference

EXAMPLE 28.6 Moving through a potential difference

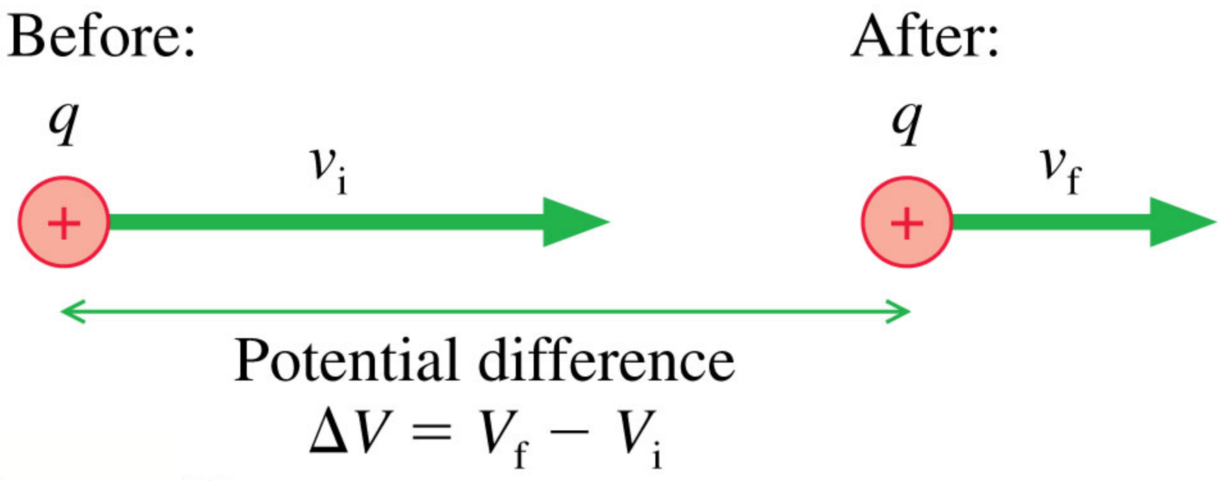
A proton with a speed of 2.0×10^5 m/s enters a region of space in which source charges have created an electric potential. What is the proton's speed after it moves through a potential difference of 100 V? What will be the final speed if the proton is replaced by an electron?

MODEL Energy is conserved. The electric potential determines the potential energy.

Example 28.6 Moving Through a Potential Difference

EXAMPLE 28.6 Moving through a potential difference

VISUALIZE The figure below is a before-and-after pictorial representation of a charged particle moving through a potential difference. A positive charge *slows down* as it moves into a region of higher potential ($K \rightarrow U$). A negative charge *speeds up* ($U \rightarrow K$).



Example 28.6 Moving Through a Potential Difference

EXAMPLE 28.6 **Moving through a potential difference**

SOLVE The potential energy of charge q is $U = qV$. Conservation of energy, now expressed in terms of the electric potential V, is $K_f + qV_f = K_i + qV_i$, or

$$K_f = K_i - q\,\Delta V$$

where $\Delta V = V_f - V_i$ is the potential difference through which the particle moves. In terms of the speeds, energy conservation is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - q\,\Delta V$$

We can solve this for the final speed:

$$v_f = \sqrt{v_i^2 - \frac{2q}{m}\Delta V}$$

Example 28.6 Moving Through a Potential Difference

EXAMPLE 28.6 **Moving through a potential difference**

For a proton, with $q = e$, the final speed is

$$(v_{\mathrm{f}})_{\mathrm{p}} = \sqrt{(2.0 \times 10^5\ \mathrm{m/s})^2 - \frac{2(1.60 \times 10^{-19}\ \mathrm{C})(100\ \mathrm{V})}{1.67 \times 10^{-27}\ \mathrm{kg}}}$$

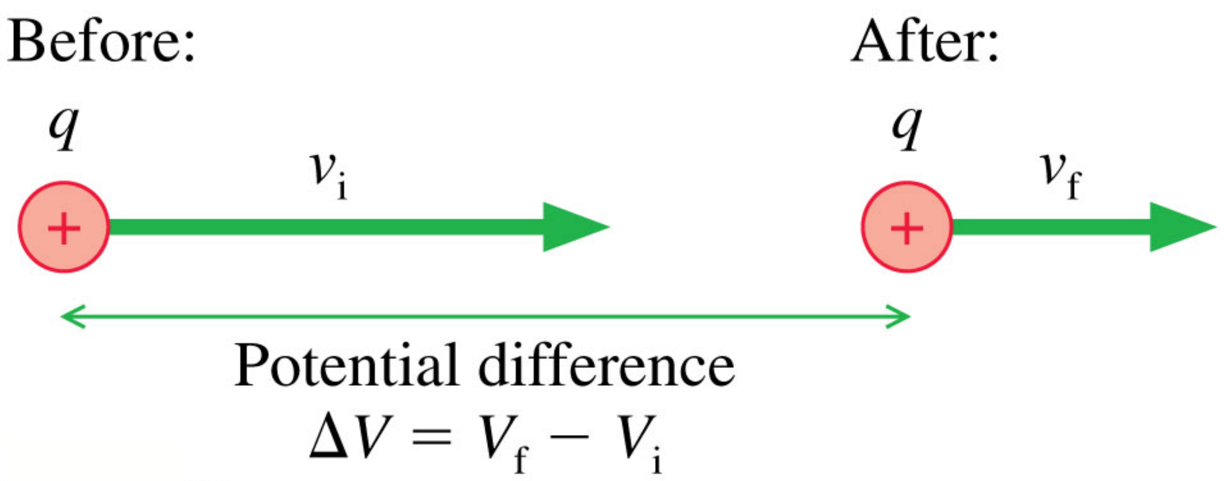
$$= 1.4 \times 10^5\ \mathrm{m/s}$$

An electron, though, with $q = -e$ and a different mass, speeds up to $(v_{\mathrm{f}})_{\mathrm{e}} = 5.9 \times 10^6\ \mathrm{m/s}$.

Example 28.6 Moving Through a Potential Difference

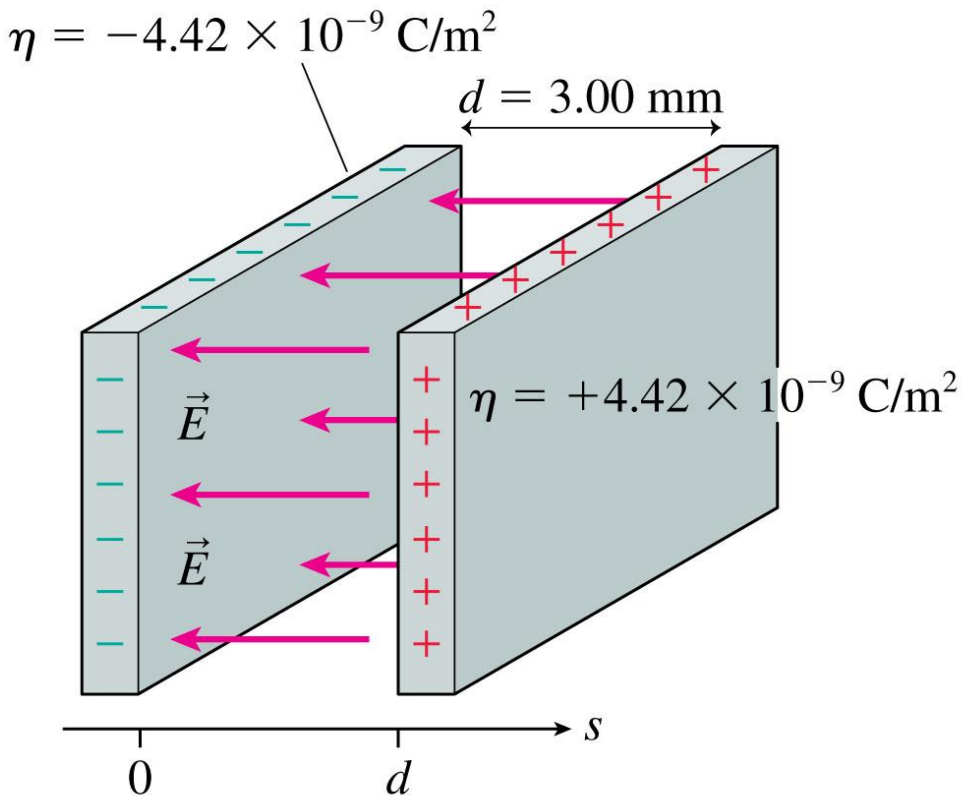
EXAMPLE 28.6 Moving through a potential difference

ASSESS The electric potential *already existed* in space due to other charges that are not explicitly seen in the problem. The electron and proton have nothing to do with creating the potential. Instead, they *respond* to the potential by having potential energy $U = qV$.



The Electric Field Inside a Parallel-Plate Capacitor

This is a review of Chapter 26.



$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive toward negative}\right)$$

$$= (500\ \text{N/C, from right to left})$$

The Electric Potential Inside a Parallel-Plate Capacitor

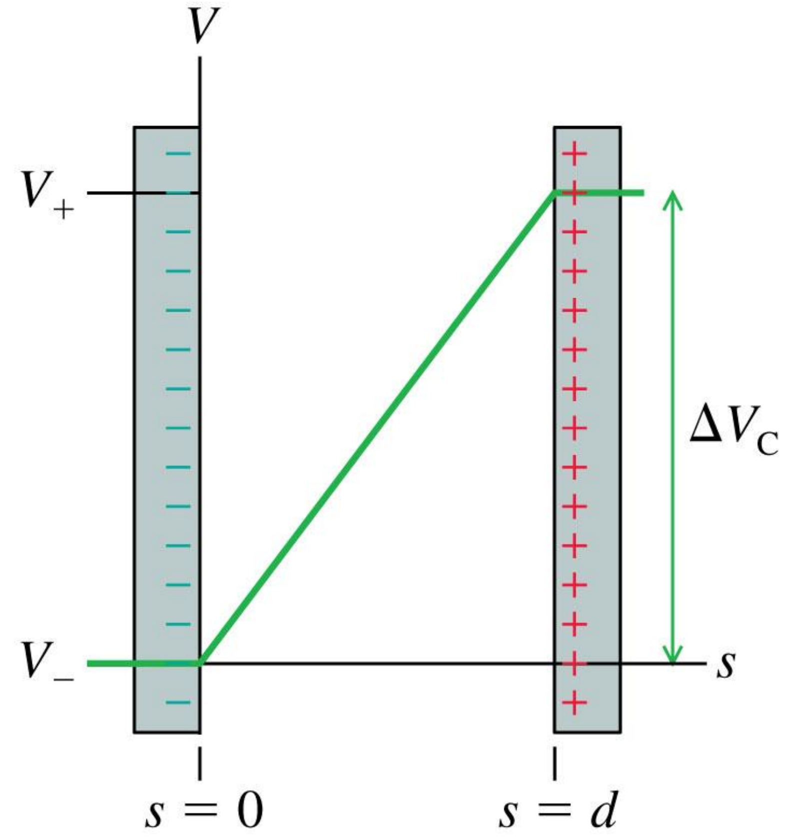
- The electric potential inside a parallel-plate capacitor is

$$V = Es \qquad \text{(electric potential inside a parallel-plate capacitor)}$$

where s **is the distance from the *negative* electrode.**

- The *potential difference* ΔV_C, or "voltage" between the two capacitor plates is

$$\Delta V_C = V_+ - V_- = Ed$$



Units of Electric Field

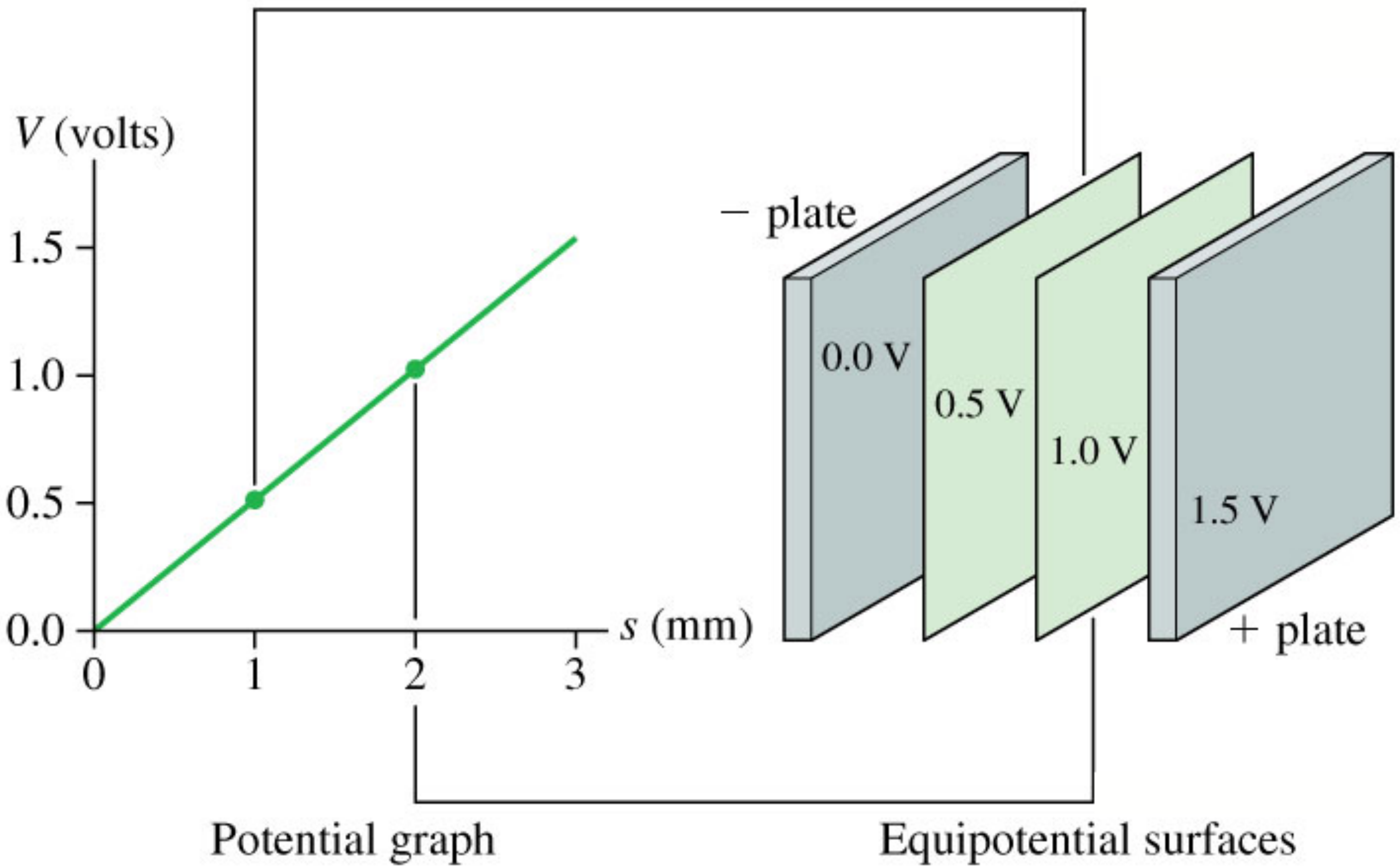
- If we know a capacitor's voltage ΔV and the distance between the plates d, then the electric field strength within the capacitor is:

$$E = \frac{\Delta V_C}{d}$$

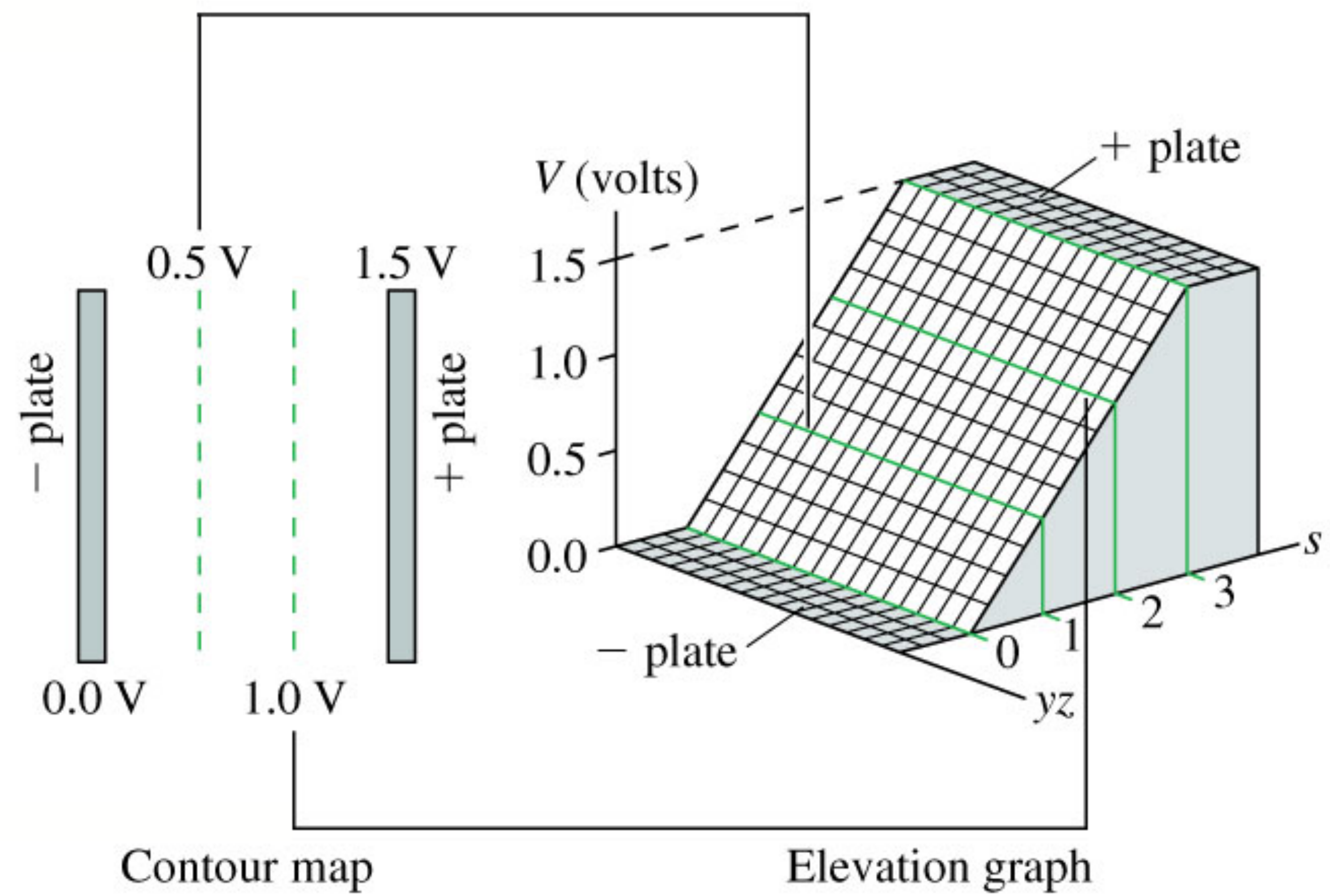
- This implies that the units of electric field are volts per meter, or V/m.
- Previously, we have been using electric field units of newtons per coulomb.
- In fact, as you can show as a homework problem, these units are equivalent to each other:

$$1 \text{ N/C} = 1 \text{ V/m}$$

The Electric Potential Inside a Parallel-Plate Capacitor

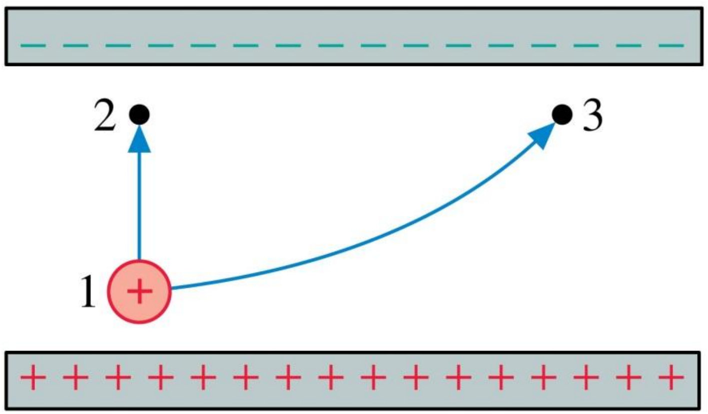


The Electric Potential Inside a Parallel-Plate Capacitor



QuickCheck 28.9

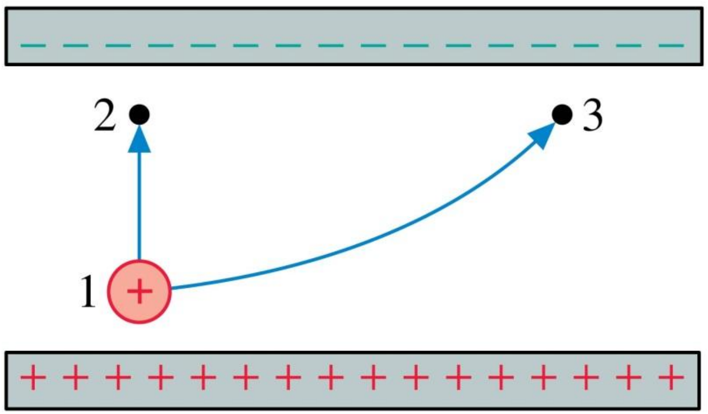
Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and 3 are related by



A. $v_2 > v_3$.
B. $v_2 = v_3$.
C. $v_2 < v_3$.
D. Not enough information to compare their speeds.

QuickCheck 28.9

Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and 3 are related by



A. $v_2 > v_3$.

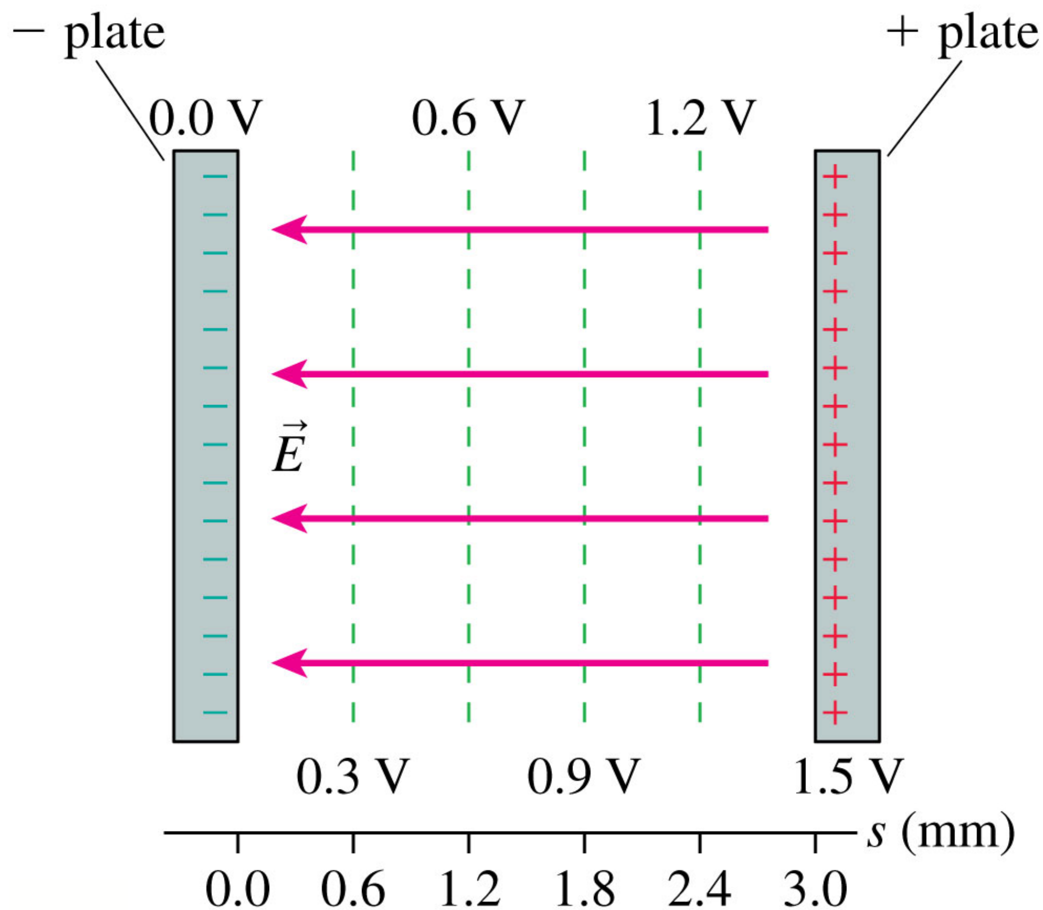
B. $\boldsymbol{v_2 = v_3}$. Energy conservation

C. $v_2 < v_3$.

D. Not enough information to compare their speeds.

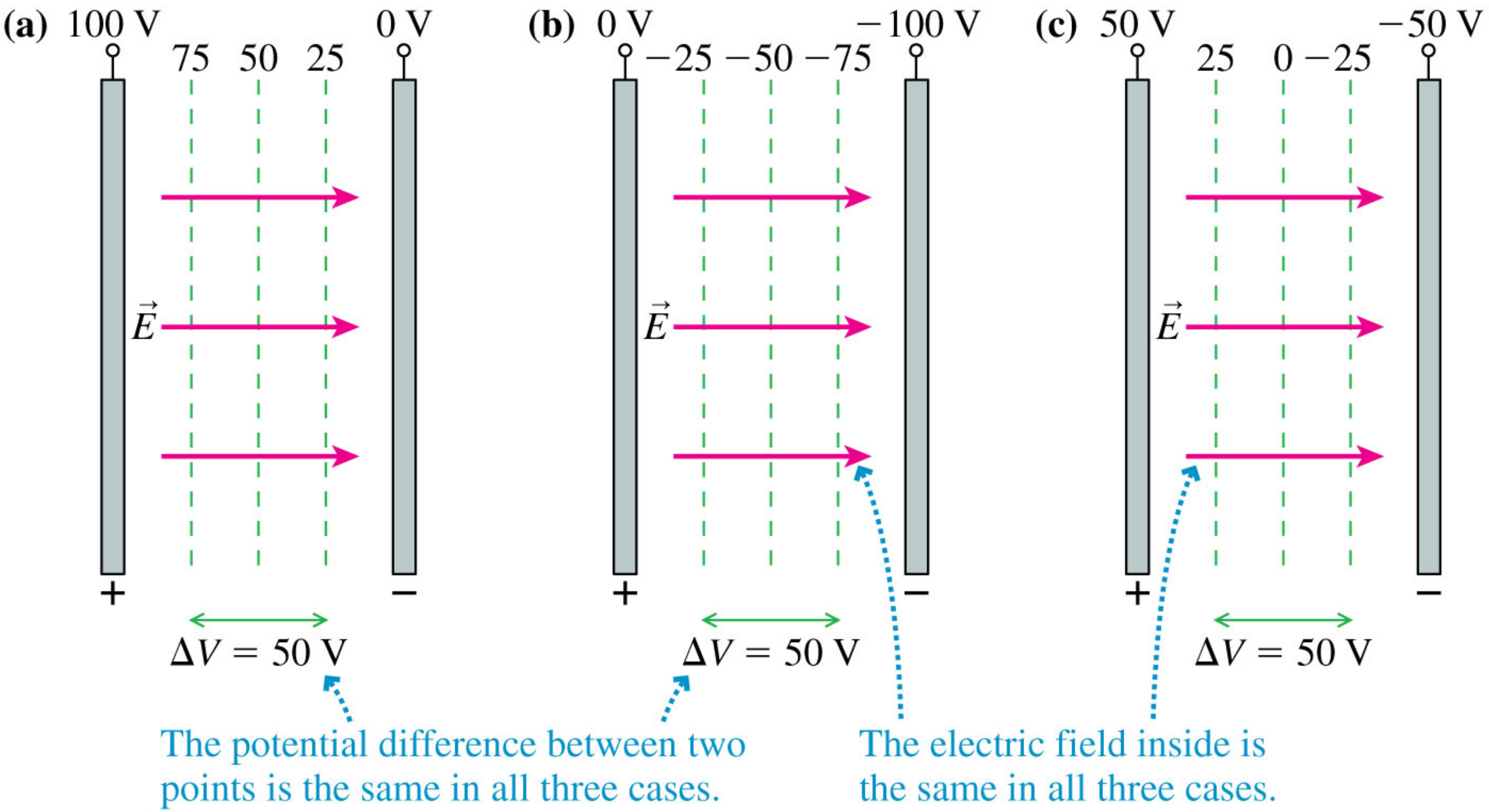
The Parallel-Plate Capacitor

- The figure shows the contour lines of the electric potential and the electric field vectors inside a parallel-plate capacitor.
- The electric field vectors are *perpendicular* to the equipotential surfaces.
- The electric field points in the direction of *decreasing* potential.



The Zero Point of Electric Potential

Where you choose $V = 0$ is arbitrary. The three contour maps below represent the *same physical situation*.



Example 28.8 The Force on an Ion

EXAMPLE 28.8 The force on an ion

Example 26.7 noted that a cell wall can be modeled as a parallel-plate capacitor, with the outer surface of the cell wall being positive while the inner surface is negative. The potential difference between the inside of the cell and the outside is called the *membrane potential*. Suppose a molecular ion with charge $5e$ is embedded within the 5.0-nm-thick wall of a cell with a membrane potential of -70 mV, typical for a nerve cell in its resting state. What is the force on the molecular ion?

Example 28.8 The Force on an Ion

EXAMPLE 28.8 The force on an ion

MODEL Model the cell wall as a parallel-plate capacitor with the inner surface being the negative plate. Although the walls are actually curved, and not large flat planes, the parallel-plate approximation is valid if the wall thickness is much less than the radius of the cell. The capacitor voltage is $\Delta V_C = 70\text{ mV} = 0.070\text{ V}$. The membrane potential is negative because the potential inside the cell is less than the potential outside, but ΔV_C, the capacitor voltage, is the *magnitude* of the potential difference and thus always positive.

Example 28.8 The Force on an Ion

EXAMPLE 28.8 **The force on an ion**

SOLVE The force on a charged particle is $\vec{F} = q\vec{E}$. The electric field strength inside the parallel-plate capacitor of the cell wall is

$$E = \frac{\Delta V_C}{d} = \frac{0.070\ \text{V}}{5.0 \times 10^{-9}\ \text{m}} = 1.4 \times 10^7\ \text{V/m}$$

Notice that we're now using V/m rather than N/C as the units of electric field. Because the field points from positive to negative, the field vector is $\vec{E} = (1.4 \times 10^7\ \text{V/m, toward inside})$. Thus the force on an ion with $q = 5e = 8.0 \times 10^{-19}$ C is

$$\vec{F} = q\vec{E} = (1.1 \times 10^{-11}\ \text{N, toward inside})$$

Example 28.8 The Force on an Ion

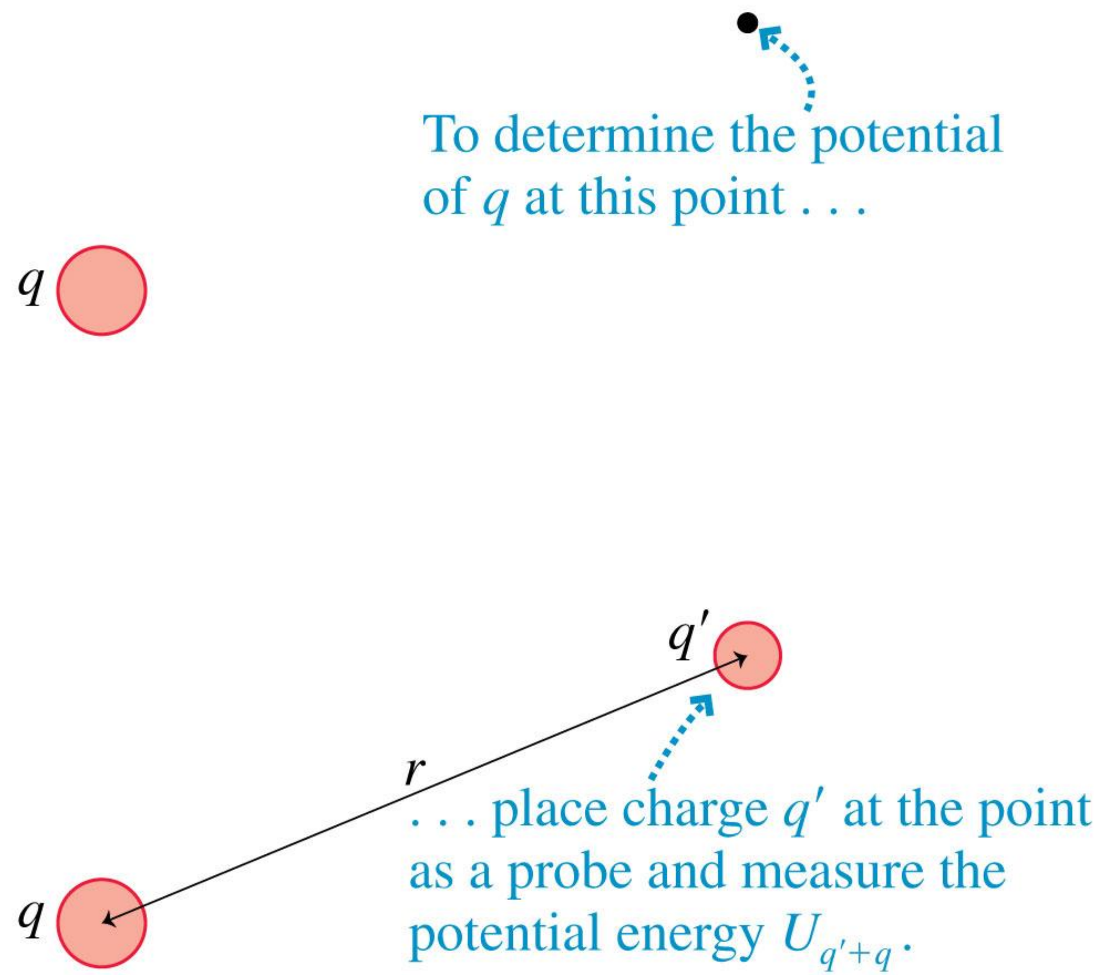
EXAMPLE 28.8 **The force on an ion**

ASSESS For cells to function, a steady flow of molecules must pass back and forth through the cell wall. Although the details of how this happens are very complex, a key idea is that a potential difference between the inside and outside of the cell creates an electric field that pushes positive ions toward the inside, negative ions toward the outside.

The Electric Potential of a Point Charge

- Let q in the figure be the source charge, and let a second charge q', a distance r away, probe the electric potential of q.
- The potential energy of the two point charges is

$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r}$$



The Electric Potential of a Point Charge

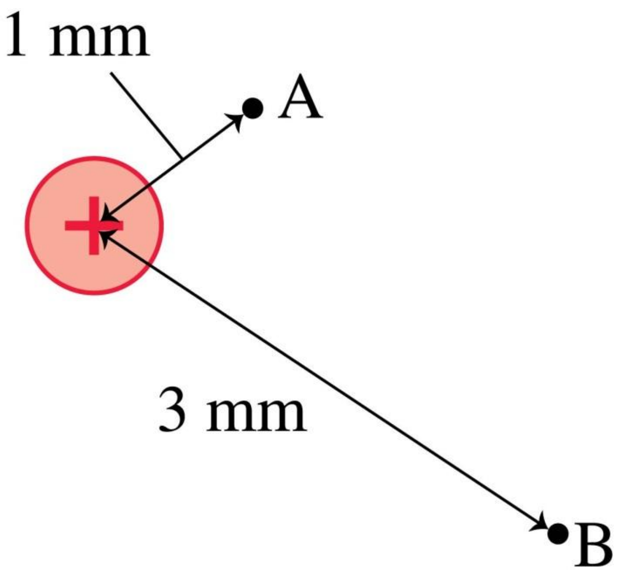
- The electric potential due to a point charge q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{(electric potential of a point charge)}$$

- The potential extends through all of space, showing the influence of charge q, but it weakens with distance as $1/r$.
- This expression for V assumes that we have chosen $V = 0$ to be at $r = \infty$.

QuickCheck 28.10

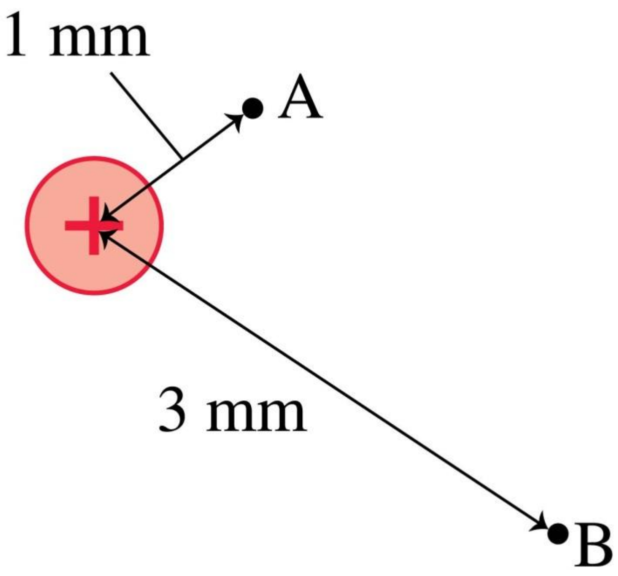
What is the ratio V_B/V_A of the electric potentials at the two points?



A. 9.
B. 3.
C. 1/3.
D. 1/9.
E. Undefined without knowing the charge.

QuickCheck 28.10

What is the ratio V_B/V_A of the electric potentials at the two points?



A. 9.
B. 3.
C. **1/3.** Potential of a point charge decreases inversely with distance.
D. 1/9.
E. Undefined without knowing the charge.

Example 28.9 Calculating the Potential of a Point Charge

EXAMPLE 28.9 Calculating the potential of a point charge

What is the electric potential 1.0 cm from a +1.0 nC charge? What is the potential difference between a point 1.0 cm away and a second point 3.0 cm away?

Example 28.9 Calculating the Potential of a Point Charge

EXAMPLE 28.9 Calculating the potential of a point charge

SOLVE The potential at $r = 1.0$ cm is

$$V_{1\,\text{cm}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = (9.0 \times 10^9\ \text{N m}^2/\text{C}^2)\,\frac{1.0 \times 10^{-9}\ \text{C}}{0.010\ \text{m}}$$

$$= 900\ \text{V}$$

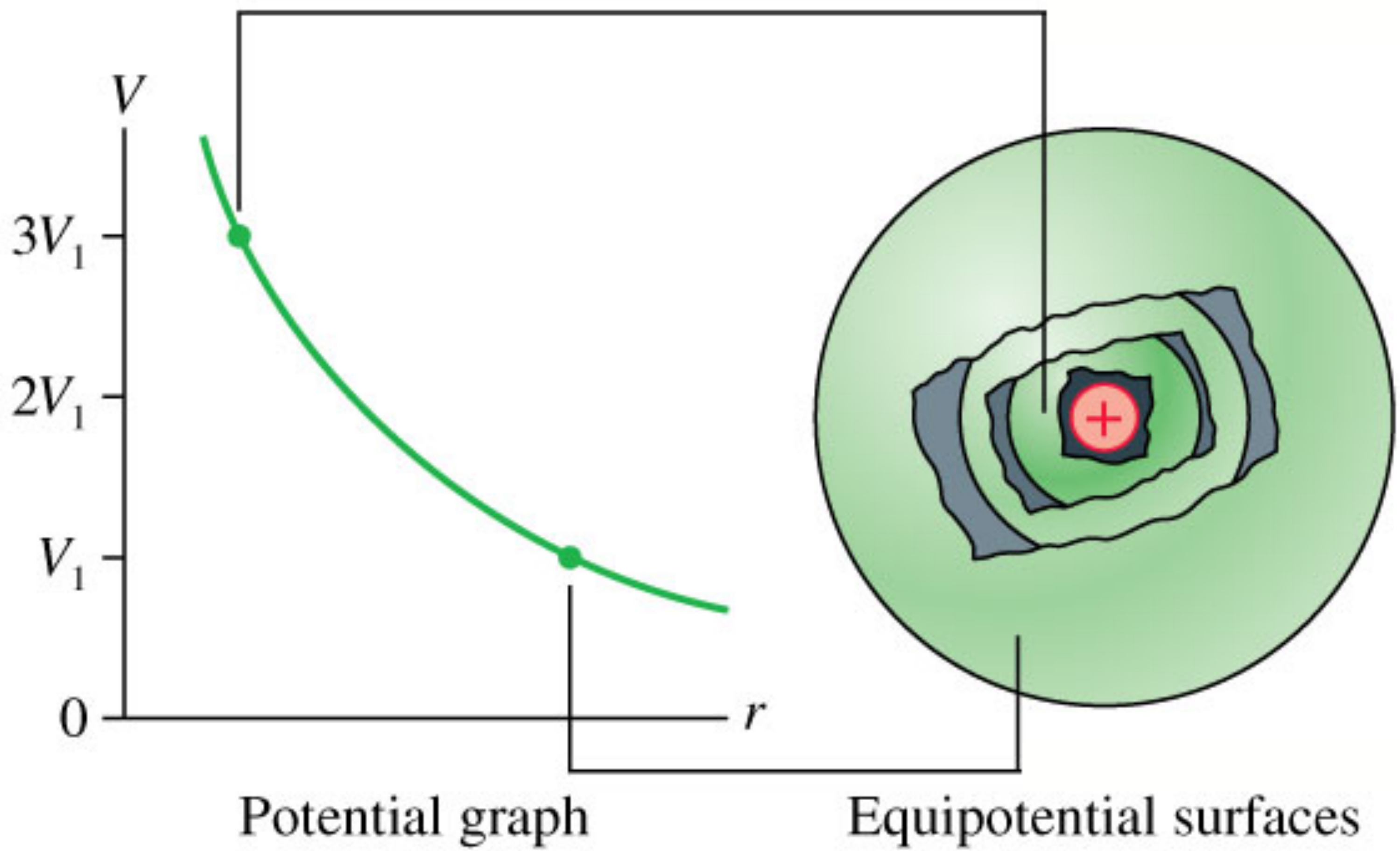
We can similarly calculate $V_{3\,\text{cm}} = 300$ V. Thus the potential difference between these two points is $\Delta V = V_{1\,\text{cm}} - V_{3\,\text{cm}} = 600$ V.

Example 28.9 Calculating the Potential of a Point Charge

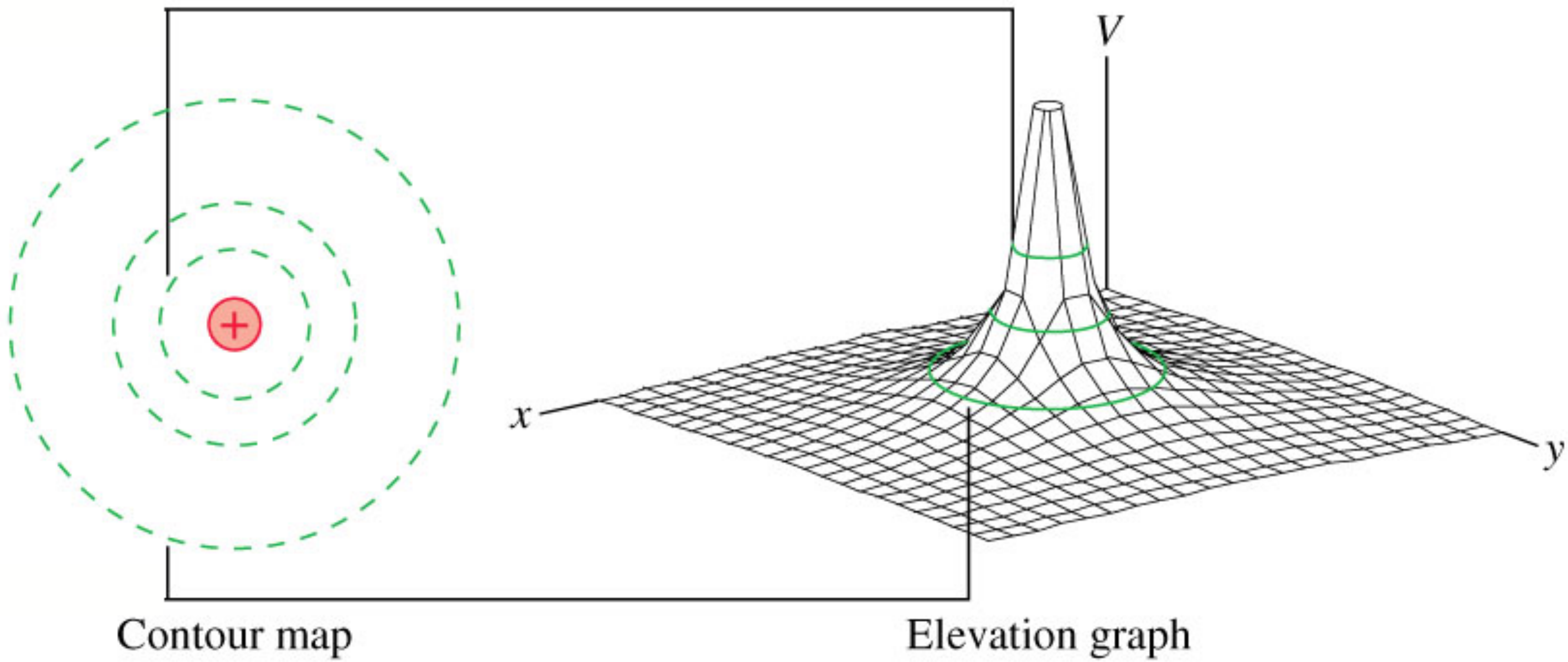
EXAMPLE 28.9 **Calculating the potential of a point charge**

ASSESS 1 nC is typical of the electrostatic charge produced by rubbing, and you can see that such a charge creates a fairly large potential nearby. Why are we not shocked and injured when working with the "high voltages" of such charges? The sensation of being shocked is a result of current, not potential. Some high-potential sources simply do not have the ability to generate much current. We will look at this issue in Chapter 31.

The Electric Potential of a Point Charge



The Electric Potential of a Point Charge



The Electric Potential of a Charged Sphere

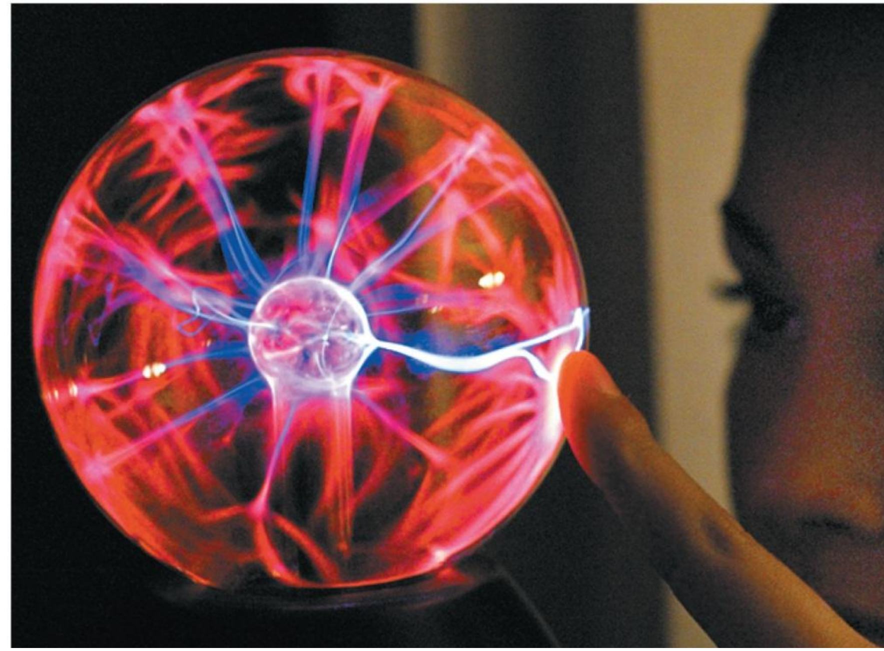
Outside a uniformly charged sphere of radius R, the electric potential is identical to that of a point charge Q at the center.

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

where r • R.

If the potential at the surface V_0 is known, then the potential at r • R is:

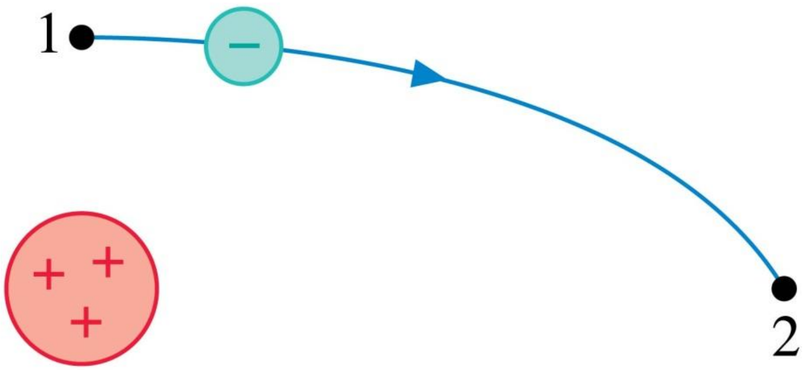
$$V = \frac{R}{r}V_0$$



A *plasma ball* consists of a small metal ball charged to a potential of about 2000 V inside a hollow glass sphere filled with low-pressure neon gas. The high voltage of the ball creates “lightning bolts” between the ball and the glass sphere.

QuickCheck 28.11

An electron follows the trajectory shown from point 1 to point 2. At point 2,



A. $v_2 > v_1$.

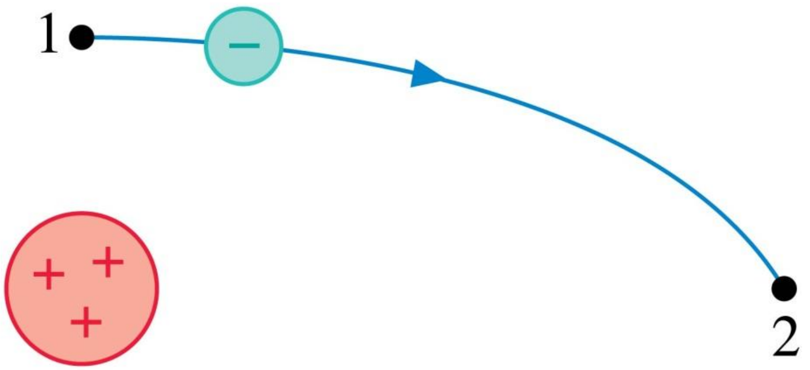
B. $v_2 = v_1$.

C. $v_2 < v_1$.

D. Not enough information to compare the speeds at these points.

QuickCheck 28.11

An electron follows the trajectory shown from point 1 to point 2. At point 2,



A. $v_2 > v_1$.

B. $v_2 = v_1$.

C. $\boldsymbol{v_2 < v_1}$.

D. Not enough information to compare the speeds at these points.

Increasing PE (becoming less negative) so decreasing KE

Example 28.10 A Proton and a Charged Sphere

EXAMPLE 28.10 **A proton and a charged sphere**

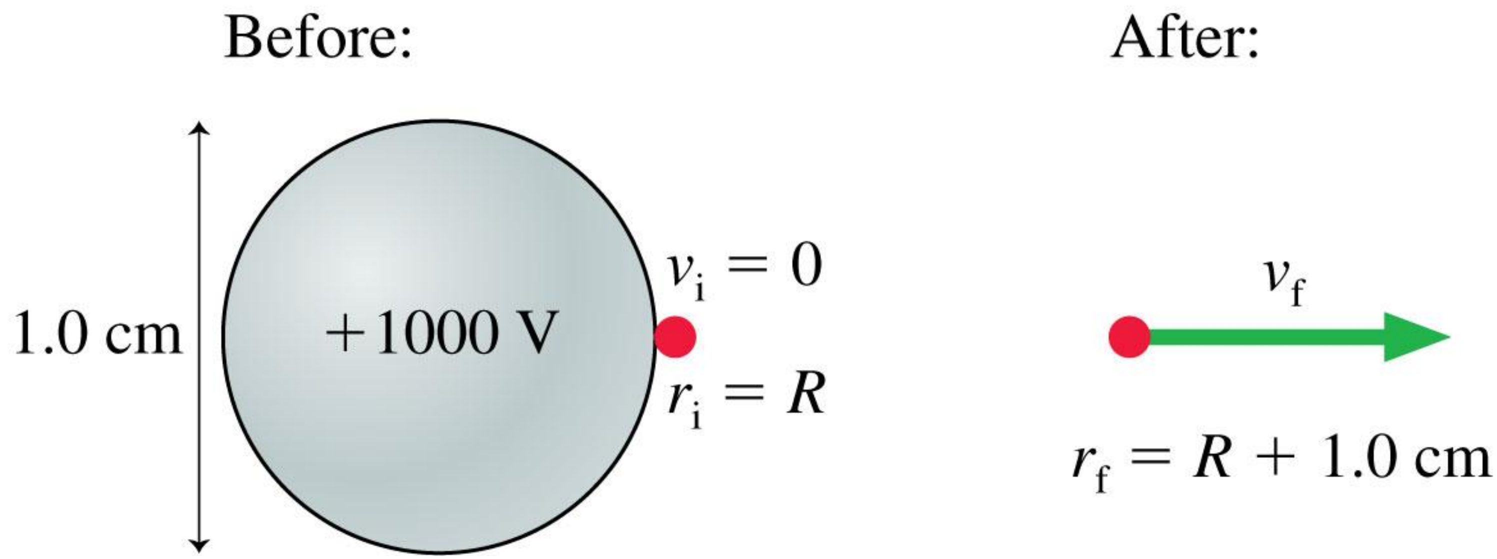
A proton is released from rest at the surface of a 1.0-cm-diameter sphere that has been charged to +1000 V.

a. What is the charge of the sphere?
b. What is the proton's speed at 1.0 cm from the sphere?

MODEL Energy is conserved. The potential outside the charged sphere is the same as the potential of a point charge at the center.

Example 28.10 A Proton and a Charged Sphere

VISUALIZE



EXAMPLE 28.10 A proton and a charged sphere

SOLVE a. The charge of the sphere is

$$Q = 4\pi\epsilon_0 R V_0 = 0.56 \times 10^{-9}\ \text{C} = 0.56\ \text{nC}$$

Example 28.10 A Proton and a Charged Sphere

EXAMPLE 28.10 A proton and a charged sphere

b. A sphere charged to $V_0 = +1000$ V is positively charged. The proton will be repelled by this charge and move away from the sphere. The conservation of energy equation $K_f + eV_f = K_i + eV_i$, with Equation 28.34 for the potential of a sphere, is

$$\frac{1}{2}mv_f^2 + \frac{eR}{r_f}V_0 = \frac{1}{2}mv_i^2 + \frac{eR}{r_i}V_0$$

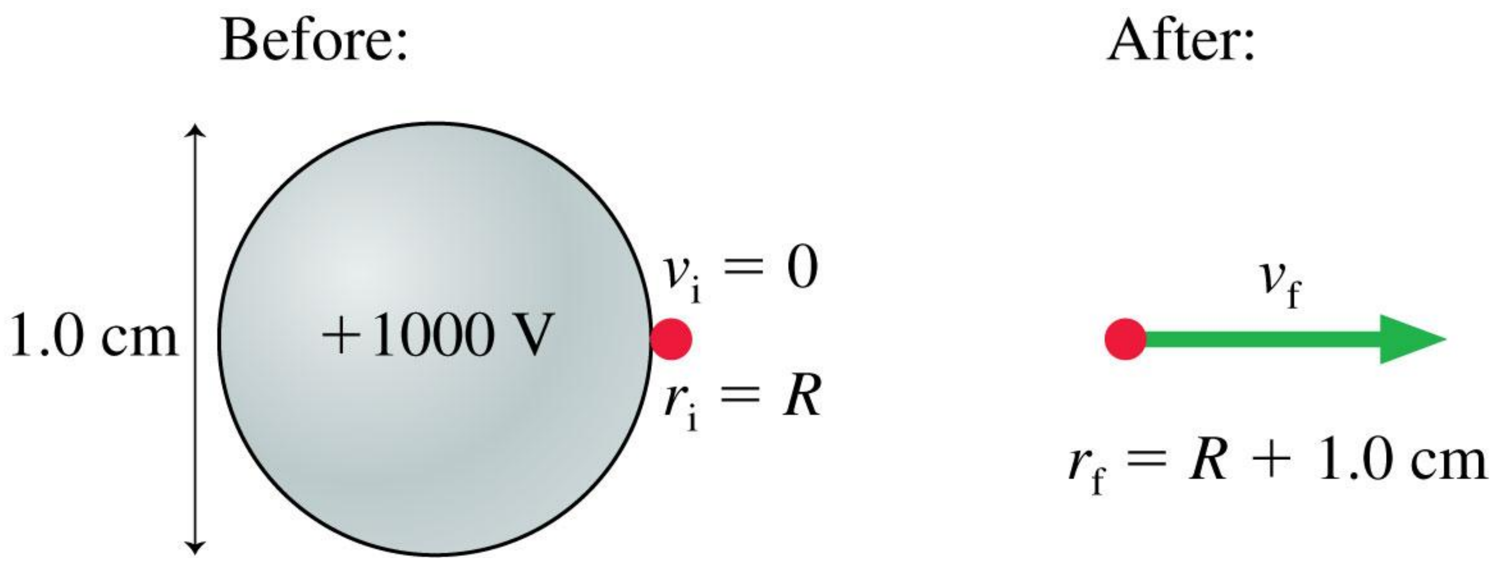
The proton starts from the surface of the sphere, $r_i = R$, with $v_i = 0$. When the proton is 1.0 cm from the *surface* of the sphere, it has $r_f = 1.0\text{ cm} + R = 1.5$ cm. Using these, we can solve for v_f:

$$v_f = \sqrt{\frac{2eV_0}{m}\left(1 - \frac{R}{r_f}\right)} = 3.6 \times 10^5 \text{ m/s}$$

Example 28.10 A Proton and a Charged Sphere

EXAMPLE 28.10 A proton and a charged sphere

ASSESS This example illustrates how the ideas of electric potential and potential energy work together, yet they are *not* the same thing.



The Electric Potential of Many Charges

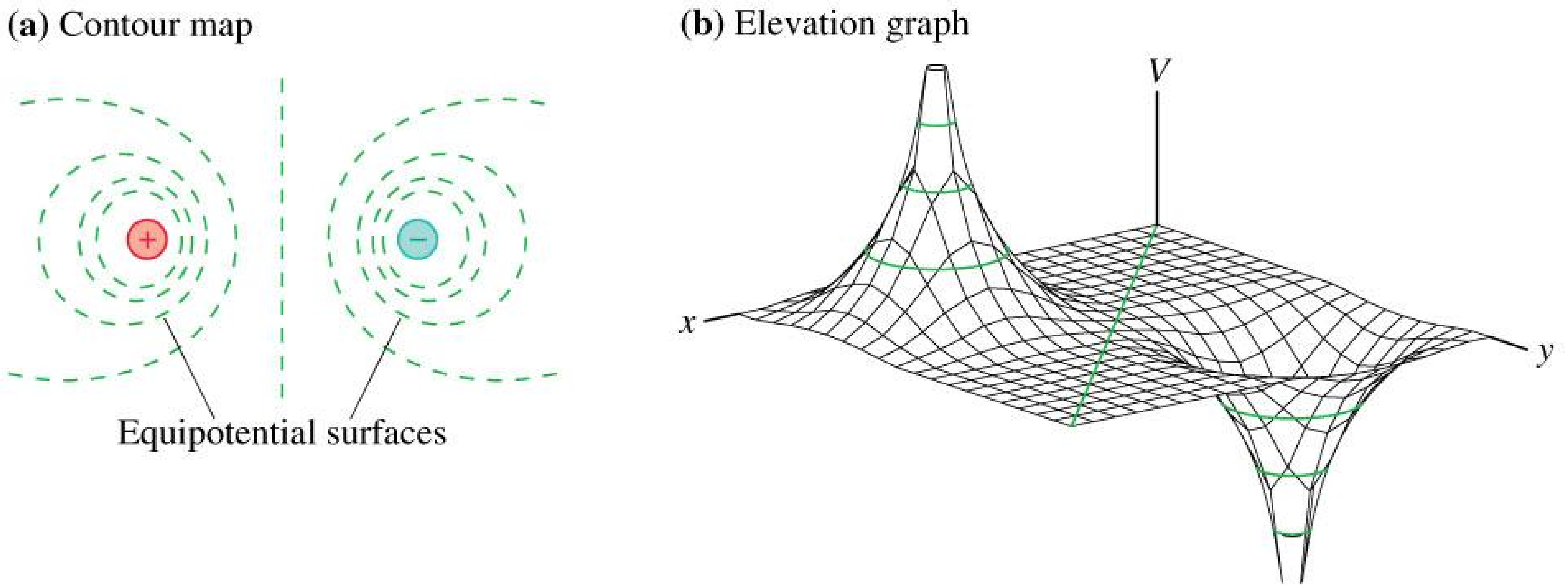
- The electric potential V at a point in space is the sum of the potentials due to each charge:

$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

where r_i is the distance from charge q_i to the point in space where the potential is being calculated.

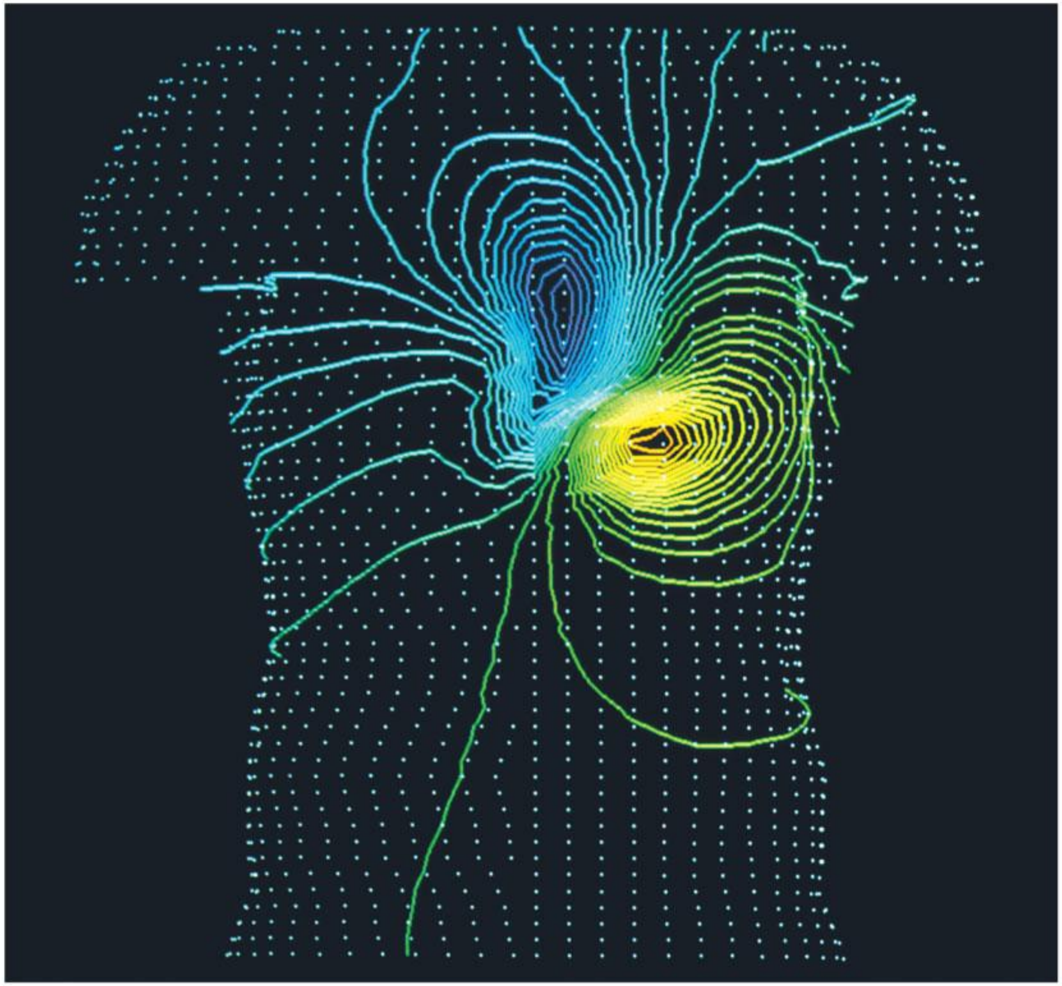
- **The electric potential, like the electric field, obeys the principle of superposition.**

The Electric Potential of an Electric Dipole



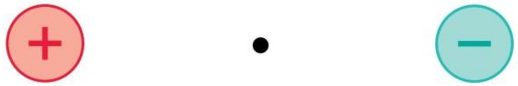
The Electric Potential of a Human Heart

- Electrical activity within the body can be monitored by measuring equipotential lines on the skin.
- The equipotentials near the heart are a slightly distorted but recognizable *electric dipole.*



QuickCheck 28.12

At the midpoint between these two equal but opposite charges,



A. $\vec{E} = \vec{0}$; $V = 0$.

B. $\vec{E} = \vec{0}$; $V > 0$.

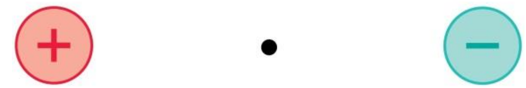
C. $\vec{E} = \vec{0}$; $V < 0$.

D. $\vec{E}$ points right; $V = 0$.

E. $\vec{E}$ points left; $V = 0$.

QuickCheck 28.12

At the midpoint between these two equal but opposite charges,



A. $\vec{E} = \vec{0}$; $V = 0$.

B. $\vec{E} = \vec{0}$; $V > 0$.

C. $\vec{E} = \vec{0}$; $V < 0$.

D. $\boldsymbol{\vec{E}}$ points right; $\boldsymbol{V = 0}$.

E. $\vec{E}$ points left; $V = 0$.

QuickCheck 28.13

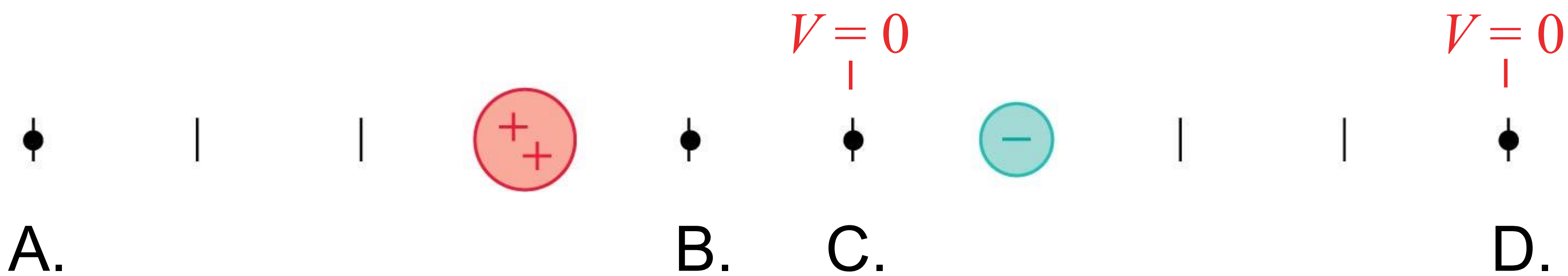
At which point or points is the electric potential zero?

A. B. C. D.

E. More than one of these.

QuickCheck 28.13

At which point or points is the electric potential zero?



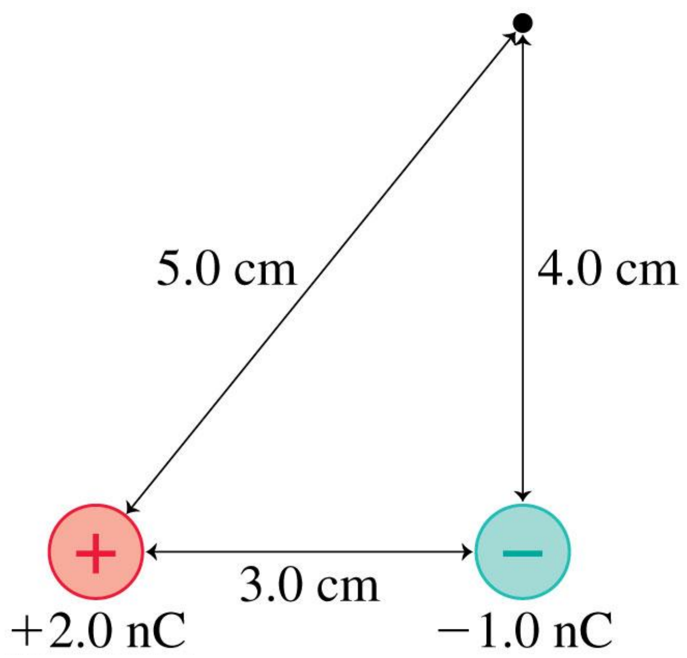
✔ **E. More than one of these.**

Example 28.11 The Potential of Two Charges

EXAMPLE 28.11 **The potential of two charges**

What is the electric potential at the point indicated in the figure?

MODEL The potential is the sum of the potentials due to each charge.



Example 28.11 The Potential of Two Charges

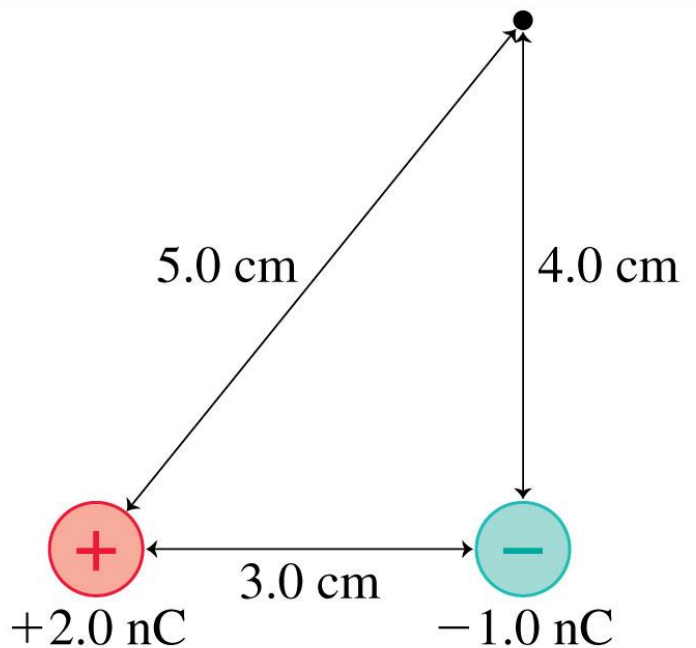
EXAMPLE 28.11 The potential of two charges

SOLVE The potential at the indicated point is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

$$= (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\left(\frac{2.0 \times 10^{-9}\text{ C}}{0.050\text{ m}} + \frac{-1.0 \times 10^{-9}\text{ C}}{0.040\text{ m}}\right)$$

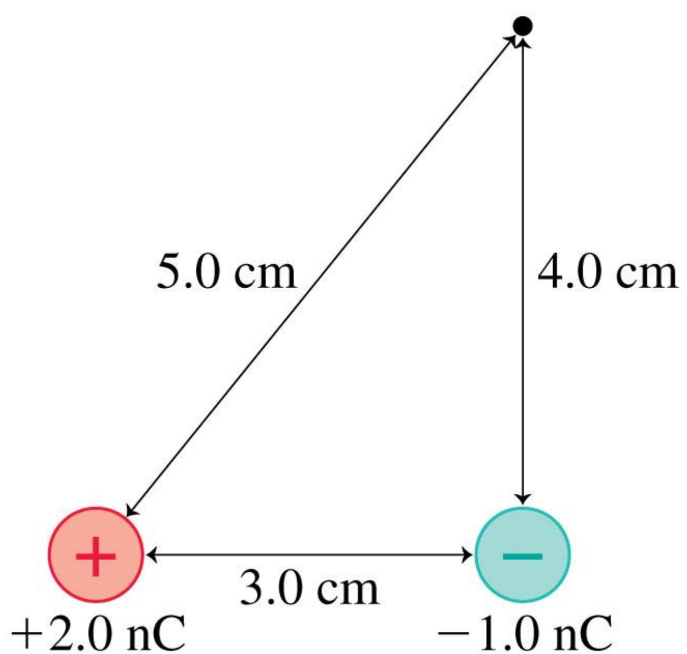
$$= 135 \text{ V}$$



Example 28.11 The Potential of Two Charges

EXAMPLE 28.11 The potential of two charges

ASSESS The potential is a *scalar,* so we found the net potential by adding two numbers. We don't need any angles or components to calculate the potential.



Problem-Solving Strategy: The Electric Potential of a Continuous Distribution of Charge

PROBLEM-SOLVING STRATEGY 28.2 **The electric potential of a continuous distribution of charge**

MP

MODEL Model the charges as a simple shape, such as a line or a disk. Assume the charge is uniformly distributed.

VISUALIZE For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the electric potential.
3. Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine V. This division is often, but not always, into point charges.
4. Identify distances that need to be calculated.

Problem-Solving Strategy: The Electric Potential of a Continuous Distribution of Charge

PROBLEM-SOLVING STRATEGY 28.2 **The electric potential of a continuous distribution of charge**

MP

SOLVE The mathematical representation is $V = \sum V_i$.

- Use superposition to form an algebraic expression for the potential at P.
- Let the (x, y, z) coordinates of the point remain as variables.
- Replace the small charge ΔQ with an equivalent expression involving a *charge density* and a *coordinate,* such as dx, that describes the shape of charge ΔQ. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- All distances must be expressed in terms of the coordinates.
- Let the sum become an integral. The integration will be over the coordinate variable that is related to ΔQ. The integration limits for this variable will depend on the coordinate system you have chosen. Carry out the integration and simplify the result.

ASSESS Check that your result is consistent with any limits for which you know what the potential should be.

Exercise 29

Example 28.12 The Potential of a Ring of Charge

EXAMPLE 28.12 The potential of a ring of charge

A thin, uniformly charged ring of radius R has total charge Q. Find the potential at distance z on the axis of the ring.

MODEL Because the ring is thin, we'll assume the charge lies along a circle of radius R.

Example 28.12 The Potential of a Ring of Charge

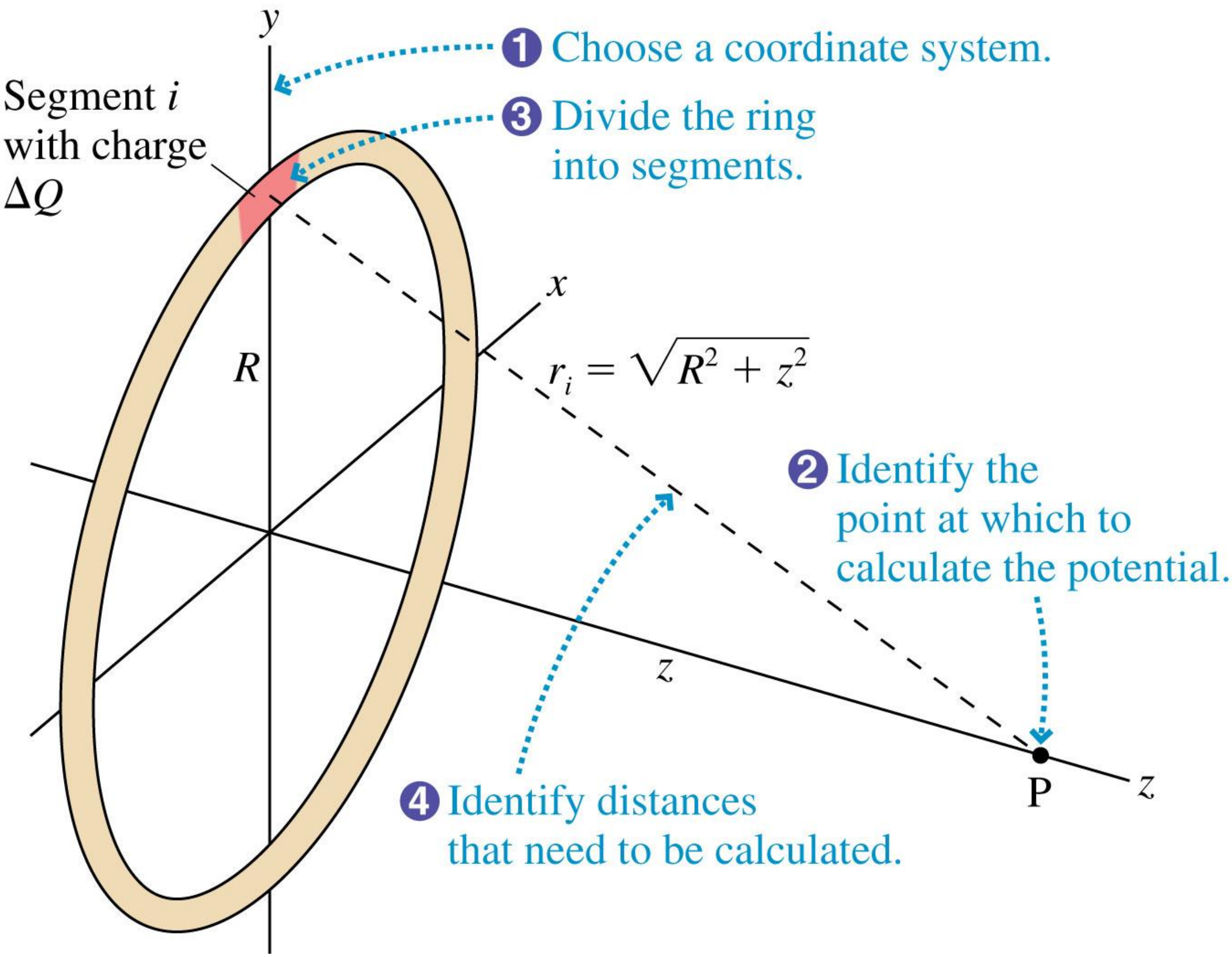
EXAMPLE 28.12 **The potential of a ring of charge**

VISUALIZE The figure on the next slide illustrates the four steps of the problem-solving strategy. We've chosen a coordinate system in which the ring lies in the xy-plane and point P is on the z-axis. We've then divided the ring into N small segments of charge ΔQ, each of which can be modeled as a point charge. The distance r_i between segment i and point P is

$$r_i = \sqrt{R^2 + z^2}$$

Note that r_i is a constant distance, the same for every charge segment.

Example 28.12 The Potential of a Ring of Charge



Example 28.12 The Potential of a Ring of Charge

EXAMPLE 28.12 **The potential of a ring of charge**

SOLVE The potential V at P is the sum of the potentials due to each segment of charge:

$$V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \sum_{i=1}^{N} \Delta Q$$

We were able to bring all terms involving z to the front because z is a constant as far as the summation is concerned. Surprisingly, we don't need to convert the sum to an integral to complete this calculation. The sum of all the ΔQ charge segments around the ring is simply the ring's total charge, $\sum(\Delta Q) = Q$; hence the electric potential on the axis of a charged ring is

$$V_{\text{ring on axis}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}$$

Example 28.12 The Potential of a Ring of Charge

EXAMPLE 28.12 **The potential of a ring of charge**

ASSESS From far away, the ring appears as a point charge Q in the distance. Thus we expect the potential of the ring to be that of a point charge when $z \gg R$. You can see that $V_{\text{ring}} \approx Q/4\pi\epsilon_0 z$ when $z \gg R$, which is, indeed, the potential of a point charge Q.

Chapter 28 Summary Slides

General Principles

Sources of *V*

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

General Principles

Sources of V

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

General Principles

Consequences of V

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + qV_f = K_i + qV_i$$

General Principles

Consequences of V

The potential energy of **two point charges** separated by distance r is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$