04 - BNF (Backus-Naur Form) and Parse Trees

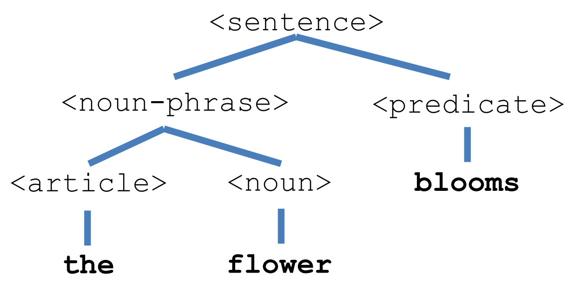
Consider again the simple grammar shown previously in section 2.0 –

```
<sentence>    ::= <noun-phrase> <predicate>
<noun-phrase> ::= <article> <noun>
<article>     ::= the | a | an
<noun>        ::= cat | flower
<predicate>   ::= jumps | blooms
```

We showed how sentences can be derived by a series of replacements:

```
<sentence>
 ⟹ <noun-phrase> <predicate>
 ⟹ <article> <noun> <predicate>
 ⟹ the flower blooms
```

Usually, derivations are more useful if they are done as parse trees.
The same derivation of "the flower blooms", expressed as a parse tree, is:



Some things to notice about Parse Trees:

- the start symbol is always at the root of the tree,
- every leaf node is a terminal,
- every interior node is a non-terminal, and
- the sentence appears in a left-to-right traversal of the leaves.

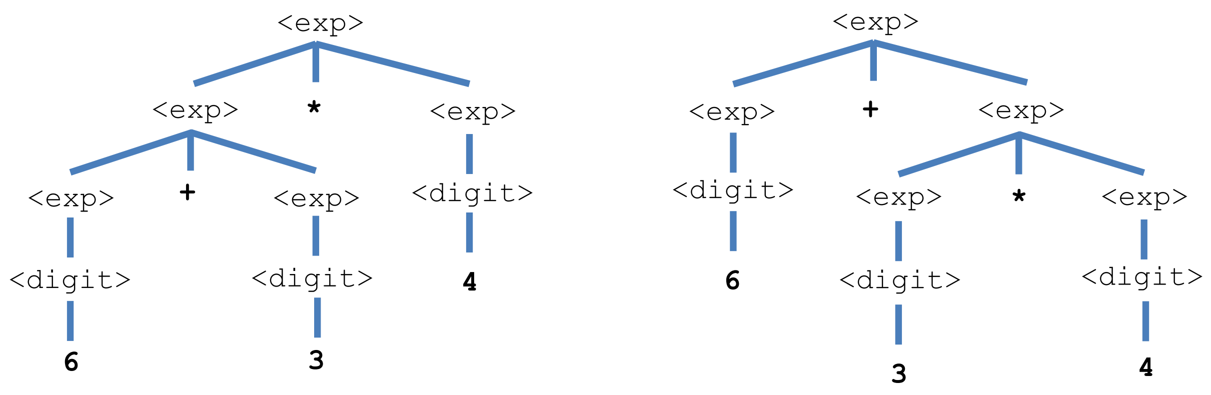
When using BNF to specify a programming language, the **terminals** of the grammar are comprised of the **tokens** produced by the lexical scanner.

Therefore, Parse Trees reveal the *syntactic structure* of the sequence of tokens that make up a computer program. It is very important that the grammar not be ambiguous. If the grammar were ambiguous, programs wouldn't be portable because there would be more than one way that compilers could translate them. Here's an example:

Consider the following grammar that expresses parenthesized expressions of digits, including both addition and multiplication:

```
<exp>    ::= <exp> + <exp> | <exp> * <exp> | ( <exp> ) | <digit>
<digit>  ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

This grammar is capable of representing a wide variety of expressions, such as `3+(4*4+(2*7))` and many, many others. However, this grammar is ambiguous. For example, there are *two* parse trees for `6+3*4`



The left tree implies a result of `(6+3)*4` which is `36`.
The right tree implies a result of `6+(3*4)` which is `18`.
(see how the parse tree already reveals some of the semantics!)
Thus the above specification is ambiguous, and therefore is an inadequate specification of the subtraction operator.

The problem is that the grammar does not express operator precedence. Most would agree that the tree on the right is the correct one (multiplication has higher precedence than addition). Also note that in the correct tree, the operator with lower precedence (+) is expanded before the operator with higher precedence (*). This leads us to correct the grammar by adding additional non-terminals so as to *require* lower precedence operators to be expanded before higher precedence operators, thusly:

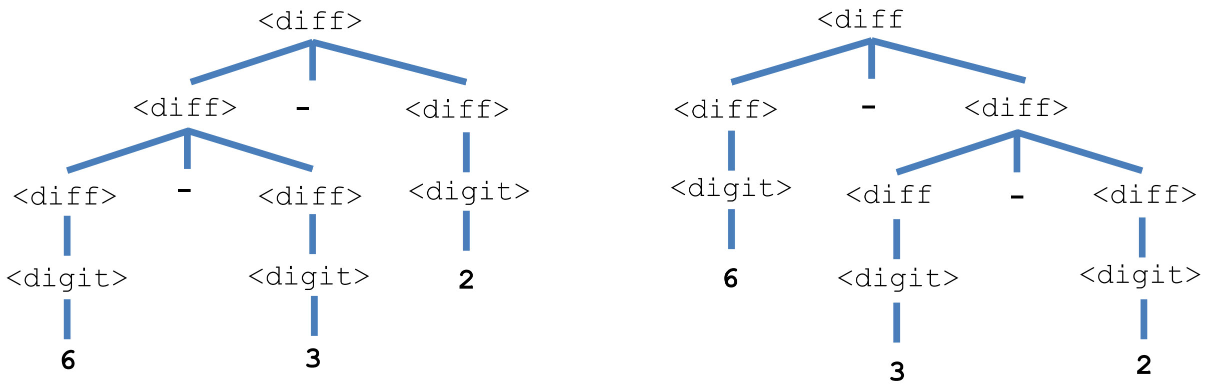
```
<exp>    ::= <exp> + <exp> | <term>
<term>   ::= <term> * <term> | <factor>
<factor> ::= ( <exp> ) | <digit>
<digit>  ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Note that the parentheses now act as a mechanism for overriding operator precedence, as is customary. However, our grammar is still ambiguous!

Consider the following BNF grammar for subtraction of digits:

```
<diff>  ::= <diff> - <diff> | <digit>
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Notice that, using this grammar, there are two parse trees for `6-3-2`



The left tree implies a result of `(6-3)-2` which is `1`.
The right tree implies a result of `6-(3-2)` which is `5`.

Thus the above specification is ambiguous, and therefore is an inadequate specification of the subtraction operator. *(note that the previous grammar contained the same ambiguity in the + and * operators).* The solution is to rewrite it so that it limits recursion to one side or the other, but not both:

```
<diff>  ::= <diff> - <digit> | <digit>
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

The above example illustrates the use of BNF to specify *left-associative* operators (such as +, -, *, /, etc.). There are also *right-associative* operators such as the exponent (^). Can you see how that would be written?

Combining what we have learned from the two examples above, here is an unambiguous grammar for mathematical expressions of integers. It has both left and right-associative operators. It uses different levels of non-terminals to express operator precedence, and also parentheses so that the programmer can use nesting and override precedence:

```
<exp>    ::= <exp> + <term> | <exp> - <term> | <term>
<term>   ::= <term> * <power> | <term> / <power> | <power>
<power>  ::= <factor> ^ <power> | <factor>
<factor> ::= ( <exp> ) | <int>
<int>    ::= <digit> <int> | <digit>
<digit>  ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```