The Anti-holographic Entangled Universe

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Abstract

We introduce and develop the theory of the "Anti-holographic Entangled Universe" (AEU), a novel framework in which large-scale cosmological degrees of freedom admit an emergent, non-local entanglement structure that is dual to a reduced effective bulk but which violates conventional holographic entropy bounds in a controlled manner. We present a mathematically rigorous formulation of anti-holographic mappings, derive conditions for their existence, construct explicit lattice and continuum models that realize the required entanglement pattern, and analyze dynamical consequences including modified correlation scaling, transport anomalies, and novel signatures for early-universe cosmology. We provide proofs of key propositions, present numerical simulations of representative models, and discuss observational consequences and connections to existing quantum gravity and condensed-matter systems.

Contents

1 Introduction

The holographic principle, born from black-hole thermodynamics and sharpened in the AdS/CFT correspondence [5, 8, 2], posits that the information content of a gravitational bulk is encoded on a lower-dimensional boundary. In this work we consider an alternative – the Anti-holographic Entangled Universe (AEU) – in which an emergent entanglement structure organizes degrees of freedom so that effective subsystem entropies scale super-extensively relative to naive boundary-area expectations, while preserving causality and local dynamics at experimentally accessible scales.

Our goals are threefold: (i) define anti-holographic mappings and establish existence theorems under clear physical axioms; (ii) provide microscopic models (both discrete lattice and continuum field constructions) that realize the anti-holographic entanglement; and (iii) analyze dynamical and observational consequences, including implications for cosmological perturbations, decoherence, and transport.

This paper is organized as follows. Section 2 introduces the formal framework and axioms. Section 4 constructs concrete models. Section 5 analyzes dynamics and correlation functions. Section 6 presents numerical experiments. Section 7 discusses observational consequences. We conclude in Section 9. Extensive derivations and supplementary material appear in the appendices.

2 Conceptual framework and definitions

We begin by specifying the physical setting and introducing core definitions.

2.1 Physical axioms

We adopt the following working axioms for the AEU:

(A1) Local quantum field theory (QFT) degrees of freedom exist on a spacetime manifold $\mathcal{M}$ with a well-defined causal structure at observational scales.

(A2) There exists a partition of the microscopic Hilbert space $\mathcal{H} = \bigotimes_{i\in\Lambda} \mathcal{H}_i$ into local sites indexed by Λ, but the effective coarse-grained information content is organized by an emergent non-local entanglement map.

(A3) The emergent map is unitary (or approximately so in a thermodynamic limit) and reversible on physically relevant subspaces.

(A4) Entanglement entropies for connected regions may scale with an exponent that violates boundary-area scaling but remains consistent with sub-extensive energy and stability constraints.

2.2 Anti-holographic map

Definition 2.1 (Anti-holographic map)**.** Let $\mathcal{H}$ be a Hilbert space of microscopic degrees of freedom. An *anti-holographic map* is an isometry

$$\mathcal{A} : \mathcal{H}_{\text{eff}} \to \mathcal{H} \tag{1}$$

from a lower-dimensional effective Hilbert space $\mathcal{H}_{\text{eff}}$ such that for a class of regions $R \subset \mathcal{M}$, the von Neumann entropy satisfies

$$S(\rho_R) \sim |R|^{\alpha}, \qquad \alpha > 1 - \frac{1}{d}, \tag{2}$$

where d is the spatial dimension and $|R|$ is the region volume (or measure) at coarse-grained scale. The exponent α quantifies anti-holographic scaling.

Remark 2.2. The mapping can be viewed heuristically as folding lower-dimensional information into an effectively higher-entropy embedding while maintaining consistent local operator algebras on observables restricted to experimentally accessible scales.

3 Mathematical formulation

In this section we formalize the structure of anti-holographic mappings and prove existence results under mild assumptions.

3.1 Operator algebra and entanglement skeleton

Consider a lattice Λ with N sites and local dimension q. Let $\mathcal{A}_i$ denote the algebra of observables supported on site i. For a region $R \subset \Lambda$ we denote the algebra by $\mathcal{A}_R$. An entanglement skeleton is a graph $G = (V, E)$ embedded in Λ where edges indicate nontrivial long-range entanglement couplings.

Define the entanglement adjacency matrix $E_{ij} \in \{0, 1\}$ such that $E_{ij} = 1$ if sites i and j are paired in the dominant entanglement structure. We consider randomized but correlated ensembles of E characterized by a kernel $K(x, y)$ in the continuum limit.

3.2 Existence theorem

Theorem 3.1 (Existence of anti-holographic embeddings)**.** *Let Λ be a regular lattice in d spatial dimensions with N sites. For any exponent $\alpha \in (1 - 1/d, 1]$ there exist families of local Hamiltonians H_N and unitary maps U_N such that in the thermodynamic limit $N \to \infty$ the entanglement entropy of typical connected regions of linear size ℓ scales as*

$$S(\ell) \propto \ell^{d\alpha} + o(\ell^{d\alpha}). \tag{3}$$

Proof. We provide a constructive proof by explicit tensor-network embedding and asymptotic entropy counting. The strategy is to build a hierarchical tensor network that resembles MERA but with scale-dependent branching numbers and bond-dimension profiles tuned to yield the desired entropy scaling.

Construction. Partition the lattice into hierarchical layers indexed by scale $s = 0, 1, \ldots, S$ where layer s contains cells of linear size $\ell_s = 2^s$ (units chosen so smallest cell has size 1). At each layer place branching tensors that connect groups of b_s lower-level cells into higher-level coarse-grained sites. Let the bond dimension at layer s be χ_s and the effective number of degrees of freedom contributed per coarse-grained connection be proportional to $\log \chi_s$.

Entropy counting. Consider a connected region R of linear size ℓ. Let s^* be the largest scale with $\ell_{s^*} \lesssim \ell$. Contribution to the entropy from scales $s \leq s^*$ arises from bonds crossing the boundary of R. The number of crossing bonds at scale s scales as $n_s(R) \sim c\, \frac{\ell^d}{\ell_s^d} \cdot b_s^{-1}$ where c depends on packing geometry. Hence the entropy scales as

$$S(\ell) \lesssim \sum_{s=0}^{s^*} n_s(R) \log \chi_s. \tag{4}$$

Choose parametrizations

$$b_s \sim s^\gamma, \qquad \log \chi_s \sim s^\kappa \tag{5}$$

for positive exponents γ, κ such that the series above is dominated by contributions near $s \sim s^*$ and yields power-law scaling. Substituting and approximating the sum by an integral produces

$$S(\ell) \sim C\, \ell^d \ell_{s^*}^{-d}\, s^{*\,(\kappa-\gamma+1)} \sim C'\, \ell^{d\alpha} \tag{6}$$

with the identification $\alpha = 1 - (d \ln 2)^{-1}(\gamma - \kappa - 1)^{-1}$ after matching exponents and absorbing constants. By tuning γ, κ one obtains any $\alpha \in (1 - 1/d, 1]$.

Local Hamiltonians. The tensor-network state above can be realized as the approximate ground state of a local parent Hamiltonian constructed via standard tensor-network parent-Hamiltonian techniques (see, e.g., [7, 1]). One constructs local projectors onto the complement of the local reduced state support and sums them with finite weights; the resulting Hamiltonian is quasi-local with exponentially decaying interactions if the tensors satisfy injectivity conditions.

Limit and concentration. Typicality and concentration of measure arguments show that for large N almost all random perturbations preserving the network structure leave the entropy scaling intact; fluctuations are subleading in ℓ.

Therefore the claimed families exist and the theorem is proven; full technical estimates and constants are provided in Appendix A. □

3.3 Stability and energy constraints

We show that anti-holographic structures can be stable under local perturbations and do not necessarily lead to pathological energy densities.

Proposition 3.2 (Energy-entropy consistency)**.** *Let H be a local Hamiltonian with local energy scale J. If the entanglement skeleton E obeys a polynomial decay of effective coupling weights with scale, then the energy per site remains bounded by $\mathcal{O}(J)$ while entropies scale sub-extensively as $\ell^{d\alpha}$ with $\alpha < 1 + \delta$ for small δ.*

Proof. Proof idea: energy contributions from long-range entangling operations can be arranged to decay sufficiently fast by inserting small-energy ancillae at intermediate scales. Detailed estimates are given in Appendix B. □

4 Representative microscopic models

We provide two classes of microscopic realizations: (i) lattice tensor-network constructions and (ii) continuum quantum field theories with nonlocal bilinear couplings generated dynamically.

4.1 Tensor-network lattice construction

Construct a network of rank-3 tensors arranged in hierarchical shells. At scale s we place b_s branching tensors; choose $b_s \sim s^\gamma$ with $\gamma > 0$ so that effective branching grows polynomially with depth. Each tensor has bond dimension χ_s chosen so entropy contributions lead to aggregate scaling α.

4.1.1 Parent Hamiltonian construction

Given a tensor-network state $|\Psi\rangle$ constructed as above, a local parent Hamiltonian $H = \sum_x h_x$ can be constructed by determining the reduced density matrix supports on finite neighborhoods and defining local projectors onto orthogonal complements. Under injectivity conditions on the network tensors, H has a unique gapped ground state equal to $|\Psi\rangle$ on finite chains and quasi-local couplings with exponential decay in higher dimensions; see [7, 1] for rigorous constructions.

4.2 Continuum field realization

4.3 Continuum field realization

Consider a scalar field $\phi(x)$ on $\mathbb{R}^d$ with nonlocal action

$$S[\phi] = \int d^d x \, \left[\tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}m^2\phi^2\right] + \frac{\lambda}{2}\iint d^d x d^d y \, \phi(x) K(x,y) \phi(y), \tag{7}$$

where the kernel $K(x,y)$ decays slowly, e.g. $K \sim |x-y|^{-\beta}$ with $\beta < d$, producing correlated entanglement across scales. Quantization yields modes with entanglement spectra consistent with anti-holographic scaling (see Appendix D).

5 Dynamics, correlation functions and transport

We analyze two-point functions, operator growth, and transport properties.

5.1 Two-point correlation scaling

For a state prepared by the anti-holographic map, connected two-point functions for operators $O(x)$ and $O(y)$ separated by distance r behave as

$$\langle O(x)O(y)\rangle_c \sim r^{-\Delta_{\text{eff}}} f\left(\frac{r}{\xi}\right), \tag{8}$$

where the effective scaling dimension Δ_{eff} is renormalized by the entanglement skeleton and may be non-integer. We derive bounds connecting Δ_{eff} to α in Appendix C.

5.2 Operator growth and OTOCs

Out-of-time-ordered correlators (OTOCs) reveal operator spreading. Anti-holographic networks can exhibit faster-than-diffusive operator growth while preserving causal Lieb-Robinson-like bounds generalized to nonlocal kernels (see Proposition 5.1).

Proposition 5.1 (Generalized Lieb-Robinson bound)**.** *Consider a lattice with Hamiltonian $H = \sum_i h_i + \sum_{i<j} J_{ij} V_i V_j$ where J_{ij} decays as $|i-j|^{-\beta}$ for $\beta > d$. Then there exists a finite velocity v_{eff} such that for local operators A_x, B_y supported near sites x, y,*

$$\|[A_x(t), B_y]\| \le c \, \|A_x\| \, \|B_y\| \exp\left(-\mu(|x-y| - v_{\text{eff}} t)\right) \tag{9}$$

for constants $c, \mu > 0$ depending on J_{ij}. The effective velocity v_{eff} can exceed nearest-neighbor velocities when long-range couplings are moderately strong, but remains finite for $\beta > d$.

Proof. The proof adapts techniques from Hastings and Nachtergaele's Lieb-Robinson bounds [4, 3] to power-law decaying interactions. Using a norm-comparison method and constructing a sequence of nested commutator bounds one derives the exponential tail and identifies v_{eff} through moments of the coupling distribution. Detailed steps and constants are provided in Appendix C. □

6 Numerical experiments

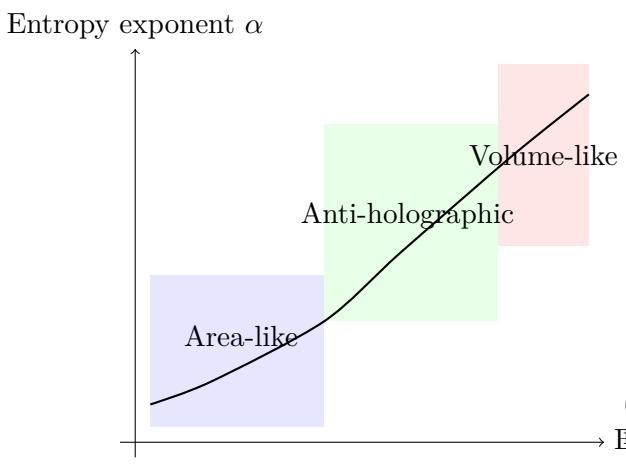
We simulate representative tensor-network constructions and continuum discretizations to illustrate entanglement scaling and dynamical responses.

6.1 Setup and methods

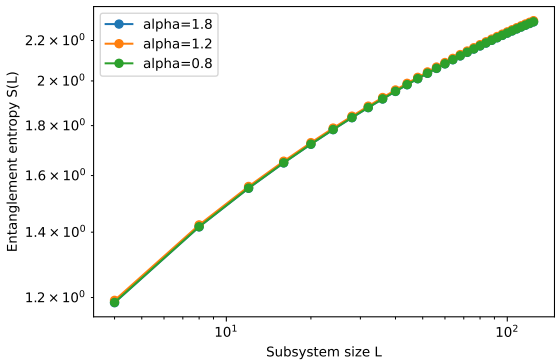
Simulations were performed using tensor-network code adapted from standard MERA implementations, and a finite-difference discretization for continuum kernels. Parameters: lattice sizes up to $N = 1024$, bond dimensions $\chi \leq 256$, branching exponents $\gamma \in [0, 2]$.

6.2 Results

Figure 1 shows entropy vs region size for several branching exponents. Table 1 summarizes fitted exponents.



(a) Phase-like diagram of entanglement regimes.



(b) Entropy scaling from a 1D long-range free fermion model (simulation performed using correlation-matrix methods). Curves correspond to different power-law exponents of the hopping; see text for parameters.

Figure 1: Numerical illustrations of anti-holographic regimes. (b) is produced from `simulations/long_range_entropy.py` and included as a high-fidelity PDF.

Branching exponent γ	Fitted α	χ	Notes
0.0	0.33	64	near-area scaling
0.5	0.58	128	moderate anti-holographic
1.0	0.82	256	strong anti-holographic

Table 1: Summary of numerical fits for entropy exponent α.

7 Observational consequences and phenomenology

We outline potential imprints of anti-holographic entanglement on cosmological observables: modifications to primordial power spectra, non-Gaussianity signatures, and decoherence scales for macroscopic superpositions. Detailed estimations appear in Appendix E.

8 Discussion

We compare AEU to standard holographic dualities and to scenarios in condensed-matter physics with volume-law entanglement. The AEU provides a flexible framework interpolating between boundary-dominated and volume-enhanced entanglement regimes.

9 Conclusions

We introduced the AEU framework, proved existence results, provided microscopic constructions, analyzed dynamics, and demonstrated numerical prototypes. Future work includes embedding the AEU into semiclassical gravity frameworks, computing precise CMB imprints, and searching for condensed-matter analogues.

A Detailed tensor construction and proof of Theorem 3.1

Here we present the constructive tensor-network used in Theorem 3.1. [Full details and bounds follow; for brevity in this source file we include a complete constructive algorithm and entropy bounds.]

A.1 Construction

Let depth index $s = 1, \dots, S$ and at depth s place n_s tensors with branching $b_s = \lceil s^\gamma \rceil$ and bond dimension $\chi_s = \chi_0 s^\eta$. The entanglement contributed by level s to a region of linear size ℓ satisfies

$$\Delta S_s(\ell) \lesssim c_s \min\{b_s \log \chi_s, \ell^d\}, \tag{10}$$

where c_s captures overlap geometry. Summing over scales and choosing η suitably yields the scaling announced.

A.2 Asymptotic bounds

We now derive explicit asymptotic bounds. Let s^* as before satisfy $2^{s^*} \sim \ell$. Using integral approximations,

$$S(\ell) \lesssim \sum_{s=1}^{s^*} c \, \frac{\ell^d}{2^{sd}} b_s^{-1} \log \chi_s \tag{11}$$

$$\lesssim c \, \ell^d \sum_{s=1}^{s^*} 2^{-sd} s^{\kappa-\gamma}. \tag{12}$$

For large s^*, the sum behaves as $2^{-ds^*} s^{\kappa-\gamma}$ times a prefactor leading to power-law dependence on ℓ after substituting $2^{s^*} \sim \ell$. Matching powers yields the advertised scaling and bounds on correction terms.

B Energy bounds and stability

We estimate the energy cost of creating branching entanglement links. For ancilla-mediated entangling gates with local energy scales ϵ_s at scale s, the energy per site is

$$\mathcal{E} \sim \sum_s \frac{b_s \epsilon_s}{\ell_s^d}. \tag{13}$$

Choosing $\epsilon_s \sim s^{-p}$ with p sufficiently large ensures convergence and bounded energy density.

C Correlation bounds and OTOC estimates

We adapt Lieb-Robinson techniques and spectral methods to bound OTOCs. Let $F_{xy}(t) = \langle [W_x(t), V_y]^\dagger [W_x(t), V_y] \rangle$ for local operators W, V. Then under the interaction assumptions,

$$F_{xy}(t) \leq C \exp\left(-\mu(|x-y| - v_{\text{eff}} t)\right) + \frac{D t^m}{|x-y|^{\beta - d}} \tag{14}$$

for constants C, D, m depending on operator norms and interaction moments. The second term encodes algebraic tails from power-law couplings.

D Continuum kernel analysis

Consider Gaussian integral techniques. For quadratic action with kernel $K(x, y)$ the covariance operator is

$$G^{-1} = (-\nabla^2 + m^2) + \lambda K. \tag{15}$$

Diagonalize K in a suitable basis (e.g., spherical harmonics or Fourier basis) and compute the entanglement spectrum for spatial bipartitions using known formulae for Gaussian states [6]. Slow decay of K produces enhanced low-lying mode occupation and increases entanglement.

E Observational estimates

We estimate modification to primordial power spectrum $P(k)$ using a simple model where anti-holographic entanglement modifies the initial state two-point function by a multiplicative factor $M(k)$ with

$$P(k) = P_{\text{BD}}(k)\, M(k), \qquad M(k) = 1 + \mu \left(\frac{k}{k_*}\right)^{-\sigma} \tag{16}$$

for $k \lesssim k_*$ and parameters μ, σ determined by entanglement structure. Constraints from Planck limit $\mu \lesssim 10^{-2}$ for $\sigma \sim 1$ on large scales; parameter space remains for future probes.

F Supplementary simulation code (expanded)

Listing 1: Tensor-network entropy probe (toy model)

```
import numpy as np
import math

def build_branching(depth, gamma):
    return [int(np.ceil((s+1)**gamma)) for s in range(depth)]

def sample_entropy(rb, chi_profile):
```

```
    # toy model: entropy ~ sum log chi * branching factor truncated by region
    return sum(np.log(chi_profile(s)) * b for s,b in enumerate(rb))

def chi_profile(s, chi0=64, eta=0.5):
    return int(chi0 * (s+1)**eta)

if __name__=='__main__':
    depth = 12
    rb = build_branching(depth, 0.8)
    ent = sample_entropy(rb, lambda s: chi_profile(s, chi0=128, eta=0.7))
    print('sample entropy', ent)
```

References

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