$\boxed{1}$

$$F'(x) = \frac{F(x+h) - F(x)}{h}$$

$F(x) = e^x$

$F'(1.5) = e^{1.5} = 4.48168907$

$h = 0.01$:

$$F'(1.5) \simeq \frac{e^{1.51} - e^{1.5}}{0.01} = 4.50417$$

Ca 0.02 Feil $(2 \cdot 10^{-2})$

$h = 0.001$:

$$F'(1.5) \approx \frac{e^{1.501} - e^{1.5}}{0.001} = 4.48393$$

Ca 0.002 Feil $(2 \cdot 10^{-3})$

$h = 0.0001$:

$$F'(1.5) \simeq \frac{e^{1.5001} - e^{1.5}}{0.0001} = 4.4819$$

ca 0.0002 Feil $(2 \cdot 10^{-4})$

$h = 0.00001$:

$$f'(1,5) \approx \frac{e^{1.50001} - e^{1,5}}{0.00001} = 4,481711$$

ca $2 \cdot 10^{-5}$ Feil

$h = 0.000001$:

$$f'(1,5) \approx \frac{e^{1,500001} - e^{1,5}}{0.000001} = 4,48169131$$

osv →

$h = 10^{-12}$: da begynner kalkulatoren å avrunde svaret, og det blir ikkje nøyaktig lenger.

$\boxed{2}$ Gjort i python

Bevis for at Feilen $\propto h^2$:

$$\frac{F(x+h) - F(x-h)}{2h}$$

$$F(x+h) = F(x) + F'(x)h + \frac{F''(x)}{2}h^2 + \frac{F'''(x)}{6}h^3 + O(h^4)$$

$$F(x-h) = F(x) - F'(x)h + \frac{F''(x)}{2}h^2 - \frac{F'''(x)}{6}h^3 + O(h^4)$$

Setter dei inn i $\frac{F(x+h) - F(x-h)}{2h}$:

$$\frac{\left(\cancel{F(x)} + F'(x)h + \cancel{\frac{F''(x)}{2}h^2} + \frac{F'''(x)}{6}h^3 + \cancel{O(h^4)}\right) - \left(\cancel{F(x)} - F'(x)h + \cancel{\frac{F''(x)}{2}h^2} - \frac{F'''(x)}{6}h^3 + \cancel{O(h^4)}\right)}{2h}$$

$$= \frac{2hF'(x) + 2h^3\frac{F'''(x)}{6}}{2h} = F'(x) + \frac{h^2}{6}F'''(x)$$

$\rightarrow$ Feilen blir $\frac{h^2}{6}F'''(x)$

$\boxed{4}$ Eksplisitt:

$$\frac{v_{i,j+1} - v_{ij}}{k} = \frac{v_{i+1,j} - 2v_{ij} + v_{i-1,j}}{h^2} \quad \Big| \cdot k \ , \ + v_{ij}$$

$$v_{i,j+1} = v_{ij} + \frac{k}{h^2}\left(v_{i+1,j} - 2v_{ij} + v_{i-1,j}\right)$$

der $\frac{k}{h^2} = r$

$\boxed{5}$ implisitt:

$$\frac{v_{i,j+1} - v_{ij}}{k} = \frac{v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1}}{h^2} \quad \Big| \cdot h^2 k$$

$$h^2 v_{i,j+1} - v_{ij} h^2 = v_{i+1,j+1} k - 2k v_{i,j+1} + v_{i-1,j+1} k \quad \Big| \begin{array}{l} +2k v_{i,j+1} \\ + v_{ij} h^2 \end{array}$$

$$h^2 v_{i,j+1} + 2k v_{i,j+1} = v_{i+1,j+1} k + v_{ij} h^2 + v_{i-1,j+1} k$$

$$v_{i,j+1}(h^2 + 2k) = v_{i+1,j+1} k + v_{ij} h^2 + v_{i-1,j+1} k$$

$$v_{i,j+1} = \frac{v_{i+1,j+1} k + v_{ij} h^2 + v_{i-1,j+1} k}{h^2 + 2k}$$