For a **fixed** integer $n \geq 2,$ consider the matrices

$$A = \begin{bmatrix} n-1 & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{1}{2} & n-2 & \cdots & -\frac{1}{2} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2} & -\frac{1}{2} & \cdots & 0 \end{bmatrix} \quad \text{and} \quad \frac{1}{n}A = \begin{bmatrix} \frac{n-1}{n} & -\frac{1}{2n} & \cdots & -\frac{1}{2n} \\ -\frac{1}{2n} & \frac{n-2}{n} & \cdots & -\frac{1}{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2n} & -\frac{1}{2n} & \cdots & 0 \end{bmatrix}.$$

For $k = 1, 2, 3, \ldots,$ let

$$\left(\frac{1}{n}A\right)^k = \begin{bmatrix} \frac{n-1}{n} & -\frac{1}{2n} & \cdots & -\frac{1}{2n} \\ -\frac{1}{2n} & \frac{n-2}{n} & \cdots & -\frac{1}{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2n} & -\frac{1}{2n} & \cdots & 0 \end{bmatrix}^k = \begin{bmatrix} a_{1,1}^{(k)}(n) & a_{1,2}^{(k)}(n) & \cdots & a_{1,n}^{(k)}(n) \\ a_{2,1}^{(k)}(n) & a_{2,2}^{(k)}(n) & \cdots & a_{2,n}^{(k)}(n) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1}^{(k)}(n) & a_{n,2}^{(k)}(n) & \cdots & a_{n,n}^{(k)}(n) \end{bmatrix},$$

i.e., $a_{i,j}^{(k)}(n)$ is the (i,j)-th entry of matrix $\left(\frac{1}{n}A\right)^k$.

Obviously, $a_{i,j}^{(k)}(n)$ is a rational function of $n,$ whose denominator (polynomial of n) has degree greater than or equal to the degree of its numerator (also polynomial of n). In particular,

- if $i \neq j,$ then the degree of the denominator of $a_{i,j}^{(k)}(n)$ is k and the degree of the numerator of $a_{i,j}^{(k)}(n)$ is less than $k,$ and
- if $i = j,$ then the entry $a_{i,i}^{(k)}(n)$ is of the form $a_{i,i}^{(k)}(n) = \left(\frac{n-i}{n}\right)^k + b_{i,i}^{(k)}(n),$ where the degree of the denominator of $b_{i,i}^{(k)}(n)$ is k and the degree of the numerator of $b_{i,i}^{(k)}(n)$ is less than $k.$

As a consequence, for any i and $j,$ it holds that $\lim_{k \to +\infty} a_{i,j}^{(k)}(n) = 0,$ where the diagonal entries converge to 0 “relatively slowly” when n is “large enough”.
Hence, it follows that

$$\lim_{k \to +\infty} \left(\frac{1}{n}A\right)^k = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

and that for n “large enough”,

$$\left(\frac{1}{n}A\right)^k \cong \begin{bmatrix} \left(\frac{n-1}{n}\right)^k & 0 & \cdots & 0 \\ 0 & \left(\frac{n-2}{n}\right)^k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \text{ (almost diagonal).}$$