PREFACE

It was on March 20, 1984, that I wrote to Herb Ryser and proposed that we write together a book on the subject of combinatorial matrix theory. He wrote back nine days later that "I am greatly intrigued by the idea of writing a joint book with you on combinatorial matrix theory. ... Ideally, such a book would contain lots of information but not be cluttered with detail. Above all it should reveal the great power and beauty of matrix theory in combinatorial settings. ... I do believe that we could come up with a really exciting and elegant book that could have a great deal of impact. Let me say once again that at this time I am greatly intrigued by the whole idea." We met that summer at the small Combinatorial Matrix Theory Workshop held in Opinicon (Ontario, Canada) and had some discussions about what might go into the book, its style, a timetable for completing it, and so forth. In the next year we discussed our ideas somewhat more and exchanged some preliminary material for the book. We also made plans for me to come out to Caltech in January, 1986, for six months in order that we could really work on the book. Those were exciting days filled with enthusiasm and great anticipation.

Herb Ryser died on July 12, 1985. His death was a big loss for me.[1] Strange as it may sound, I was angry. Angry because Herb was greatly looking forward to his imminent retirement from Caltech and to our working together on the book. In spite of his death and as previously arranged, I went to Caltech in January of 1986 and did some work on the book, writing preliminary versions of what are now Chapters 1, 2, 3, 4, 5 and 6. As I have been writing these last couple of years, it has become clear that the book we had envisioned, a book of about 300 pages covering the basic results and methods of combinatorial matrix theory, was not realistic. The subject, at least as I view it, is too vast and too rich to be compressed into 300 or so pages. So what appears in this volume represents only a

[1] My article "In Memoriam Herbert J. Ryser 1923–1985" appeared in the *Journal of Combinatorial Theory, Ser. A*, Vol. 47 (1988), pp. 1–5.

portion of combinatorial matrix theory. The choice of chapters and what has been included and what has been omitted has been made by me. Herb contributed to Chapters 1, 2 and 5. I say all this not to detract from his contribution but to absolve him of all responsibility for any shortcomings. Had he lived I am sure the finished product would have been better.

As I have written elsewhere,[2] my own view is that "*combinatorial matrix theory* is concerned with the use of matrix theory and linear algebra in proving combinatorial theorems and in describing and classifying combinatorial constructions, and it is also concerned with the use of combinatorial ideas and reasoning in the finer analysis of matrices and with intrinsic combinatorial properties of matrix arrays." This is a very broad view and it encompasses a lot of combinatorics and a lot of matrix theory. As I have also written elsewhere[3] matrix theory and combinatorics enjoy a symbiotic relationship, that is, a relationship in which each has a beneficial impact on the other. This symbiotic relationship is the underlying theme of this book. As I have also noted[4] the distinction between matrix theory and combinatorics is sometimes blurred since a matrix can often be viewed as a combinatorial object, namely a graph.

My view of combinatorial matrix theory is then that it includes a lot of graph theory and there are separate chapters on matrix connections with (undirected) graphs, bipartite graphs and directed graphs, and in addition, a chapter on special graphs, most notably strongly regular graphs. In order to efficiently obtain various existence theorems and decomposition theorems for matrices of 0's and 1's, and more generally nonnegative integral matrices, I have included some of the basic theorems of network flow theory. In my view latin squares form part of combinatorial matrix theory and there is no doubt that the permanent of matrices, especially matrices of 0's and 1's and nonnegative matrices in general, is of great combinatorial interest. I have included separate chapters on each of these topics. The final and longest chapter of this volume is concerned with generic matrices (matrices of indeterminates) and identities involving both the determinant and the permanent that can be proved combinatorially.

Many of the chapters can be and have been the subjects of whole books. Thus I have had to be very selective in deciding what to put in and what to leave out. I have tried to select those results which I view as most basic. To some extent my decisions have been based on my own personal interests. I have included a number of exercises following each section, not viewing the exercises as a way to further develop the subject but with the more

[2] "The many facets of combinatorial matrix theory," *Matrix Theory and Applications*, C. R. Johnson ed., Proceedings of Symposia in Applied Mathematics, Vol. 40, pp. 1–35, Amer. Math. Soc., Providence (1990).

[3] "The symbiotic relationship of combinatorics and matrix theory," *Linear Algebra and Its Applications*, to be published.

[4] Ibid.

modest goal of providing some problems for readers and students to test their understanding of the material presented and to force them to think about some of its implications.

As I mentioned above and as the reader has no doubt noticed in my brief chapter description, the topics included in this volume represent only a part of combinatorial matrix theory. I plan to write a second volume entitled "Combinatorial Matrix Classes" which will contain many of the topics omitted in this volume. A tentative list of the topics in this second volume incudes: nonnegative matrices; the polytope of doubly stochastic matrices and related polytopes; the polytope of degree sequences of graphs; magic squares; classes of (0,1)-matrices with prescribed row and column sums; Costas arrays, pseudorandom arrays (perfect maps) and other arrays arising in information theory; combinatorial designs and solutions of corresponding matrix equations; Hadamard matrices and related combinatorially defined matrices; combinatorial matrix analysis including, for instance, the role of chordal graphs in Gaussian elimination and matrix completion problems; matrix scalings of combinatorial interest; and miscellaneous topics such as combinatorial problems arising in matrices over finite field, connections with partially ordered sets and so on.

It has been a pleasure working these last several years with David Tranah of Cambridge University Press. In particular, I thank him and the series editor Gian-Carlo Rota for their understanding of my desire to make a two-volume book out of what was originally conceived as one volume. I prepared the manuscript using the document preparation system LaTeX, which was then edited by Cambridge University Press. I wish to thank Wayne Barrett and Vera Pless for pointing out many misprints. My two former Ph.D. students, Tom Foregger and Bryan Shader, provided me with several pages of comments and corrections. During the nearly five years in which I have worked on this book I have had, and have been grateful for, financial support from several sources: the National Science Foundation under grants No. DMS-8521521 and No. DMS-8901445, the Office of Naval Research under grant No. N00014-85-K-1613, the National Security Agency under grant No. MDA904-89 H-2060, the University of Wisconsin Graduate School under grant No. 160306 and the California Institute of Technology. For the last 25 years I have been associated with the Department of Mathematics of the University of Wisconsin in Madison. It's been a great place to work and I am looking forward to the next 25.

I wish to dedicate this book to the memory of my parents:

Mollie Verni Brualdi
Ulysses J. Brualdi

Richard A. Brualdi
Madison, Wisconsin