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Exponential random graph modeling of a faculty hiring network: The IEOR case

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ABSTRACT

Faculty hiring networks consist of academic departments in a particular field (vertices) and directed edges from the departments that award Ph.D. degrees to students to the institutions that hires them as faculty. Study of these networks has been used in the past to find a hierarchy, or ranking, among departments, but they can also help reveal sociological aspects of a profession that have consequences in the dissemination of educational innovations and knowledge. In this article, we propose to use a new latent variable Exponential Random Graph Model (ERGM) to study faculty hiring networks. The model uses hierarchy information only as an input to the ERGM, where the hierarchy is obtained by modification of the Minimum Violation Ranking (MVR) method recently suggested in the literature. In contrast to single indices of ranking that can only capture partial features of a complex network, we demonstrate how our latent variable ERGM model provides a clustering of departments that does not necessarily align with the hierarchy as given by the MVR rankings, permits to simplify the network for ease of interpretation, and allows us to reproduce its main characteristics including its otherwise difficult to model presence of directed self-edges, common in faculty hiring networks. Throughout the paper, we illustrate our methods with application to the Industrial/Systems/Operations Research (IEOR) faculty hiring network, not studied before. The IEOR network is contrasted with those previously studied for other related disciplines, such as Computer Science and Business.

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1. Introduction

A faculty hiring network is represented by a graph $G = (V, E)$ composed of vertices or nodes $v_i \in V, i = 1, \dots, n$ denoting university departments in a given academic discipline, and directed arcs or edges $e_{ij} \in E, i, j = 1, 2, \dots, n$ whose integer value attribute denotes the number of faculty hired by department v_j who received their Ph.D. in department v_i. A department hiring a faculty member who received his/her Ph.D. from another department generates a directed edge in the network going from the sender department to the receiver department. The network can be represented by an adjacency matrix (or "sociomatrix") $\mathbf{Y} = [Y_{ij}] \in \mathbb{Z}^{n \times n}$ that denotes the number of faculty generated by department i currently hired by department j. In the pre-internet era, the study of faculty hiring networks was confined to departments of Sociology (Schichor, 1970; Burris, 2004) where directories with the necessary faculty information existed in print. Today, with most departments and faculty posting their information in personal web pages, studying a faculty hiring network has become easier to do, even if no directories are available, provided the faculty information is gathered. There now exist analyses of faculty hiring networks in a broad range of disciplines in the US, such as Political Science (Fowler *et al.*, 2007; Schmidt and Chingos, 2007), Mathematics (Myers *et al.*, 2011), Communication (Mai *et al.*, 2015), Business (Clauset *et al.*, 2015), Computer Science (Clauset *et al.*, 2015; Huang *et al.*, 2015), Law (Katz *et al.*, 2011), and History (Clauset *et al.*, 2015).

Researchers have studied the hiring and placement patterns of their academic fields for a variety of reasons. One concept these studies attempt to clarify is the *prestige* of a department, a rather elusive and inadequately theorized concept (Burris, 2004), but popular among university administrators. Departmental prestige can be studied as an effect due to the position of the department in a hierarchy existing in the faculty hiring network, as these positions provide a ranking within a discipline. Some authors have considered hiring networks for the purpose of determining the inequality in the production of Ph.D. holders (Clauset *et al.*, 2015), an apparently general phenomena in which a small number of departments produce a large fraction of the Ph.Ds hired as professors. Others have studied these networks to determine inequalities in the hiring process (e.g., of women, see Way *et al.* (2016)), or to study the sociological aspects of a discipline (Fowler *et al.*, 2007; Katz *et al.*, 2011), e.g., whether communities or a dominance hierarchy exists among departments. Hiring and placing of Ph.D. students is also studied in some areas to understand how new ideas are disseminated through a profession.

In this article we propose new methodology for the study of a faculty hiring network and illustrate it with application

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to the Industrial/Systems/Operations Research (from now on, IEOR) hiring faculty network of the USA. The network is composed of departments/programs/schools (from now on, "departments") within these three inter-related fields. Appendix 1 gives a list of all 83 IEOR departments considered, how they were selected, and the different criteria and assumptions that were used in collecting the departmental and faculty data. The IEOR dataset was compiled in summer 2016. The main assumption of our approach is that a faculty hiring network may imply a hierarchy between the departments, and this hierarchy can be used to develop a random graph model that explains the topology of the network. We therefore first perform statistical tests on the existence and steepness of a linear dominance hierarchy. Given evidence of such a hierarchy, we introduce the notion of a Minimum Violation and Strength (MVS) measure of departmental prestige, and use it to compute a near linear hierarchy, which can be contrasted with other measures of network importance, including published rankings (US News and NRC).

Although single measures of department prestige provide an inherently incomplete description of a complex network, we show how the MVS rankings are useful as covariates in an Exponential Random Graph Model (ERGM) in which the positions of each department in a social space of relations are latent variables to be determined. The model allows us to consider the uncertainty and sparseness in the edge data and permits us to find groups of related departments. Our main contribution consists in demonstrating how with the latent variable ERGM model it is possible to reduce or simplify a faculty hiring network, and to reproduce its main features. Throughout this article, we use the IEOR faculty hiring network to illustrate the methods and compare it with similar networks in related academic disciplines. We conclude with a summary and discussion of the implications of our findings. We first start by describing the main characteristics of the IEOR network.

2. Descriptive statistics of the IEOR network compared to those from related disciplines.

IEOR faculty data were collected during May–June of 2016 from faculty web pages working in institutions in the USA. In total, 1179 faculty from 83 IEOR departments were considered in the analysis (Appendix 1). Table 1 lists general descriptive statistics of the IEOR network compared with those of two closely related fields, Computer Science (CS) and Business schools, networks analyzed (under similar data collection assumptions than us) by Clauset *et al.* (2015), excluding their "Earth" (outside US) department and links from and to this vertex. The IEOR network is considerably smaller, with an average department size of 14.2 faculty compared with 21.4 faculty for CS and the much bigger schools of Business with an average number of 70.1 faculty. Two distinctive characteristics of the IEOR network relative to CS and Business (see Table 1) are: (i) its much larger proportion of self-hiring, given by the self-edges in the network (14% of all faculty, almost three times that in Business and close to twice that of CS); and (ii) its lower degree assortativity, explained below. Consistent with previous studies of faculty hiring networks of other disciplines (e.g., Clauset *et al.* 2015), we define a *self-hire* in the wide sense of a department hiring a faculty member that received his/her Ph.D. from the same institution as the department in question, but not necessarily from the same department within the institution. Furthermore, not all self-hires are necessarily hired immediately after graduation. For IEOR, 47.5% of all self-hires were immediate (there are no immediate self-hiring data available for Business and CS). Figure 1 (top) depicts the number of self-hires among the 83 IEOR departments, sorted in a hierarchy that will be explained below. Note how prevalent self-hiring is: 74% of the IEOR departments have (as of summer 2016) self-hired. This is similar to Business schools (74%) but higher than in CS departments (58%). Almost 14% of all IEOR faculty are hired by their originating institution. In some departments, self-hires account for more than 40% of the faculty (see Figure 1, bottom). Our dataset, however, has only limited information about the intermediate departments where a faculty worked prior to returning to her alma mater.

Table 1. Descriptive statistics of the IEOR network compared with that of CS departments and Schools of Business. Assortativity is a measure of how much vertices with similar values of an attribute connect together (the Minimum Violation Ranking (MVR) is explained in Section 4.)

					Assortativity		
Network	*Vertices*	*Edges (% Female)*	*Self-edges/ edges*	*Density*	*Degree*	*MVR*	*Reciprocity*
IEOR	83	1179 (19.5)	0.1399	0.1121	0.1452	0.4614	0.1538
CS	205	4388 (25.6)	0.0711	0.0688	0.2964	0.5327	0.1264
Business	112	7856 (16.8)	0.0556	0.2738	0.2661	0.4330	0.2197

The *assortativity* index is a correlation measure of how much vertices with similar values of a given scalar attribute tend to be connected together (for this and other descriptive network statistic definitions, see, e.g., Newman (2010)). Shown in Table 1 are the assortativity measurements with respect to two vertex attributes. The first one, assortativity based on the total degree of each vertex (department) indicates a lower tendency for IEOR departments, compared with those in CS and Business, to establish hiring-placement connections with departments with a similar total number of hires and placements. The assortativity with respect to the MVR indices is an indication of how likely departments establish hiring-placement relations with departments equally ranked in a hiring hierarchy to be discussed in Section 4. For IEOR, this assortativity is between that of CS and Business.

Other descriptive statistics in the IEOR network shown in Table 1 are also somewhere between those of the CS and the Business networks. The density or connectance of the IEOR network, defined as the number of non-zero entries in the adjacency matrix $\mathbf{Y}$ divided by the maximum number of possible edges, is between CS (which has a very sparse network) and Business (which is densest). As it will be shown below, the IEOR faculty hiring network is actually quite sparse except for a group of departments, those at the top of a hiring hierarchy, which connect more often among them.

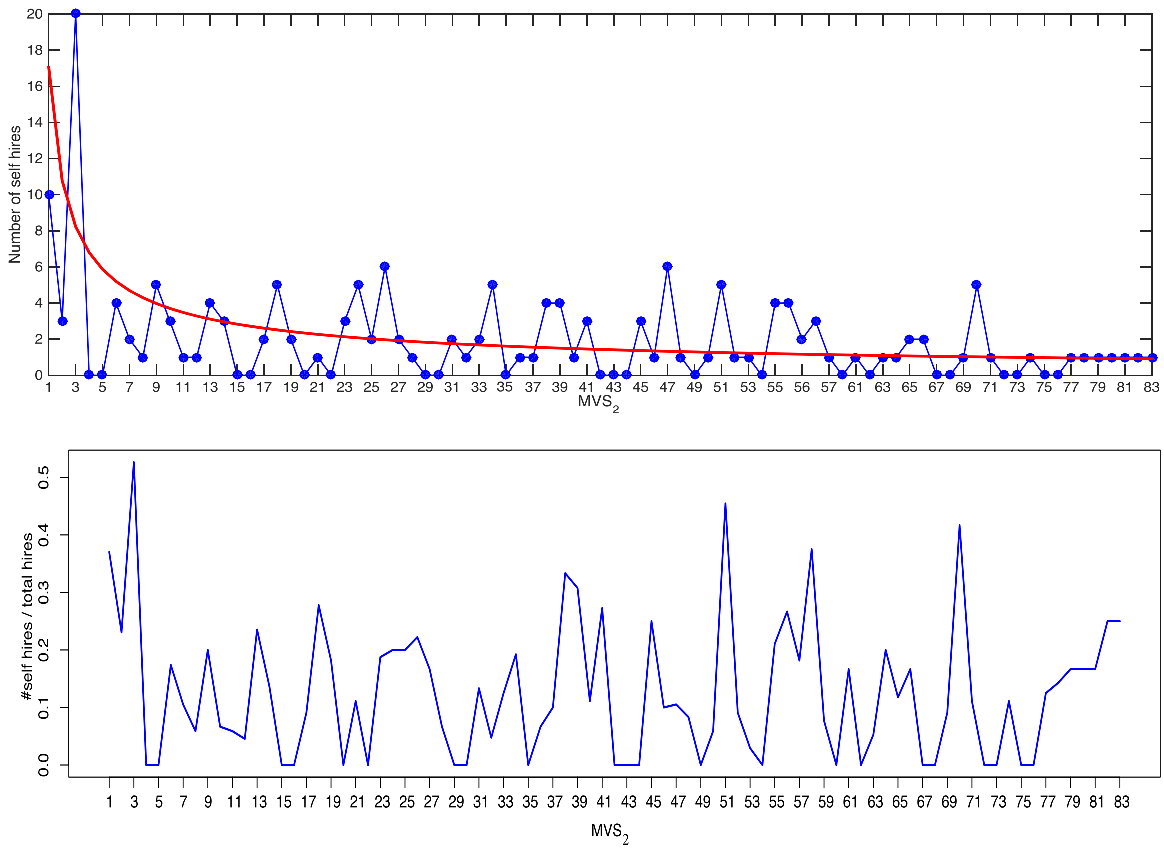

Figure 1. Top: in blue, observed number of self-hires among the 83 IEOR departments sorted by MVS_2 hierarchy index described later in this article. In red, expected number of self-hires (μ_{ii}) as predicted by the latent location ERGM model described in Section 5. Bottom: self hires as a percentage of all faculty in each department, sorted by MVS_2 index. While self-hires are more numerous in the top departments, self-hires as a proportion of total faculty are significant across many of the IEOR departments.

The proportion of female faculty in IEOR is 19.5%, higher than in CS (16.8%) but lower than in Business schools (25.6%), an inequality that calls for further analysis that is beyond the present study. Also shown in Table 1 is the *reciprocity*, the proportion of departments with exchanges of mutual hires. The reciprocity of the IEOR network is also between that of CS and Business.

Finally, the inequality in the production of Ph.D holders among IEOR departments, although quite large, is also between that of CS departments and Business schools. Figure 2 shows the Lorenz curves for the proportion of Ph.D. holders produced. Approximately, 10% of the IEOR departments generate about 50% of the faculty, although this is not as pronounced as in Business schools, which have a very steep Lorenz curve near zero, with around 3% schools generating 40% of the faculty.

Some additional descriptive statistics, in particular, the correlations between vertex ranks based on common network centrality measures are given in Section 4.

3. Existence of a linear dominance hierarchy

Given a faculty hiring network, a question of interest is if it is possible to order the departments in such a way to form a *linear dominance hierarchy*. In this section we review methods to determine the existence of a hierarchy in a social network, as we wish to use such hierarchy information to model a faculty network in later sections. The concept of dominance, popular in social networks in ecology and competitive sports, refers to an attribute of the pattern of repeated interactions between two individuals, characterized by a consistent outcome in favor of the same individual in a dyad (De Vries, 1995). The consistent winner is dominant and the loser subordinate. Individuals form a *linear* dominance hierarchy if and only if: (i) for every dyad (i, j) either i dominates j or j dominates i; and (ii) every triad is transitive, i.e., for any individuals i, j, and k it is true that if i dominates j and j dominates k, then i dominates k. Transitivity is equivalent to a dominance hierarchy that is acyclic (Ali *et al.*, 1986). The notion of a dominance hierarchy has been studied in faculty networks by Clauset *et al.* (2015), Huang *et al.* (2015), and Mai *et al.* (2015). Applied to faculty hiring networks, if a department v_j hires more Ph.D. holders from department v_i than the number v_i hires from v_j, the relationship between these two vertices implies a "dominance" relation of department i with respect to department j, given that j wishes more strongly to have access to the students produced in department i and not *vice versa* (ties are therefore allowed). In his study of animal societies, Landau (1951) defined a score structure $W = (w_i, w_2, \ldots, w_n)$ where each w_j equals the number of individuals dominated by element j. A *hierarchy* is then a score structure

$$W = (w_1 = n-1, w_2 = n-2, \ldots, w_n = 0)$$

so that the members of the society can be ordered as $1 \succ 2 \succ 3 \succ \ldots \succ n$, with each dominating all the members

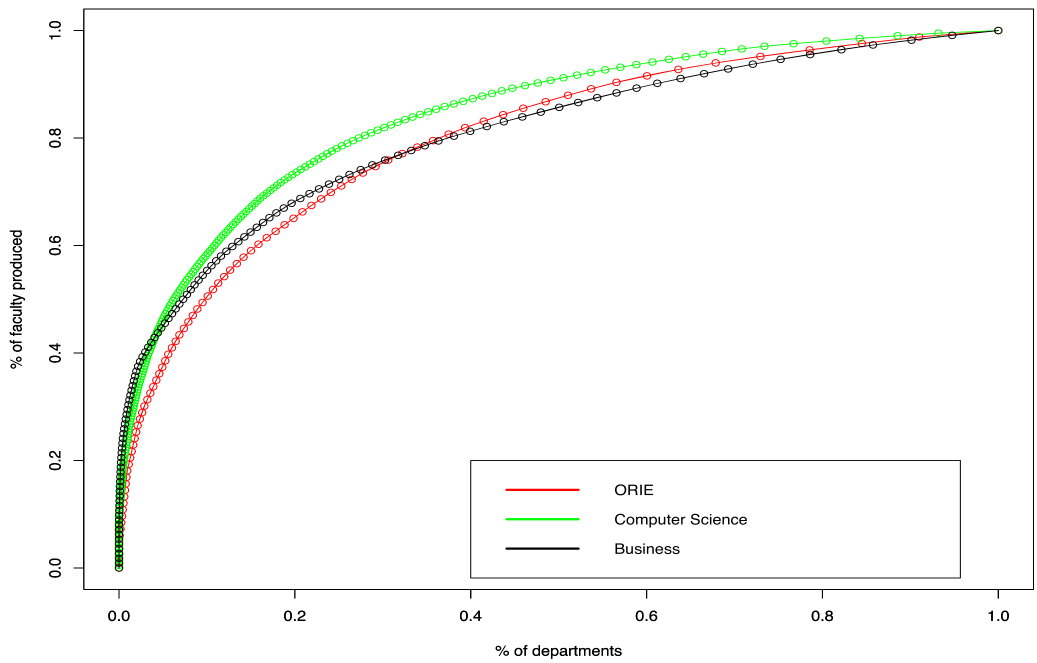

Figure 2. Lorenz curves indicating the inequality in the percentage of faculty production in IEOR, CS, and Business as a percentage of the number of departments. About 10% of IEOR departments generate close to half of all IEOR faculty.

below it, and being dominated by all those above. At the opposite extreme is an "egalitarian" society where assuming n odd,

$$W = \left(w_1 = \frac{n-1}{n}, w_2 = \frac{n-1}{n}, \ldots, w_n = \frac{n-1}{n} \right).$$

Landau (1951) then introduced his "hierarchy index" h, a measure of the variability of the w_js normalized so that $h=0$ implies equality (egalitarian dominance) and $h=1$ implies a perfect linear hierarchy, with h equal to

$$h = \frac{12}{n(n^2-1)} \sum_{j=1}^{n} \left(w_j - \frac{n-1}{2} \right)^2.$$

Of course, a perfect linear hierarchy may not exist in a society, and some violations to a perfect linear hierarchy may exist, a topic we discuss in Section 4.

A practical difficulty when determining if a linear hierarchy exists in a society (and in general, for any other inference one desires to attempt based on observed social interactions) is that we may have few data points about individuals interacting, with many not interacting during the observation period. The basic approach to deal with the spareness of the observed adjacency matrix $\mathbf{Y}$ is to treat it as a realization of a stochastic process, and to use nonparametric tools for statistical inference. Along this approach is De Vries (1995), who introduces a randomization test for the hypothesis of no linear hierarchy based on Landau's h statistic that takes into account unknown tied relationships (in our case, these are departments that have never hired each other's Ph.D. students). To perform the test, each dyad (i, j) is randomized m times and the h statistic is computed for each random sociomatrix giving a set of numbers $\{h_s\}_{s=1}^{m}$. h is also computed for the observed sociomatrix. If the empirical p-value of the test, defined as $1-|\{h_s>h\}|/m$ (where $|\cdot|$ indicates cardinality) is small, this is evidence that the observed linear hierarchy is statistically different than that of a random matrix, which has no hierarchy.

Ordering individuals in a linear or near-linear hierarchy is justified only if there is statistical evidence in favor of such a hierarchy comparing the existing hierarchy to what could be obtained from a random matrix. Finding a near-linear hierarchy $(h<1)$ is a hard combinatorial problem about which we comment below. Figure 3 shows results of the De Vries tests applied to the IEOR, CS and Business networks (data for CS and Business are from 2015 taken from Clauset *et al.* (2015)). The empirical p-values are zero for all three disciplines, implying there is a statistically significant linear hierarchy in each of the three networks.

If a linear or near-linear hierarchy is significant, a related question is how *steep* is the hierarchy. In Appendix 2 we statistically test for the significance of the steepness of a linear hierarchy (De Vries *et al.*, 2006), and find that the IEOR, CS and Business networks all have a hierarchy with a statistically significant slope.

The tests for the significance of a linear hierarchy and for a significant slope were repeated for the top IEOR departments (sorted according to the MVS_2 index in Table A1 in Appendix 1, further discussed below). Table 2 shows the results of these tests applied to an increasing number of (sorted) departments. The slope is quite steep and significant only for the first 10 departments. Overall, for the 83 IEOR departments the slope is statistically different from zero, but its actual numerical value is rather small.

Similar to the tests in this section, in the following we will again treat the observed network as a noisy realization of possible networks, and following Clauset *et al.* (2015) we will use a "bootstrapping" approach to network creation to help perform an analysis that considers other possible (sparse) networks that could have been observed by chance.

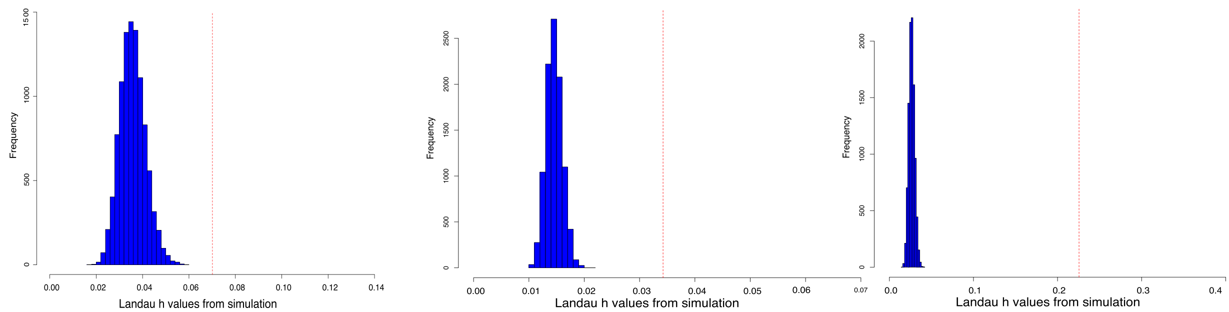

Figure 3. De Vries' test for the presence of a linear dominance hierarchy in the complete ($n = 83$) IEOR network (Left), the CS network (Middle, $n = 205$) and the Business network (Right, $n = 112$). Randomization distribution (10 000 simulations) under the null hypothesis of random graph of Landau's h values. Red vertical lines are the observed h values. In all cases, the empirical p-values are 0.0, indicating strong evidence of the presence of a hierarchy in each of three faculty networks.

Table 2. Dominance hierarchy linearity and steepness test results for different IEOR subnetworks. The hierarchy is steeper closer to the top, where it is also much denser. Hierarchy considered is the MVS_2 hierarchy explained in Section 4.

No. of departments	*Density (connectance)*	*Observed h*	*p-value*	*Slope*	*p-value*
Top 10	0.6000	0.5590	0.0307	−0.4281	0.0000
Top 20	0.4025	0.3036	0.0098	−0.2061	0.0000
Top 50	0.1912	0.1328	0.0004	−0.0688	0.0000
All 83	0.1121	0.0701	0.0000	−0.0278	0.0000

3.1. A caveat: "observational zeroes" in a sparse sociomatrix

The preceding analysis indicates that the 83 IEOR departments can be ranked according to a statistically significant near linear hierarchy using hire-placement data, even though the hierarchy appears to be quite flat for most of the n departments except at the very top of the hierarchy. The de Vries' steepness test (Appendix 2) depends on the number of interactions between the actors or individuals in the network. In animal behavior, the ideal way to find a hierarchy is to observe pairwise competitions in a balanced (designed) "tournament" (David, 1987), but in the field of social network data an abundance of "observational zeroes" makes it more difficult to determine a hierarchy. Although no formal power analysis is available, De Vries (1995) gives some numerical evidence to suggest that the power of the linearity test to detect an existing hierarchy goes down when the frequency of observational zeroes increases, and it is quite possible that the same occurs in the steepeness test. Table 2 shows how the connectance (density) of the departments among the top of the hierarchy is much higher than among the rest, and this implies the test statistics have more information about placements and hires within this group that among dyads that include at least one individual ranked lower in the hierarchy.

The predominance of zeroes in a sociomatrix may not necessarily indicate there are no dominance relations among the dyads. As warned by De Vries *et al.* (2006) one should be careful when interpreting linearity and steepness tests for societies in which there are very few pairwise interactions recorded. This warning is by extension worth keeping in mind when trying to find a dominance relation or ranking in a faculty hiring network. We can distinguish between three types of "observational zeroes" in the adjacency matrix (or sociomatrix):

1. In animal behavior, if two individuals are not observed to interact, it may be because there is a dominance relation present, with the subordinate individual avoiding the dominant individual. We will call the similar case of departments avoiding hiring from higher ranked departments *deference* or *avoidance* zeroes.
2. There may simply be no dominance-subordinate relation between two individuals that "respect" each other, a case analogous to departments that do not have hiring-placement relations simply due to their finite capacity, which limits the possibility of observing more hiring-placement relations, a case we will call *equality* zeroes.
3. It could happen that we have not observed long enough the network and the question of whether there is a dominance between two individuals is unresolved. This would be the case when relatively young departments have not established hiring-placement relations with many departments simply due to their young age. This case will be called *unresolved* zeroes.

It is not possible, however, to determine from the data what kind of "observational zeroes" one is dealing with in a particular hiring network.

With these precautionary notes we now proceed to find a near linear hierarchy in the IEOR network. For the CS and Business networks, see Clauset *et al.* (2015). The presence of "observational zeroes" and the low density of the IEOR network justifies the bootstrapping approach of the next section. Likewise, the different connectance among strata of the hierarchy and its corresponding changes in steepness also justifies the search for groups (communities) of departments, a task we address in Section 5, where we incorporate the hierarchy information into a latent variable ERGM.

4. The minimum violations and minimum violations and strength indices

Clauset *et al.* (2015) define a hierarchical index of a faculty hiring network by a permutation π that induces the minimum number of edges that "point up" the hierarchy. This is found by

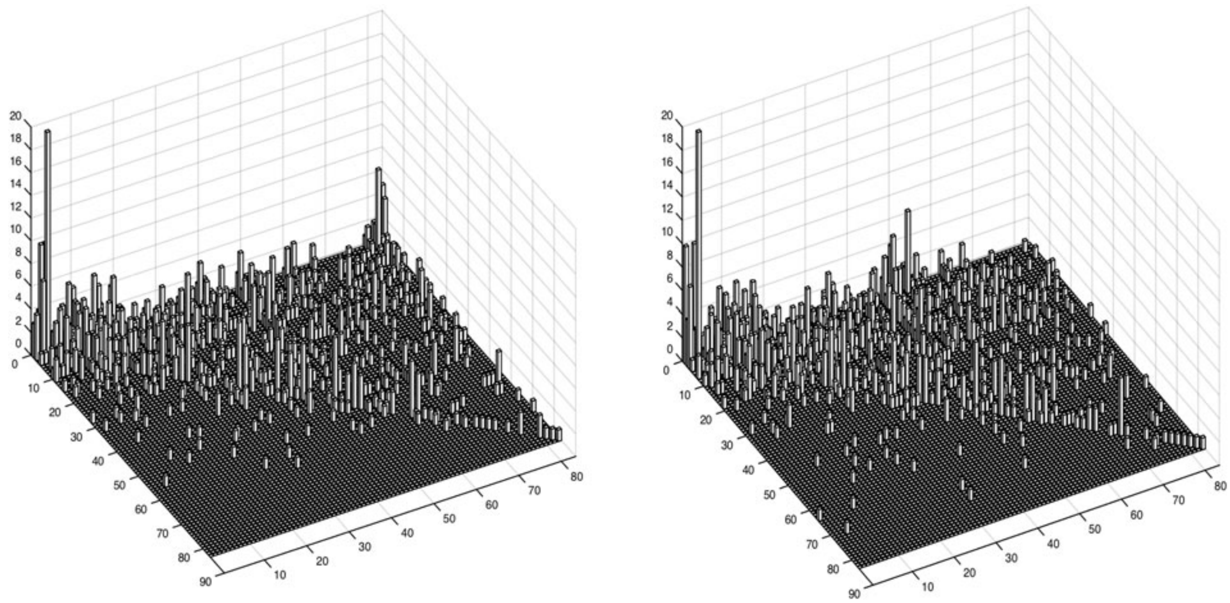

Figure 4. Left: matrix bar plot of the $\mathbf{Y}$ adjacency matrix under the MVS_1 rankings, which considers the strength of a type 1 violation (unexpected placement of a faculty from a lower to a higher ranked department). Right: matrix bar plot of the $\mathbf{Y}$ adjacency matrix under the MVS_2 rankings, which considers the strength of a type 1 violation and also of type 2 violations (unexpected number of hires from lower ranked departments). The plot on the right is closer to "hillside" than the one on the left, a consequence of the different objectives. Note the peak in the upper right corner in the plot on the left, these are departments that hire an unexpected number of faculty from the top departments, yet ended up, under the MVS_1 rankings, in a very low position. The MVS_2 rankings ameliorate this situation to a certain degree.

$$\min_{\pi(\mathbf{Y})} S(\pi(\mathbf{Y})) = \min_{\pi(\mathbf{Y})} \sum_{i>j}\sum Y_{ij}\mathrm{sign}\big(\pi(i)-\pi(j)\big) \qquad (1)$$

Thus, if there is an edge from i to j ($Y_{ij}>0$) and $\mathrm{sign}(\pi(i)-\pi(j)) = +1$ (i.e., $\pi(i)>\pi(j)$) this means a lower ranked department in the hierarchy has placed a faculty at a higher ranked department (recall rank one is highest), in other words, we have a "violation" of the hierarchy implied by π and this will increase $S(\pi(\mathbf{Y}))$. Similarly, if there is an edge from i to j and $\mathrm{sign}(\pi(i)-\pi(j)) = -1$ (i.e., $\pi(i)<\pi(j)$), this is not a violation, and will make $S(\pi(\mathbf{Y}))$ decrease. A permutation $\pi(\mathbf{Y})$ that minimizes Equation (1) is called a *Minimum Violation Ranking* (MVR) which has been proposed as the optimal way of rankings players in a round-robin tournament (Ali *et al.*, 1986). For networks where a perfect linear hierarchy does not exist ($h<1$, thus violations are unavoidable) solving Equation (1) is a hard combinatorial problem. Problem (1) is in particular equivalent to reordering the columns and rows of the adjacency matrix such that we get an upper triangular matrix, a problem which has been proved to be NP-complete (Charon and Hudry, 2010). For the 83 departments in the IEOR network, however, an exact algorithm for finding the MVRs, such as Pedings *et al.* (2012) binary linear integer programming formulation, is computationally feasible. Note that multiple optimal MVR rankings with the same number of violations may exists in a complex network. Although we will argue that the MVRs do not fully reflect the hierarchy of a faculty hiring network, Table A1 in Appendix 1 lists the MVR rankings of the 83 IEOR departments considered in this study for completeness.

Rather than solving Equation (1), which considers only the number of violations, we could also consider the *strength* of each violation. To obtain a dominance relation in a society of animals, De Vries (1998) defines the strength of the violations in a sociomatrix as:

$$\min_{\pi(\mathbf{Y})} S_{MVS_1}(\pi(\mathbf{Y})) = \min_{\pi(\mathbf{Y})} \sum_{i>j}\sum (i-j)Y_{ij} \qquad (2)$$

where the difference $(i-j)$ (with $i>j$) measures the strength of a violation in the ranking (which exists if $Y_{ij}>0$). In our case, entries under the diagonal are *unexpected faculty hires*, where department j hires a number of faculty from a lower ranked department i. We will refer to the resulting rankings from solving Equation (2) the *Minimum violations and strength* rankings and will denote them by MVS_1. To obtain these rankings, we modify the stochastic search algorithm in Clauset *et al.* (2015) to account for the strength of a violation (see Appendix 3). The IEOR MVS_1 rankings are shown in Table A1 in Appendix 1.

A problem with both the MVR and MVS_1 rankings previously proposed in the literature on hiring networks is that a department that places few of its own former students, despite hiring from the top departments, may be ranked very low. For instance, this is the case, under both MVR and MVS_1 rankings, of Naval Postgraduate School OR dept., which received among the lowest rankings using these two indices. A department that consistently hires from top ranked departments should be ranked higher than one that not only does not place its Ph.D. holders but also does not have any hiring interactions with the top ranked group. To illustrate, a bar graph of the adjacency matrix $\mathbf{Y}$ sorted according to the MVS_1 rankings indicates the nature of the problem (Figure 4, left): while both Equation (1) and Equation (2) tend to minimize the number of entries below the diagonal, the entries above the diagonal are not considered. Note the large Y_{ij} values in the upper right corner of the matrix plot; these correspond to low ranked departments under the MVS_1 criterion that hired repeatedly from the very top departments in the ranking, yet they ended up with a very low MVS_1 ranking. To correct this anomaly, we propose to also account for this type of secondary violation, which we will call *unexpected placements* i.e., when a top department places too many Ph.D. holders in other lower ranked departments that should not be that attractive for their graduating students. Penalizing this kind of violation (and its strength) will result in an improved ranking for a department that "hires high". A compound criterion, including the strength of both unexpected hires and unexpected placements is:

$$\min_{\pi(\mathbf{Y})} S_{\text{MVS}_2}(\pi(\mathbf{Y})) = \min_{\pi(\mathbf{Y})} \sum_{i>j}\sum (i-j)Y_{ij} + (i-j)Y_{ji} \tag{3}$$

where Y_{ji} in the second term considers abnormal entries above the diagonal (since $i > j$) and $(i-j)$ measures the strength of this type of violation, similarly to Equation (2). It would be wrong, however, to use this compound criteria as it gives equal weight to the two types of violations, whereas unexpected hires (or violations, i.e., entries below the diagonal) should be accounted more severely than unexpected placements (above the diagonal). Assigning weights to each objective is *ad-hoc* and therefore not a solution. Instead, to give primary importance to violations (unexpected hires), the pairwise stochastic swapping algorithm that we utilize (Appendix 3) in solving Equation (2) was defined to accept a new ranking (i.e., exchanging the rankings of two departments in question) if either:

1. the number of (unexpected hire) violations is lower after the switch, or
2. the number of (unexpected hire) violations is the same as before the switch, but the sum of the strengths of the two types of violations, $S_{\text{MVS}_2}(\pi(\mathbf{Y}))$, is lower after the switch.

We refer to the resulting rankings as MVS_2 rankings. They are reported in Appendix 1, Table A1.

A concept related to the two types of violations, unexpected hires and unexpected placements, found in the area of sport teams rankings, is that of an adjacent matrix in "hillside" form (Pedings *et al.*, 2012). In sport team rankings, the adjacency matrix contains the point or goal differential between all pairs of teams in a tournament. A sports tournament sociomatrix $\mathbf{Y}$ is in hillside form if it is ascending along all rows and descending along all columns, i.e., if $Y_{ij} \le Y_{ik}, \forall i, \forall j \le k$, and $Y_{ij} \ge Y_{kj}, \forall j, \forall i \le k$ (Pedings *et al.*, 2012). Associated with this definition there are two types of violations: "upsets", nonzero entries below the diagonal matrix, and "weak wins", entries above the diagonal matrix that do not follow a hillside pattern, i.e., when team i did not score as many points as it would have been expected when playing team j, with $j > i$.

Although the concept of weak wins and upsets in a sports tournament is similar to that of unexpected hires and placements, there are important differences. In a sports tournament, the sociomatrix contains goal differentials, and therefore the hillside form as defined above is the ideal form of a hierarchy. Contrary to a tournament, where every team plays against all other teams and therefore a *complete comparison network* can be formed, we do not have pairwise comparisons for all dyads in a faculty hiring network, i.e., not all edges are observed as we have "observational zeroes" as mentioned before, and this requires special consideration. In addition, in a faculty hiring network the definition of a *hillside form* adjacency matrix needs to be modified to account for both unexpected hiring and unexpected placements, since we wish to find a hierarchy such that we have both descending columns *and descending rows* as well:

$$Y_{ij} \ge Y_{ik}, \quad \forall i, \forall j \le k, \quad \text{and} \quad Y_{ij} \ge Y_{kj}, \quad \forall j, \forall i \le k.$$

That is, instead of winning by more goals against decreasingly ranked opponents, higher ranked departments are expected to place fewer faculty at decreasingly lower ranked departments. According to our definition, the MVS_1-ranked matrix (see Figure 4, left) is not in hillside form since there is a "peak" on the upper right cell of the matrix.

Figure 4 (right) shows the adjacency matrix for the IEOR departments sorted according to the MVS_2 indices. Note how the matrix is closer to hillside form, i.e., a matrix with both rows and columns closer to being monotonically descending (Naval Postgraduate School then deservedly ranks much higher, and the peak on the left plot is moved considerably further to the left of the matrix).

4.1. Bootstrapping the observed network

Given the sparseness typical of a faculty hiring network due to observational zeroes, we provide more robust MVS_2 (and MVS_1) indices by optimizing 1000 "bootstrapped", or randomly sampled (with replacement) networks (using significantly less than 1000 bootstrap resamples did not provide clearly estimated rank distributions although unfortunately this increased the computing time necessary to obtain each index to 15.4 hours. on a quad core IMac running Matlab). The bootstrapped networks are obtained by sampling edges, with replacement, from the observed faculty hiring network, such that the probability of sampling an edge is proportional to the number of faculty Y_{ij} in that edge. Each bootstrapped network $\mathbf{Y}_b$ was then optimized according to the $S_{\text{MVS}_1}(\pi(\mathbf{Y}_b))$ or $S_{\text{MVS}_2}(\pi(\mathbf{Y}_b))$ criteria using the stochastic search method of Appendix 3, and the resulting MVS values were recorded for each department. The results for MVS_2 across the ensemble of bootstrapped replications for the IEOR network are shown in Figure 5. Note how there is less uncertainty in the MVS_2 values in the extremes of the hierarchy, and more uncertainty for those ranked in the middle, a phenomenon also reported for CS and Business networks (Clauset *et al.*, 2015). The final indices MVS_1 and MVS_2 reported on Table A1 are the average ranks computed from the 1000 bootstrapped and optimized networks. Using medians or the so-called Kemeny–Young scores for finding an aggregate ranking are other possible ways to obtain a single ranking from the ensemble, although we used means for compatibility with the MVR statistics in Clauset *et al.* (2015). The resulting indices provide a linear ordering which does not allow for ties. If desired, but inconsequential to the

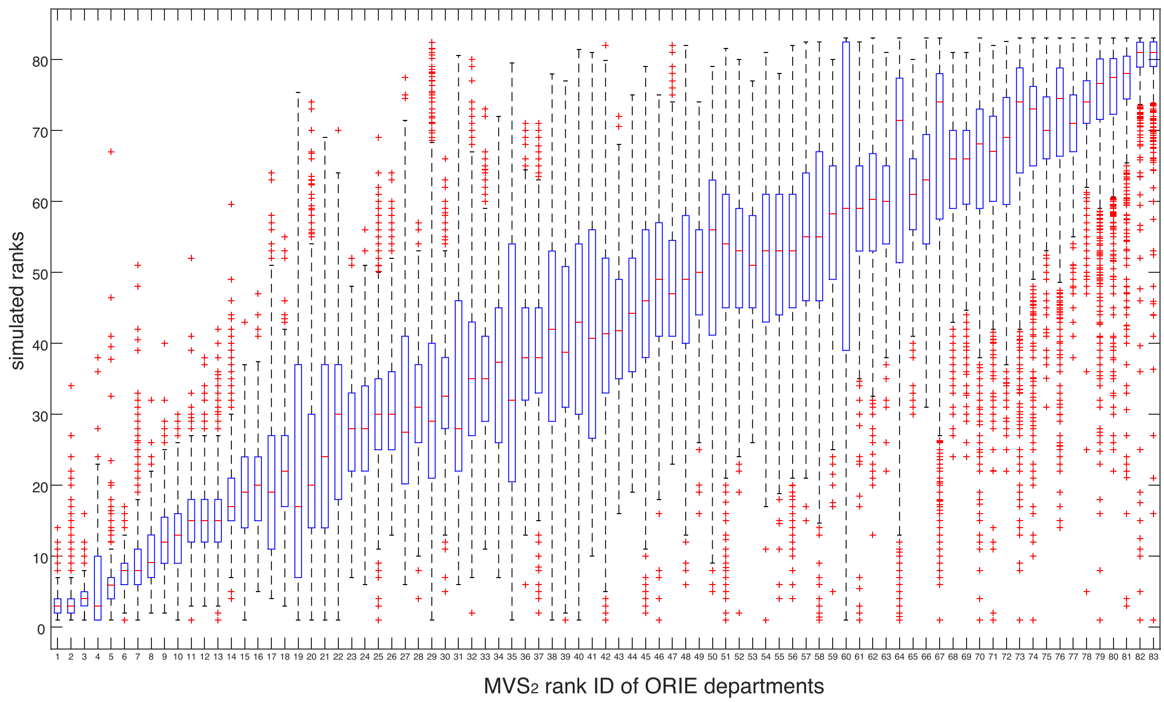

Figure 5. Boxplots of the boostrapped Minimum Violation and Strength (MVS_2) ranks for all 83 departments in the IEOR network, sorted by mean MVS_2 value (1000 bootstrapped networks). The reported MVS_2 ranks correspond to the average values. Note how the extreme ranks are less uncertain, a characteristic also reported for other disciplines (Clauset *et al.*, 2015).

rest of this article, we could define ties in MVS_2 ranks between two departments if, for instance, their bootstrapped distributions displayed in Figure 5 are such that the differences between their means are not statistically significant. A network representation of the top 15 MVS_2 IEOR departments is shown in Figure 6. This is the most dense part of the IEOR network.

4.2. Correlations between different ranks for the IEOR network.

Table 3 shows the Kendall rank correlation coefficients between various rankings for the IEOR departments, including two published rankings (NRC and US News & World Report, see Appendix 1) and the MVR, MVS_1, and MVS_2 rankings described earlier (Kendall correlations are more appropriate than Pearson's as the data are ranks, and they are less sensitive to noise than Spearman's correlations). All these rankings are shown in Table A1 in Appendix 1. Not surprisingly, the MVR, MVS_1, and MVS_2 rankings are highly correlated. The Hub, Out degree, PageRank, and (left) Eigenvalue rankings we computed are also correlated, because in all these "centrality" measures importance is bestowed by out of degree. In a social network, *Hubs* and *Authorities* are types of vertices defined in an intertwined manner: a Hub is a vertex that points to many other vertices with high authority, and an authority is a vertex pointed to by many hubs (Newman, 2010). In a faculty hiring network, vertices with high hub ranking are therefore departments that "feed" faculty to departments sought after by faculty candidates, thus Hub importance refers to *placement capacity* to departments that in turn are important (similarly, the authority ranks refer to *hiring capacity*, the ability to attract and hire faculty from important departments). The MVS_2, MVS_1, and MVR

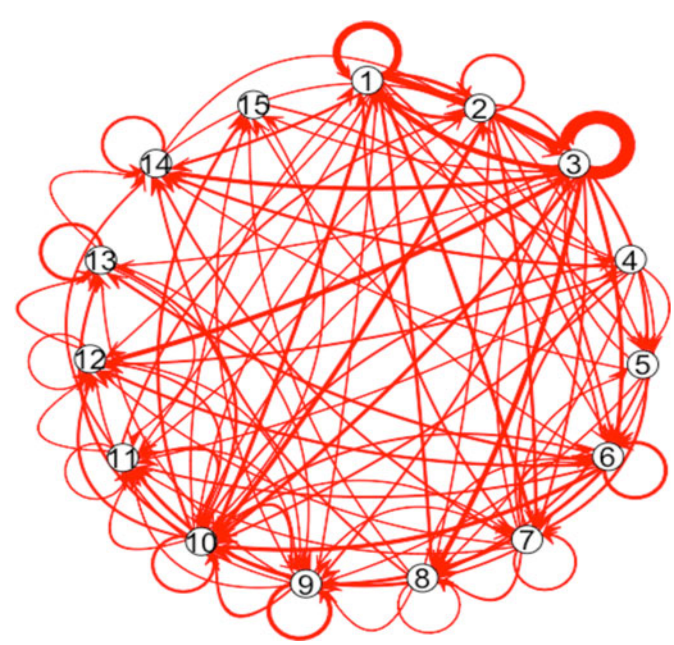

Figure 6. IEOR faculty hiring network for the top 15 departments sorted according to MVS_2 index. Edge width is proportional to number of hires. The edge density (or connectance) of the IEOR network is high only for the departments at the top of the hierarchy, but otherwise it is quite sparse, see Table 2.

rankings are more correlated with the out-degree related rankings (out-degree, hub, PageRank and Eigenvalue) than with the *Betweenness* rankings (a centrality ranking measure of a network based on how many shortest paths between every pair of vertices passes through the vertex in question), a matter to which we return in our ERGM model presented below.

Although there is significant correlation between published rankings and the MVS_2 and MVR indices, there are significant differences especially among the top 20

Table 3. Kendall's rank correlation coefficients between the ranks of common measures of vertex importance, including public rankings, in the observed IEOR network. Only pairs of entries with no NA values were considered. (USN = US News 2016 ranks, NRC = 2011 NRC ranks).

	MVS_2	MVS_1	MVR	USN	NRC	In-deg	Out-deg	Eigen	PgRank	Bet.	Hub	Auth.
MVS_2	1.000											
MVS_1	0.778	1.000										
MVR	0.823	0.854	1.000									
USN	0.692	0.611	0.613	1.000								
NRC	0.661	0.634	0.647	0.716	1.000							
In-deg	0.506	0.482	0.459	0.438	0.389	1.000						
Out-deg	0.577	0.543	0.535	0.533	0.494	0.427	1.000					
Eigen	0.491	0.478	0.459	0.503	0.441	0.320	0.691	1.000				
PgRank	0.517	0.489	0.494	0.502	0.464	0.334	0.817	0.785	1.000			
Bet.	0.388	0.331	0.350	0.352	0.279	0.402	0.575	0.498	0.523	1.000		
Hub	0.553	0.555	0.538	0.556	0.485	0.419	0.793	0.758	0.760	0.458	1.000	
Auth.	0.510	0.527	0.471	0.475	0.424	0.586	0.453	0.446	0.422	0.238	0.538	1.000

departments (see Appendix 1 Table A1). Finally, we point out the correlation between the two published rankings (US News and NRC), which exists despite their significant differences in methodology and even in departments considered.

5. Latent location variables and clustering of faculty hiring networks: A latent ERGM

Descriptive statistics as those in Section 2 can only provide partial information about the structure of a complex network. Single indices such as the MVR and MVS indices (or such as published rankings) that try to capture the "prestige" of an academic department are inherently incomplete. One feature that was evident from the descriptive analysis of the IEOR network (Sections 2 and 4) was that there is a core of departments at the top of the hierarchy that form denser connections, whereas the periphery is much more sparsely connected. Also, there is evidence that the steepness of the hierarchy varies within the hierarchy (Appendix 2 and Table 2). This indicates that in order to better understand the structure of a faculty network, rather than finding a linear hierarchy based on single indices, it is worth finding groups of similar departments.

In this section, instead of simply applying clustering algorithms directly to the faculty hiring network data, we first consider the observed network as a noisy realization, or sample, from a stochastic network model. We study a particular type of ERGM in which the conditional probability of a tie between two actors, given covariates (if any), depends on the distance between actors in an unobserved or latent "social space" (Krivitsky and Handcock, 2008). As it will be shown, a vertex covariate useful to explain the network topology is the MVS_2 ranking described in previous sections.

In a latent ERGM model, the adjacency matrix $\mathbf{Y}$ is viewed as a random matrix that depends on parameters $\boldsymbol{\beta}$, covariates $\mathbf{x}$ and positions $\mathbf{Z} = \{\mathbf{Z}_i \in \mathbb{R}^d\}_{i=1}^n$ in a d-dimensional 'social space". The latent ERGM model we used is:

$$P(\mathbf{Y}=\mathbf{y}|\boldsymbol{\beta},\mathbf{x},\mathbf{Z}) = \prod_{i,j} P\left(Y_{i,j}=y_{ij}|\boldsymbol{\beta},\mathbf{x},\mathbf{Z}\right)$$

$$P\left(Y_{ij}=y_{ij}|\boldsymbol{\beta},\mathbf{x},\mathbf{Z}\right) = f\left(y_{ij}|\mu_{ij}\right) = \frac{\exp\left(-\mu_{ij}\right)\mu_{ij}^{y_{ij}}}{y_{ij}!}$$

$$\log\left(\mu_{ij}\right) = E\left[Y_{ij}|\boldsymbol{\beta},\mathbf{x},\mathbf{Z}\right] = \eta_{ij}(\boldsymbol{\beta},\mathbf{x},\mathbf{Z})$$

$$\eta_{ij}(\boldsymbol{\beta},\mathbf{x},\mathbf{Z}) = \sum_{k=1}^{p} x_{k,i,j}\beta_k - |\mathbf{Z}_i - \mathbf{Z}_j|$$

We thus adopted a Poisson density for the number of hires between two departments Y_{ij} and a link function $\eta = g(\mu) = \log\left(\mu_{ij}\right)$ which is log-linear in the covariates and in the distances in an Euclidean latent space (defined by the $\mathbf{Z}_i$s). The latent positions are assumed to follow a mixture of spherical multivariate normals in the social space, which could have in principle any dimension d:

$$\mathbf{Z}_i \overset{i.i.d.}{\sim} \sum_{g=1}^{G} \lambda_g \mathrm{N}_d\left(\boldsymbol{\mu}_g, \sigma_g^2 \mathbf{I}_d\right), \quad i = 1, \ldots, n \tag{4}$$

where the λ_gs ($0 \le \lambda_g \le 1$) are the prior probabilities a vertex belongs to each group g ($\sum_g^G \lambda_g = 1$).

Fitting the model implies finding estimates for the parameters $\boldsymbol{\beta}, (\lambda_g, \boldsymbol{\mu}_g, \sigma_g^2)$ for $g = 1, \ldots G$. A Bayesian formulation has been suggested by Krivitsky *et al.* (2009) who proposed a Markov Chain Monte Carlo (MCMC) estimation approach based on Gibbs sampling updates for $\boldsymbol{\beta}, \boldsymbol{\mu}_g, \sigma_g, \lambda_g$ and a Metropolis–Hastings update for the latent positions $\mathbf{Z}_i$. As it is standard in Bayesian estimation of mixture models (see, e.g., Bishop, 2006) the MCMC method introduces new variables $K_i, i = 1, \ldots, n$ equal to g if actor i belongs to the gth group, $g = 1, .., G$ with a multinomial prior with parameters λ_g. This method has been implemented in the `latentnet` (Krivitsky and Handcock, 2008) R package. The MCMC fitting routine is in function `ergmm`. This function permits modeling self-loops, an important characteristic of the IEOR network as discussed in Section 1. We use the method described in Krivitsky and Handcock (2008) to setup the priors for all hyperparameters.

For the MCMC estimation, for each alternative model we used a warm-up period of 50 000 iterations, then ran the MCMC routine for 5 000 000 more iterations, and finally collected statistics only every 100 iterations over 50 000 more iterations. Such high numbers of iterations are necessary due to the slow mixing in these type of models during the MCMC computations (see Handcock and Raftery, 2007).

There are not many techniques available for ERGM model selection. We selected a final model comparing the alternative models' Bayesian Information Criterion (BIC), considering whether the MCMC estimation procedure converged satisfactorily or not for a given model, and looking at model diagnostics involving the prediction performance of each model considered. Models with two-dimensional ($d = 2$) location vectors $\mathbf{Z}_i$ had difficulty converging; the chains did not show adequate mixing. For $d = 3, G = 2$ (two groups or clusters in three dimensions), BIC = 5171 but the β_j coefficients failed to converge; for $d = 3$ and $G = 3$, BIC = 5221 (somewhat worse) but it had satisfactory MCMC convergence for all its parameters (see Figures 16 and 17 in Appendix 4).

The fitted final model for the IEOR network, obtained from the posterior means of all parameters, is:

$$\begin{aligned} \log(\mu_{ij}) &= \log\left(E\left[Y_{ij}|\boldsymbol{\beta}, \mathbf{x}, Z\right]\right) = \beta_0 + \beta_1 x_{ij} + \beta_2 x_i \\ &\quad + \beta_3 x_j - |\mathbf{Z}_i - \mathbf{Z}_j|, i, j = 1, \ldots, n \\ &= \underbrace{2.8363}_{(2.52,3.10)} + \underbrace{0.1722}_{(0.09,0.24)} x_{ii} \underbrace{-1.1190}_{(-1.21,-1.02)} x_i + \underbrace{0.2828}_{(0.19,0.37)} x_j - |\mathbf{Z}_i - \mathbf{Z}_j| \end{aligned} \quad (5)$$

where $x_{ij} = \log(\text{MVS}_2(i))$ if $i = j$ and $x_{ii} = 0$ for $i \neq j$ is a self "loop" covariate, modeling self-hires, $x_i = \log(\text{MVS}_2(i))$ is a "sender" covariate and $x_j = \log(\text{MVS}_2(j))$ is a "receiver" covariate (Krivitsky and Handcock, 2008). Numbers in the under braces are the 95% credible posterior intervals for each coefficient, note how they all exclude zero. The sender and receiver covariates model the propensity of a department to either provide or receive faculty. The motivation for this model was that the MVS_2 indices can provide an indication of the number of faculty flowing between departments. Taking the log(MVS_2) implies we assume effects are linear, given this is a log-linear model. For instance, suppose $\text{MVS}_2(1) = 1$ and $\text{MVS}_2(2) = 2$. Then the fitted model predicts $\mu_{11} = 16.94$ self hires on average for the top IEOR department.

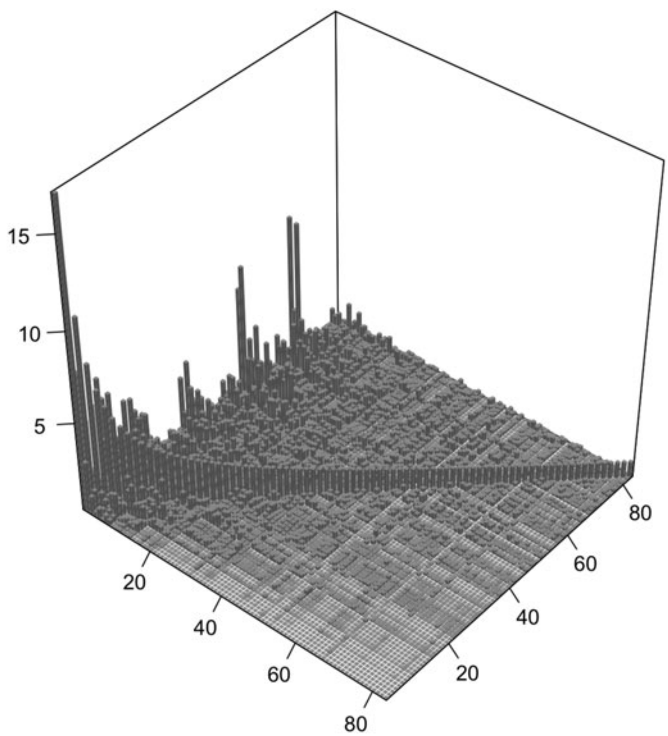

Figure 7. Predicted mean number of faculty hires in the IEOR network ($\hat{\mu}_{ij}$) given by the fitted latent location ERGM model (5). Height equals expected number of faculty, axes are the 83 departments sorted according to MVS_2. Compare with the observed number of hires, the plot on the right of Figure 4.

5.1. Model prediction performance

The fitted latent location ERGM model (5) adequately predicts the expected value of the number of hires within and between IEOR departments. Using $i = j$ in Equation (5), the predicted mean self-hire values μ_{ii} are obtained, and these are contrasted with the observed self-hired values in the IEOR network in the top plot of Figure 1 (red line), showing overall good agreement, accurately predicting the increase of self-hires at the top of the hierarchy and its decline towards the bottom of the hierarchy. Figure 7 diagrammatically shows the fitted expected number of hires between all 83 IEOR departments, which can be compared to the plot on the right of Figure 4, again demonstrating a behavior close to that in the observations. More specific model diagnostics for ERGM models are based on Monte Carlo simulations of the fitted network and comparison of the observed statistics to those in the ensemble of simulations.

We performed posterior checks based on 1000 simulated networks from the posterior of the fitted model using the `latentnet` R package. We compared posterior properties of the simulated networks against the corresponding statistics of the observed IEOR faculty network. Figure 8 contrasts the actual observed values for the in (8 left) and out degree (8 right) distribution (bold red) with the interquartile range of the simulated values from the posteriors (blue boxplots). Both distributions can approximately generate the

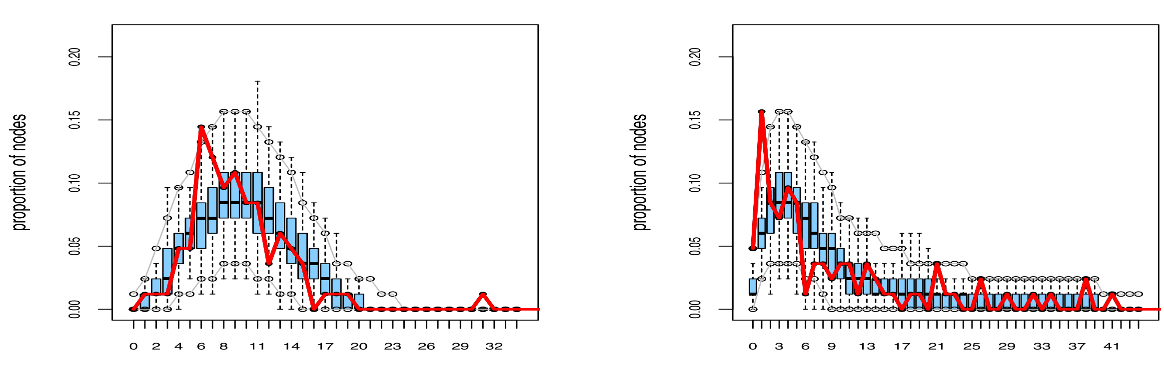

Figure 8. Diagnostic plots for the ERGM model (5) based on 1000 simulations. Left: in-degree distribution. Right: out-degree distribution. Boxplots give the interquartile ranges and the whiskers the extreme values simulated from the posterior of the fitted model (5), red continuous lines are the observed values. With exception of a slight over-generation of vertices with in-degree six and seven and out-degree equal to one, the fitted model reproduces the observed degree distributions.

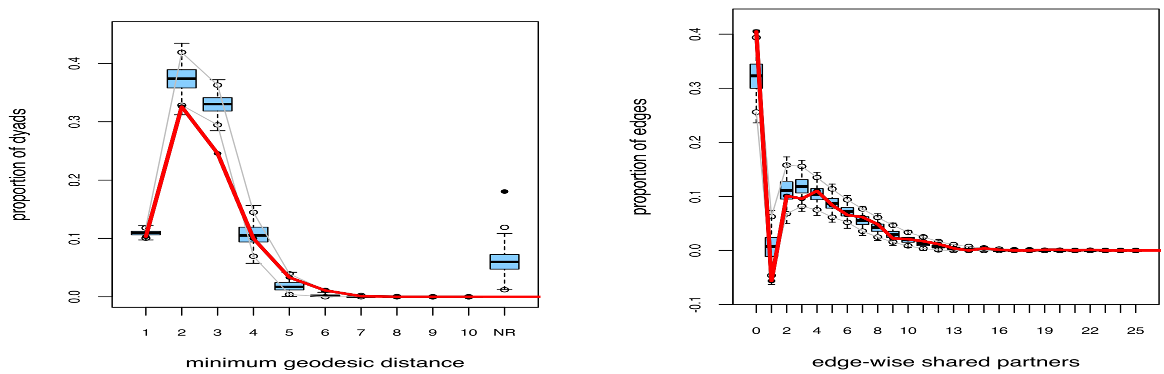

Figure 9. Diagnostic plots for the latent ERGM model (5) based on 1000 simulations. Left: geodesic distance distribution. Right: edge-wise shared partner distribution. The number of edge-wise shared partners is defined as the number of edges e_{ij} such that both i and j have k common neighbors ("shared partners"). Boxplots give the interquartile range and the whiskers the extreme values simulated from the posterior of the fitted model (5), red continuous lines are the observed values. The fitted model is able to reproduce the observed statistics. Note how almost 40% of the edges in the IEOR network have no common neighbors.

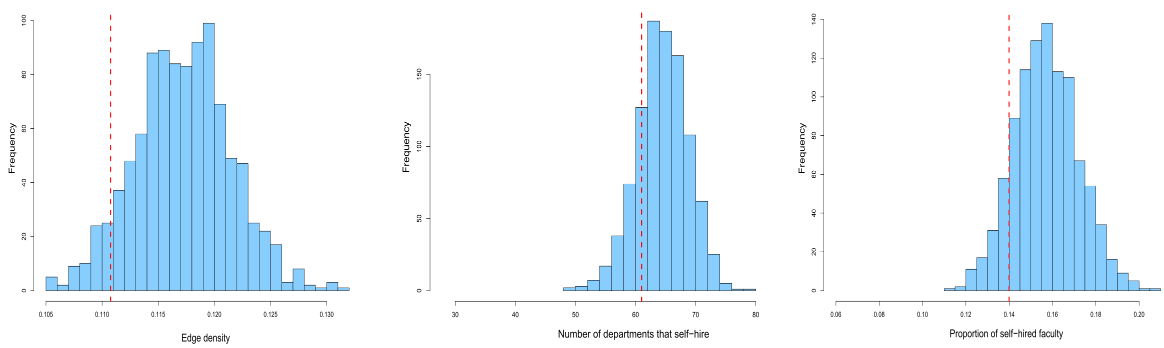

Figure 10. Diagnostic plots for the latent location ERGM model (5) based on 1000 simulations. Left: edge density distribution. center: number of departments that self-hire. Right: proportion of self-hired faculty. Observed values in IEOR network are given by red vertical lines. In all cases, the fitted model reproduces the observed values in the actual network.

observations, including the very skewed out-degree distribution. There is some over-generation of nodes with in-degrees equal to six and seven, and over-generation of nodes with out-degrees equal to one edge. Figure 9 (left) contrasts posterior simulations of the minimum geodesic distances between the vertices and the actual values, which are reproduced well by the model. The plot on the right contrasts the posterior simulations versus actual values for the edge-wise shared partners, defined as the number of edges e_{ij} such that both i and j have k common neighbors. The peculiar shape of this distribution is very well reproduced by the latent ERGM model. Finally, in Figure 10 (left) we first computed the posterior edge density distribution of the simulated networks and contrasted it with the actual value (red dotted line), which falls among the simulated values, indicating the model can reproduce networks with the correct density of edges. The number of departments that self-hire ranges from around 49 to 78 in the posterior simulated networks, and this includes the observed value (61) in the real IEOR network (red dotted line in Figure 10, center). Finally, the proportion of Ph.Ds self hired by their original department ranges from about 12% to 20%, again including the observed 14% (red dotted line, Figure 10, right).

We conclude from these "goodness of fit" posterior simulations that the general structure of the IEOR faculty hired network is captured by the fitted latent ERGM (5).

5.2. Determination of groups of departments

The latent ERGM model permits us to determine G groups in which the nodes are clustered in the location space $\mathbf{Z}$ via the posterior of the K_i variables obtained from the MCMC estimation. We found, however, that the $3 = G$ clusters thus formed do not have an easy interpretation, as the separation between the clusters (measured by the number of edges between clusters) is not large enough. The sometimes poor clustering behavior of the latent ERGM models is well known in the literature (see, e.g., the discussion to the paper by Handcock and Raftery, 2007) given the identifiability issues that exist in this type of models, especially for sparse networks and the assumption of spherical normal distributions in Equation (4).

Instead, to select groups of departments, we took the mean of the posterior of the latent locations $\mathbf{Z}_i$, $i = 1, \ldots, 83$ from the fitted ERGM model and ran the PAM (Partitioned around medoids) algorithm (Kaufman and Rousseeuw, 2008) to find three groups (clusters) of departments. This provided a much better group separation between the clusters than using the posterior of the K_i as the basis of clustering. An objective measure of quality of clustering is the "Gap" statistic (Tibshirani *et al.*, 2001). According to this statistic, the best number of IEOR clusters is either one (no clustering) or three (see Figure 11). Although not clustering

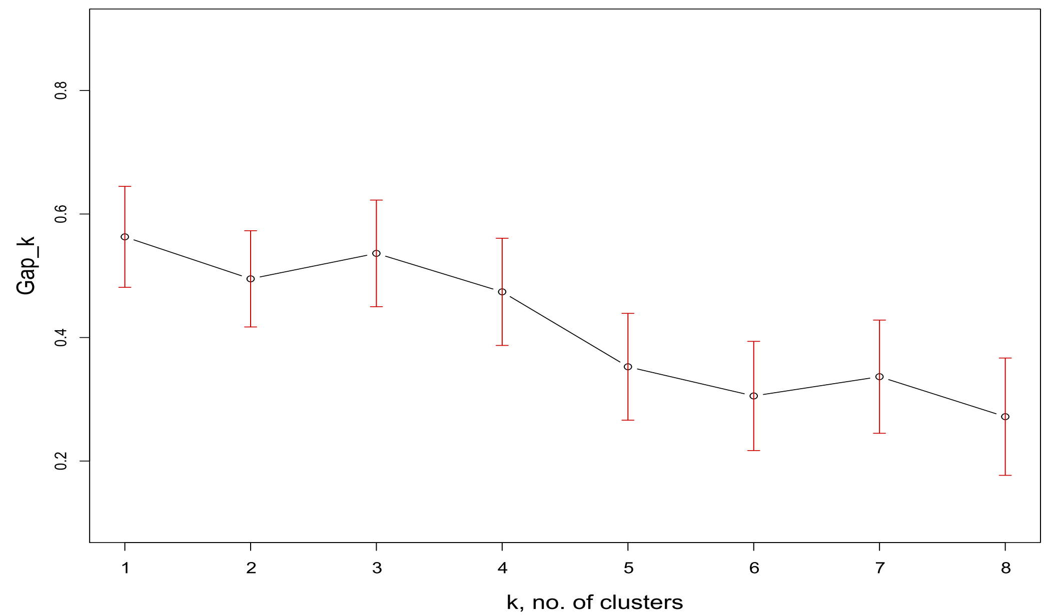

Figure 11. Simulated "gap" statistics (Tibshirani *et al*., 2001) to determine the number of clusters in the latent variables $\mathbf{Z}_i$ identified in the exponential random graph model (5). Either one or three clusters are identified.

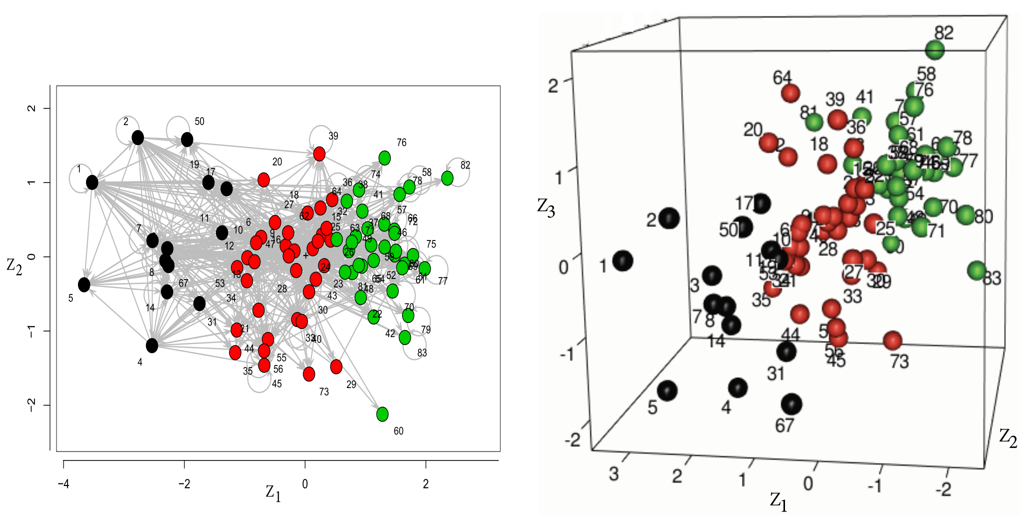

Figure 12. Latent Positions Z_1, Z_2, Z_3 after fitting model (5). Vertices are colored according to the three identified groups (red = group A, black = group B, and green = group C). Left: plot over first two latent variables, Right: plot over the three-dimensional latent space (edges not shown). Numbers correspond to MVS_2 indices, see Table A1. Evident features are the few connections between groups B and C, and how group A contains departments with a high degree of "betweenness" (Newman, 2010).

the departments does not provide any further insights about the IEOR network, choosing a model with three clusters coincides with the best ERGM latent model we fit (for $G = 3$ groups in $3 = d$ dimensions) and provides an interesting interpretation, as we show next. The three latent groups hence formed are displayed in different colors in Figure 12 and are listed in Appendix 1 (Tables A1 and A2).

It is notable how well the first latent group includes only departments who more consistently hire from top departments in the hierarchical MVS_2 order. Using the three groups of departments, we can reduce the IEOR network to a simple aggregated network where faculty flow within and between the three groups (Figure 13). Most of the hires (62%) take place within groups, only 38% is between groups. Note how groups C and B are very thinly connected: only three faculty receiving their Ph.D. in the 34 departments in group C have been hired by the 14 departments in group B. Inversely, only 14 Ph.D. holders from group B have been hired in departments from group C. Group C has also provided few (26) faculty to group A. This indicates that the IEOR network is strongly separated in clusters, with departments in the bottom of the hierarchy producing almost no faculty for those on the top. Interestingly, the "intermediate" departments in group A (35 departments) include the top seven departments as ranked by "betweenness" and overall have the lowest (i.e., most important) betweenness average ranks (34.8 for group A, 40.0 for group B, 50.1 for group C, see Appendix 1 Table A2). It could be argued that the intermediate departments in group A keep the IEOR as a single, connected discipline. Departments in group B are mostly private universities with OR and Systems departments, which hire mostly OR faculty from other group B departments, whereas those in group A are mostly IE departments that hire IEs but also hire OR faculty, explaining some of the hiring patterns between these groups. An interesting question for further work is to determine if the degree of interaction between groups B and C is higher or lower than the interaction threshold between different (but close) disciplines, such as OR and CS, for instance. The latent ERGM methods could be used to study the structure of nominally different but apparently related disciplines.

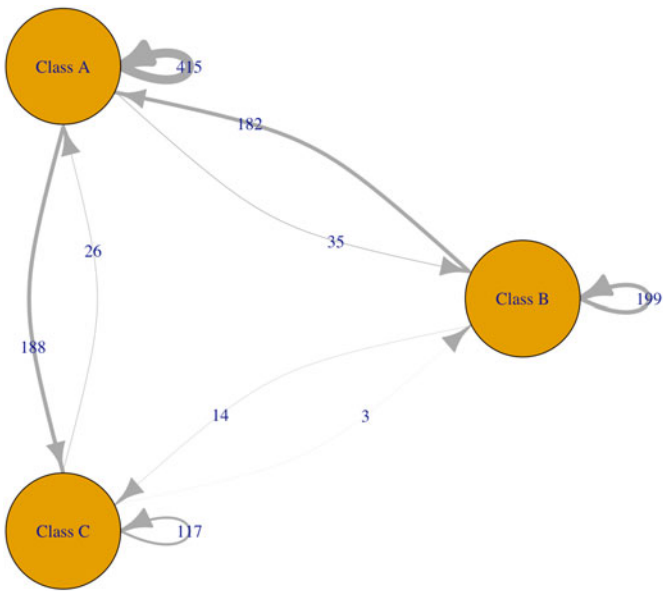

Figure 13. Simplified IEOR network, showing the faculty hires between the three identified groups of departments. Edge width is proportional to the number of faculty hires, which are the numbers shown on each edge. The bulk of the hire-placements occurs within group. Note also how there are very few connections between groups B and C, with departments in group C contributing only to three hires in departments in group B.

6. Discussion and conclusions

We have proposed new methodology for the analysis of faculty hiring networks, and demonstrated it with application to the IEOR network. We provided an approximate linear hierarchy index (Minimum Violation and Strength, MVS_2) obtained through optimizing bootstrapped networks sampled from the original faculty hiring network and therefore not sensitive to the unequal density of edges (and that stand in contrast to published rankings). Single indices of hierarchy, however, do not capture all important features in a complex network, for instance, the highly connected core of the departments in the top of the hierarchy, the high incidence of self-hires, or the skewed out degree distribution. This is the reason we propose the latent variable ERGM of Section 5 and the associated cluster analysis as a better approach for understanding the hiring patterns of a faculty hiring network. The Latent ERGM treats the MVS_2 indices as useful vertex covariates that explain the formation of edges. For the IEOR faculty network the latent ERGM results in groups that do not necessarily follow the MVS_2 ranks. The model successfully captures the main characteristics of the IEOR faculty network, including its node degree distributions, its incidence of self-hiring, and its edge density

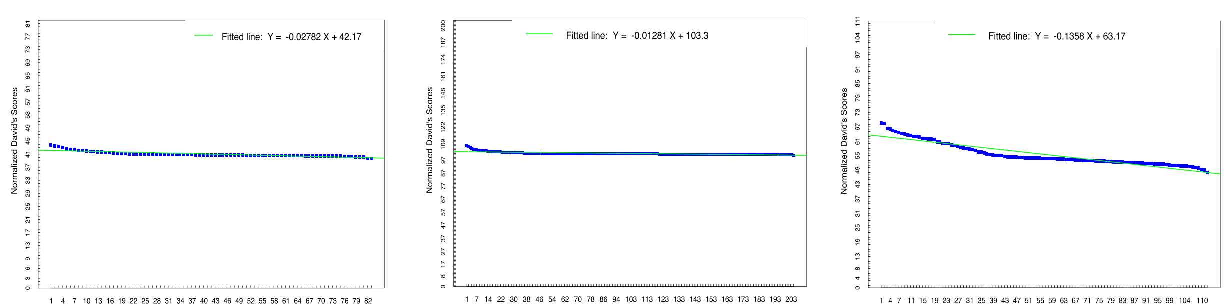

Figure 14. Fitted line and observed David's statistics for a linear hierarchy. Left: IEOR network ($n = 83$, slope $= -0.02782$). Middle: CS ($n = 205$, slope $= -0.01281$). Right: Business ($n = 112$, slope $= -0.1358$). Note how for IEOR and CS the observed absolute slope increases for the first 10 departments, indicating a steeper dominance at the top of the hierarchy relative to other departments, but otherwise the hierarchy is quite flat.

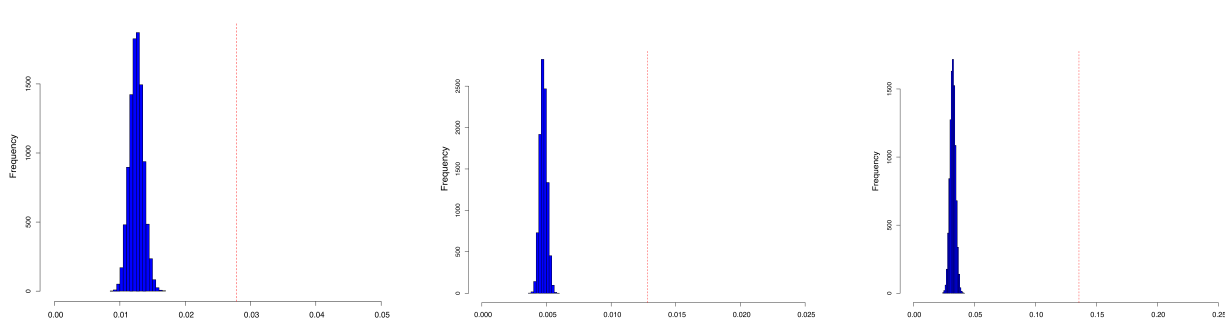

Figure 15. Randomized distribution (10 000 simulations) of the De Vries' test statistic for the significance of the steepness in the dominance hierarchy of (Left) the complete IEOR network, (Middle) the CS network, and (Right) the Business network. Red vertical lines are the observed David's D_i statistics. In all cases, the empirical p-values are 0.0, indicating significance steepness in the hierarchy of each faculty network.

distribution among others. This model allowed us to simplify the IEOR network to one with only three groups of departments, with most hiring taking place within groups, also showing the little interaction between groups at the extremes of the hierarchy. Furthermore, departments in the intermediate group (A) act as a "link" between the other two groups, keeping the network connected as a single discipline, with departments in this group having among the highest "betweenness" importance (Newman, 2010).

Some aspects of the hiring-placement process can be thought to add "noise" to the collected data, which is very sparse at lower levels of the hierarchy. This includes, for instance, the presence of "politics" in the hiring department. In our analysis, we handled the noise and sparseness via permutation tests or bootstrapping when finding evidence of a hierarchy or trying to determine an index, or through a statistical model for the network. owever, as discussed in Section 3.1, zero- valued edges do not necessarily indicate the absence of a dominance relation. Unfortunately, it is not possible to determine from the available data whether an "observational zero" in a social network implies the lack of a dominance relation or rather its presence, with the dominated party trying to avoid interactions with the dominant one (i.e., departments *avoiding* hiring from higher ranked departments).

A few of our methods have considerable wider applicability beyond faculty hiring networks. The new MVS_2 ranking method has application in social network models in Ecology to determine dominance among animals, and also for ranking teams in any sports competition, particularly competitions where not all teams play against each other, when ranking is difficult. Likewise, the new method for forming clusters in latent ERGM models can be used quite generally as a post-processing step in any directed network where it is of interest to discover location clustering in a social network space.

The analysis of faculty hiring networks presented in this article refers to data collected at a single point in time. This neglects the *dynamic* effect of hires over the years which could be modeled if more complete data about the years of subsequent employments of each single professor in the network were available (our datasets contain partial data, too incomplete to attempt such analysis and this is left for future work). If such data were collected, dynamic ERGMs could be a good way to study the dynamics of faculty hiring networks. The particular analysis of the IEOR network and the conclusions we reach from it naturally depend on the data collected and the inevitable assumptions and criteria needed for its collection (see Appendix 1).

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Appendix 1: Institution data

The list of IEOR departments was formed by merging those Ph.D. granting departments in the 2016 US News & World report" Industrial/Manufacturing/Systems Engineering" graduate rankings (accessed 5/12/2016) with those in the 2011 National Research Council (National Research Council 2011) rankings for" Operations Research, Systems Engineering and Industrial Engineering". The NRC lists both departments and programs, so an institution may appear more than once; we included each institution only once in the network (their NRC ranking shown below corresponds to the highest ranking of either program or department) and collected the faculty information as if it were a single department. Only departments with existing web pages as of May 2016 and that have a Ph.D. program were included (this excluded U. of Nebraska-Lincoln). The NRC rankings listed in Table A1 below are those given by the average of the 5% and 95% "R" rankings, listed in standard competition order. There are Industrial Engineering departments which are merged with other disciplines in a single department (e.g., Mechanical and Industrial Engineering, or Computer Systems and Industrial Engineering); an effort was made to include only those faculty in the IEOR field. It was assumed, in the absence of information, that a faculty member working in an IEOR department obtained his/her Ph.D. in the IEOR department of the listed institution (a number of faculty only lists their alma mater institution but not their alma mater's department). A good proportion of faculty working in IEOR departments (34% in our data) obtained their Ph.D. in a different discipline, such as CS, EE, and Business. We have therefore defined earlier a "self-hire" in the wider sense of hiring from the same institution, but not necessarily from the same department. This is consistent with prior studies in Business and CS faculty hiring networks (Clauset *et al.*, 2015) to which we have contrasted the IEOR data. Also, faculty that received their Ph.D. outside the USA (or outside of our list of departments) were not considered. Only faculty in tenured or tenure-track positions were included, although it was not always clear if some faculty positions were tenured or not.

It is important to point out that the edges (v_i, v_j) of the IEOR network are formed by faculty *currently* (as of summer 2016) in department j who received their Ph.D. from institution i. This evidently neglects the movement of faculty through some of other intermediate departments/institutions between i and j and is a potential source of error. Our dataset contains partial information with respect to these intermediate departments where each faculty worked before their current job, information too incomplete to attempt such an analysis which is left for further work.

The final IEOR dataset is contained in two files, one for the 83 institutions (vertices) and its attributes, and one for the 1179 faculty (edges) and its attributes. The files are available as supplementary materials to this

Table A1. Importance rank measures, IEOR departments, sorted by MVS_2 index.

	Ranks												
Institution	*MVS_2*	*MVS_1*	*MVR*	*USNews*	*NRC*	*In-deg*	*Out-deg*	*Eigen.*	*PageR.*	*Bet*	*Hub*	*Auth.*	*Group*
Stanford	1	2	2	4	1	5	2	2	2	17	2	2	B
UC Berkeley	2	1	1	2	4	36	6	3	3	26	3	18	B
MIT	3	3	3	6	3	3	1	1	1	18	1	1	B
Carnegie M.	4	4	4	*NA*	7	79	12	6	8	14	8	59	B
Princeton	5	5	5	*NA*	18	66	33	8	16	68	15	16	B
U. Michigan	6	6	6	2	5	11	3	9	5	5	4	8	A
Cornell	7	7	7	7	8	16	10	4	6	25	7	12	B
Columbia	8	8	8	11	20	22	17	16	12	58	17	6	B
Purdue	9	9	9	9	9	9	5	13	7	6	6	13	A
GA Tech	10	10	4	1	2	2	4	5	4	1	5	3	A
Northwest.	11	11	11	4	6	20	15	10	9	39	10	28	B
U. Illi. (UC)	12	14	14	15	33	13	13	17	14	28	14	5	A
U. Wis (Ma)	13	12	12	7	9	21	14	15	11	10	9	22	A
U. Penn	14	15	18	28	14	14	24	7	17	43	20	7	B
U. Florida	15	13	13	19	23	56	9	19	13	36	16	38	A
Penn St.	16	16	15	12	12	7	8	18	10	4	13	15	A
USC	17	19	19	12	29	12	28	22	22	35	25	10	B
Ohio St.	18	21	21	17	9	19	11	25	19	12	19	19	A
UT (Aus)	19	18	16	19	23	46	29	12	15	33	22	21	B
U. Minn.	20	20	20	32	46	61	39	37	38	61	29	31	A
U. Maryland	21	17	17	*NA*	16	60	22	11	21	37	11	32	A
U. Iowa	22	22	22	39	22	80	31	24	35	54	28	71	C
U. Pitt	23	24	24	23	39	25	16	32	24	19	24	20	A
VPI	24	25	25	9	15	10	7	26	18	7	12	25	A
U. Arizona	25	27	27	28	35	53	26	41	37	55	27	51	A
NC State	26	33	32	12	12	6	19	40	30	15	30	29	A
Lehigh	27	23	26	18	28	39	34	30	29	47	43	40	A
SUNY Buf	28	29	31	28	37	28	21	27	25	20	26	33	A
U. Miss (Co)	29	26	23	58	31	78	54	34	51	69	45	63	A
Rutgers	30	28	30	21	38	47	35	21	23	34	44	49	A

(*continued*)

Table A1. Continued.

	Ranks												
Institution	*MVS $_2$*	*MVS$_1$*	*MVR*	*USNews*	*NRC*	*In-deg*	*Out-deg*	*Eigen.*	*PageR.*	*Bet*	*Hub*	*Auth.*	*Group*
UNC (Ch-H)	31	35	42	*NA*	34	32	58	23	45	48	38	36	B
Texas A& M	32	31	34	15	24	15	20	20	20	2	23	30	A
U. Virginia	33	37	35	28	23	27	37	48	42	38	34	23	A
U. Mass.	34	39	46	36	50	8	25	29	31	23	18	9	A
Boston U.	35	30	28	39	30	62	60	47	57	66	53	24	A
U. Arkansas	36	40	41	39	57	31	44	39	48	40	49	43	A
U. S. Florida	37	38	36	46	54	54	27	42	34	11	33	67	C
Oklahoma St	38	34	38	39	41	40	30	38	36	45	32	44	C
RPI	39	41	37	21	19	37	36	55	41	46	39	45	A
U. Wash.	40	42	40	26	35	57	48	49	52	59	42	35	A
Kansas St.	41	50	39	46	47	49	46	62	49	65	57	48	C
U. Illi. (Chi)	42	36	33	46	45	71	52	45	53	62	48	61	C
Iowa St.	43	45	47	26	32	26	40	35	39	21	41	37	A
U. Conn.	44	59	54	*NA*	54	35	57	58	67	63	31	39	A
G. Wash. U.	45	44	51	53	53	43	42	33	40	50	37	27	A
Texas Tech	46	47	48	53	40	55	23	51	33	9	47	68	C
Arizona St.	47	48	49	23	21	1	18	28	27	3	21	11	A
U. Oklahoma	48	53	56	46	50	42	53	57	61	49	58	50	C
Clemson	49	58	58	32	47	29	51	63	58	44	61	52	C
Naval Post.	50	83	75	23	*NA*	23	69	69	78	70	60	14	B
Miss. U.	51	43	44	58	62	50	32	44	32	31	50	78	C
Auburn	52	49	52	32	42	48	41	54	47	29	52	66	C
Northeastern	53	81	82	36	27	4	70	70	79	71	51	4	A
Wayne St.	54	54	61	53	44	44	64	52	60	42	62	41	C
Stevens	55	61	74	39	*NA*	17	49	50	55	53	40	17	A
G. Mason	56	51	60	32	*NA*	30	38	36	43	22	35	42	A
U. Louisville	57	57	50	66	62	51	55	65	54	56	65	58	C
U. Alabama	58	52	45	*NA*	61	67	47	64	44	57	63	57	C
UC Florida	59	55	57	39	54	38	45	56	50	24	59	62	C
UT (Dal)	60	46	29	58	*NA*	83	74	53	77	76	55	82	C
U. Houston	61	62	55	53	65	72	50	60	56	52	56	77	C
NJIT	62	79	80	*NA*	57	34	83	82	83	82	80	26	A
Air Force IT	63	71	77	46	*NA*	18	62	72	76	51	54	47	C
Case West.	64	32	53	38	43	77	59	14	26	41	36	55	A
SUNY (Bin)	65	80	81	58	57	24	65	76	75	75	67	53	C
Oregon St.	66	77	72	46	49	41	63	73	69	72	69	69	C
Wash. U.	67	66	43	39	*NA*	69	71	71	68	67	71	34	B
Wichita St.	68	69	78	66	*NA*	33	76	67	73	60	78	56	C
W. Virginia	69	63	70	66	*NA*	52	56	59	59	13	66	64	C
UT (Arling)	70	60	66	58	*NA*	45	43	43	46	8	46	70	C
U. Tenn.	71	56	69	58	62	58	61	31	28	27	68	72	C
NCA&T St	72	78	71	66	*NA*	64	82	78	82	78	81	81	C
Worcester P	73	74	64	53	*NA*	73	80	75	80	74	82	46	A
Florida St.	74	64	79	66	*NA*	59	66	46	62	16	64	65	C
U. NC (Ch.)	75	65	73	58	*NA*	63	73	66	74	32	73	76	C
U. Wis (Mil)	76	76	65	58	*NA*	74	81	77	81	77	83	60	C
Old Dom U	77	67	76	*NA*	66	65	68	68	66	64	74	83	C
Ohio U.	78	68	83	66	*NA*	68	67	61	63	30	72	80	C
New Mex St	79	73	63	*NA*	60	76	79	83	72	83	75	73	C
U. Miami	80	75	68	66	52	75	75	79	71	79	77	79	C
U. Ark. LR	81	82	67	46	*NA*	70	72	74	70	73	70	54	C
Montana St.	82	72	59	*NA*	*NA*	82	78	81	65	81	76	74	C
Florida IT	83	70	62	*NA*	*NA*	81	77	80	64	80	79	81	C

paper. The data collection effort was undertaken in summer 2016 over a 4 week span. After data gathering, all faculty data were checked manually for errors by three persons. Table A1 contains the attributes in the institutions file, which includes ranks of various measures of vertex importance and the latent groups, see Table A2, found in Section 5.

Appendix 2: Statistical tests on the steepness of a linear hierarchy in a network

De Vries *et al.* (2006) introduces the concept of the *steepness* of a linear hierarchy, namely, the size of absolute differences between adjacently ranked individuals in their overall success in winning dominance encounters. When these differences are large, the hierarchy is said to be steep and is called shallow otherwise. DeVries' steepness measure is based on the D_i score by David (1987), a measure of the dominance success of an individual i given by the unweighted and weighted sum of the individual's dyadic proportions of "wins" (in our case, Ph.D. placed in other departments) combined with an unweighted and weighted sum of its dyadic proportion of "losses" (Ph.Ds hired from other departments). The proportions of "wins" P_{ij} are defined as $P_{ij} = y_{ij}/n_{ij}$ where y_{ij} is the (i, j) entry in the sociomatrix $\mathbf{Y}$ and n_{ij} is the total number of interactions between i and j (i.e., $n_{ij} = y_{ij} + y_{ji}$). David's (normalized) scores are defined as $D_i = (w_i + w2_i - l_i - l2_i + n(n-1)/)/n$ where $w_i = \sum_{j \neq i} P_{ij}, w2_i = \sum_{j \neq i} w_j P_{ij}$, $l_i = \sum_{j \neq i} P_{ji}$ and $l2_i = \sum_{j \neq i} l_j P_{ji}$. The factors containing the total number of individuals scale this term to make it vary between zero and $n - 1$. To measure the steepness of a hierarchy, order the individuals according to D_i. Call R the rank of the individuals. Then a simple linear regression model $D = \beta_1 R + \beta_0$ is fit to the data. The estimated coefficient $\hat{\beta}_1$ is an estimate of the steepness of the hierarchy.

To test for the significance of the steepness of a hierarchy using a permutation test, assume as null hypothesis that the steepness is that

Table A2. IEOR departments, sorted by latent groups.

Institution	*Group*	*Betweeness*	*MVS_2*
GA Tech	A	1	10
Texas A& M	A	2	32
Arizona St.	A	3	47
Penn St.	A	4	16
U. Michigan	A	5	6
Purdue	A	6	9
VPI	A	7	24
U. Wis (Ma)	A	10	13
Ohio St.	A	12	18
NC State	A	15	26
U. Pitt	A	19	23
SUNY Buf	A	20	28
Iowa St.	A	21	43
G. Mason	A	22	56
U. Mass.	A	23	34
U. Illi. (UC)	A	28	12
Rutgers	A	34	30
U. Florida	A	36	15
U. Maryland	A	37	21
U. Virginia	A	38	33
U. Arkansas	A	40	36
Case West.	A	41	64
RPI	A	46	39
Lehigh	A	47	27
G. Wash. U.	A	50	45
Stevens	A	53	55
U. Arizona	A	55	25
U. Wash.	A	59	40
U. Minn.	A	61	20
U. Conn.	A	63	44
Boston U.	A	66	35
U. Miss (Co)	A	69	29
Northeastern	A	71	53
Worcester P	A	74	73
NJIT	A	82	62
Carnegie M.	B	14	4
Stanford	B	17	1
MIT	B	18	3
Cornell	B	25	7
UC Berkeley	B	26	2
UT (Aus)	B	33	19
USC	B	35	17
Northwest.	B	39	11
U. Penn	B	43	14
UNC (Ch-H)	B	48	31
Columbia	B	58	8
Wash. U.	B	67	67
Princeton	B	68	5
Naval Post.	B	70	50
UT (Arling)	C	8	70
Texas Tech	C	9	46
U. S. Florida	C	11	37
W. Virginia	C	13	69
Florida St.	C	16	74
UC Florida	C	24	59
U. Tenn.	C	27	71
Auburn	C	29	52
Ohio U.	C	30	78
Miss. U.	C	31	51
U. NC (Ch.)	C	32	75
Wayne St.	C	42	54
Clemson	C	44	49
Oklahoma St	C	45	38
U. Oklahoma	C	49	48
Air Force IT	C	51	63
U. Houston	C	52	61
U. Iowa	C	54	22
U. Louisville	C	56	57
U. Alabama	C	57	58
Wichita St.	C	60	68
U. Illi. (Chi)	C	62	42
Old Dom U	C	64	77
Kansas St.	C	65	41
Oregon St.	C	72	66
U. Ark. LR	C	73	81
SUNY (Bin)	C	75	65
UT (Dal)	C	76	60
U. Wis (Mil)	C	77	76
NCA&T St	C	78	72
U. Miami	C	79	80
Florida IT	C	80	83
Montana St.	C	81	82
New Mex St	C	83	79

given by a randomly formed network. Fitting linear regression models $D = \beta_1 R + \beta_0$ from the randomly generated networks each with ranks R will result in an empirical distribution of the $\hat{\beta}_1$ coefficient, which can then be compared to the $\hat{\beta}_1$ estimate from the actual network. Small empirical p-values imply the hierarchy is significantly steeper than that given by a simple random graph.

Figure 14 shows the regression models fitted to the David statistics for the IEOR, CS and Business networks. The IEOR and CS networks appear to have a steep hierarchy only for approximately the first 10 departments. This is in contrast to Business schools which have a steeper slope for about a third of the departments. Figure 15 indicates, however, that the slope is significantly different to that of a random graph in all three cases. The steepness results shown are related to the connectedness (density) of the three networks, since the CS network is the most sparse, followed by IEOR and then Business.

Appendix 3: Stochastic search algorithm for finding MVS rankings

Algorithm 1 Minimum Violation and Strength (MVS) ranking of boostrapped networks

procedure optimize($\mathbf{Y}, B$, burnin, iterations, interval)
  for $b = 1$ to B **do**
    $\mathbf{Y}_b \leftarrow$ bootstrap($\mathbf{Y}$)
    $\pi_0(\mathbf{Y}_b) \leftarrow$ out degree rankings
    for $k = 1$ to burnin **do**
      $\pi(\mathbf{Y}_b) \leftarrow \text{SWAP}(\mathbf{Y}_b, \pi_0(\mathbf{Y}_b))$
    for $k = 1$ to iterations **do**
      $\pi_b(\mathbf{Y}_b) \leftarrow \text{SWAP}(\mathbf{Y}_b, \pi_0(\mathbf{Y}_b))$
      if int(k/interval)$-k$/interval $== 0$ **then** Save $\pi_b(\mathbf{Y}_b)$
    $\text{MVS}_2(\mathbf{Y}_b) \leftarrow$ average(saved $\pi_b(\mathbf{Y}_b)$'s)
  return $\text{MVS}_2 \leftarrow$ average($\text{MVS}_2(\mathbf{Y}_1), \ldots, \text{MVS}_2(\mathbf{Y}_B)$)

Algorithm 2 Stochastic swapping to improve a given ranking

procedure swap($\mathbf{Y}, \pi(\mathbf{Y})$)
  Randomly select vertices i and j and swap them to form $\pi_{new}(\mathbf{Y})$.
  if $S(\pi_{new}(\mathbf{Y})) > S(\pi(\mathbf{Y}))$ or $(S(\pi_{new}(\mathbf{Y})) = S(\pi(\mathbf{Y}))$ and $S(\pi_{new}(\mathbf{Y})) < S(\pi(\mathbf{Y})))$
  then
    Accept the swap and **return** $\pi_{new}(\mathbf{Y})$.
  else return $\pi(\mathbf{Y})$)

Algorithm 1 is a modification of the optimization approach in Clauset *et al.* (2015), who found minimum violation rankings, adapted for finding MVS rankings. They reported exchanges of more than two vertices did not improve the solutions found with swapping pairs of vertices. The algorithm takes as initial ranking that of the out-degrees of each node giving preference to departments that place faculty in other departments. In analogy with MCMC methods, the algorithm was run for a burn-in period of 10^5 iterations (that are discarded), after which 1000 ranks $\pi_b(\mathbf{Y})$ were saved every 100 iterations (i.e., iterations $= 10^5$ and interval $= 100$ in Algorithm 1). The averages of these 1000 ranks gives the MVS_1 or MVS_2 ranks. In addition, to incorporate the uncertainties related to the low density areas of the IEOR network,

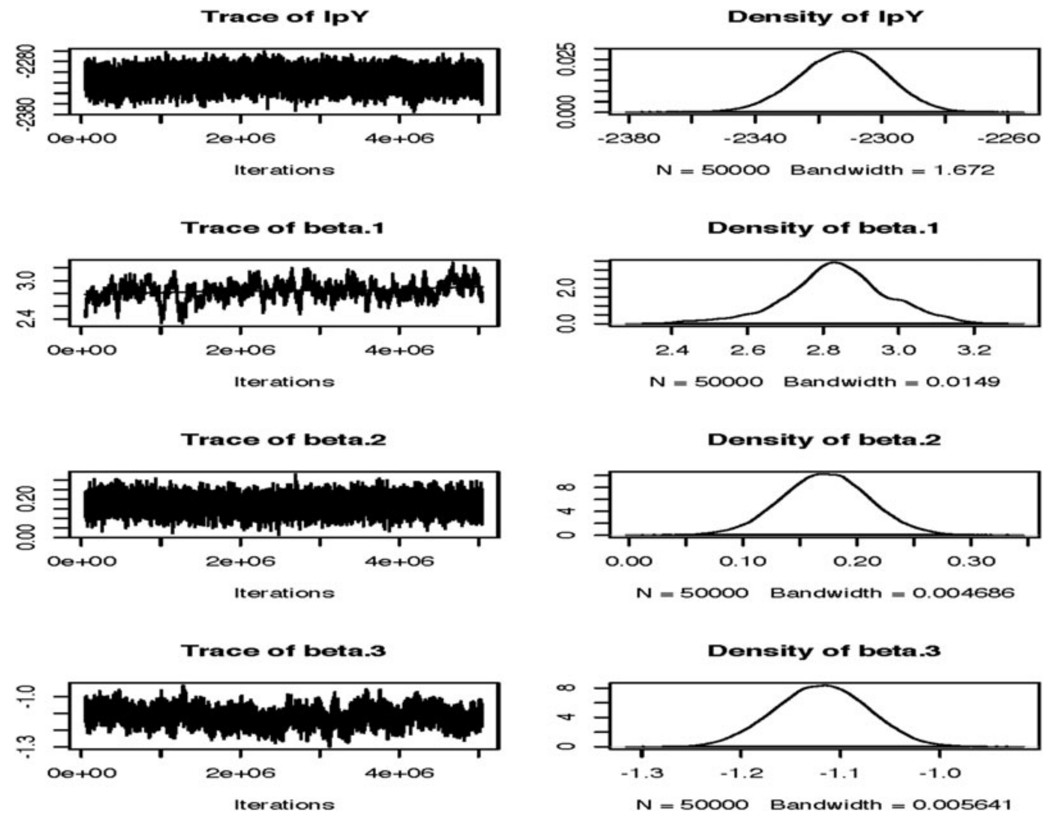

Figure 16. MCMC diagnostics for model (5): log-likelihood and values of β_0, β_1, and β_2. All convergence diagnostics are adequate, i.e., trace plots are stable and therefore parameter values converge in distribution. Traces shown for last 5 000 000 iterations, distributions shown for last 50 000 iterations.

which could be considered as "noise", the optimization was repeated for $B = 1000$ different bootstrapped networks, where the edges in each network were randomly sampled with replacement from the edges of the real IEOR network, with sampling probabilities proportional to the edge attributes Y_{ij}. The reported MVS_1 and MVS_2 ranks in Table A1 correspond to the ensemble mean of the optimal ranks obtained from each of these bootstrapped replications, using either $S_{MVS_1}(\pi(\mathbf{Y}))$ or $S_{MVS_2}2(\pi(\mathbf{Y}))$ in the SWAP procedure (Algorithm 2).

Appendix 4: MCMC convergence diagnostics for the ERGM model

Supplementary materials: IEOR datasets

Two datasets are provided in text (comma-delimited) files:

- IEvertexlist.csv. Department attributes (83 rows). The vertices are indexed by a field simply called" index", which is not a measure of rank or an attribute of a particular department. It roughly corresponds to the US News & World rankings, but since our sample of departments is broader than the USNWR ranks, our indices necessarily do not correspond with those rankings.
- IEedgelist.csv. Edge (faculty) information, using the indices in the vertex dataset (1445 rows). Fields are: U = dept. where Ph.D. was obtained (UYear = year of Ph.D.), VCurrent = current department (VCurrentYear = date of hiring current position), VPrevious = immediately previous dept., if any (and info. available, VPreviousYear = year of previous appointment), Rank=(Asst, Assoc, Full), Gender=(M/F), PhdOtherDiscipline=(TRUE/FALSE), Tenure=(TRUE/FALSE). Only faculty where Tenure = TRUE were included in the IEOR network (1179 edges).

See Appendix 1 for a discussion of its content and data collection criteria and assumptions)

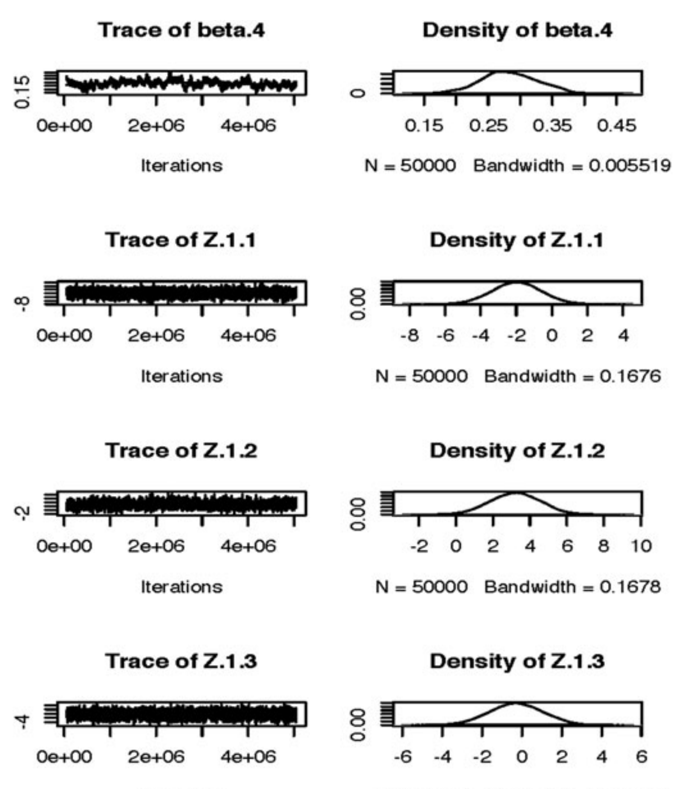

Figure 17. MCMC diagnostics for model (5), (cont.): values of β_3, Z_1, Z_2, and Z_3. All convergence diagnostics are adequate, i.e., trace plots are stable and therefore parameter values converge in distribution. Traces shown for last 5 000 000 iterations, distributions shown for last 50 000 iterations.