Rankability, Predictability, and Ellipses

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Spectral Rankability

Algorithm 1 Spectral Rankability of Graph Data Γ.

function $[r] = \text{specR}(\Gamma)$:

$n \leftarrow$ the number of vertices in Γ

$D \leftarrow$ the degree matrix of Γ

$L \leftarrow$ graph Laplacian of Γ

$s = \text{diag}\,(n-1, n-2, \ldots, 0)$

$r = \frac{\text{hd}(D,S)+\text{hd}(L,S)}{2(n-1)}$

return

Connectivity Rankability

Algorithm 2 Connectivity Rankability of Graph Data Γ.

function $[r] = \text{connR}(\Gamma)$:

$n \leftarrow$ the number of vertices in Γ

$L \leftarrow$ graph Laplacian of Γ

$\alpha = \min_{x \in S} x^T L x$, where $S = \{x \in \mathbb{R}^n : x \perp e, \|x\| = 1\}$

$\beta = \max_{x \in S} x^T L x$, where $S = \{x \in \mathbb{R}^n : x \perp e, \|x\| = 1\}$

$\tilde{\alpha} \leftarrow$ related quantity for perfect dominance graph

$\tilde{\beta} \leftarrow$ related quantity for perfect dominance graph

$r = \frac{|\alpha - \tilde{\alpha}| + |\beta - \tilde{\beta}|}{2n}$

return

Results

Year	specR	connR	Massey	Colley	Cycles
1995	0.143	0.043	0.893	0.926	7
1996	0.143	0.043	0.857	0.963	1
1997	0.185	0.069	0.679	0.714	62
1998	0.183	0.062	0.750	0.889	30
1999	0.143	0.058	0.821	0.889	23
2000	0.143	0.002	0.929	0.929	3
2001	0.143	0.001	0.857	1.000	1
2002	0.143	0.005	0.893	0.963	3
2003	0.143	0.049	0.786	0.885	18
2004	0.339	0.168	0.762	0.778	48
2005	0.162	0.013	0.821	0.885	16
2006	0.195	0.100	0.750	0.808	45
2007	0.316	0.169	0.643	0.654	205
2008	0.195	0.099	0.714	0.852	43
2009	0.143	0.049	0.786	0.857	26
2010	0.292	0.166	0.750	0.741	174
2011	0.286	0.149	0.643	0.808	125
2012	0.286	0.163	0.643	0.769	111

	Massey	Colley	Cycles
specR	-0.719	-0.785	0.826
connR	-0.802	-0.829	0.843

Results

Year	specR	connR	Massey	Colley
2002	0.294	0.174	0.579	0.654
2003	0.227	0.119	0.710	0.710
2004	0.136	0.088	0.738	0.785
2005	0.147	0.083	0.729	0.738
2006	0.119	0.060	0.711	0.722
2007	0.155	0.108	0.771	0.780
2008	0.200	0.058	0.775	0.775
2009	0.227	0.112	0.756	0.740
2010	0.227	0.147	0.697	0.706
2011	0.094	0.060	0.748	0.782
2012	0.227	0.130	0.664	0.664
2013	0.273	0.152	0.790	0.756
2014	0.173	0.100	0.745	0.776
2015	0.340	0.217	0.720	0.700

	Massey	Colley
specR	-0.339	-0.615
connR	-0.408	-0.615

Numerical Range

The algebraic connectivity is the following minimization problem:

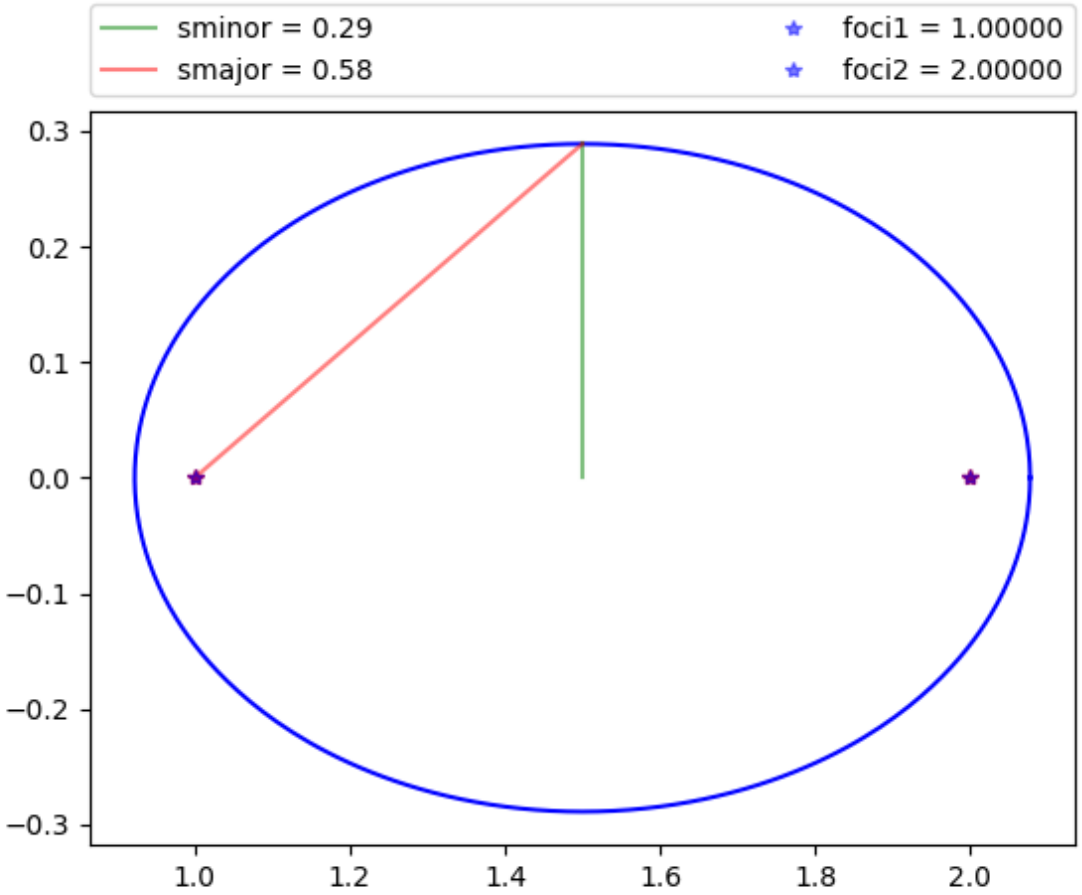
$$\alpha = \min_{\|x\|=1} Re(x^* Q^T L Q x).$$

The numerical range is defined by

$$W(Q^T L Q) = \left\{ x^* Q^T L Q x, \ \ \|x\| = 1 \right\}$$

Conjecture

The numerical range associated with the perfect dominance graph is an ellipse!



Unique Characterization?

The numerical range associated with a cycle is a polygon: