Rankability Updates

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Update

- Spectral-degree characterization of *complete dominance* graphs holds for all digraphs with weights $w_{ij} \in [0, 1]$.

- Therefore, we can model multiple games and tie situations and compare against the ideal complete dominance graph.

Perturbation Theory

- The eigenvalues of the graph Laplacian of a complete dominance graph are extremely well-conditioned.
- Therefore, for small perturbations of a complete dominance graph, the eigenvalues will only be slightly perturbed.
- Adding or deleting an edge with weight w_{ij} changes on eigenvalue by w_{ij}.
- The Hausdorff distance between the eigenvalues of a complete dominance graph and any other digraph with weights $w_{ij} \in [0, 1]$ is bounded above by $(n - 1)$.

The Algorithm

Algorithm 1 Spectral Rankability of Graph Data Γ.

function $[r] = \text{specR}(\Gamma)$:

$n \leftarrow$ the number of vertices in Γ

$D \leftarrow$ the out-degree matrix of Γ

$L \leftarrow$ graph Laplacian of Γ

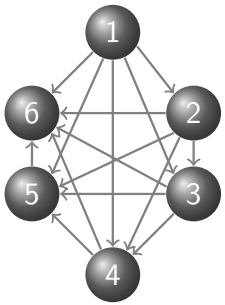
$S = \text{diag}(n-1, n-2, \ldots, 0)$

$r = 1 - \frac{\text{hd}(D,S)+\text{hd}(L,S)}{2(n-1)}$

return

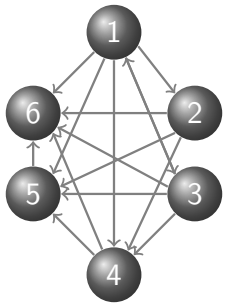
SIMOD Examples

Complete dominance



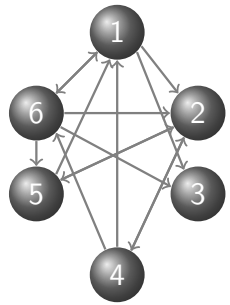
specR = 1.0000
edgeR = 1.0000

Perturbed Dominance



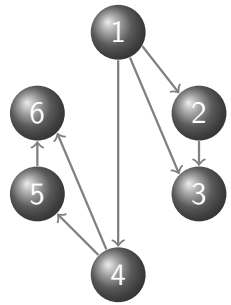
specR = 0.9382
edgeR = 0.9994

Perturbed Random C



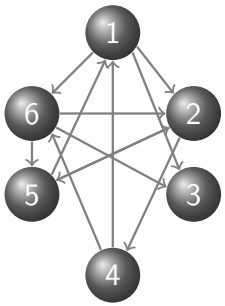
specR = 0.8202
edgeR = 0.9987

Nearly Disconnected



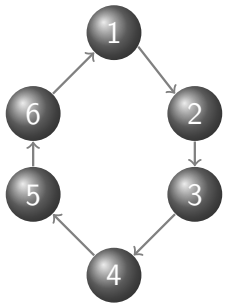
specR = 0.6000
edgeR = 0.9926

Random



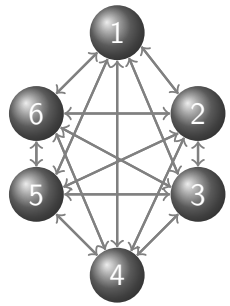
specR = 0.5891
edgeR = 0.9900

Cycle



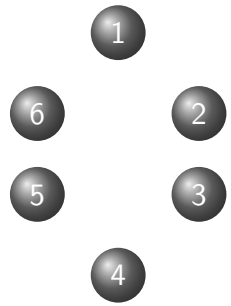
specR = 0.3000
edgeR = 0.9939

Completely Connected



specR = 0.2000
edgeR = 0.0000

Empty



specR = 0.0000
edgeR = 0.0000.

Correlation between specR and edgeR is 0.79.

Opening Remarks

- The Sinquefield Cup is an invite only round-robin chess tournament for Grand Master level players.
- The Sinquefield Cup started in 2013 and the 2019 tournament takes place this August 16 - 29.
- Throughout its history, the Sinquefield Cup has been one of the highest rated tournaments. For instance, in 2014, the average Elo rating was 2802, which is the highest average in the history of chess.

A Few More Details

- In 2013, the Sinquefield Cup was a double round-robin tournament comprised of 4 players.
- In 2014, the Sinquefield Cup was a double round-robin tournament comprised of 6 players.
- Since then, the Sinquefield Cup has been a single round-robin tournament comprised of 10 players.

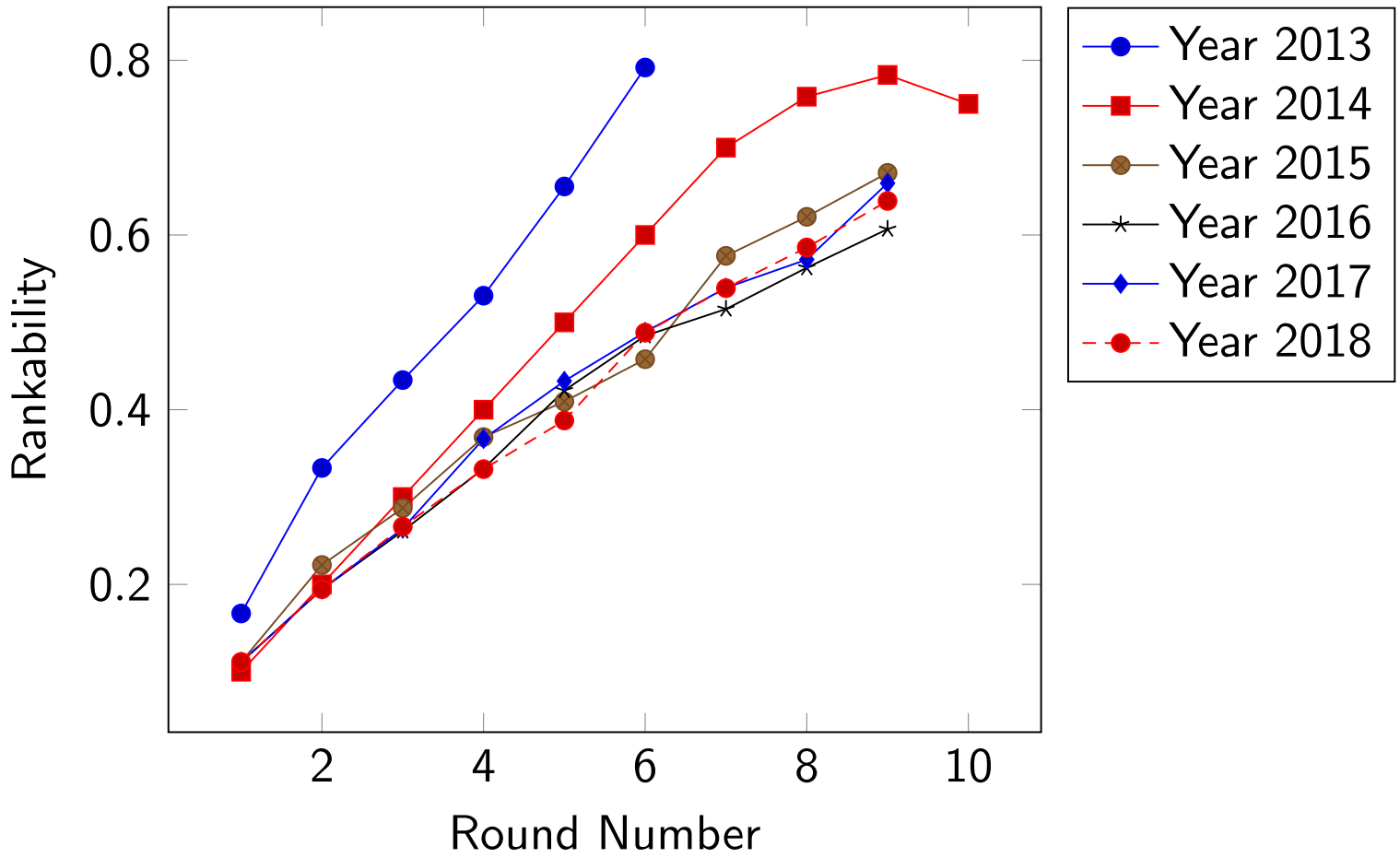
Modeling Decisions

Note that all weights w_{ij} are initially set to zero. Then, for each round we update as follows:

- If player i beats player j, then $w_{ij} = w_{ij} + 1$.
- If player j beats player i, then $w_{ji} = w_{ji} + 1$.
- If player i and j tie, then $w_{ij} = w_{ij} + 1/2$ and $w_{ji} = w_{ji} + 1/2$.

We also normalize the weights based on the number of times the pair of players have played eachother.

Round by Round Rankability



Remarks

Note that the year 2013 has the highest rankability score after 6 rounds. The scorecard for that year is shown below.

1st Sinquefield Cup, 9–15 September 2013, St. Louis, USA, Cat. XXII (2797)

	Player	Rating	1	2	3	4	Points	TPR
1	**Magnus Carlsen (Norway)**	2862		½ ½	½ 1	1 1	**4½**	2968
2	Hikaru Nakamura (United States)	2772	½ ½		1 0	1 ½	**3½**	2862
3	Levon Aronian (Armenia)	2813	½ 0	0 1		½ ½	**2½**	2735
4	Gata Kamsky (United States)	2741	0 0	0 ½	½ ½		**1½**	2623

Note the uniform distribution of total points for each player, this data is very rankable.

Remarks

The next most rankable year is 2014. The scorecard for that year is shown below.

2nd Sinquefield Cup, 27 August – 7 September 2014, St. Louis, Missouri, United States, Category XXIII (2801.7)

	Player	Rating	1	2	3	4	5	6	Points	Wins	H2H	TPR[9]
1	**Fabiano Caruana (Italy)**	2801		1 ½	1 1	1 1	1 ½	1 ½	**8½**			3103
2	Magnus Carlsen (Norway)	2877	0 ½		½ ½	½ ½	1 ½	½ 1	**5½**			2822
3	Veselin Topalov (Bulgaria)	2772	0 0	½ ½		1 ½	0 ½	1 1	**5**			2807
4	Maxime Vachier-Lagrave (France)	2768	0 0	½ ½	0 ½		1 ½	½ ½	**4**	1	1½	2738
5	Levon Aronian (Armenia)	2805	0 ½	0 ½	1 ½	0 ½		½ ½	**4**	1	½	2731
6	Hikaru Nakamura (United States)	2787	0 ½	½ 0	0 0	½ ½	½ ½		**3**			2658

Note that this is the year that Fabiano went on his historical run. However, there is a tie between 4 and 5 and not much difference between 2 and 3; hence, this year is slightly less rankable than 2013.

Remarks

The least rankable years were 2016 and 2018. Both scorecards are shown below.

4th Sinquefield Cup, 4–16 August 2016, St. Louis, Missouri, United States, Category XXII (2778.6)

	Player	Rating	1	2	3	4	5	6	7	8	9	10	Points	Wins	SB	TPR	Tour Points
1	**Wesley So (United States)**	2771		½	1	½	½	1	½	½	½	½	**5½**			2859	13
2	Levon Aronian (Armenia)	2792	½		½	½	½	1	0	½	1	½	**5**	2	21.75	2820	7.75
3	Veselin Topalov (Bulgaria)	2761	0	½		½	½	½	½	1	1	½	**5**	2	21.00	2823	7.75
4	Viswanathan Anand (India)	2770	½	½	½		½	½	1	½	½	½	**5**	1	22.25	2822	7.75
5	Fabiano Caruana (United States)	2807	½	½	½	½		½	½	½	½	1	**5**	1	21.50	2818	7.75
6	Hikaru Nakamura (United States)	2791	0	0	½	½	½		½	1	½	1	**4½**	2		2777	4.5
7	Maxime Vachier-Lagrave (France)	2819	½	1	½	0	½	½		½	½	½	**4½**	1		2774	4.5
8	Ding Liren (China)	2755	½	½	0	½	½	0	½		1	½	**4**			2738	3
9	Peter Svidler (Russia)	2751	½	0	0	½	½	½	½	0		1	**3½**			2701	2
10	Anish Giri (Netherlands)	2769	½	½	½	½	0	0	½	½	0		**3**			2654	1

6th Sinquefield Cup, 18–28 August 2018, St. Louis, Missouri, United States, Category XXII (2787.5)

	Player	Rating	1	2	3	4	5	6	7	8	9	10	Points	H2H	Wins	Black	TPR	Tour Points
1–3	**Magnus Carlsen (Norway)**	2842		½	½	½	½	½	½	½	1	1	**5½**	1	2	0	2861	15
1–3	**Fabiano Caruana (United States)**	2822	½		½	½	½	½	½	½	1	1	**5½**	1	2	0	2864	15
1–3	**Levon Aronian (Armenia)**	2767	½	½		½	1	½	½	½	½	1	**5½**	1	2	0	2870	15
4	Shakhriyar Mamedyarov (Azerbaijan)	2801	½	½	½		½	½	½	1	½	½	**5**		1	0	2829	10
5	Alexander Grischuk (Russia)	2766	½	½	0	½		½	½	½	1	½	**4½**	1	1	1	2790	6
6–7	Maxime Vachier-Lagrave (France)	2779	½	½	½	½	½		½	½	½	½	**4½**	1	0	0	2788	6
6–7	Viswanathan Anand (India)	2768	½	½	½	½	½	½		½	½	½	**4½**	1	0	0	2790	6
8	Wesley So (United States)	2780	½	½	½	0	½	½	½		½	½	**4**		0	0	2745	3
9–10	Hikaru Nakamura (United States)	2777	0	0	½	½	0	½	½	½		½	**3**	½	0	0	2664	1.5
9–10	Sergey Karjakin (Russia)	2773	0	0	0	½	½	½	½	½	½		**3**	½	0	0	2664	1.5

Remarks

- There is a 4 way tie for second place in 2016. Furthermore, each place is only separated by half a point.
- Similarly, in 2018 there was a 3 way tie for first place. Interestingly, the winners agreed to split the prize and not deal with elimination games.
- Certainly, these years are less rankable than 2013 and 2014.

Definition

The ranking fit refers to how well a ranking "fits" or "matches" the data from a certain system.

We measure the ranking fit based on the number of changes made to the ranking when new data from a system is considered.

Sinquefield Cup Ranking Fit

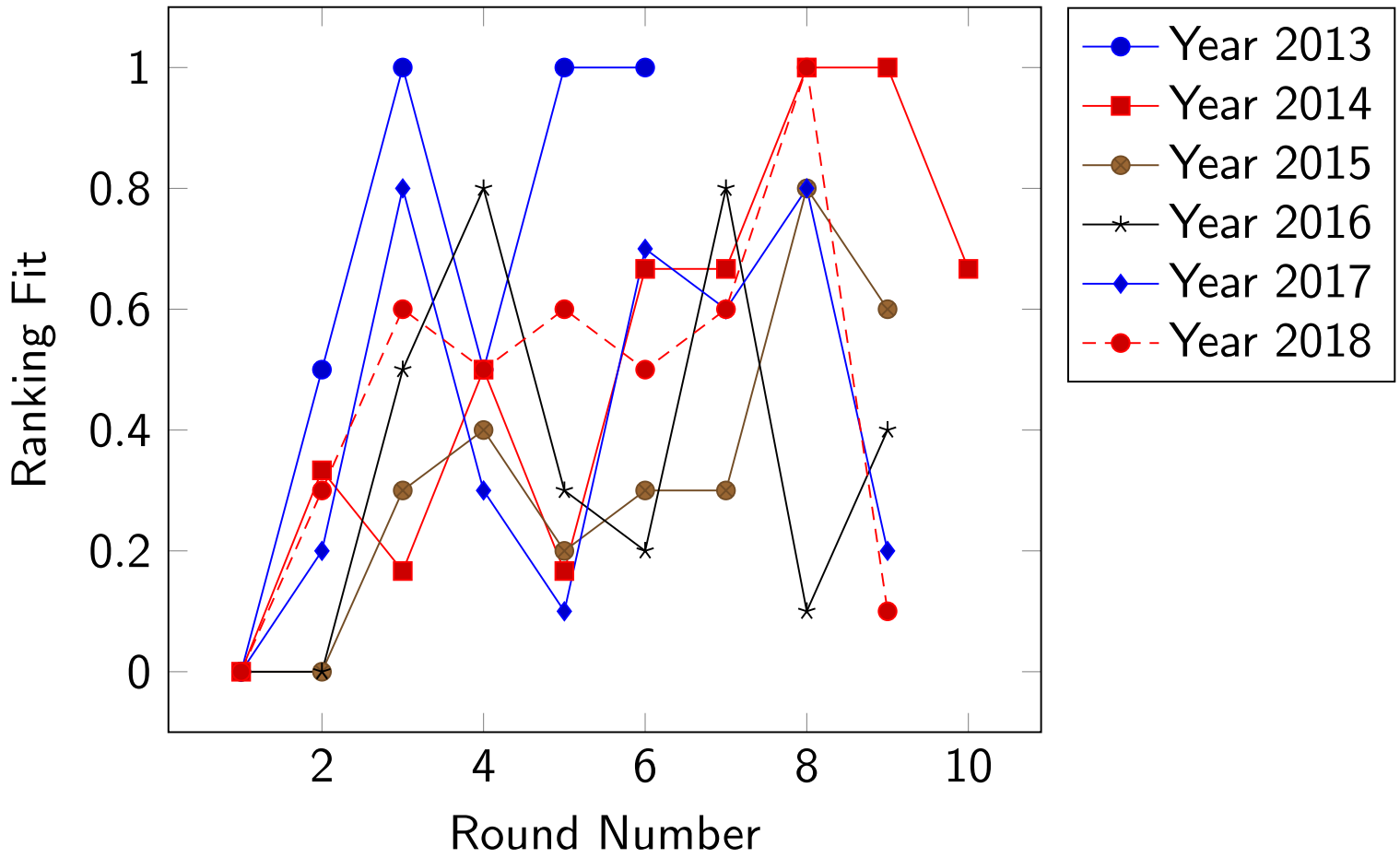
- We rate the players in the tournament based on their "local" Elo rating, where each player starts at zero and is updated based on their performance in that round.

- We rank the players in the tournament by their local Elo rating.

- Let k denote the number of changes in our ranking from the previous round, and let n denote the total number of players in the tournament. Then, the ranking fit is defined by

$$1 - \frac{k}{n}.$$

Round by Round Ranking Fit



Remarks

- Something very exciting happened in the last round of 2018; indeed, only one player had the same ranking in round 9 as they did in round 8.
- If you compare the last round Ranking Fit and the last round Rankability, the correlation is 0.83.
- Ranking Fit seems (to me) very related to predictability.