Rankability, Predictability, and Ellipses

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Spectral Rankability

Algorithm 1 Spectral Rankability of Graph Data Γ.

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function $[r] = \text{specR}(\Gamma)$:

$\quad n \leftarrow$ the number of vertices in Γ

$\quad D \leftarrow$ the degree matrix of Γ

$\quad L \leftarrow$ graph Laplacian of Γ

$\quad s = \text{diag}\,(n-1, n-2, \ldots, 0)$

$\quad r = \frac{\text{hd}(D,S) + \text{hd}(L,S)}{2(n-1)}$

$\quad$**return**

Connectivity Rankability

Algorithm 2 Connectivity Rankability of Graph Data Γ.

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function $[r] = \text{connR}(\Gamma)$:

  $n \leftarrow$ the number of vertices in Γ

  $L \leftarrow$ graph Laplacian of Γ

  $\alpha = \min_{x \in S} x^T L x$, where $S = \{x \in \mathbb{R}^n \colon x \perp e, \|x\| = 1\}$

  $\beta = \max_{x \in S} x^T L x$, where $S = \{x \in \mathbb{R}^n \colon x \perp e, \|x\| = 1\}$

  $\tilde{\alpha} \leftarrow$ related quantity for perfect dominance graph

  $\tilde{\beta} \leftarrow$ related quantity for perfect dominance graph

  $r = \frac{|\alpha - \tilde{\alpha}| + |\beta - \tilde{\beta}|}{2n}$

  return

Results

Year	specR	connR	Massey	Colley	Cycles
1995	0.143	0.043	0.893	0.926	7
1996	0.143	0.043	0.857	0.963	1
1997	0.185	0.069	0.679	0.714	62
1998	0.183	0.062	0.750	0.889	30
1999	0.143	0.058	0.821	0.889	23
2000	0.143	0.002	0.929	0.929	3
2001	0.143	0.001	0.857	1.000	1
2002	0.143	0.005	0.893	0.963	3
2003	0.143	0.049	0.786	0.885	18
2004	0.339	0.168	0.762	0.778	48
2005	0.162	0.013	0.821	0.885	16
2006	0.195	0.100	0.750	0.808	45
2007	0.316	0.169	0.643	0.654	205
2008	0.195	0.099	0.714	0.852	43
2009	0.143	0.049	0.786	0.857	26
2010	0.292	0.166	0.750	0.741	174
2011	0.286	0.149	0.643	0.808	125
2012	0.286	0.163	0.643	0.769	111

	Massey	Colley	Cycles
specR	-0.719	-0.785	0.826
connR	-0.802	-0.829	0.843