Update: Algebraic Connectivity

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May 28, 2019

Algebraic Connectivity

Let Γ be a directed graph and L be the graph Laplacian.

Define
$$S = \{x \in \mathbb{R}^n \colon x \perp e, \|x\| = 1\}.$$

Then, the algebraic connectivity of Γ is

$$\alpha(\Gamma) = \min_{x \in S} x^T L x,$$

and a related quantity is

$$\beta(\Gamma) = \max_{x \in S} x^T L x.$$

Let Γ be a perfect dominance graph on n vertices. Then,

- $\alpha(\Gamma) + \beta(\Gamma) = n$.

- $0 < \alpha(\Gamma) \leq 1$ and $n - 1 \leq \beta(\Gamma) < n$.

Snapshot of Results

Year	specR	αR	βR	Year	specR	αR	βR
1995	0.143	0.006	0.408	2004	0.339	0.669	0.993
1996	0.143	0.001	0.408	2005	0.162	0.095	0.065
1997	0.185	0.153	0.565	2006	0.195	0.680	0.543
1998	0.183	0.093	0.530	2007	0.316	1.000	1.000
1999	0.143	0.902	0.012	2008	0.195	0.680	0.531
2000	0.143	0.013	0.008	2009	0.143	0.680	0.058
2001	0.143	0.003	0.006	2010	0.292	0.947	1.000
2002	0.143	0.080	0.003	2011	0.286	0.680	1.000
2003	0.143	0.090	0.408	2012	0.286	0.912	1.000

■ Most Rankable

□ Least Rankable