Round-Robin Rankability

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Opening Remarks

- The spectral-degree characterization of perfect dominance graphs holds for all directed graphs with weights

$$0 \leq w_{ij} \leq 1.$$

Therefore, we can weight ties with $1/2$.

- There is an analogous characterization for perfect dominance graphs associated with double, triple, etc. round-robin tournaments. Therefore, we can naturally handle multiple games.

SpecR Algorithm

For a single round-robin tournament we measure rankability using the following algorithm.

Algorithm 1 Spectral Rankability of for Round-Robin.

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function $[r] = \text{SpecR}(\Gamma)$:
    $n \leftarrow$ the number of vertices in Γ
    $D \leftarrow$ the out-degree matrix of Γ
    $L \leftarrow$ graph Laplacian of Γ
    $S = \text{diag}(n-1, n-2, \ldots, 0)$
    $r = 1 - \frac{\text{hd}(D,S) + \text{hd}(L,S)}{2(n-1)}$
    return

Note that this algorithm works for all digraphs with weights

$$0 \le w_{ij} \le 1.$$

SpecDR Algorithm

Four a double round-robin tournament we measure rankability using the following algorithm.

Algorithm 2 Spectral Rankability for Double Round-Robin.

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function $[r] = \text{SpecDR}(\Gamma)$:
    $n \leftarrow$ the number of vertices in Γ
    $D \leftarrow$ the out-degree matrix of Γ
    $L \leftarrow$ graph Laplacian of Γ
    $S = \text{diag}\,(2(n-1), 2(n-2), \ldots, 0)$
    $r = 1 - \frac{\text{hd}(D,S) + \text{hd}(L,S)}{4(n-1)}$
    return

Note that this algorithm works for all digraphs with weights

$$0 \leq w_{ij} \leq 2.$$

Opening Remarks

- The Sinquefield Cup is an invite only round-robin chess tournament for Grand Master level players.

- The Sinquefield Cup started in 2013 and the 2019 tournament takes place this August 16 - 29.

- Throughout its history, the Sinquefield Cup has been one of the highest rated tournaments. For instance, in 2014, the average Elo rating was 2802, which is the highest average in the history of chess.

A Few More Details

- In 2013, the Sinquefield Cup was a double round-robin tournament comprised of 4 players.

- In 2014, the Sinquefield Cup was a double round-robin tournament comprised of 6 players.

- Since then, the Sinquefield Cup has been a single round-robin tournament comprised of 10 players.

Modeling Decisions

Note that all weights w_{ij} are initially set to zero. Then, for each round we update as follows:

- If player i beats player j, then $w_{ij} = w_{ij} + 1$.

- If player j beats player i, then $w_{ji} = w_{ji} + 1$.

- If player i and j tie, then $w_{ij} = w_{ij} + 1/2$ and $w_{ji} = w_{ji} + 1/2$.

Round by Round Analysis
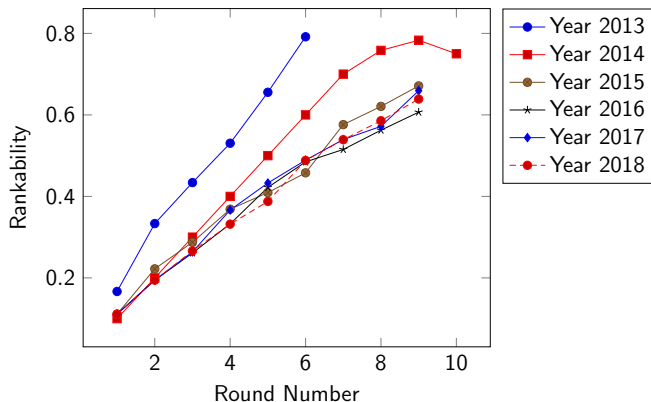
Closing Remarks

Note that the year 2013 has the highest rankability score after 6 rounds. The scorecard for that year is shown below.

1st Sinquefield Cup, 9–15 September 2013, St. Louis, USA, Cat. XXII (2797)

	Player	Rating	1	2	3	4	Points	TPR
1	🇳🇴 **Magnus Carlsen** (Norway)	2862		½ ½	½ 1	1 1	**4½**	2968
2	🇺🇸 Hikaru Nakamura (United States)	2772	½ ½		1 0	1 ½	**3½**	2862
3	🇩🇪 Levon Aronian (Armenia)	2813	½ 0	0 1		½ ½	**2½**	2735
4	🇺🇸 Gata Kamsky (United States)	2741	0 0	0 ½	½ ½		**1½**	2623

Note the uniform distribution of total points for each player, this seems very rankable to me!

Closing Remarks

The next most rankable year is 2014. The scorecard for that year is shown below.

2nd Sinquefield Cup, 27 August – 7 September 2014, St. Louis, Missouri, United States, Category XXIII (2801.7)

	Player	Rating	1	2	3	4	5	6	Points	Wins	H2H	TPR[9]
1	🇮🇹 **Fabiano Caruana** (Italy)	2801		1 ½	1 1	1 1	1 ½	1 ½	**8½**			3103
2	🇳🇴 Magnus Carlsen (Norway)	2877	0 ½		½ ½	½ ½	1 ½	½ 1	**5½**			2822
3	🇧🇬 Veselin Topalov (Bulgaria)	2772	0 0	½ ½		1 ½	0 ½	1 1	**5**			2807
4	🇫🇷 Maxime Vachier-Lagrave (France)	2768	0 0	½ ½	0 ½		1 ½	½ ½	**4**	1	1½	2738
5	🇦🇲 Levon Aronian (Armenia)	2805	0 ½	0 ½	1 ½	0 ½		½ ½	**4**	1	½	2731
6	🇺🇸 Hikaru Nakamura (United States)	2787	0 ½	½ 0	0 0	½ ½	½ ½		**3**			2658

Note that this is the year that Fabiano went on his historical run. However, there is a tie between 4 and 5 and not much difference between 2 and 3; hence, this year is slightly less rankable than 2013.

Closing Remarks

The least rankable years were 2016 and 2018. Both scorecards are shown below.

4th Sinquefield Cup, 4–16 August 2016, St. Louis, Missouri, United States, Category XXII (2778.6)

	Player	Rating	1	2	3	4	5	6	7	8	9	10	Points	Wins	SB	TPR	Tour Points
1	Wesley So (United States)	2771		½	1	½	½	1	½	½	½	½	5½			2859	13
2	Levon Aronian (Armenia)	2792	½		½	½	½	1	0	½	1	½	5	2	21.75	2820	7.75
3	Veselin Topalov (Bulgaria)	2761	0	½		½	½	½	1	1	½		5	2	21.00	2823	7.75
4	Viswanathan Anand (India)	2770	½	½	½		½	1	½	½	½	1	5	1	22.25	2822	7.75
5	Fabiano Caruana (United States)	2807	½	½	½	½		½	½	½	½	1	5	1	21.50	2818	7.75
6	Hikaru Nakamura (United States)	2791	0	0	½	½	½		½	1	1	½	4½	2		2777	4.5
7	Maxime Vachier-Lagrave (France)	2819	½	1	½	0	½	½		½	½	½	4½	1		2774	4.5
8	Ding Liren (China)	2755	½	½	0	½	½	0	½		½	1	4			2738	3
9	Peter Svidler (Russia)	2751	½	0	0	½	½	½	½	0		1	3½			2701	2
10	Anish Giri (Netherlands)	2769	½	½	½	0	0	½	½	0	0		3			2654	1

6th Sinquefield Cup, 18–28 August 2018, St. Louis, Missouri, United States, Category XXII (2787.5)

	Player	Rating	1	2	3	4	5	6	7	8	9	10	Points	H2H	Wins	Black	TPR	Tour Points
1–3	Magnus Carlsen (Norway)	2842		½	½	½	½	½	½	½	1	1	5½	1	2	0	2861	15
1–3	Fabiano Caruana (United States)	2822	½		½	½	½	½	½	½	½	1	5½	1	2	0	2864	15
1–3	Levon Aronian (Armenia)	2767	½	½		0	1	½	½	½	½	1	5½	1	2	0	2870	15
4	Shakhriyar Mamedyarov (Azerbaijan)	2801	½	½	½		½	½	½	1	1	½	5		1	0	2829	10
5	Alexander Grischuk (Russia)	2766	½	½	0	½		½	½	½	½	1	4½	1	1	1	2790	6
6–7	Maxime Vachier-Lagrave (France)	2779	½	½	½	½	½		½	½	½	½	4½	1	0	0	2788	6
6–7	Viswanathan Anand (India)	2768	½	½	½	½	½	½		½	½	½	4½	1	0	0	2790	6
8	Wesley So (United States)	2780	½	½	½	0	½	½	½		½	½	4		0	0	2745	3
9–10	Hikaru Nakamura (United States)	2777	0	½	½	0	½	½	½	½		½	3	½	0	0	2664	1.5
9–10	Sergey Karjakin (Russia)	2773	0	0	½	½	0	½	½	½	½		3	½	0	0	2664	1.5

Closing Remarks

Note that there is a 4 way tie for second place in 2016.
Furthermore, each place is only separated by half a point.
Similarly, in 2018 there was a 3 way tie for first place. Interestingly, the winners agreed to split the prize and not deal with elimination games. Certainly, these years are less rankable than 2013 and 2014.

What's Next?

- Now that the spectral rankability can deal with multiple games, I would like college basketball conference data where the modeling decision is similar to the one chosen here.

- We can also deal with ties, so I would like college sports data (conferences only) where ties are allowed. I think baseball allows for ties?

- I would also like to do "week by week" analysis like was done here (round by round). This should be easy with college football. Can we do other sports?

- I am still on the lookout for dominance data from animal populations, or plurality voting data. Perhaps we should talk to an expert?