College Football Rankability

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Spectral Rankability

The Hausdorff distance between the sets x and y is defined by

$$d(x,y) = \max\{\max_i \min_j |x_i - y_j|, \max_i \min_j |y_i - x_j|\}.$$

The spectral rankability measure is denoted by rankS, where

$$\begin{aligned}
&\text{function } [r] = \text{rankS}(A):\\
&\quad n = \text{size}(A)\\
&\quad x = [\text{sum}(A[i,:]) \text{ for } i \text{ in range}(n)]\\
&\quad D = \text{diag}(x)\\
&\quad L = D - A\\
&\quad s = [n-k \text{ for } k \text{ in range}(1, n+1)]\\
&\quad e = \text{eigvals}(L)\\
&\quad r = \tfrac{d(e,s)+d(x,s)}{2(n-1)}
\end{aligned}$$

Algebraic Rankability

The algebraic connectivity of a directed graph is defined by

$$\alpha(L) = \lambda_{min}\left(\frac{1}{2}Q^T(L + L^T)Q\right),$$

where L is the Laplacian of the associated graph.

The algebraic rankability measure is computed by rankA, where

function $[r] = \text{rankA}(A)$:
$n = \text{size}(A)$
Let $\mathcal{L}$ be laplacian of dominance graph of size n
Let L be laplacian of the graph associated with A.
$r = |\alpha(\mathcal{L}) - \alpha(L)|$

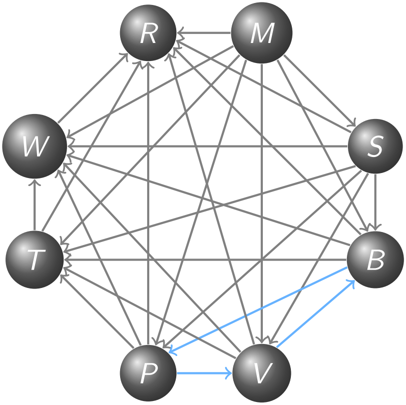
Results

Year	rankH	rankA	rankH+rankA
2000	0.143	0.914	1.057
2001	0.143	0.013	0.156
2002	0.143	0.081	0.224
2003	0.143	0.092	0.234
2004	0.339	0.678	1.017
2005	0.162	0.096	0.259
2006	0.195	0.690	0.885
2007	0.316	1.014	1.330
2008	0.195	0.690	0.885
2009	0.143	0.690	0.833
2010	0.292	0.960	1.252
2011	0.286	0.690	0.975
2012	0.286	0.925	1.211

Most Rankable

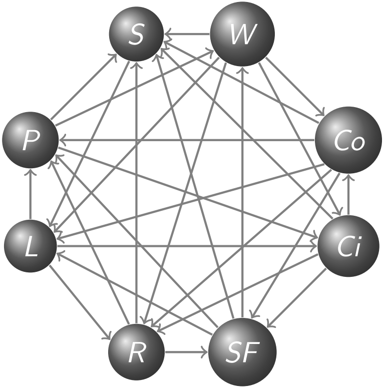
Least Rankable

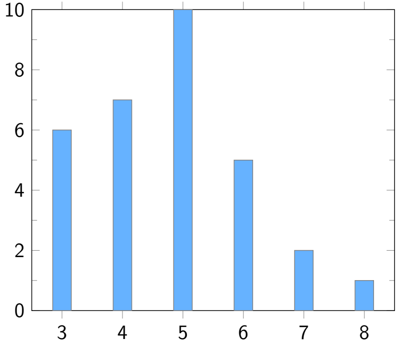
Year 2001



Cycle of length 3

Year 2007





Results

Year	rankH	rankA	rankH+rankA
2000	0.949	2.166	3.114
2001	0.945	2.169	3.114
2002	0.941	1.901	2.843
2003	0.937	2.027	2.963
2004	0.945	2.002	2.947
2005	0.949	1.932	2.880
2006	0.946	1.832	2.778
2007	0.950	1.930	2.880
2008	0.946	1.952	2.898
2009	0.943	1.924	2.867
2010	0.947	2.084	3.030
2011	0.943	1.955	2.898
2012	0.947	2.041	2.988

Most Rankable

Least Rankable