Quantitative Developer - Technical Assignment

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1 Combinatorics

Each glass can hold 0 to 10 pens, as is not stated otherwise. The question is in how many ways can you assign 5 glasses to 10 pens. 1 pen can be assigned to 5 glasses, and pen assignments are independent, as glasses do not have a maximum capacity. Thus,

$$\text{total combinations} = 5^{10} = 9,765,625$$

2 Calculus

The idea is that we can always find a function that satisfies the given conditions and which can get arbitrarily close to 5. That is, we know that $f(x) \geq 0$, $f'(x) > 0\ \forall x \in \mathbb{R}$ and $f(2) = 5$, thus we can see that this function is always increasing and non-negative, hence the upper bound for the function evaluated at 0 is $f(0) < 5$. To show that we can get arbitrarily close, we may consider the following example. We are looking for a function that is $f(2) = 5$, always increasing and positive. One such example is

$$f(x) = 5 \times 10^{(x-2)}$$

this function is equal to $f(0) = \frac{5}{100}$. However, by changing the exponent, we increase the value of the function evaluated at 0. Consider the following:

$$f_k(x) = 5 \times 10^{\frac{1}{k}(x-2)}, \quad \text{where } k = 1, 2, 3, ...$$

we can see that:

$$f_1(0) = \frac{5}{100} = 0.05, \quad f_2(0) = \frac{5}{10} = 0.5, \quad f_3(0) = 1.077..., \quad f_4(0) = 1.581...$$

and so on. In fact, if we take the limit, we get:

$$\lim_{k\to\infty} f_k(0) = 5 \lim_{k\to\infty} 10^{\frac{1}{k}(0-2)} = 5, \quad \text{since } \lim_{k\to\infty} \frac{1}{k}(0-2) = \lim_{k\to\infty} \frac{-2}{k} = 0$$

Therefore, we conclude that there will always be a function that satisfies these conditions and has $f(0)$ arbitrarily close to 5. Hence, the supremum is 5. (However, technically the maximum value that a particular function could take is not 5, since such a function would fail the condition that $f'(x) > 0 \quad \forall x$, since it would have to be constant on $x \in [0, 2]$).

3 Linear Algebra

The given matrix is:

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The matrix A is given in its row echelon form, and we can see that all of its pivots are not zero. Therefore, all three rows / columns are linearly independent, which means that the rank of the matrix A is $\text{rank}(A) = 3$.

4 Stochastic Calculus

X is a Geometric Brownian Motion with constant drift and diffusion coefficients. The differential of X is:

$$\mathrm{d}X(t) = \mu(t)\,X(t)\,\mathrm{d}t\, + \sigma(t)\,X(t)\,\mathrm{d}W(t) \tag{1}$$

where $\mu(t) = \mu$ and $\sigma(t) = \sigma$.
To find the differential of X^2 we apply Ito's calculus by setting $f(t,x) = x^2$. Then, according to Ito's Lemma:

$$\mathrm{d}f(t,x) = f_t\,\mathrm{d}t\, + f_x\,\mathrm{d}X(t)\, + \tfrac{1}{2} f_{xx}\,\mathrm{d}X(t)\,\mathrm{d}X(t)$$

In our case $f_t(t, X(t)) = 0$, $f_x(t, X(t)) = 2X(t)$, and $f_{xx}(t, X(t)) = 2$. Substituting these values into the equation above, we can find the differential for X^2:

$$\begin{aligned} \mathrm{d}X^2(t) &= 2X^2(t)\,[\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W(t)] + \sigma^2 X^2(t)\,\mathrm{d}t \\ &= 2X^2(t)\,\left[\left(\mu + \tfrac{1}{2}\sigma^2\right)\mathrm{d}t + \sigma\,\mathrm{d}W(t)\right] \end{aligned} \tag{2}$$

5 Portfolio Allocation

We are given two equities X and Y, with returns r_x and r_y, respectively. Additionally, we are given the following:

$$1)\quad \mathbb{E}[r_x] = \mu_x \tag{3}$$
$$2)\quad \mathbb{E}[r_y] = \mu_y \tag{4}$$
$$3)\quad \mathrm{Var}(r_x) = \sigma_x \tag{5}$$
$$4)\quad \mathrm{Var}(r_y) = \sigma_y \tag{6}$$
$$5)\quad \frac{\mathrm{Cov}(r_x, r_y)}{\sqrt{\mathrm{Var}(r_x)\,\mathrm{Var}(r_y)}} = \rho \tag{7}$$

Using (5), (6), and (7) we can construct a covariance matrix for the equities X and Y:

$$\Sigma = \begin{bmatrix} \sigma_x & \rho\sqrt{\sigma_x \sigma_y} \\ \rho\sqrt{\sigma_x \sigma_y} & \sigma_y \end{bmatrix}$$

We can also define a vector of quantities held in each of the equities. I define it as follows:

$$q = \begin{bmatrix} q_x \\ q_y \end{bmatrix}$$

Then the variance of the portfolio return $\mathrm{Var}(r_p)$ is:

$$\mathrm{Var}(r_p) = q^{\mathrm{T}}\Sigma q = q_x^2 \sigma_x + q_y^2 \sigma_y + 2\rho\sqrt{\sigma_x \sigma_y}\, q_x q_y$$

We can minimize $\mathrm{Var}(r_p)$ w.r.t. q_y by solving the following:

$$\frac{\mathrm{d}\,\mathrm{Var}(r_p)}{\mathrm{d}\,q_y} = 0$$

$$\frac{\mathrm{d}\,\mathrm{Var}(r_p)}{\mathrm{d}\,q_y} = 2q_y\sigma_y + 2\rho\sqrt{\sigma_x \sigma_y}\, q_x = 0$$

After rearranging, we find the solution:

$$q_y = -\frac{2\rho\sqrt{\sigma_x \sigma_y}\, q_x}{2\sigma_y} = -\rho\sqrt{\frac{\sigma_x}{\sigma_y}}\, q_x$$