COMP 340 **Analysis of Algorithms**

Problem Set 1

Assigned: January 19, 2018 *Due: February 2, 2018*

Always provide explanations and show as much work as possible. Solutions to odd-numbered exercises are available at `http://www.algorist.com/algowiki/index.php/The_Algorithms_Design_Manual_(Second_Edition)`. If you are stuck on an even problem, try to find a similar odd-numbered exercise.

1. Exercise 1-2 from Skiena.
2. 1-5.
3. The version of the knapsack problem in exercise 1-5 asks for a subset that sums exactly to a given parameter T. Notice that sometimes there is no solution for a given value of T. You are given an algorithm A that solves this problem in time $T(n)$. Consider a variant of this problem: find a subset of S that sums to the highest possible value $\leq T$. Give an algorithm that uses A as a procedure to solve this variant in time $O(T(n) \cdot \log T)$. Briefly explain (a formal proof is not necessary) its correctness and running time.
4. Your classmate claims that all jelly beans are the same color. They give the following proof:

   Proof. (By induction.) Base case: when we have a single jelly bean it can only be one color.

   Inductive hypothesis: Assume that all sets of n jelly beans are the same color. Consider a set of $n+1$ jelly beans. Choose a subset A of size n from this set; these beans must be the same color by our inductive hypothesis.

   Now consider the bean you did not pick. Swap this bean with any bean from the set A. This forms a set B of n beans, and so must be the same color by our inductive hypothesis.

   Sets A and B share $n-1$ beans, hence each set must be the same color. In other words, all $n+1$ beans are the same color. □

   What is wrong with this proof?
5. 1-8
6. 1-12
7. 1-17
8. 1-20. Interviewers love to ask estimation questions like this. The important part here is your *process*, not the actual answer you get. Make some simplistic assumptions and ballpark it.
9. 2-2
10. 2-6
11. 2-8
12. 2-12
13. 2-18
14. 2-23
15. 2-36