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# Lab 10 Question 1A and 1B

1A. Recursive Trace of `gcd( 8, 6 )`:

```
gcd( 8, 6 );
--> gcd( 6, 8%6 );
--> gcd( 6, 2 );

gcd( 6, 2 );
--> gcd( 2, 6%2 );
--> gcd( 2, 0 );

gcd( 2, 0 );
return 2;
```

1B. Please reason the correctness of the above recursive function to calculate the value of `gcd( a, b ).`

We must start our inductive analysis by first proving that the base case is true.
Since a and b are both integers greater than or equal to 0, we take the smallest value for each parameter which is $a = 0$ and $b = 0$.

By plugging in $a$ and $b$ into the method gcd $(a, b)$, the method's return will be $\gcd(0,0) = 0$, which is correct as the result of $\gcd(a, 0) = a = \ (0)$. By this equivalency, the base case returns true, and if the base case returns true, we can assume that every $k + 1$ value is true for this function.

Now we move onto the inductive steps.
Let $a_n$ and $b_n$ be the $a$ and $b$ in gcd $(a, b)$ after N recursive calls, maintaining the restriction of $N > 0$.

Let $q$ and $r$ be the quoetient and the remainder of dividing $a_n$ into $b_n$, so that

$a_n = qb_n + r$ and $0 \leq r < b_n$

since $r = a_n \% b_n$.

Plug it into the equation, and work out the function, considering the gcd $(a, b)$'s steps within the method. For each step it takes in the method:

$$\gcd(a_n, b_n) = \gcd(b_n, a_n)$$

We can substitute in our inductive hypothesis: $a_n = qb_n + r$

$$\gcd(b_n, a_n) = \gcd(b_n, qb_n + r)$$

Finally, we can take our range that: $0 \leq r < b_n$

$$\gcd(b_n, qb_n + r) = \gcd\,(b_n, r)$$

When the next recursive call is made, which is the resulting values of the previous being $n - 1$,

$$\gcd\,(a_{n-1}, b_{n-1}) = \gcd\,(b_n, r)$$

By proof of induction, the method is true for all values of $N \geq 0$.