

Can On-the-Job Training Bring Back Wage Growth in the UK?

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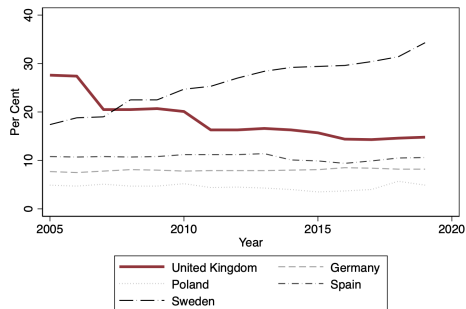
June 9, 2024

Motivation

Stagnation of UK Productivity & Wages

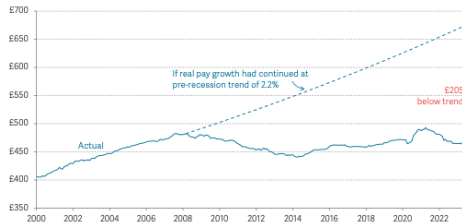
The UK has seen its **training participation and real wage growth stagnate**.

(a) Participation Rate in Education and Training.



Source: Eurostat Unit F3, Labour Market and Lifelong Learning.

(b) Real Average Weekly Earnings in the UK: Actual and Pre-Recession Trend.



1 Introduction

2 The Model

3 Data and Calibration

4 Training Incentive Policy

- Stylised Example
- Aggregate Results
- Extending the Policy to All Firms

5 Conclusion

Research Question

What are the **labour market consequences of subsidised training programmes?**

- 1 I expand the framework in [Guner and Ruggieri \(2021\)](#).
- 2 I use it as a quantitative laboratory to analyse how a training programme:
 - affects the wage distribution and life-cycle wage growth;
 - impacts the firm productivity and training cost distribution; and
 - changes the training and hiring decisions of firms.
- 3 The policy is the 2017 *Apprenticeship Levy* in the UK.

Introduction

Facts on the UK Training & The 2017 Apprenticeship Levy

The common features of the UK on-the-job training:

- 1 less on-the-job training for the past 20 years (De Lyon and Dhingra, 2020); and
- 2 participation in on-the-job training correlates with higher earnings (Britton et al., 2020).

To tackle these issues, the ***Apprenticeship Levy*** was introduced in 2017.

	LARGE FIRMS (>£3M PAYROLL OR >50 EMPLOYEES)	SMALL FIRMS
EXTRA TAX	Collect 0.5% of the Total Wage Bill	No Extra Tax
WHAT THE MONEY DOES	Tax Proceeds Can Be Used to Cover 90% of Apprenticeship Costs	No Subsidy
TIME CONSTRAINTS	Money Can Be Used for 2 Years	NA

Results

- 1 The *Apprenticeship Levy* **increases wage growth and average wage**.
- 2 It does so via three mechanisms:
 - *the training effect*: raising overall human capital,
 - *the search value effect*: rising the value of search for employed workers, and
 - *the distribution effect*: cutting the share of low-productivity and high-training-cost firms.
- 3 Extending the policy to all firms will likely weaken the effects.

Introduction

Innovation & Literature Contribution

My work adds to **three strands of economic literature**.

- 1 I add to the literature studying the **links between labour frictions and training**:
 - Flinn et al. (2017) and Fu (2011) combine on-the-job training with search frictions; and
 - Guner and Ruggieri (2021) analyse labour frictions and incentives to invest in training.
- 2 The macro and public literature study the **causes of the UK productivity puzzle**:
 - Turrell et al. (2021) and Patterson et al. (2016) focus on macro-labour mismatches; and
 - The IFS Deaton Review of Inequalities, e.g. Giupponi and Machin (2022).
- 3 The labour literature analyses **human capital and labour market outcomes**:
 - Becker (1965)'s argument on firm-specific human capital;
 - Acemoglu and Pischke (1998)'s seminal argument on training provision; and
 - Arellano-Bover (2024) studies labour impacts of firm heterogeneity.

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The Model

Modelling Labour Markets & Training

Modelling Objectives

I expand the framework in Guner and Ruggieri (2021) by:

- 1 adding on-the-job search; and
- 2 including the government training subsidy.

The Model

Key Elements of the Model

Modelling the UK's labour markets, I seek to **capture the relationships between:**

- 1 training incidence and wage growth;
- 2 the impact of training on the value functions of employment and a vacancy; and
- 3 the subsidy's influence on training provision.

I also expect a trade-off between wages and training akin to [Flinn et al. \(2017\)](#).

The Model

Why Is Training Subsidy Needed?

Why Should Firms Pay for Training?

The classical view is that workers should pay for their training (Becker, 1965).

However, firms have an **incentive to provide training**:

- 1 in the presence of **labour market frictions** (Fu, 2011); and
- 2 with information asymmetry and skill heterogeneity (Acemoglu and Pischke, 1998).

Why Is It Desirable to Subsidise Training?

- Firms **do not internalise** the aggregate human capital gains (Flinn et al., 2017).
- Firms **do not internalise the future matches' profits** (Lentz and Roys, 2024).
- Firms **under-provide training** in the presence of on-the-job search (Fu, 2011).

The Model

Environment: Heterogeneous Workers and Firms

► Production Function

There is a **unit measure of heterogeneous workers**. Each of them:

- 1 has exogenous **ex-ante skills**, $a \in \mathcal{A}$, with the dynamics governed by three processes:
 - depreciation while non-employed (at rate p^d),
 - appreciation via on-the-job **learning** (at rate p^e), and
 - appreciation via on-the-job **training** (at rate p^t);
- 2 could be employed or non-employed; and
- 3 faces a stochastic life-cycle in the labour market.

There is an **endogenously-determined measure of firms**:

- 1 with **productivity** z and **training costs** ξ drawn from ψ_z and ψ_ξ , respectively;
- 2 facing exogenous exit rate δ_f and both endo- and exogenous **worker separation**; and
- 3 with their growth being bounded by convex vacancy costs.

The model includes **standard DMP matching frictions**:

$$m(S, v) = Sv (S^\eta + v^\eta)^{-\frac{1}{\eta}}, \quad (1)$$

where the **total measure of searchers** is $S = U + \gamma E$.

- $\eta > 0$ governs the elasticity of new matches w.r.t. the pool of searchers.
- $\gamma > 0$ captures the relative efficiency of employed searchers.

Consequently, the **contact rates** are:

$$\phi_w^u = (1 + \theta^{-\eta})^{-\frac{1}{\eta}}; \quad (\text{Non-Employed})$$

$$\phi_w^e = \gamma (1 + \theta^{-\eta})^{-\frac{1}{\eta}} = \gamma \phi_w^u; \text{ and} \quad (\text{Employed})$$

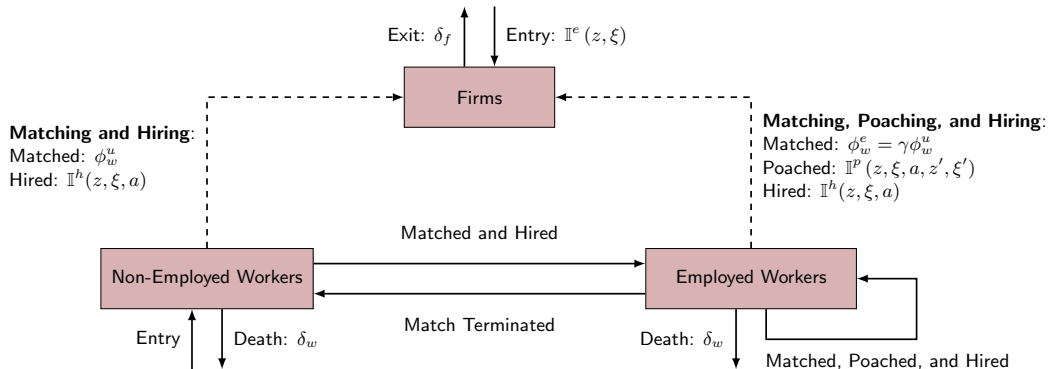
$$\phi_f = (1 + \theta^\eta)^{-\frac{1}{\eta}}, \quad (\text{Firms})$$

where I define the **labour market tightness** as $\theta \equiv \frac{v}{S}$. Upon matching, a worker-firm pair engage in *Nash* bargaining.

The Model

Frictional Labour Market: Structure

► Hiring Decision



The Model

Worker-Firm Dynamics

▸ Timing

▸ Workers (End)

▸ Workers (Beginning 1)

▸ Workers (Beginning 2)

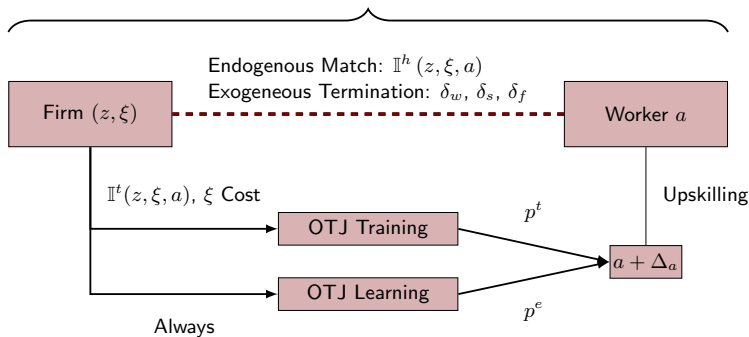
▸ Firm Values

▸ Nash Bargaining

▸ Hiring and Training

Joint Match Value: $M(z, \xi, a) = J^e(z, \xi, a) + V(z, \xi, a)$

Surplus Value: $S(z, \xi, a) = M(z, \xi, a) - J^u(a)$



The government **imposes tax** $\tau(z)$ on firm revenues:

$$\tau(z) = \tau \times \mathbb{I}\{z \geq \hat{z}\}, \quad (3)$$

where \hat{z} is calibrated such that the levy is imposed on firms with 50 or more employees.

- The tax proceeds are used to subsidise training programmes at the levy-paying firms.
- The **government covers a proportion of training costs**: $\lambda(z) = \lambda \times \mathbb{I}\{z \geq \hat{z}\}$.

The balanced budget condition determines the subsidy level:

$$\begin{aligned} \iiint_{(z, \xi, a) \in \mathcal{Z} \times \mathcal{E} \times \mathcal{A}} \tau(z) r(z, \xi, a) \psi_a(a) \psi_v(z, \xi) da dz d\xi = \\ \iiint_{(z, \xi, a) \in \mathcal{Z} \times \mathcal{E} \times \mathcal{A}} \lambda(z) \xi \mathbb{I}^t(z, \xi, a) \psi_a(a) \psi_v(z, \xi) da dz d\xi, \end{aligned} \quad (4)$$

where $\mathbb{I}^t(z, \xi, a)$ is the training decision.

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Data & Calibration

Section Goals

Key Message

- 1 I calibrate the pre-subsidy model to the UK data for 2010 - 2016.
- 2 I discuss the crucial moments for analysing the training programme.
- 3 I show that my model fits the data well.

To calibrate the model, I will use the **UK data between 2010 and 2016** from:

- the income statistics from the Five-Quarter Longitudinal Labour Force Survey (LFS);
- Claimant Count and Vacancies from the Office of National Statistics (ONS); and
- the Labour Force Survey flows estimates from the ONS.

Data and Calibration

Training Moments

Important Note

I ensure the training-related moments match similar results in the labour literature.

I review the literature on training to **assess the magnitude of the key training moments**:

- for the UK (Dearden et al., 2006);
- for other developed countries (Bartel, 1995; Gregory, 2020); and
- based on meta-studies (Card et al., 2018).

- 1 The model's overall fit is satisfactory.
- 2 The firm size distribution is precisely matched.
- 3 The training incidence is matched well, but slightly overestimated for the largest firms.

I also check the **sensitivity of my results** to changes in the key parameters by:

- 1 analysing the Jacobian of the distance function; and
- 2 conducting Jørgensen (2023)'s sensitivity analysis of the estimates of p^t , $\underline{\xi}$, $\bar{\xi}$, and λ_2 .

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Training Incentive Policy

Apprenticeship Levy: Results

► Matching at Medium Cost

► Matching at High Cost

► Matching at Low Cost

Key Results

The *Apprenticeship Levy* leads to:

- 1 **more workers being offered training** from their employers;
- 2 no changes in who gets hired;
- 3 fewer high-cost and low-productivity firms;
- 4 a minor **right-ward shift in the wage distribution**; and
- 5 a **higher average wage inequality**.

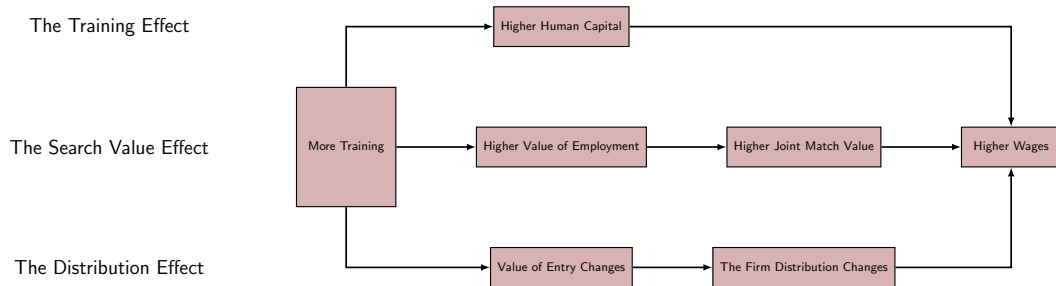
Reminder: The *Apprenticeship Levy*:

- imposes a 0.5 per cent payroll tax on firms with more than 50 employees; and
- uses the levy's proceeds to subsidise training at those firms.

Training Incentive Policy

Mechanism

► Indifference Curves



Training Incentive Policy

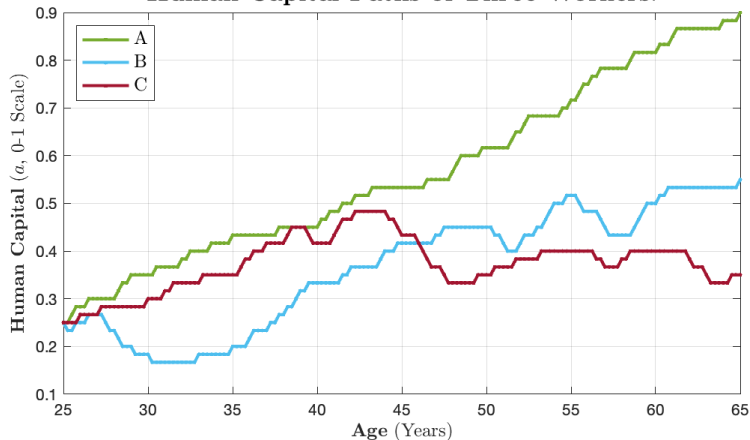
Stylised Example: Human Capital Paths

► Firm Productivity Paths

► Training Cost Paths

Consider workers with **identical ex-ante entry human capital**.

Human Capital Paths of Three Workers.



Training Incentive Policy

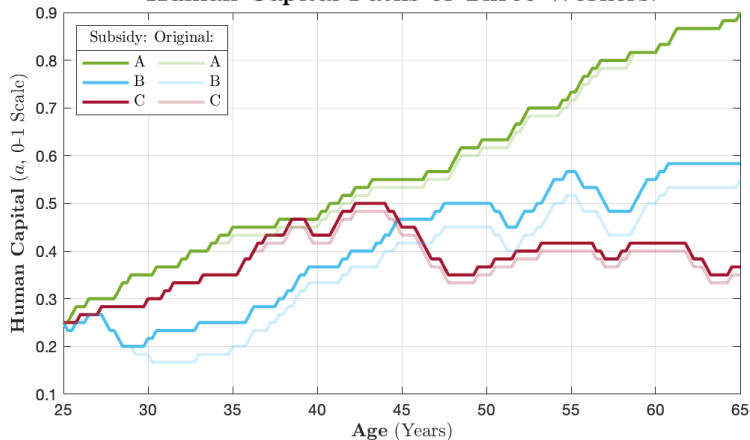
Stylised Example: Human Capital Paths

► Firm Productivity Paths

► Training Cost Paths

The policy **benefits those in the “middle”** of the human capital distribution most.

Human Capital Paths of Three Workers.



Training Incentive Policy

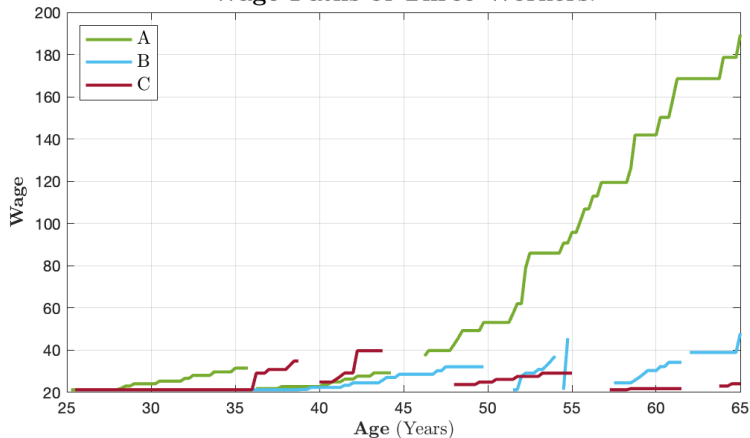
Stylised Example: Wages Paths

► Firm Productivity Paths

► Training Cost Paths

Before the policy is introduced, their **wages differ a lot**.

Wage Paths of Three Workers.



Training Incentive Policy

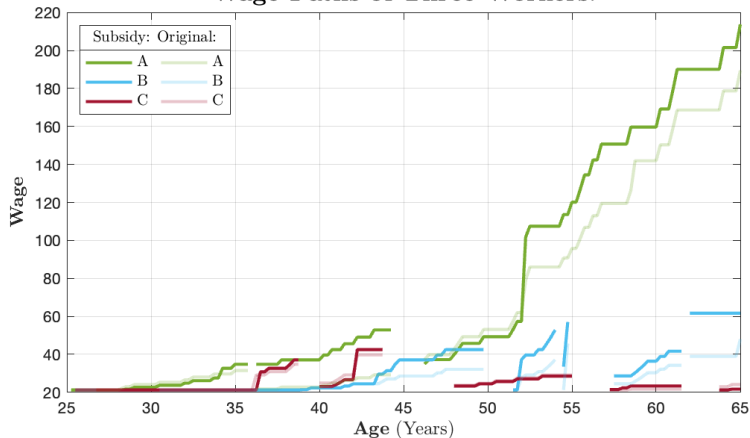
Stylised Example: Wages Paths

► Firm Productivity Paths

► Training Cost Paths

Those in the **middle and at the top** see the biggest gains.

Wage Paths of Three Workers.



Training Incentive Policy

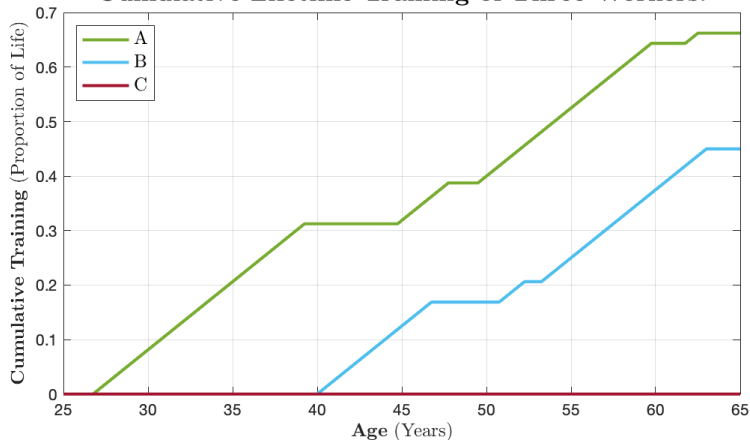
Stylised Example: Cumulative Training Paths

► Firm Productivity Paths

► Training Cost Paths

There is strong **heterogeneity** in the training provision.

Cumulative Lifetime Training of Three Workers.



Training Incentive Policy

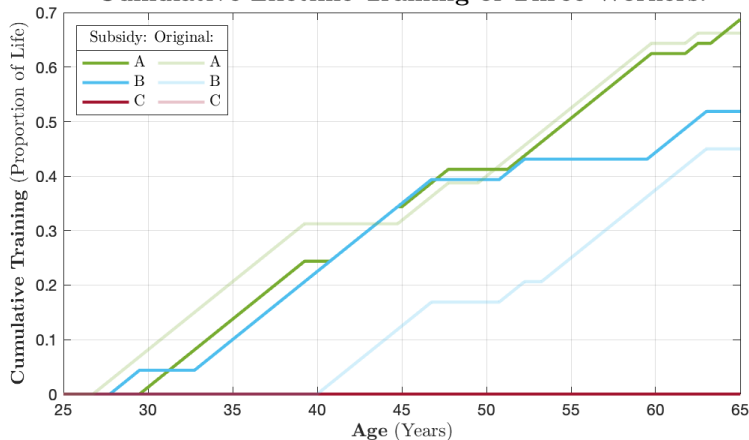
Stylised Example: Cumulative Training Paths

► Firm Productivity Paths

► Training Cost Paths

The “**middle**” worker sees the largest **lifetime training increase**.

Cumulative Lifetime Training of Three Workers.



Training Incentive Policy

Stylised Example: What Do We Learn?

► Indifference Curves

Before the policy is introduced:

- 1 the ex-ante identical workers can have different human capital and wage paths; and
- 2 the training provision depends on:
 - firm productivity, especially for the low-skilled workers and
 - the training costs for all workers.

The Policy's Impact

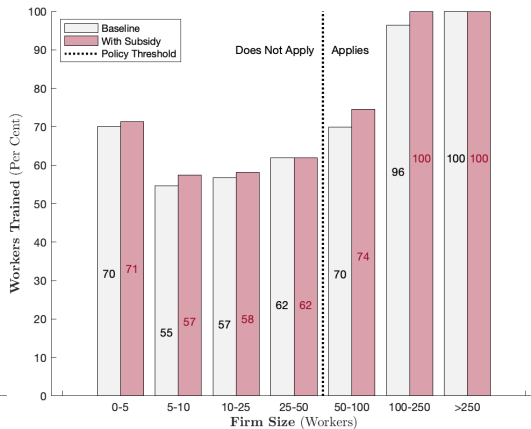
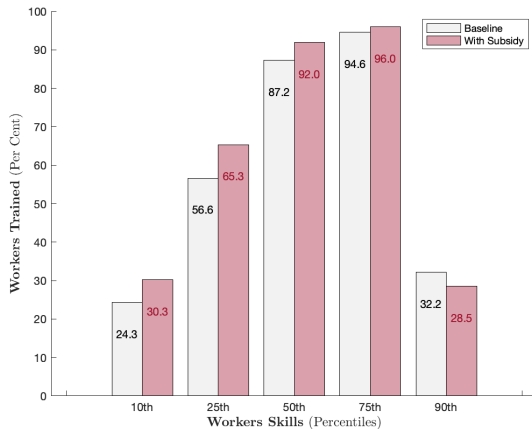
- 1 Those in the “middle” see their training increase most.
- 2 The policy has a **positive impact on all workers' labour outcomes** through:
 - a higher training provision for those in the middle of the skill distribution;
 - an increased employment value through a higher option value of job search.

Training Incentive Policy

Aggregate Results: Training Offered

► Training Contour

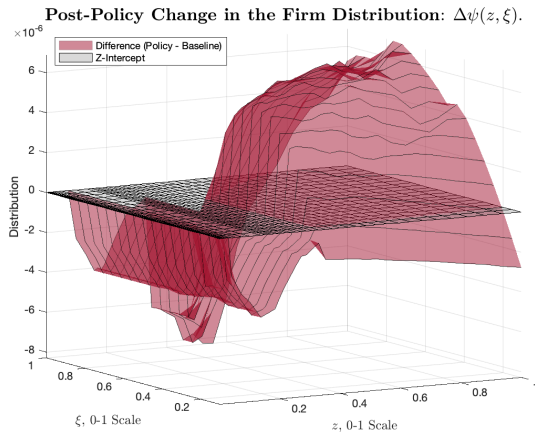
With the subsidy, **firms offer more training** (even those not subsidised).



Training Incentive Policy

Aggregate Results: Firm Distribution Affected

Low-productivity and high-cost firms are pushed out of the market!



Training Incentive Policy

Aggregate Results: Workers

The *Apprenticeship Levy*:

- increases the average entry wage; and
- modestly **boosts the average pay and wage growth**, but lowers employment.

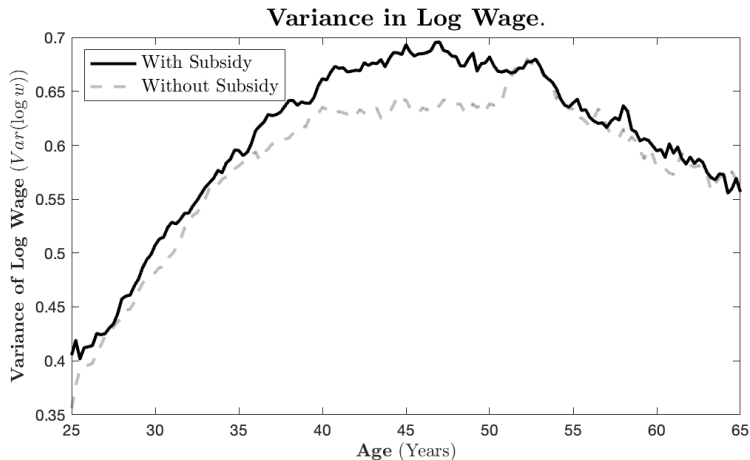
Worker Variable	Baseline	With Subsidy
Average Entry Wage (Proportion of Baseline)	1.000	1.120
Average Wage (Proportion of Baseline)	1.000	1.024
Wage Growth (Entry - 5 Yrs.)	0.163	0.167
Wage Growth (Entry - 10 Yrs.)	0.397	0.403
Wage Growth (Entry - 25 Yrs.)	0.929	0.915
Employment Rate	0.644	0.640

Training Incentive Policy

Aggregate Results: Wage Inequality

► Lower Skill Inequality

Earnings inequality increases.



Training Incentive Policy

Aggregate Results: Further Inequality Statistics

Two mechanisms impact the wage distribution:

- workers in the “middle” get trained more;
- workers at the top benefit from the larger option value of search.

The latter effect dominates, **increasing the P90-50 ratio!**

Inequality Statistic	Baseline	With Subsidy
Gini	0.405	0.407
P90-50	2.655	2.694
P50-10	4.688	4.675
Mean-Median	1.213	1.227
Var. of Log Wage	0.806	0.812

Training Incentive Policy

Extending the Policy to All Firms: Summary of Additional Results

I investigate **one policy adjustment**: making the tax and subsidy universal for all firms.

Further Results: Expanding the Policy to All Firms

- Less training is provided to the least and most skilled workers.
- Firms provide **more training**.
- The average wage remains constant.
- **Wage growth and employment are lower.**

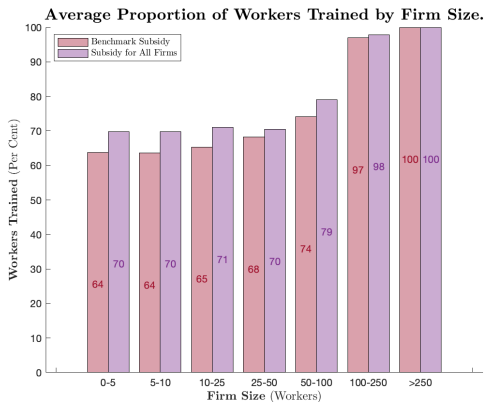
⇒ This likely indicates the counteracting influences of all three mechanisms.

Note: The policy remains budget-neutral.

Training Incentive Policy

Extending the Policy to All Firms: Who Receives More Training?

- The **least** and **most skilled workers now receive less training.**
- Firms provide marginally more training.



Training Incentive Policy

Extending the Policy to All Firms: Workers

Extending the policy to all firms **does not improve outcomes:**

- wage growth drops below the baseline levels;
- average wage remains the same while the average worker skill is much higher.

This likely indicates the influence of the *search value* and *distribution effects*.

Worker Variable	Baseline	Benchmark Subsidy	Subsidy for All Firms
Average Entry Wage (Proportion of Baseline)	1.000	1.120	1.110
Average Wage (Proportion of Baseline)	1.000	1.024	1.024
Wage Growth (Entry - 5 Yrs.)	0.163	0.167	0.140
Wage Growth (Entry - 10 Yrs.)	0.397	0.403	0.362
Wage Growth (Entry - 25 Yrs.)	0.929	0.915	0.903
Employment Rate	0.644	0.640	0.639

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Conclusion

Key Messages and Further Steps

Key Conclusions

- 1 The **policy positively impacts wage growth in the UK.**
- 2 It does so by:
 - increasing human capital;
 - boosting the value of employment; and
 - changing the firm distribution.
- 3 Various policy adjustments are likely to change the outcome.

Further steps include:

- writing the paper;
- quantifying alternative policy transmission mechanisms; and
- improving the model's calibration.

Bibliography I

- Acemoglu, D. and Pischke, J.-S. (1998). Why Do Firms Train? Theory and Evidence. *The Quarterly Journal of Economics*, 113(1):79–119.
- Arellano-Bover, J. (2024). Career Consequences of Firm Heterogeneity for Young Workers: First Job and Firm Size. *Journal of Labor Economics*.
- Bartel, A. P. (1995). Training, Wage Growth, and Job Performance: Evidence from a Company Database. *Journal of Labor Economics*, 13(3):401–425.
- Becker, G. S. (1965). *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*. Midway reprint, New Brunswick (Me.), 2nd ed edition.
- Britton, J., Espinoza, H., McNally, S., Speckesser, S., Tahir, I., and Vignoles, A. (2020). Post-18 Education - Who Is Taking the Different Routes and How Much Do They Earn? Technical Report 013, Centre for Vocational Education Research.
- Card, D., Kluve, J., and Weber, A. (2018). What Works? A Meta Analysis of Recent Active Labor Market Program Evaluations. *Journal of the European Economic Association*, 16(3):894–931.
- De Lyon, J. and Dhingra, S. (2020). Firm investments in Skills and Capital in the UK Services Sector. OECD Economics Department Working Papers 1632. Series: OECD Economics Department Working Papers Volume: 1632.

Bibliography II

- Dearden, L., Reed, H., and Van Reenen, J. (2006). The Impact of Training on Productivity and Wages: Evidence from British Panel Data*. *Oxford Bulletin of Economics and Statistics*, 68(4):397–421.
- Flinn, C., Gemici, A., and Laufer, S. (2017). Search, Matching and Training. *Review of Economic Dynamics*, 25:260–297.
- Fu, C. (2011). Training, Search and Wage Dispersion. *Review of Economic Dynamics*, 14(4):650–666.
- Giupponi, G. and Machin, S. (2022). Labour Market Inequality. Technical report, Institute for Fiscal Studies.
- Gregory, V. (2020). Firms as Learning Environments: Implications for Earnings Dynamics and Job Search. Technical report.
- Guner, N. and Ruggieri, A. (2021). Misallocation and Inequality. Technical report, University of Nottingham, Centre for Finance, Credit and Macroeconomics (CFCM). Issue: 2021/01.
- Jørgensen, T. H. (2023). Sensitivity to Calibrated Parameters. *The Review of Economics and Statistics*, 105(2):474–481.

Bibliography III

- Lentz, R. and Roys, N. (2024). Training and Search On the Job. *Review of Economic Dynamics*, 53:123–146.
- Patterson, C., Şahin, A., Topa, G., and Violante, G. L. (2016). Working Hard in the Wrong Place: A Mismatch-Based Explanation to the UK Productivity Puzzle. *European Economic Review*, 84:42–56.
- Turrell, A., Speigner, B., Copple, D., Djumalieva, J., and Thurgood, J. (2021). Is the UK's Productivity Puzzle Mostly Driven by Occupational Mismatch? An Analysis Using Big Data on Job Vacancies. *Labour Economics*, 71:102013.

The Model Appendix

Environment: Production

Firm (ξ, z) has a workforce of ℓ , with worker i contributing:

$$g(z, i) = \kappa z a(i), \quad (5)$$

where κ is the aggregate productivity.

The **production function** is linear:

$$y(z, \xi, \ell, \psi) = \int_0^\ell g(z, i) \psi(i \mid z, \xi, \ell) di \implies \quad (6a)$$

$$y(z, \xi, \ell, \psi) = \kappa z \bar{a}(z, \xi, \ell, \psi) \ell, \quad (6b)$$

where \bar{a} is the average human capital at the firm.

The Model Appendix

Value Functions: Simplifying Notation

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Without loss of generality, I **split each period into**:

- 1 the beginning of the period (e.g, the endogenous decisions such as training); and
- 2 the end of the period when all the exogenous processes happen (e.g., dying).

The Model Appendix

Value Functions: The End-of-the-Period Workers

► Back to Dynamics Slide

The **values of unemployment and employment at the end of the period** are:

$$J^{u,h}(a) = b + \frac{1 - \delta_w}{1 + r} J^u(a); \text{ and} \quad (7a)$$

$$J^{e,h}(z, \xi, a) = w(z, \xi, a) + \frac{1 - \delta_w}{1 + r} \left\langle \overbrace{[\delta_f + (1 - \delta_f) \delta_s] J^u(a)}^{\text{Exogenous Separation}} + \{1 - [\delta_f + (1 - \delta_f) \delta_s]\} \right. \\ \times \left. \left\{ \underbrace{p^h(z, \xi, a) J^e(z, \xi, a + \Delta_a)}_{\text{Upskilling}} + \underbrace{[1 - p^h(z, \xi, a)] J^e(z, \xi, a)}_{\text{Same Skills}} \right\} \right\rangle, \quad (7b)$$

where J^u and J^e are unemployment and employment values at the beginning of the period and:

$$p^h(z, \xi, a) \equiv p^e + \mathbb{I}^t(z, \xi, a) p^t. \quad (8)$$

The Model Appendix

Value Functions: The Beginning-of-the-Period Workers (1)

► Back to Dynamics Slide

The **values of unemployment and employment at the beginning of the period**:

$$\begin{aligned} J^u(a) = & (1 - \phi_w) \overbrace{\left[\underbrace{p^d J^{u,h}(a - \Delta_a)}_{\text{Skill Deterioration}} + \underbrace{(1 - p^d) J^{u,h}(a)}_{\text{Same Skills}} \right]}^{\text{Remaining Unemployed}} + \\ & + \phi_w \underbrace{\int \int_{(z,\xi) \in \mathcal{Z} \times \mathcal{E}} \left\{ \underbrace{\mathbb{I}^h(z, \xi, a) J^{e,h}(z, \xi, a)}_{\text{Match Formed}} + \underbrace{[1 - \mathbb{I}^h(z, \xi, a)] J^{u,h}(a)}_{\text{Match Not Formed}} \right\}}_{\text{Forming a Match}} \psi_v(z, \xi) d\xi dz, \end{aligned} \tag{9}$$

where $\mathbb{I}^h(\cdot)$ is the indicator variable of the match being formed. $\psi_v(\cdot, \cdot)$ is the joint PDF.

The Model Appendix

Value Functions: The Beginning-of-the-Period Workers (2)

► Back to Dynamics Slide

The **value of being employed at the beginning of the period** is given by:

$$\begin{aligned} J^e(z, \xi, a) = & \phi_w^e \mathbb{I}^h(z, \xi, a) \iint_{(z', \xi') \in \mathcal{Z} \times \mathcal{E}} \left\langle \overbrace{\mathbb{I}^h(z', \xi', a) \max \{ J^{e,h}(z', \xi', a), J^{e,h}(z, \xi, a) \}}^{\text{Prospective Firm Wants to Match}} + \right. \\ & \left. \overbrace{[1 - \mathbb{I}^h(z', \xi', a)] J^{e,h}(z, \xi, a)}^{\text{Prospective Firm Rejects the Match}} \right\rangle \psi_v(\xi', z') d\xi' dz' + \overbrace{(1 - \phi_w^e) \mathbb{I}^h(z, \xi, a) J^{e,h}(z, \xi, a)}^{\text{No New Offer, Match Continued}} + \\ & \overbrace{[1 - \mathbb{I}^h(z, \xi, a)] J^{u,h}(z, \xi, a)}^{\text{Match Not Preserved}}. \end{aligned} \quad (10)$$

The Model Appendix

Value Functions: Firms

[▶ Back to Dynamics Slide](#)

The **match values for the firm at the beginning and end of the period** are:

$$V(z, \xi, a) = \mathbb{I}^h(z, \xi, a) V^h(z, \xi, a); \text{ and} \quad (11a)$$

$$V^h(z, \xi, a) = \overbrace{r(z, a) [1 - \tau(z)] - w(z, \xi, a)}^{\text{Net Revenue}} + \frac{1 - \delta}{1 + r} \left\{ \overbrace{-\mathbb{I}^t(z, \xi, a) \xi [1 - \lambda(z)]}^{\text{Training Cost}} \right. \\ \left. + \underbrace{p^h(z, \xi, a) V(z, \xi, a + \Delta_a)}_{\text{Upskilling}} + \underbrace{[1 - p^h(z, \xi, a)] V(z, \xi, a)}_{\text{Same Skills}} \right\}. \quad (11b)$$

The Model Appendix

Decisions: Nash Bargaining

[▶ Back to Dynamics Slide](#)

Bargaining between employees and employers takes place every period. **Wages** are given by:

$$w(z, \xi, a) = \arg \max_w \left\{ \underbrace{[J^{e,h}(z, \xi, a; w) - J^{u,h}(a)]^\beta}_{\text{Worker Surplus}} \underbrace{[V^h(z, \xi, a; w)]^{1-\beta}}_{\text{Firm Surplus}} \right\}, \quad (12)$$

where

- $\beta \in (0, 1)$ is the workers' bargaining power;
- $J^{e,h} \equiv$ the end-of-period value of working;
- $J^{u,h} \equiv$ the end-of-period value of unemployment; and
- $V^h \equiv$ the end-of-period value of the match.

The Model Appendix

Decisions: Hiring & Training

► Back to Structure Slide

► Back to Dynamics Slide

A match between worker $a \in \mathcal{A}$ and firm $(z, \xi) \in \mathcal{Z} \times \mathcal{E}$ is formed whenever the **match surplus is positive**:

$$\mathbb{I}^h(z, \xi, a) = \begin{cases} 1 & \text{if } S^h(z, \xi, a) > 0 \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

where $S^h(z, \xi, a)$ is the difference between the value of an active match and the value of unemployment.

Training provision is governed by:

$$\mathbb{I}^t(z, \xi, a) = \begin{cases} 1 & p^t \left[\overbrace{M(z, \xi, a + \Delta_a)}^{\text{Value of Active Match}} - M(z, \xi, a) \right] > \xi [1 - \lambda(z)] \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

The Model Appendix

Subsidy Algorithm: Procedure

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► [Maths Tricks](#)

I find the **optimal subsidy level** for revenue tax $\tau(z)$:

- 1 guess $\lambda_i(z)$;
- 2 calculate the government revenue and expenditure; and
- 3 update $\lambda_i(z)$:
 - if revenue $>$ expenditure, then $\lambda_{i+1}(z) > \lambda_i(z)$;
 - if revenue $<$ expenditure, then $\lambda_{i+1}(z) < \lambda_i(z)$;
 - if revenue \simeq expenditure, then stop.

The Model Appendix

Subsidy Algorithm: Mathematical Simplifications

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[▶ Return to Algorithm](#)

To avoid creating many extra variables, I **leverage the steady-state relationship between $w(z, \xi, a)$ and $r(z, \xi, a)$** :¹

$$w(z, \xi, a) = \beta r(z, \xi, a) + (1 - \beta)b \implies \quad (15a)$$

$$\text{Average Tax Revenue Per Firm-Worker Pair} = \frac{\tau}{\beta(1 - \tau)} [\bar{w} - (1 - \beta)b] \implies \quad (15b)$$

$$\text{Total Tax Revenue} = N^f \times \bar{l} \times \text{Average Tax Revenue Per Firm-Worker Pair}, \quad (15c)$$

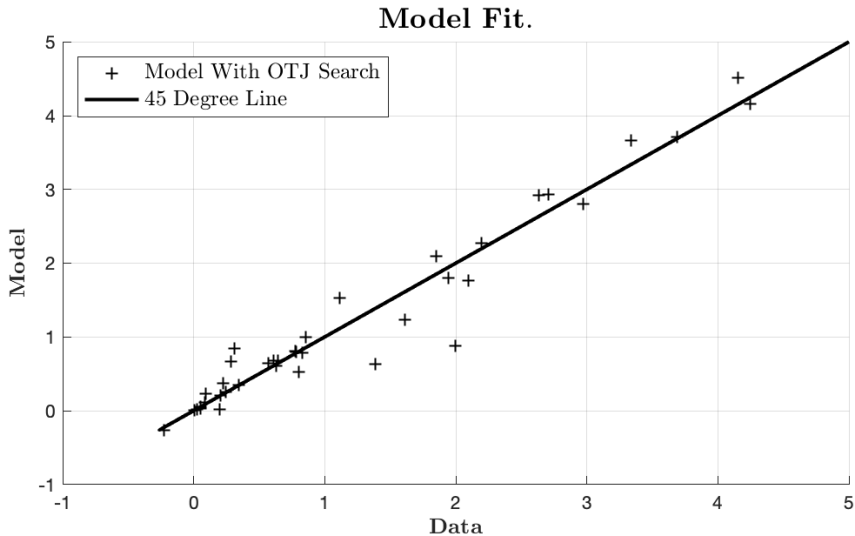
where N^f and \bar{l} are the average number of firms in the economy and the average firm size, respectively.

¹ $r(z, \xi, a)$ is the **after-tax** revenue!

Calibration and Data Appendix

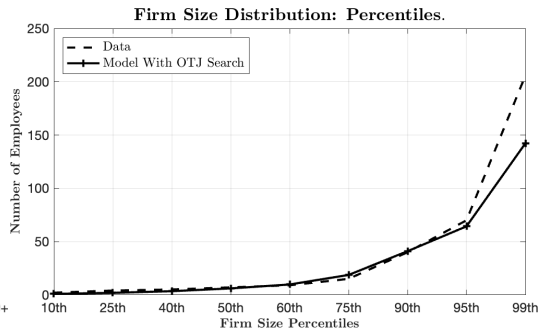
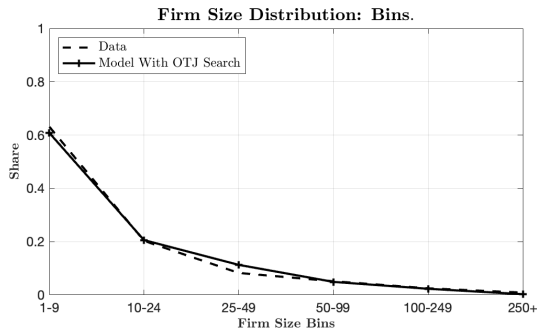
Overall Model Fit

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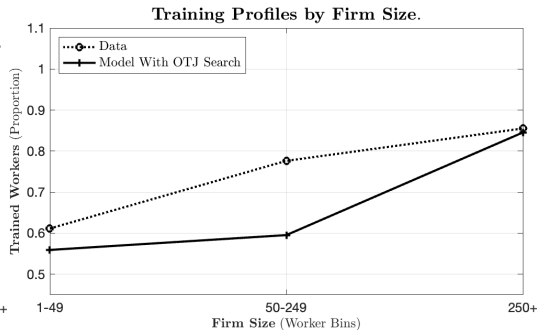
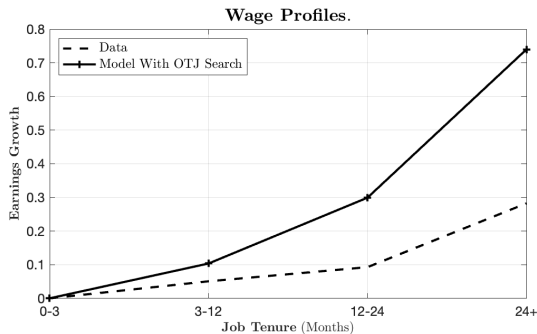
Calibration and Data Appendix

Firm Distribution Fit

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Calibration and Data Appendix

Worker Earnings and Training Fit

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Calibration and Data Appendix

All Firm Moments and Fit

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Moment	Data	Model
$\mathbb{E}(\log l)$, Annual	2.10	1.77
St. Dev($\log l$), Annual	1.11	1.53
$\mathbb{E}l$, Annual	19.55	16.53
#Firms, 1-9 Emp	0.63	0.61
#Firms, 10-24 Emp	0.20	0.21
#Firms, 25-49 Emp	0.08	0.11
#Firms, 50-99 Emp	0.05	0.05
#Firms, 100-249 Emp	0.02	0.02
#Firms, 250+ Emp	0.01	0.00
Firm Size Pct, 10th	2.00	0.88
Firm Size Pct, 25th	4.00	1.88
Firm Size Pct, 50th	7.00	6.03
Firm Size Pct, 75th	15.00	18.67
Firm Size Pct, 90th	40.00	40.98
Firm Size Pct, 95th	70.00	64.31
Firm Size Pct, 99th	206.00	142.31
Firm Size Pct, 40th	5.00	3.43
Firm Size Pct, 60th	9.00	9.73
Training Firms, Share	0.65	0.68
Training Firms, Share 1-49	0.61	0.67
Training Firms, Share 50-249	0.78	0.81
Training Firms, Share 250+	0.86	1.00
Trained Workers within Firms, Share	0.44	0.56

Calibration and Data Appendix

All Worker Moments and Fit

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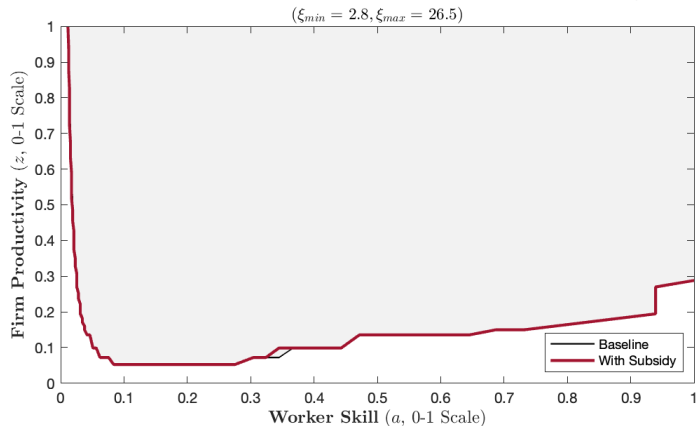
Moment	Data	Model
Non-Employment	0.23	0.37
Avg. Tenure, Quarters	8.33	6.65
Benefits-Wage Ratio	0.35	0.35
Avg. Log Wage at Entry	-0.24	-0.56
Avg. Log Wage after 20 Years	0.07	0.12
Avg. Re-Emp Log Wage	-0.23	-0.26
Diff. Return from Tenure: 4-12 m. - 0-3m	0.05	0.03
Diff. Return from Tenure: 12-24 m. - 0-3m	0.09	0.22
Diff. Return from Tenure: 24+ m. - 0-3m	0.28	0.67
Std. Log Wage at Entry	0.57	0.65
Std. Log Wage after 25y.	0.78	0.79
Std. Log Ee-Emp Wage	0.83	0.79
Training Premium	0.20	0.25
Job-to-Job Transitions, Rate	0.02	0.02

Training Incentive Policy Appendix

Accepted Matches: Medium Training Costs

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Matching Policy Function at Medium Training Cost ($\xi = 11.5$).

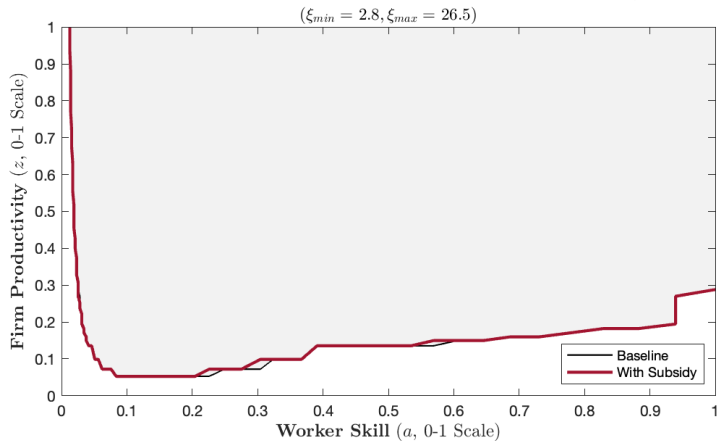


Training Incentive Policy Appendix

Accepted Matches: High Training Costs

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Matching Policy Function at High Training Cost ($\xi = 24.0$).

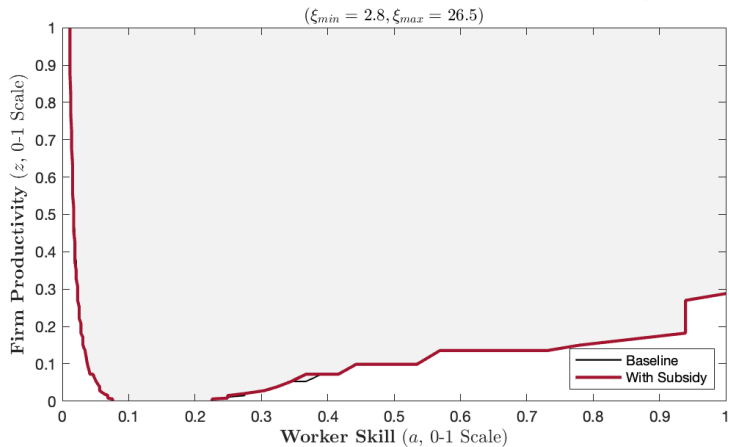


Training Incentive Policy Appendix

Accepted Matches: Low Training Costs

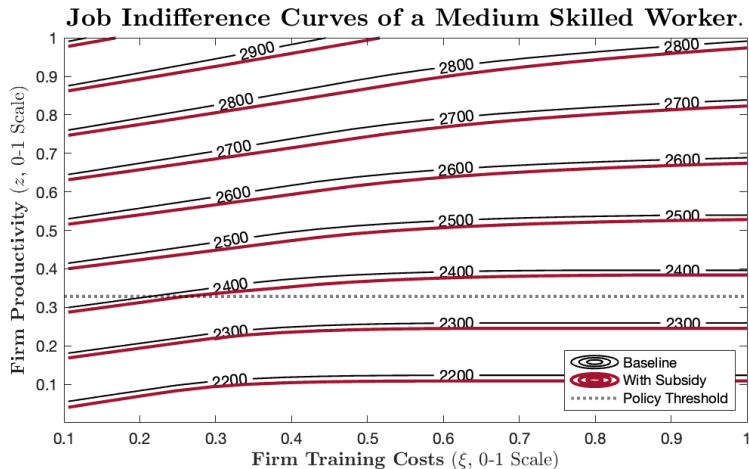
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Matching Policy Function at Low Training Cost ($\xi = 4.0$).



Training Incentive Policy Appendix

Job Value Indifference Curves

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Training Incentive Policy Appendix

Stylised Example: Firm Productivity

▸ Paths: HC

▸ Paths: Wages

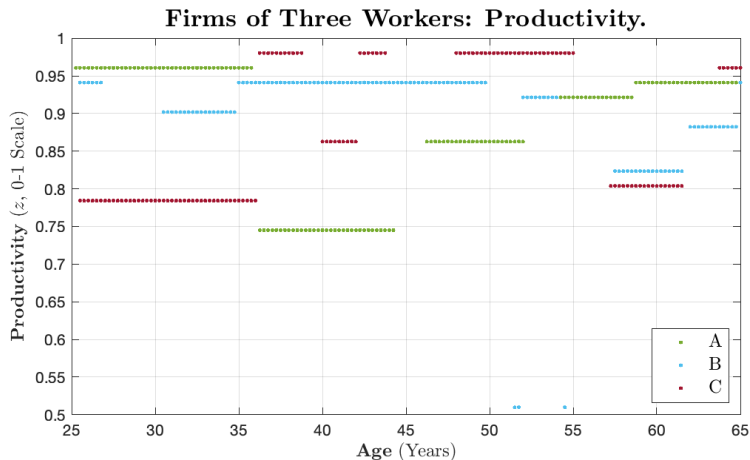
▸ Paths: Training

▸ Paths: HC Policy

▸ Paths: Wages Policy

▸ Paths: Training Policy

High-productivity firms employ the chosen workers.



Training Incentive Policy Appendix

Stylised Example: Training Costs

▸ Paths: HC

▸ Paths: Wages

▸ Paths: Training

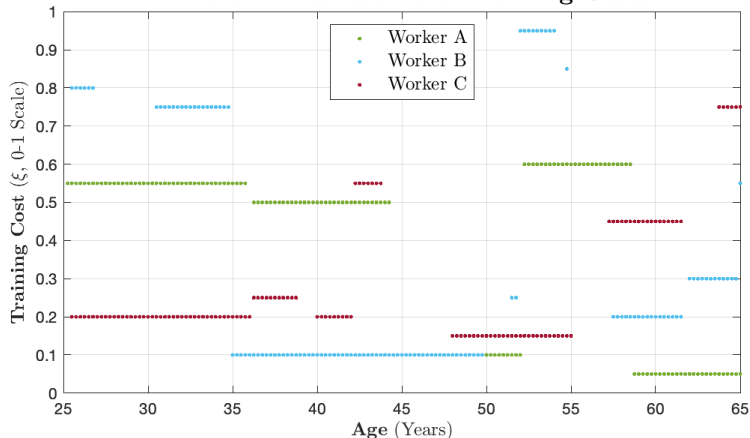
▸ Paths: HC Policy

▸ Paths: Wages Policy

▸ Paths: Training Policy

High-productivity firms employ the chosen workers.

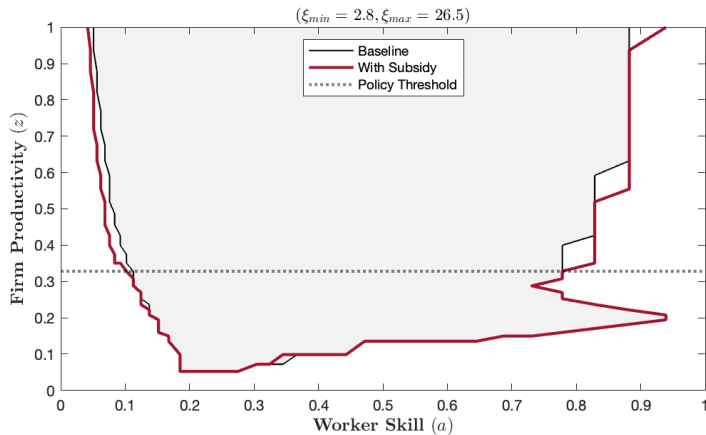
Firms of Three Workers: Training Costs.



Training Incentive Policy Appendix

Training Incidence

Training Policy Function at Medium Training Cost ($\xi = 11.5$)



Training Incentive Policy Appendix

Human Capital Inequality

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The **inequality in human capital decreases!**

