Economics of Banking - Course Notes*

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Contents

*The notes are based on the *Economics of Banking* (taught by Javier Suarez).

1 The Industrial Organisation View of Banks

1.1 Introduction

Assets	Liabilities
R	D
L	I
	K

Table 1.1: A Stylised Balance Sheet of a Bank. (Note: *The convention is that you list the asset side in the descending order of liquidity and the liability side in the descending order of seniority.*)

For many years, the core economic theory **ignored financial intermediation and banking**. If anything, banking activities were tangentially introduced as ad hoc institutions in specific strands of literature:

- in **macroeconomics**, when discussing on money supply and monetary policy transmission; and
- in **finance**, in some inventory and portfolio-selection models adapted so as to capture the regulatory and institutional constraints faced by banks when solving their liquidity and portfolio management problems.

Banks were incorporated to mainstream microeconomics and industrial organisation (IO) in the late 1960s through the so-called **Klein-Monti approach**.

1.1.1 Klein-Monti Approach

Definition 1.1 (Banks in the Klein-Monti Approach)**.** In the Klein-Monti approach, banks are **multi-product firms that are the exclusive suppliers of loans and deposits**. They appear in their balance sheets together with other assets and liabilities:

- cash, reserves, interbank deposits, other debt assets, equities, real assets, etc.; and
- interbank borrowing, other debt liabilities, equity capital, etc.

Banks operate under some specific structures of intermediation costs and legal constraints.

These models focus on: (1) the **determination of equilibrium prices** (interest rates, fees) and **quantities** under a given market structure; and (2) the **determination of the equilibrium market structure** under different technological and regulatory environments (e.g., entry barriers (natural or legal), scale, scope or network economies, switching costs, interest rate ceilings).

In principle, these models **abstract from providing a rationale for financial intermediation, the intertemporal dimension of banking activities, and risk** (credit risk, liquidity risk, etc.).

The basic **model assumptions are** as follows.

1. Deposits and loans are "goods" that banks supply and some other agents demand.
2. Both goods are homogeneous (except, perhaps, by location or some other aspect of product differentiation).

3. Banks are profit maximisers.

4. Banks are risk-free.

The basic models are **still useful** as possible **building blocks** in a fuller model and in **providing intuition** on questions orthogonal to what is left aside. Some recent models in this tradition incorporate some of the issues left aside such as depositor and borrower heterogeneity, (endogenous) credit and liquidity risk, — details regarding deposit and loan contracts, banks' monitoring and risk management roles, risk taking incentives, bank capital, and prudential issues.

1.2 The Basic Perfect Competition Model and Its Extensions

1.2.1 Main Model Ingredients

This is a **static model, without uncertainty**, with **many profit maximising banks**. You can think of a continuum of banks. Banks take **interest rates of their products as given**:

- 🏛 **loans**, L, L yield interest rate r_l, with the market demand $L(r_l)$ such that: $L' < 0$; and
- 🏛 **deposits**, D, L yield interest rate r_d, with the market demand $D(r_d)$ such that: $D' > 0$.

There exists the **interbank market where banks can borrow and lend**, without limits, at an **(exogenous) interest rate $r \geq 0$**. $I \equiv$ **the net interbank borrowing position is.**[1.1]

There is an **unremunerated reserve requirement** obliges banks to hold a minimal **fraction ϕ** of deposits as central bank reserves R.

$$R \geq \phi D \tag{1.1}$$

We can relax the assumption of unremuneration (done as an exercise later). We assume there are no intermediation costs and that there is no equity capital.

1.2.2 Solving a Bank's Problem

The **balance sheet** imposes:

$$\underbrace{R + L = D + I}_{\text{Constraint!}}. \tag{1.2}$$

For any remuneration lower than r, it is **strictly optimal for the bank not to keep excess reserves**, so equation (1.1) implies $R = \phi D$. Hence:

$$\phi D + L = D + I, \tag{1.3}$$

can be re-arranged to:

$$I = L - (1 - \phi)D. \tag{1.4}$$

[1.1] This is **closely related to the risk-free rate**. Historically, banks lend to one another at rates close to the risk-free counterpart. Note that this is no longer the case during financial crises. A good indicator of an upcoming financial crisis is the spread between the interbank and risk-free rates.

Using that, we can establish the **profit expression**.

$$\Pi \equiv (1+r_l)L + R - (1+r_d)D - (1+r)I \implies \tag{1.5a}$$

$$\Pi = r_l L - r_d D - rI \implies \tag{1.5b}$$

$$\Pi = (r_l - r)L + [(1-\phi)r - r_d]\, D \tag{1.5c}$$

The maximisation programme is as follows.

$$\max_{L,D} \{(r_l - r)L + [(1-\phi)r - r_d]\, D\} \tag{1.6}$$

The profits are originated from the **intermediation margin,** $r_l - r$. For the deposit side of the activity, the intermediation margin is $[(1-\phi)r - r_d]$. Analysing the programme, we can notice:

$$L = \begin{cases} \infty & \text{if} \quad r_l > r \\ [0,\infty) & \text{if} \quad r_l = r \\ 0 & \text{if} \quad r_l < r \end{cases}, \text{ and} \tag{1.7a}$$

$$D = \begin{cases} 0 & \text{if} \quad r_d > r(1-\phi) \\ [0,\infty) & \text{if} \quad r_d = r(1-\phi) \\ \infty & \text{if} \quad r_d < r(1-\phi) \end{cases}. \tag{1.7b}$$

The optimality conditions imply:[1.2]

$$r_l^\star = r, \text{ and} \tag{1.8}$$

$$r_d^\star = (1-\phi)r \tag{1.9}$$

From conditions (1.8) and (1.9) we can clearly see that:

$$\frac{dr_l^\star}{dr} = 1; \text{ and} \tag{1.10}$$

$$\frac{dr_d^\star}{dr} = 1 - \phi \simeq 1. \tag{1.11}$$

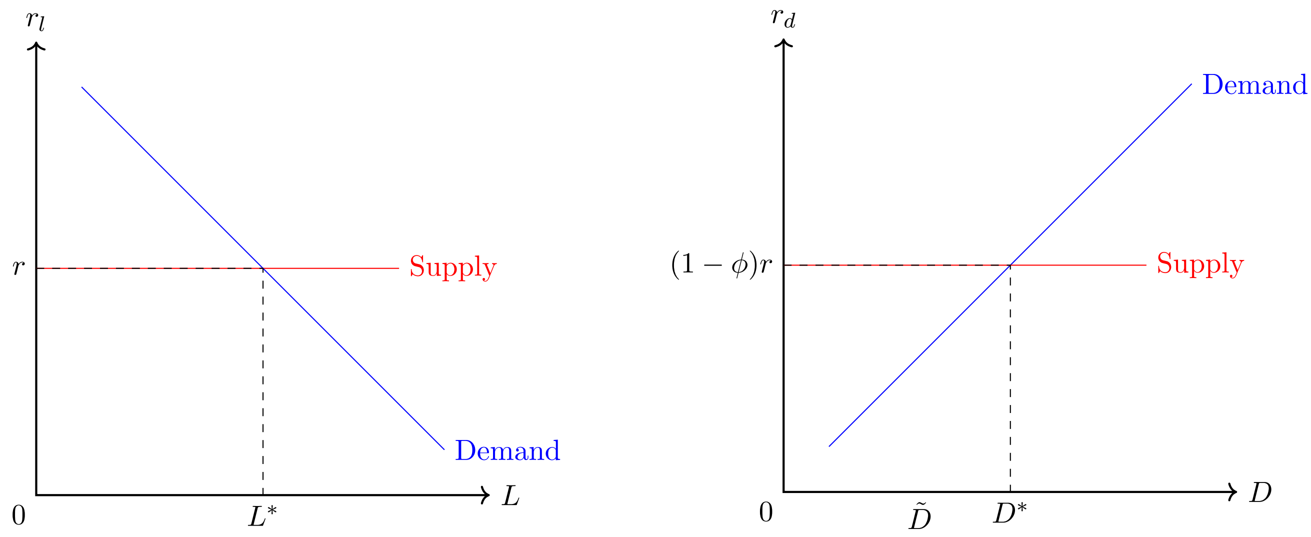
The equilibrium quantities can be obtained **recursively**: $L^\star = L\left(r_l^\star\right)$ and $D^\star = D\left(r_d^\star\right)$. That is, we can obtain them separately, as visible in the figure 1.1.

One can observe to important implications:

1. **separability**; and
2. **recursivity**: the supply side determines the prices and the demand side determines the equilibrium quantities.

If r is endogenous, separability breaks down!

[1.2] We can also obtain it from the supply-side analysis. This hints at the perfectly elastic supply of the loans.



(a) Characterising Loan Market.

(b) Characterising Deposit Market.

Figure 1.1: Separate Loans and Deposit Markets.

1.2.3 Introducing Intermediation Costs

We are including an extra assumption. That is, assume that **costs are given by an increasing and convex function** $C(L, D)$:

$$\Pi = (r_l - r)L + [(1-\phi)r - r_d]\, D - C(L, D) \tag{1.12}$$

The new first-order conditions imply:

$$r_l - r = \frac{\partial C}{\partial L}, \text{ and} \tag{1.13}$$

$$(1-\phi)r - r_d = \frac{\partial C}{\partial D}. \tag{1.14}$$

Therefore, we have $L^s(r_l, r_s)$ and $D^s(r_l, r_s)$. This implies that **intermediation margins are positive**. Moreover, if $\frac{\partial^2 C}{\partial D \partial L} \neq 0$, the problem is **no longer separable**.[1.3]

Consider the case when:

$$C(L, D) \equiv c_L L + c_D D \tag{1.15}$$

Then, quickly plugging it into conditions (1.13) and (1.14), we can obtain the following expressions:

$$r_l^\star = r + c_L; \text{ and} \tag{1.16}$$

$$r_d^\star = (1-\phi)r - c_D. \tag{1.17}$$

We can see that the changes in r produce "parallel" shifts in supply schedules. The quantities are determined (recursively) as $L(r_l^\star)$ and $D(r_d^\star)$.

Institutional remark. There is a long tradition of empirical work trying to determine the importance of scale, scope, and network economies in banking. The traditional banking activities may not carry very large

[1.3] If there is a constant marginal cost for each of two products, we have linearity and, as a result, separability. If the second derivative of the cost function w.r.t. to loans is positive (convexity), the supply curve on the left panel of figure 1.1 becomes positive and convex. Conversely, in this case, the supply of deposits (red curve on the right panel), becomes decreasing and concave.

economies of scale. However, there seem to be more important scale economies in wholesale and investment banking.

1.2.4 Introducing Equity Capital

In speaking about **"equity capital,"** we refer to equity financing, own funds, or capital. Assume a **regulatory requirement** of the type:

$$K \geq kL, \tag{1.18}$$

where $K \equiv$ equity capital and $k \equiv$ capital requirement. Further, let $\rho \equiv$ the opportunity costs of equity capital, such that $\rho \geq r$. ρ could, for example, equity returns, such as dividends. Alternatively, interpret ρ as the shareholders' marginal opportunity cost.

$$\Pi \equiv (1 + r_l)L + R - (1 + r_d)D - (1 + r)I - (\rho - r)K \implies \tag{1.19a}$$

$$\Pi = (r_l - r)L + [(1 - \phi)r - r_d]\, D - (\rho - r)K \tag{1.19b}$$

Notice that separability is preserved.

- If $\rho = r$, any $K \geq kL$ is optimal.
- If $\rho > r$, then $K = kL$ which implies the following:

$$r_d^\star = (1 - \phi)r; \text{ and} \tag{1.20}$$

$$r_l^\star = (1 - l)r + \rho k. \tag{1.21}$$

In his analysis, he treats a lump-sum tax on extraordinary profits as something that increases ρ.

Institutional remark. The Basel agreements on capital requirements (1989, 2004) make k a function of the composition & risk of bank assets.

1.3 The Monopolist Bank Model (Klein-Monti)

It is identical to the basic competitive model except a **single bank** **monopolising loans & deposits markets**. The maximisation programme is:

$$\max_{r_l, r_d} \left\{ (r_l - r)L(r_l) + [(1 - \phi)r - r_d]\, D(r_d) \right\}, \tag{1.22}$$

where $L' < 0$ and $D' > 0$. The first-order conditions are:

$$\frac{\partial \Pi}{\partial r_l} = (r_l - r)L'(r_l) + L(r_l) = 0 \implies \tag{1.23a}$$

$$\frac{r_l - r}{r_l} = \underbrace{\varepsilon_L^{-1} \equiv \frac{(-1)}{r_l}\frac{L}{L'}}_{\text{Lerner Index}} \tag{1.23b}$$

$$\frac{\partial \Pi}{\partial r_d} = [(1-\phi)r - r_d]\, D'(r_d) - D(r_d) = 0 \tag{1.23c}$$

$$\frac{(1-\phi)r - r_d}{r_d} = \underbrace{\varepsilon_D^{-1} \equiv \frac{1}{r_d}\frac{D}{D'}}_{\text{Lerner Index}} \tag{1.23d}$$

Definition 1.2 (Elasticity of Demand)**.**

$$\varepsilon_{D,r_d} \equiv \frac{\partial D}{\partial r_d}\frac{r_d}{D} = -r_d \frac{D'}{D} \tag{1.24}$$

It is similar for the elasticity of supply (we put -1 there, as a convention).

The **Lerner Indices** are just inverse elasticities. Note that we can obtain the perfect competition case when:

$$\varepsilon_L \to \infty \quad \wedge \quad \varepsilon_D \to \infty. \tag{1.25}$$

1.4 Imperfect Competition Models

1.4.1 Cournot Competition

Now, consider an **oligopoly made up of n (symmetric) banks**. Note that, for the sake of convenience, we use inverse market demand functions:

$$L(r_l): \; L' < 0 \implies r_l(L): \; r_l' < 0, \text{ and} \tag{1.26a}$$

$$D(r_d): \; D' > 0 \implies r_d(D): \; r_d' > 0. \tag{1.26b}$$

Define the i-th bank's objective function:

$$\Pi_i = \left[r_l \left(L_i + \sum_{j \neq i} L_j \right) - r \right] L_i + \left[(1-\phi)r - r_d \left(D_i + \sum_{j \neq i} D_j \right) \right] D_i. \tag{1.27}$$

Remember that banks **solve their objective functions over quantities**. Now, take the first-order conditions to obtain the **price-setting rules**.

$$\frac{\partial \Pi_i}{\partial L_i} = (r_l - r) + r_l' L_i = 0 \implies \tag{1.28a}$$

$$(r_l - r)\, n + r_l' L = 0 \implies \tag{1.28b}$$

$$\frac{r_l - r}{r_l} = \frac{1}{n\varepsilon_L} \tag{1.28c}$$

where $\varepsilon_L \equiv -\frac{1}{L}\frac{r_l}{r_l'(L)}$. Further, we have:

$$\frac{\partial \Pi_i}{\partial D_i} = [(1-\phi)r - r_d] - r_d' D_i = 0 \tag{1.29a}$$

$$[(1-\phi)r - r_d]\, n - r_d' D = 0 \tag{1.29b}$$

$$\frac{(1-\phi)r - r_d}{r_d} = \frac{1}{n\varepsilon_D} \tag{1.29c}$$

where $\varepsilon_D \equiv \frac{1}{D}\frac{r_d}{r_d'(D)}$. This is equivalent to what we define before.

Lemma 1.1. *The elasticity of deposit demand can be rewritten in the following form.*

$$\varepsilon_l = r_d \frac{D'}{D} = \frac{-1}{D}\frac{r_d}{r_d'}. \tag{1.30}$$

The same can be done for the elasticity of loan demand.

Proof.

$$r_d \frac{D'}{D} = r_d \frac{\frac{\partial D}{\partial r_d}}{D} = \frac{1}{D}\frac{r_d}{\frac{\partial r_d}{\partial D}} = \frac{1}{D}\frac{r_d}{r_d'}, \tag{1.31}$$

as required. The same follows for the elasticity of loan demand ☐

Notice that:

$$n \to \infty \implies r_l \to r \wedge r_d \to (1-\phi)r. \tag{1.32}$$

- The **limit cases** are equivalent to monopoly ($n = 1$) and perfect competition ($n \to \infty$). Small elasticities or small n widen the margins.[1.4]

- In the constant-elasticity case, intermediation margins are increasing in the interbank rate r:

$$r_l^\star = \underbrace{\frac{1}{1 - \frac{1}{n\varepsilon_L}}}_{\text{Mark-up}} r \implies \frac{dr_l^\star}{dr} > 1, \text{ and} \tag{1.33a}$$

$$r_d^\star = \underbrace{\frac{1}{1 - \frac{1}{n\varepsilon_D}}}_{\text{Mark-up}} (1-\phi)r \implies \frac{dr_d^\star}{dr} < 1 - \phi. \tag{1.33b}$$

  This effectively means that the loan rate increases more than in the perfect competitive case. Conversely, the deposit rates grow at a slower rate than the reference rate. Empirically, due to contractual inertia, the substitution effects are different.

- Price setting occurs at market level subsequently to banks' quantity decisions. In principle, explicit competition in prices seems a much closer description of reality, but the Cournot competition could be a reduced form of a two-stage competition situation in which banks decide: (1) on capacity (number of branches, employees, capital) and (2) on interest rates.

[1.4] Note that we can use this information to endogenise entry of new banks. That is, we will have new banks enter the market as long as the Cournot result yields non-zero profits.

1.4.2 Bertrand Competition

Price competition is more "plausible" than quantity competition, but the **plain Bertrand model has serious limitations**.

1. **Unrealistic predictions**. By standard price-cutting arguments, with a perfectly competitive inter-bank market, the prediction is as in the perfect competition case.

2. **Multiplicity of equilibria**. In the absence of an interbank market, having simultaneous competition in the two sides of the balance sheet poses technical problems and can produce counterintuitive results (Yanelle, 1997).

The plain Bertrand model predictions are little robust to introducing frictions (e.g., switching or transport cost) or product differentiation. Monopolistic competition model may be a more sensible choice.

1.5 Monopolistic Competition

1.5.1 Salop's Model

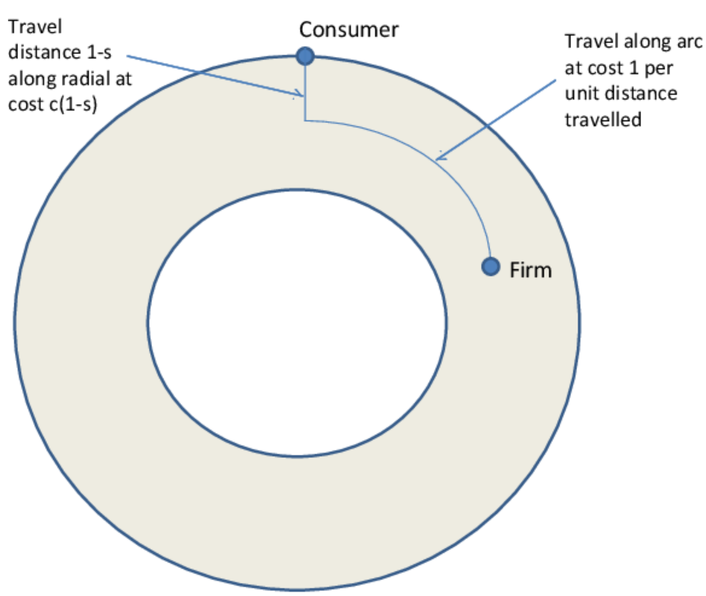


Figure 1.2: Visualising the Salop Model.

There are two dates ($t = 0, 1$), with no uncertainty.

Depositors. We consider a measure- one continuum of depositors uniformly distributed along a circumference of unit length (see figure 1.2 for a visualisation). The consumers are endowed with a unit of funds at $t = 0$ and wish to consume at $t = 1$. They incur a transportation cost αx when moving to adistance x.

Banks. There are $n \geq 2$ banks are symmetrically located along the circumference. They supply deposits at a rate r_i. They also invest the proceeds at a rate r. Importantly, depositors can only invest in bank deposits, which obliges them to move to a bank once.

1.5.2 Symmetric Equilibrium

All banks offer the **same deposit rate** $r_i = r_d \; \forall i$, from which no **profitable unilateral deviation exists**. Each depositor chooses to deposit his funds in the bank that offers him the highest net return (that is, interest payments net of transport costs). In practice they only need to check the rates offered by the two banks at each side of a depositor's location. A bank's **marginal depositor** can be identified from such a depositor's indifference condition.

Definition 1.3 (Marginal Depositor)**.** If bank i offers r_i and its competitors offer r_d, the **marginal depositor** will be at a distance x such that:

$$\underbrace{r_i - \alpha x}_{LHS} = \underbrace{r_d - \alpha\left(\frac{1}{n} - x\right)}_{RHS} \tag{1.34}$$

- $LHS \equiv$ the cost of using bank i; and
- $RHS \equiv$ the cost of using the alternative bank.

We can rearrange condition (1.34) to obtain the distance:

$$x = \frac{1}{2n} + \frac{r_i - r_d}{2\alpha} \tag{1.35}$$

Note that we cannot yet equate $r_i = r_d$ as we need to rule out the existence of a more profitable deviation. This leads us to the **demand function**:

$$D_i\left(r_i, r_d\right) = 2x = \frac{1}{n} + \frac{r_i - r_d}{\alpha}, \tag{1.36}$$

whose elasticity is decreasing in α. The **objective function** is as follows:

$$\Pi_i = \left(r - r_i\right) \times D_i\left(r_i, r_d\right). \tag{1.37}$$

Note that it is concave in r_i. The solution comes from the following first-order condition.

$$\left(r - r_i\right)\frac{1}{\alpha} - \frac{1}{n} + \frac{r_i - r_d}{\alpha} = 0 \implies \tag{1.38a}$$

$$\underbrace{r_i = r_d}_{\text{Symmetry}} \implies r_d^{\star} = r - \frac{\alpha}{n}. \tag{1.38b}$$

The **market power** is related to the transport cost, which can be interpreted as **product differentiation**. If $\alpha \to 0$ or $n \to \infty$, we have that $r_d \to r$ (perfect competition result). Notice that the **equilibrium profits** are increasing inα and decreasing in n:

$$\Pi^{\star} = \frac{r - r_d^{\star}}{n} = \frac{\alpha}{n^2} \implies n\Pi^{\star} = r - r_d^{\star} = \frac{\alpha}{n}, \tag{1.39}$$

with the second equation representing industry profits. If you shut down the branches, then you transfer the profit to the depositors.

1.5.3 Free Entry Equilibrium

Suppose each bank incurs a **fixed cost** F ($\neq$ entry cost). Following entry (or exit), all banks relocate so as to keep equidistant. In the equilibrium n will be the integer such that the following condition is satisfied.

$$\frac{\alpha}{(n+1)^2} \leq F \leq \frac{\alpha}{n^2} \implies \tag{1.40a}$$

$$n \simeq n^\star \equiv \sqrt{\frac{\alpha}{F}} \implies \tag{1.40b}$$

$$r - r_d^\star \simeq \sqrt{\alpha F} \tag{1.40c}$$

We can interpret it as the determinant of the intermediation margin. For example, we should expect the inventions like internet should decrease F, effectively decreasing intermediation costs too.[1.5]

Is this free-entry equilibrium efficient? There is a **trade-off between entry costs and transport costs**. Socially, we would love to minimise the aggregate cost of two things: (1) the total fixed cost paid by the banks; and (2) the aggregate transportation cost incurred by the depositors. The aggregate net returns would be maximised by:

$$n^{FB} = \arg\left\{\min\left[nF + 2n\int_0^{\frac{1}{2n}} \alpha x dx\right]\right\}. \tag{1.41}$$

Note, however, that:

$$2n\int_0^{\frac{1}{2n}} \alpha x dx = \frac{\alpha}{4n}. \tag{1.42}$$

Combine the first-order condition with condition (1.40b).

$$n^{FB} = \frac{1}{2}n^\star. \tag{1.43}$$

There are **two important results**:

- excess entry (each entering bank ignores its negative effect on other banks' profit); and
- possible argument for entry regulation (in banking?).

1.5.4 Effects of Introducing Regulatory Ceilings to Deposit Rates

Start with the unregulated free-entry equilibrium with $n = n^\star$. **Consider the effects of introducing a binding ceiling:** $\overline{r}_d < r_d^\star$. First, note that it can be proven that:

$$\left.\frac{\partial \Pi_i}{\partial r_i}\right|_{r_i = r_d = \overline{r}_d} > 0. \tag{1.44}$$

Then, we have a unique equilibrium with $n = n^\star$ and $r_d^{\star\star} = \overline{r}_d$. Further, $\Pi_i > F$ is implied by $\overline{r}_d < r_d^\star$. The new entry would lead to the following number of banks:

$$n^{\star\star} = \frac{r - \overline{r}_d}{F} > n^\star > n^{FB} \tag{1.45}$$

[1.5] Note that this could also be interpreted as the distance between branches going to 0.

Long story short, ceilings plus free entry is a bad combination. **Deposit rates decrease and the number of banks (or branches) increases.**

2 Banks' Risk Taking

This is a **model of risk taking with endogenous franchise values**.

2.1 Introduction

There are **two effects of bank capital on risk-taking incentives**:

1. the **capital at risk effect** (capital $\uparrow \implies$ losses for shareholders $\uparrow \implies$ risk-taking incentives $\downarrow$); and

2. the **franchise value effect** (capital $\uparrow \implies$ franchise values $\downarrow \implies$ risk-taking incentives $\uparrow$).

These two effects happen at the same time, potentially counteracting one another.

We focus on the framework of Repullo (2004).

- He conducts a joint analysis of capital requirements, market power, and risk-taking in a model featuring: (1) **dynamic considerations** (endogenous franchise value); and (2) **explicit formalisation of imperfect competition**.

- He revisits the discussion in Hellmann et al. (2000), which models imperfect competition in a reduced form.

The **main ingredients of the model** are (1) circular road specification of the deposit market; and (2) two types of assets (prudent and gambling). The **main results** are as follows:

1. **competition and risk-taking**: (I) high (low) intermediation margins $\rightarrow$ prudent (gambling) equilibrium and (II) intermediation margins $\rightarrow$ prudent + gambling equilibrium; and

2. **regulation**: (I) capital requirements and deposit rate ceilings can both ensure prudent equilibrium, possibly implying low deposit rates and (II) if informationally feasible, risk-based capital requirements can dissuade risk-taking without affecting equilibrium deposit rates.

2.2 The Model of Repullo (2004)

Setting. We face an infinite time horizon: $t \in \{1, 2, ...\}$. There are $n > 2$ banks symmetrically located around a unit circle. Each bank j receives a license at $t = 0$. The license is withdrawn when bank is insolvent. If so, a new bank enters the market.

Banks. Start with the funding. Banks **compete for deposits** offering rates r_j. Banks can raise capital from their owners, who require rate ρ. The model imposes a bespoke regulatory background. Namely, banks must **hold minimum capital k per unit of deposits**. Bank deposits are fully insured (at a zero premium; deposit insurance is funded with lump-sum taxes). There are **two mutually exclusive investment opportunities**:

1. **prudent asset** (P) with return μ_m; and

2. **gambling asset** (G) with high return μ_h with probability $1 - \pi$ and low return π_l with probability π.

We have to make some important parameter assumptions:

$$\mu_h > \mu_m > (1-\pi)\mu_h + \pi\mu_l; \quad \text{and} \tag{2.1a}$$

$$\rho > \mu_m. \tag{2.1b}$$

Importantly, we focus on the **symmetric equilibrium** (namely, each bank gets $\frac{1}{n}$ deposits).

Definition 2.1 (Risk Shifting)**. Risk shifting** is the transfer of risk(s) from one party to another party. Risk shifting can take on many forms, from purchasing an insurance policy to hedging investment positions to corporations moving from defined-benefit pensions to defined-contribution retirement plans like 401(k)s.

Consider it in banking. Gambling is bad for asset returns. However, thanks to limited liability distortions, banks are happy to gamble.

2.3 Characterisation of Equilibrium: Prudent Asset Only Baseline

Choice. At each date t each bank j chooses capital $k_j \geq k$ per unit of deposits and deposit rate r.

The **demand for deposits** of bank j when other banks offer r is given by the **indifference condition**:

$$r_j - \alpha x = r - \alpha\left(\frac{1}{n} - x\right) \Rightarrow x\left(r_j, r\right) = \frac{1}{2n} + \frac{r_j - r}{2\alpha} \tag{2.2}$$

This leads to the **demand function**:

$$D\left(r_j, r\right) = 2x\left(r_j, r\right) = \frac{1}{n} + \frac{r_j - r}{\alpha} \tag{2.3}$$

The problem of bank j:

$$V_P = \max_{k_j \geq k, r_j} \left\langle -k_j D\left(r_j, r\right) + \frac{1}{1+\rho}\left\{\underbrace{\left[\mu_m - r_j + \left(1+\mu_m\right)k_j\right]}_{\text{Profit for Shareholders}} D\left(r_j, r\right) + V_P\right\}\right\rangle \tag{2.4}$$

where $\frac{1}{1+\rho}$ represents the discount rate. Notice that increasing the capital requirement pushes the shareholders' return.

Proposition 2.1 (Capital)**.** *It's trivial to see that $k_j = k$ (corner solution).*

Proof. It's trivial. $\square$

Proposition 2.2 (Deposit Rate)**.** *The solution is as follows.*

$$r_j = r_P(k) \equiv \mu_m - \frac{\alpha}{n} - \delta_P k, \ \textit{where} \tag{2.5a}$$

$$\delta_P \equiv \rho - \mu_m > 0 \tag{2.5b}$$

Proof. Take the first-order condition of programme (2.4).

$$-\frac{k}{\alpha} + \frac{1}{1+\rho}\left[-\left(\frac{1}{n} + \frac{r_j - r}{\alpha}\right) + \frac{\mu_m - r_j + \left(1+\mu_m\right)k}{\alpha}\right] = 0 \Longrightarrow \tag{2.6}$$

Use the symmetry assumption by making $r = r_j$ and solve for r_j:

$$r_j = r_P(k) \equiv \mu_m - \frac{\alpha}{n} - \delta_P k. \tag{2.7}$$

You can obtain V_P quite easily.

$$\left\{-k + \frac{1}{1+\rho}\left[\mu_m - r_P(k) + (1+\mu_m)\,k\right]\right\}\frac{1}{n} = \frac{1}{1+\rho}\frac{\alpha}{n^2} \implies \tag{2.8a}$$

$$V_P = \left[\frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \frac{1}{(1+\rho)^3} + \dots\right]\frac{\alpha}{n^2} = \frac{\alpha}{\rho n^2} \tag{2.8b}$$

We're done with the proof. □

2.4 Characterisation of Equilibrium: Gambling Asset Only Baseline

Start with the **problem of bank j**:

$$V_G = \max_{k_j \geq k, r_j} \left\langle -k_j D\left(r_j, r\right) + \frac{1-\pi}{1+\rho}\left\{\left[\mu_h - r_j + (1+\mu_h)\,k_j\right] D\left(r_j, r\right) + V_G\right\}\right\rangle \tag{2.9}$$

Proposition 2.3 (Capital)**.** *We, again, obtain the **corner solution**.*

$$k_j = k \tag{2.10}$$

Proposition 2.4 (Deposit Rate)**.** *We have the following result:*

$$r_j = r_G(k) = \mu_h - \frac{\alpha}{n} - \delta_G k, \quad \textit{where} \tag{2.11a}$$

$$\delta_G \equiv \frac{1+\rho}{1-\pi} - (1+\mu_h) > 0. \tag{2.11b}$$

The key insight is that the gambling gains are **passed on to the depositors due to competition**.

Proposition 2.5. *In equilibrium ,the **NPV of gambling only is lower than of behaving in a prudent manner**:*

$$V_P > V_G. \tag{2.12}$$

Proof. In the prudent and gambling equilibria, the **NPV generated per period is**:

$$V_P^\star = \frac{\alpha}{\rho n^2} \tag{2.13a}$$

$$\left\{-k + \frac{1-\pi}{1+\rho}\left[\mu_h - r_G(k) + (1+\mu_h)\,k\right]\right\}\frac{1}{n} = \frac{1-\pi}{1+\rho}\frac{\alpha}{n^2} \implies \tag{2.13b}$$

$$V_G = \left[\frac{1-\pi}{1+\rho} + \left(\frac{1-\pi}{1+\rho}\right)^2 + \dots\right]\frac{\alpha}{n^2} = \frac{(1-\pi)\alpha}{(\rho+\pi)n^2} < V_P \tag{2.13c}$$

This means that, in the long term, gambling is worse than staying prudent.[2.1] □

[2.1] It's basically like drinking. In the long run, you would be better off not drinking. However, every Friday you are tempted to get wasted, regretting it later on Saturday.

2.5 The General Case (Endogenous Asset Choice)

Solving Strategy. Set $k_j = k$ and **check for 2 possible types of symmetric equilibrium**.

Proposition 2.6 (Existence of Prudent Equilibrium)**.** *From the basic game theory, we know that a* ***prudent equilibrium exists if no bank finds it profitable to deviate to*** (G, r_j') *for one period:*

$$\max_{r_j} \left\{ -kD\left(r_j, r_P(k)\right) + \frac{1-\pi}{1+\rho} \left[\left(\mu_h - r_j + (1+\mu_h)\, k\right) D\left(r_j, r_P(k)\right) + V_P \right] \right\} \leq V_P. \tag{2.14}$$

We can repeat the same for the gambling equilibrium.

Proposition 2.7 (Existence of Gamling Equilibrium)**.** *From the basic game theory, we know that a* ***gambling equilibrium exists if no bank finds it profitable to deviate to*** (G, r_j') *for one period:*

$$\max_{r_j} \left\{ -kD\left(r_j, r_G(k)\right) + \frac{1-\pi}{1+\rho} \left[\left(\mu_m - r_j + (1+\mu_m)\, k\right) D\left(r_j, r_G(k)\right) + V_G \right] \right\} \leq V_G. \tag{2.15}$$

Proposition 2.8 (Critical Values)**.** *There are* ***two critical values****:*

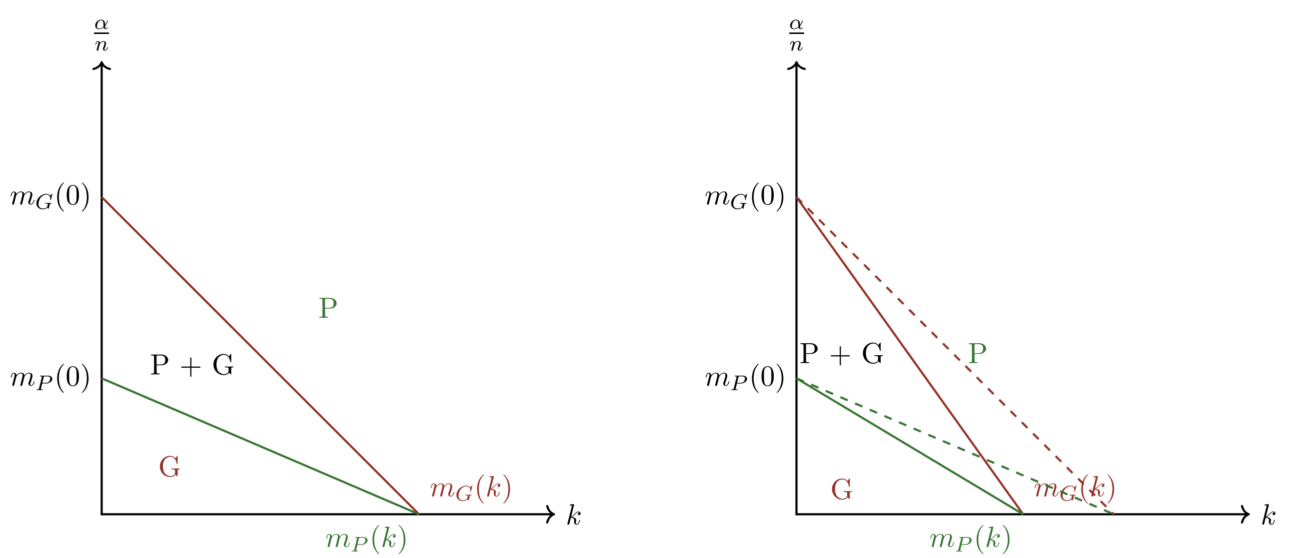
$$m_P(k) = \frac{\mu_h - \mu_m - (\delta_G - \delta_P)\, k}{2(h-1)}; \text{ and} \tag{2.16a}$$

$$m_G(k) = h m_P(k), \text{ where} \tag{2.16b}$$

$$h = \sqrt{\frac{\rho + \pi}{(1-\pi)\rho}} > 1, \tag{2.16c}$$

such that:

1. *a* ***prudent equilibrium exists if*** $\frac{\alpha}{n} \geq m_P(k)$*; and*
2. *a* ***gambling equilibrium exists if*** $\frac{\alpha}{n} \leq m_G(k)$.



(a) What Equilibria We Obtain.

(b) Equilibria with Risk-Based Capital Requirement.

Figure 2.1: Representing Types of Equilibria in the Repullo (2004) Framework.

2.6 Extension of the Model

Risk-based Capital Requirements. Consider the situation where we have risk-based capital requirements:

$$\begin{cases} k_P = 0 \\ k_G = k. \end{cases} \tag{2.17}$$

Proposition 2.9 (Critical Values with Risk-Based Requirements). *There are* ***two critical values:***

$$m'_P(k) = \frac{\mu_h - \mu_m - \delta_G k}{2(h-1)}; \text{ and} \tag{2.18a}$$

$$m'_G(k) = h m'_P(k), \text{ where} \tag{2.18b}$$

$$h = \sqrt{\frac{\rho + \pi}{(1-\pi)\rho}} > 1, \tag{2.18c}$$

such that:

1. *a* ***prudent equilibrium exists if*** $\frac{\alpha}{n} \geq m'_P(k)$*; and*

2. *a* ***gambling equilibrium exists if*** $\frac{\alpha}{n} \leq m'_G(k)$.

The risk-based capital requirement **expands the region of existence of prudent equilibrium without reducing the prudent equilibrium deposit rates**.

Deposit Rate Ceilings. Before the crisis, there were no deposit rates on the state-guaranteed accounts (so that the government doesn't insure profit making). There are **two types of ceilings**:

- **non-binding**:

$$\bar{r} \geq r_P = \frac{\mu_m^2 \mu_h}{h^2 - 1}; \text{ and} \tag{2.19}$$

- **binding**

$$\bar{r} \leq r_P. \tag{2.20}$$

Proposition 2.10 (Critical Values w/ Interest Caps). *If* $\bar{r} \geq r_P$, ***there are two critical values*** $M_P(\bar{r})$ *and* $M_G(\bar{r})$ *such that:*

1. *a* ***prudent equilibrium exists*** *if* $\frac{\alpha}{n} \geq M_P(\bar{r})$*; and*

2. *a* ***gambling equilibrium exists*** *if* $\frac{\alpha}{n} \leq M_P(\bar{r})$.

Further, $\bar{r} \leq r_P$*, a prudent equilibrium exists for all* $\frac{\alpha}{n}$.

2.7 Discussion

Main Conclusion.

- The effect of capital requirements on equilibrium deposit rates can partly offset (here, fully offset) the **effect of capital regulation on franchise values**. This is neglected by Hellmann et al. (2000); there, the charter value is zero.

- **Flat-rate capital requirements can control excessive risk-taking**, but **risk-based capital requirements**, if feasible, are better!
- **Deposit rate ceilings can also be useful**, but they have other well-known problems (over-investment in services, excess entry, etc.).
- Supervisor **could use level of deposit rates** (say, relative to market rates) as a signal of excessive risk taking

Special assumptions. We assume a **fixed aggregate demand for deposits** (no other assets), **perfectly elastic supply of prudent and gambling assets**, and **perfectly elastic supply of bank capital**. The implications are as follows:

- There is a **full pass through** of the effects to depositors; and
- Less competition is **not welfare decreasing**!

3 Credit Market Imperfections and Debt Contracts

3.1 Credit Rationing (Stiglitz and Weiss, 1981)

The model consists of imperfect information and credit rationing.

Definition 3.1 (Credit Rationing)**.** Credit rationing happens when there are some customers who are trustworthy, who still do not receive loans. Do not confuse it with red lining (excluding unworthy customers).

We cover the adverse selection of Stiglitz and Weiss (1981) which comments on moral hazard and features dichotomous outcome formation.

3.1.1 Imperfect Information & Credit Rationing

In **credit markets**, borrowers obtain funds from lenders in exchange for promises of a specified stream of future interest and principal. The typical support for such transactions is a debt contract (e.g., mortgage, loan, line of credit, bond, commercial paper, etc.).

Debt is sometimes described as a **fixed-income security** (akin to a government bond), but indexation, floating rates, embedded options, and, above all, default make most debt more complicated. Its full description must include: principal, maturity, repayment path, interest rate (if floating, the benchmark rate), negotiability, covenants, buyback provisions, explicit guarantees, etc.

For years, the **credit market was modelled as one for risk-free debt**. That is, from the investors' perspective, a perfect substitute for government bonds, whose price can then be derived from the term structure of interest rates.

Today, most applications are explicit about default. However, default per se does not make debt special as the **Modigliani-Miller theorem may still apply**![3.1] Debt is special because of taxes, financial distress costs, information asymmetries, conflicts of interest, incentive problems, etc. Financing conditions, including contract details, may alter the cash flows available for distribution among agents. Debt contracts may or may not be an optimal response to the underlying problem(s).

Basically, we need to introduce **frictions to obtain rationing**.

3.1.2 The Contribution of Stiglitz and Weiss (1981)

First, we analyse the equilibrium analysis of the credit market under imperfect information. We model a market for one-period loans with adverse selection (hidden types) or moral hazard (hidden action) related to default risk. **We obtain credit rationing as an equilibrium phenomenon!**

The key insights about a loan's promised repayment, B (principal + interest):

- then the **expected repayment, $R(B)$**, is a **non-monotonic function of B**, (most plausibly it's single peaked);
- we get **adverse selection**: high B attracts "bad risks;"

[3.1] Long story short, a firm's capital structure does not matter for its valuation (as long as certain assumptions apply; e.g., there are frictionless capital markets).

- we get **moral hazard**: high B encourages risk taking;
- with an **upward sloping supply of funds, the credit market may feature equilibrium credit rationing**: among apparently identical applicants, some receive a loan, others don't;
- the **rejected applicants wouldn't receive a loan even if they promised a larger B**!

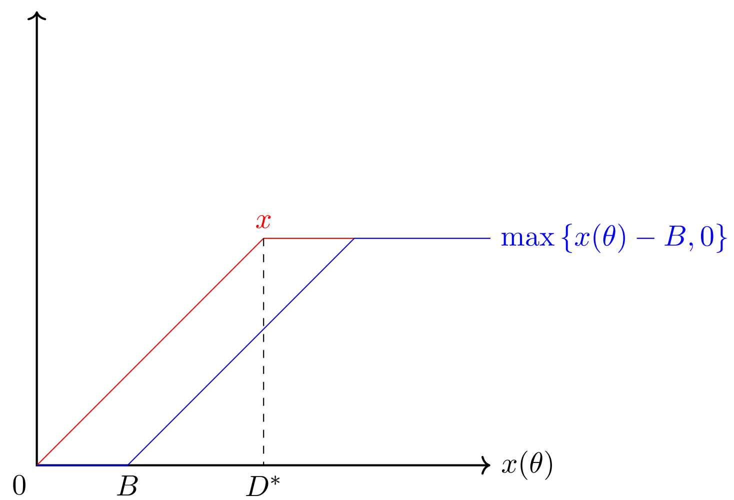


Figure 3.1: Entrepreneur's Payoff Graph.

3.1.3 The Adverse Selection Model

Setting. There are two dates ($t = 0, 1$) and **three classes of risk neutral agents**: depositors, borrowers, and banks.

1. Depositors are **characterised by a supply of funds**, $S(R_d)$, where R_d is the gross deposit interest rate.
2. **Borrowers are a measure-I continuum** of penniless entrepreneurs, with projects characterised by a parameter $\theta \in [\underline{\theta}, \overline{\theta}]$, with CDF $G(\theta)$.
   (a) They need a unit of investment at $t = 0$.
   (b) They deliver independent cash flow $x(\theta) \in [0, \infty)$ at $t = 1$, with CDF $F(x, \theta)$.
   (c) They undertake a project that has a $t = 1$ opportunity costs C (e.g., lost labour income) for each entrepreneur.
   (d) Each entrepreneur **knows their own θ, but nobody else does**.
   (e) The key assumption is that **projects differ in their risk**:

$$\theta_1 < \theta_2 \implies x(\theta_1) \underset{SSD}{\succsim} x(\theta_2). \tag{3.1}$$

3. Banks are **intermediaries that compete a la Bertrand**.
   (a) They **offer deposits at a (chosen) rate R_d**.

(b) They use the funds to offer limited-liability loans to entrepreneurs. Namely, they offer **one unit of funds at $t = 0$, in exchange for a (chosen) promised repayment B at $t = 1$.**

Theorem 1. *The* ***second-order stochastic dominance (SSD) implies****:*

$$\mathbb{E}\, x(\theta_1) = \mathbb{E}\, x(\theta_2) = \bar{x};\ \textit{and} \tag{3.2a}$$

$$\int_0^y \left[F(x, \theta_1) - F(x, \theta_2)\right] dx \leq 0\ \forall\, y \geq 0. \tag{3.2b}$$

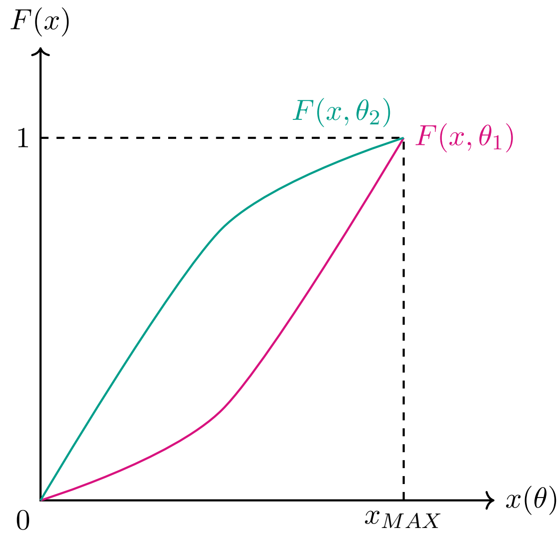


Figure 3.2: Representing Second Order Stochastic Dominance.

Comment. **First, loans are risky!** Long story short, entrepreneurs and banks get:

$$B: \quad \min\{x(\theta), B\};\ \text{and} \tag{3.3a}$$

$$E: \quad \max\{0, x(\theta) - B\}. \tag{3.3b}$$

We can calculate an average payment from a loan (banks' perspective):

$$P(B, \theta) = \int_0^B x dF(x, \theta) + [1 - F(B, \theta)] \implies \tag{3.4a}$$

$$P(B, \theta) = B - \int_0^B F(x, \theta) dx. \tag{3.4b}$$

$P(B, \theta)$ has **three properties: it's lower than B, increases in B, and decreases in θ**. Further, note that equation (3.4b) implies that $P(B, \theta)$ is B less than the area under the graph until B. Try to understand it based on an example. That is, if you borrow very little, then it is almost certain you will earn above that $(F(x, \theta) \simeq 0)$ so $P(B, \theta) \simeq B$.

Second, we have a **Bertrand-type bank competition**. Banks maximise profits under a constant returns to scale (CRS) technology. They undercut each other unless profits are zero. This implies that the equilibrium profits are driven to zero.

Third, banks are safe! Since $x(\theta)$ is independent across projects, **credit risk idiosyncratic and diversifiable**. **Diversification** makes the return on a bank's loan portfolio deterministic.

3.1.4 Who Demands a Given Loan?

Suppose **B is the lowest repayment offered by banks**. If entrepreneur with risk θ takes a loan with B, their payoff at $t=1$ is: $\max\{0, x(\theta) - B\}$. Hence, on average, we have:

$$E(B,\theta) = \bar{x} - B + \int_0^B F(x,\theta)dx. \tag{3.5}$$

This decreases in B and increases in θ. The former implies that you **appropriate more of the cashflow if you are a riskier customer**! Notice that bankruptcy has no cost in this model. Now, suppose $\exists\, C$:

$$E(B,\underline{\theta}) < C < E\left(B,\bar{\theta}\right), \tag{3.6}$$

which implies:

- $\exists\, \hat{\theta}(B) \in \left(\underline{\theta}, \overline{\theta}\right)$ such that: $E\left(B,\hat{\theta}\right) = C$; and
- All **entrepreneurs with $\theta > \hat{\theta}(B)$ apply for this loan**.

3.1.5 How Much Does the Bank Get from a Loan?

If the loan is randomly granted to its applicants, then **the bank gets the following expected value**:

$$R(B) = \mathbb{E}\left[P(B,\theta)|\theta \geq \hat{\theta}\right] \tag{3.7}$$

$R(B)$ is **likely to be non-monotonic in B** (which worsens applicants pool).

- With a **finite number of types**, $R(B)$ is surely non-monotonic. $\hat{\theta}(B)$ reaches points were subsets of entrepreneurs drop out. $R(B)$ discontinuously falls at those point.
- With a **continuous distribution**, we have:

$$R'(B) = \underbrace{\mathbb{E}\left[P_1(B,\theta)|\theta \geq \hat{\theta}\right]}_{\text{Standard Effect}} - \underbrace{\left\{P(B,\hat{\theta}) - \mathbb{E}\left[P(B,\theta)|\theta \geq \hat{\theta}\right]\right\} \frac{g\left(\hat{\theta}(B)\right)}{1 - G\left(\hat{\theta}(B)\right)} \frac{d\hat{\theta}(B)}{dB}}_{\text{Adverse Selection Effect}}. \tag{3.8}$$

This is what leads to the **hump shape of the repayment graph**!

3.1.6 Equilibrium

Proposition 3.1 (Symmetric Bertrand Equilibrium)**.** *Pair $(R_d^\star, B^\star)$ **constitutes an equilibrium** if banks meet the following conditions.*

1. *They **break-even**:*[3.2]

$$R_d^\star = R\left(B^\star\right). \tag{3.9}$$

[3.2] The free entry condition guarantees the zero profit property of the system (that is, the Bertrand competition driving the profits to zero).

2. *They find no profitable deviation:*

   (a) *in* ***loan making***

$$R(B) \leq R\left(B^{\star}\right) \quad \forall B < B^{\star}; \tag{3.10}$$

   (b) *in* ***deposit-taking***

$$S\left(R_d^{\star}\right) \leq \left[1 - G\left(\hat{\theta}(B)\right)\right] I, \tag{3.11}$$

   where $I \equiv$ *the number of customers and* $S(\cdot) \equiv$ *the supply function of deposits (this is equivalent to the market clearing condition); and*

   (c) *in the* ***case of rationing****:*

$$S\left(R_d^{\star}\right) < \left[1 - G\left(\hat{\theta}(B)\right)\right] I \implies R(B) \leq R\left(B^{\star}\right) \quad \forall B > B^{\star}. \tag{3.12}$$

Proof. The proof covers every point separately.

Breaking Even. With $R_d^{\star} > R\left(B^{\star}\right)$, no banks would operate. With $R_d^{\star} < R\left(B^{\star}\right)$, undercutting would be profitable.

No Profitable Deviation. Start with loan making. Suppose $\exists B_0 < B^{\star}$ such that $R\left(B_0\right) > R\left(B^{\star}\right)$. Then, a bank could earn extra profits by attracting deposits with $R_d = R^{\star} + \varepsilon$ and offering loans with $B = B_0$. This breaks the assumption there are no profitable loans at lower B.

Deposit Taking. Suppose:

$$S\left(R_d^{\star}\right) > \left[1 - G\left(\hat{\theta}(B)\right)\right] I. \tag{3.13}$$

Then, a bank could make profits by attracting deposits at some $R_d < R_d^{\star}$ and using the proceeds to make loans with $B = B^{\star} - \varepsilon$. In plain words, **there is no excess supply of funds**.

The Case of Rationing. Suppose:

$$S\left(R_d^{\star}\right) < \left[1 - G\left(\hat{\theta}(B)\right)\right] I, \tag{3.14}$$

but $\exists\, B^1 > B^{\star}$ such that $R\left(B^1\right) > \left(B^{\star}\right)$. Then, a bank could make profits by attracting deposits with:

$$R_d = R_d^{\star} + \varepsilon \tag{3.15}$$

and offering a loan with $B = B^1$. Hence, **credit rationing can occur only at peaks**! □

3.1.7 Graphical Representation of Equilibrium

Define the function:

$$H(B) \equiv S^{-1}\left(\left[1 - G\left(\hat{\theta}(B)\right)\right] I\right), \tag{3.16}$$

which implies that:

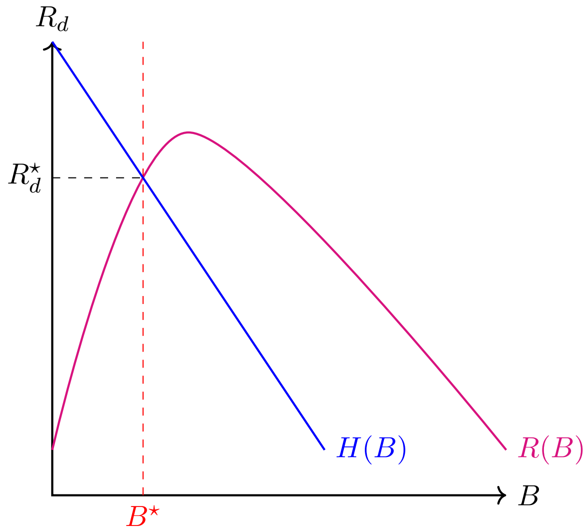
- **LHS**: deposit supply at $R_d = H(B)$; and
- **RHS**: loan demand at B.

It is **non-negative** & **monotonically decreasing**. This way, we can rewrite condition (3.11):

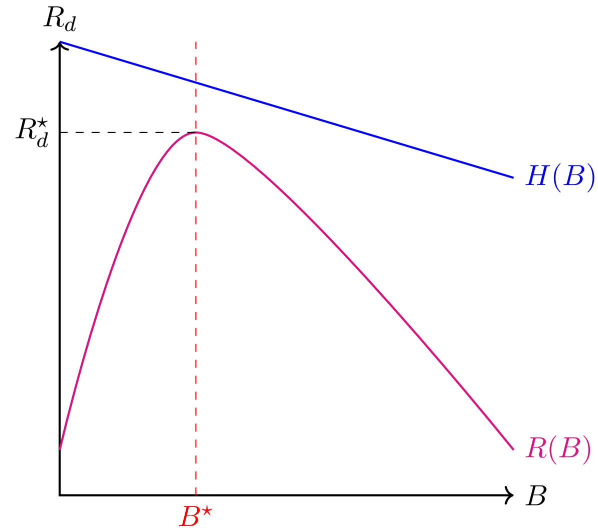
$$R_d^\star \leq H\left(B^\star\right). \tag{3.17}$$

Now, we have two relevant curves in the (B, R_d) space:

1. the **market clearing curve**: $R_d = H(B)$; and
2. the **zero-profit curve**: $R_d = R(B)$



(a) Standard Equilibrium without Credit Rationing.



(b) Standard Equilibrium without Credit Rationing Case.

Figure 3.3: Two Versions of the Equilibrium.

There are **two cases that we can analyse** (both shown on figure 3.3):

1. $H(B)$ **crosses the upward sloping section of** $R(B)$. This happens at most once. This unique equilibrium features market clearing. See, figure 3.3a.
2. The unique equilibrium features **credit rationing**! We have the following conditions satisfied:

$$B^\star = \arg\max_B R(B); \text{ and} \tag{3.18a}$$

$$R_d^\star = R\left(B^\star\right) < H\left(B^\star\right). \tag{3.18b}$$

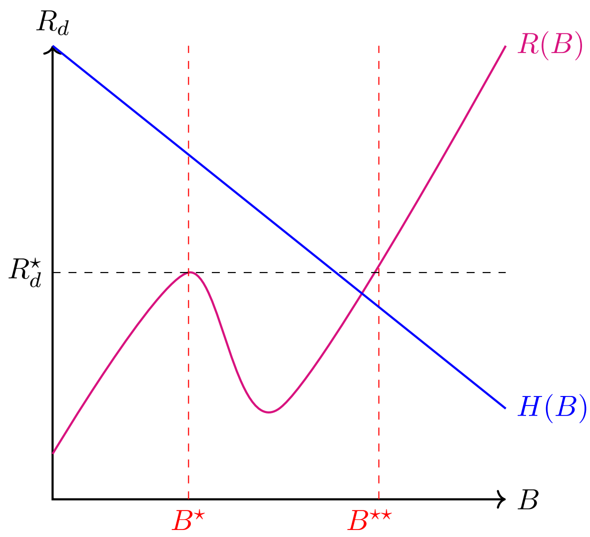


Figure 3.4: Complex Case of Mixed Equilibrium.

Consider this **complex case** on figure 3.4. In this case, we have a mixed equilibrium, where banks offer a random combination of $B^\star$ and $B^{\star\star}$.

Discussion.

- **Rationing and equilibrium are compatible**. Due to "imperfections;" here, adverse selection: with observable project types rationing will not occur.
- There are **two different equilibrium regimes**. A change in $S(R_d)$ produces a pure "quantity" effect. If monetary policy changes $S(R_d)$, large effects on investment may go with minor (or no) effects on interest rates.

3.1.8 Discussion

Screening vs. Rationing. Bester (1985) **discusses separating equilibria when some screening device (such as collateral) is available**. Recall that the entrepreneurs willing to provide personal assets as guarantees for their loans.

SSD vs. FSD. De Meza and Webb (1987) show that with **FSD-ranked projects**, rationing never occurs. Loan applicants are those with better projects. Increasing B improves the selection of borrowers, which implies that $R(B)$ is monotonic.

Perfectly Elastic Supply of Funds. Mankiw (1986) argues that equilibrium credit rationing does not emerge. Instead, the credit market collapses (no trade).

Equity Contracts. With equity contracts, there is no rationing.

3.1.9 Comments on Moral Hazard

Stiglitz and Weiss (1981) **model moral hazard along the lines of a risk-shifting problem**, associating riskier projects with lower expected cash flows. In doing so, they show that **moral hazard can induce non-monotonicity in $R(B)$**.

Most moral hazard problems in corporate finance exhibit such a feature:

- When there is effort (or similar) involved because **lenders appropriate part of the return to effort, while the entrepreneur incurs all the effort cost**.
- And even when there is no effort involved (but excessive risk involves lower expected cash flow), **because debtors have incentives to take excessive risk**, like in Jensen and Meckling (1976).

Adverse selection and moral hazard models have been extensively used as building blocks in models of banks' "screening" and "monitoring" roles, respectively.

3.1.10 The Dichotomous Outcome Formulation

Many papers in the literature simplify the analysis by describing project outcomes as a dichotomous success/-failure random variable and by normalising the failure return to zero.

Consider **adverse selection**:

$$x(\theta) = \begin{cases} 1 + \alpha(\theta) & \text{w.p. } \theta, \\ 0 & \text{w.p. } 1 - \theta. \end{cases} \tag{3.19}$$

To prevent θ to be inferred from the success outcome, it is commonly assumed success/failure is observable but the project's payment in case of success is not. This also **justifies the focus on debt contracts**.

Now, consider **moral hazard in a risk-shifting version of the model**:

$$x(p) = \begin{cases} 1 + \alpha(p) & \text{w.p. } p, \\ 0 & \text{w.p. } 1 - p, \end{cases} \tag{3.20}$$

where p is an unobservable action of the borrower. Allen and Gale (2001) assume that $\alpha(p)$ is positive and decreasing in p while

$$p\,[1 + \alpha(p)] \tag{3.21}$$

is concave and maximised at some $p^\star \in (0, 1)$.

Finally, consider **moral hazard in a risk-shifting version of the model**:

$$x(p) = \begin{cases} X & \text{w.p. } p, \\ 0 & \text{w.p. } 1 - p, \end{cases} \tag{3.22}$$

where p is an unobservable action of the borrower involving an increasing private costs $\psi(p)$ or some decreasing private benefits $b(p)$. Holmstrom and Tirole (1997) consider two choices of p. Repullo and Suarez (1998, 2000) consider a continuous p.

3.2 Verifiability and the Debt Contract (Gale and Hellwig, 1985)

In this chapter, we review a framework building towards a **theory of debt**, highlighting the specification of Gale and Hellwig (1985). The model contains **unverifiable cash flows and strategic defaults**.

3.2.1 Towards a Theory of Debt

For long, **financial contracts were taken as given**. Agents were supposed to choose among a restricted set of standard contracts. However, many results are contract dependent (e.g., Stiglitz and Weiss (1981) credit rationing equilibrium). If inefficiencies are due to the form of the contract, why don't agents change the contract?

The **security design approach** postulates that contracts should be an implication, rather than a primitive of the analysis. This brings the problem one step backwards. What friction determines the optimal contract? Do we accept arbitrarily complex solutions?

Two Distinct Traditions.

1. **Complete contracts**: by application of the revelation principle, we should generally consider the optimal fully-contingent contract (under truth-telling constraints); e.g., in Gale and Hellwig (1985) we have costly state verification and in Innes (1990) we have moral hazard combined with the monotonicity requirement.

2. **Incomplete contracts**: real-world contracts are simpler than the mechanism design literature suggests and unspecified frictions make fully-contingent contracts too costly to enforce. Examples include Aghion and Bolton (1992), Bolton and Dewatripont (2005), Dewatripont and Tirole (1994), and Hart and Moore (1998).

The incomplete contracts approach lacks sufficiently convincing and tractable micro-foundations. That is, **many authors consider that assuming that some information is observable but not verifiable is an unacceptable short-cut**.

Yet the **incomplete contracts paradigm has several attractive features**:

- control rights are important;
- ex-post bargaining may occur; and
- institutions matter (e.g., by constraining the allocation of control rights or structuring the bargaining process).

The following sections illustrate how the debt contract has been justified by specific contributions within these two traditions.

3.2.2 Costly State Verification

Gale and Hellwig (1985) inaugurated the security design approach in corporate finance. This contrasts with capital structure models, where debt and equity are taken as given.

The standard debt contract is shown to be optimal in a **costly state verification setup**.

1. **Contingent contracts are costly to enforce** due to the cost of verifying the relevant contingencies. An entrepreneur costlessly observes their cash flow. Lenders can observe it at a cost.

2. Townsend (1979) was the first to analyse optimal contracting in the presence of verification costs.

In order to finance the entrepreneur at the minimum cost **optimal contract should minimise verification costs**. Debt accomplishes this by setting a fixed repayment! If honoured, no verification is needed! Then, verification only occurs upon default. This is similar to being paid a regular wage every month! Long story short, this gives a **rationale for debt and bankruptcy**.

Extending the approach to understand other contracts is not easy. Results are little robust. Optimal mechanisms are frequently too complicated. Recent progress with optimal dynamic contracts under a recursive formulation is remarkable. For example, see DeMarzo and Fishman (2007).

3.2.3 The Model of Gale and Hellwig (1985)

Setting. There are two dates ($t = 0, 1$) and zero riskless rate. There are **two classes of risk neutral agents**: an entrepreneur and many potential lenders The entrepreneur has **no wealth and wants to undertake an investment project**:

- one unit of investment at $t = 0$; and
- random cashflow $x \in [0, \infty)$ at $t = 1$ with CDF $F(x)$.

We observe **limited liability**: To oblige an agent to pay, it is necessary to prove that he has enough cash. At $t = 1$, only the entrepreneur observes x for free. Lenders can verify x at $\phi(x)$.[3.3]

Contract. By the Revelation Principle, we can focus on direct mechanisms that induce truth-telling. We observe a **contract between the entrepreneur and a lender**. There are:

1. an announcement of x by the entrepreneur;
2. a verification rule:
$$Q(x) = \begin{cases} 0 & \text{if } x \text{ is not verified;} \\ 1 & \text{if } x \text{ is verified; and} \end{cases} \tag{3.23}$$
3. a payment from the entrepreneur to the lender $P(x)$.

Pair (Q, P) constitutes a contract!

Optimal Contract. A contract is optimal if it solves:

$$\begin{aligned} \max_{(Q,P)} \quad & \{\mathbb{E}\left[x - P(x)\right]\} & \\ \text{s.t.} \quad & P(x) \le x & \text{(LL)} \\ & \mathbb{E}\left[P(x) - q(x)Q(x)\right] \ge 1 & \text{(FC)} \\ & P\left(x\right) \le P\left(x'\right) \ \forall\, x, x' : Q\left(x'\right) = 0 & \text{(IC)}, \end{aligned} \tag{3.24}$$

where LL $\equiv$ limited liability, FC $\equiv$ financing constraint (lender's PC), and IC $\equiv$ incentive compatibility. The incentive compatibility constraint **entails the entrepreneur announcing the truth to announcing something that that will not lead the truth to be revealed**:

$$x - P(x) \ge x - P\left(x'\right) \quad \forall x, x' : Q\left(x'\right) = 0. \tag{3.25}$$

[3.3] The idea of signing a smart contract is to reduce $1 + \mathbb{E}\,\phi(x)$.

This implicitly implies that if **one is caught lying, the payment is zero**!

3.2.4 Solving the Model

Remember that we assume that the **solution is deterministic**!

Lemma 3.1. ***Contract*** (Q, P) ***satisfies (IC)*** *iff* $\exists B \in [0, \infty)$ *such that:*

1. *if* $Q(x) = 0 \implies P(x) = B$*; and*
2. *if* $Q(x) = 1 \implies P(x) \leq B$*.*

Proof. ($\Longrightarrow$) We prove it by contradiction.

- Suppose $x, x' : Q(x) = Q(x') = 0 \wedge P(x) > P(x')$. Hence, the entrepreneur prefers x' to x, which is a contradiction.
- Suppose $x, x' : Q(x) = 1, Q(x') = 0 \wedge P(x) > B$. Hence, the entrepreneur prefers x' to x, which is a contradiction.

($\Longleftarrow$) (1) and (2) imply the IC. The proof is concluded. □

Lemma 3.2. ***IF*** (Q, P) ***is optimal, (FC) holds with equality!***[3.4]

Proof. We prove it by contradiction again. Suppose:

$$\mathbb{E}\left[P(x) - q(x)Q(x)\right] = 1 + \varepsilon, \tag{3.26}$$

with $\varepsilon > 0$. Define a new contract (Q', P') such that:

$$Q'(x) = Q(x) \quad \forall x; \text{ and;} \tag{3.27a}$$

$$P'(x) = \max\left\{P(x) - \varepsilon, 0\right\} \quad \forall x. \tag{3.27b}$$

This contract dominates the original version, which constitutes a contradiction. □

Remark. *Hence, I* ***can modify the optimal contract to the following form****:*

$$\begin{aligned}
\min_{(Q,P)} \quad & \left\{\mathbb{E}\left[q(x)Q(x)\right]\right\} \\
s.t. \quad & P(x) \leq x && \textit{(LL)} \\
& \mathbb{E}\left[P(x) - q(x)Q(x)\right] = 1 && \textit{(FC')} \\
& P(x) \leq P(x') \ \forall\, x, x' : Q(x') = 0 && \textit{(IC).}
\end{aligned} \tag{3.28}$$

Standard Debt Contract.

Definition 3.2 (Standard Debt Contract)**.** A **SDC is a pair** (Q, P) **for which** $\exists\, B \in [0, \infty)$ such that:

$$Q(x) = \begin{cases} 0 & \text{if } x \geq B \\ 1 & \text{if } x < B, \end{cases} \tag{3.29}$$

[3.4] There is no interest to leave any rest to the lenders. It would not be optimal. If there were a slack in the contract, we would either be able to make the lender happier or make the verification cost lower.

and

$$P(x) = \min\{B, x\}. \tag{3.30}$$

Long story short, there is some promised payment B. A default occurs when $x < B$. Upon default, verification takes place and x is paid. Otherwise, verification does not take place and B is paid.

Lemma 3.3. ***Any SDC satisfies (LL) and (IC).***

Proof. Trivial (see the graph below). □

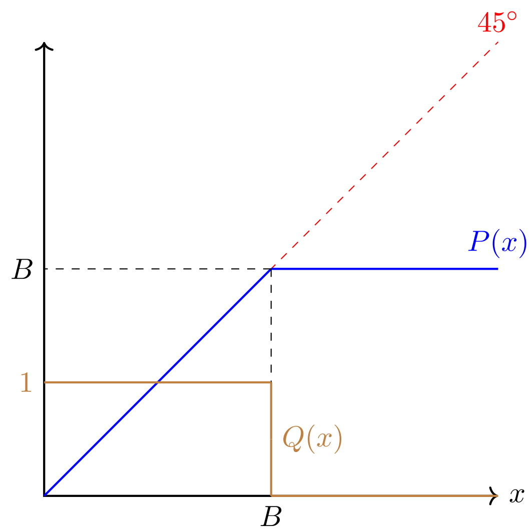


Figure 3.5: Graphical Representation of a SDC.

When Is a SDC Feasible? By (FC'), the lender's expected repayment must be 1:

$$R(B) = 1, \tag{3.31}$$

where, using integration by parts:

$$R(B) = \int_B^\infty B dF(x) + \int_0^B (x - \phi(x))\, dF(x) \implies \tag{3.32a}$$

$$R(B) = B - \int_0^B F(x)dx - \int_0^B \phi(x)dF(x). \tag{3.32b}$$

Clearly, we can see that:

$$R(0) = 0; \text{ and} \tag{3.33a}$$

$$\lim_{B\to\infty}\{R(B)\} = \mathbb{E}\, x - \mathbb{E}\, \phi(x). \tag{3.33b}$$

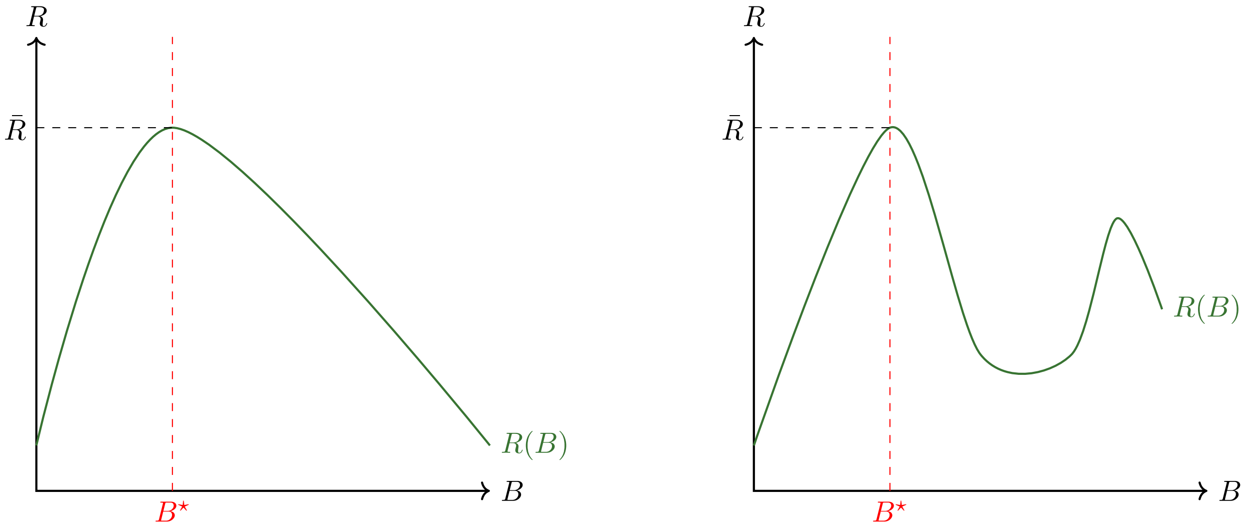
$R'(B)$ may be positive or negative. Finally, $\exists B : R(B) = 1$ **iff the maximum debt capacity is larger than 1**:

$$\bar{R} = \sup_{B\in\mathbb{R}_+}\{R(B)\} \geq 1. \tag{3.34}$$

What Is the Best of the Feasible SDCs? Provided that condition (3.34) is satisfied, the SDC with:

$$B^\star = \min\{B|R(B) = 1\} \tag{3.35}$$

is the **best of the feasible SDCs**. Choosing the **lowest feasible B minimises verification costs**!



(a) Representing $\bar{R}$.

(b) Representing $\bar{R}$ in a More Complicated Version

Figure 3.6: Add Q(x) functions.

Proposition 3.2. ***If an optimal contract exists, a SDC is optimal.***

Proof. Let (Q, P) be an optimal contract. I prove it by contradiction. Define the SDC, (Q', p'), whose promised payment B' equals fixed payment B of the original contract when $Q(x) = 0$ (the state of no verification).

Clearly, (Q', P'):

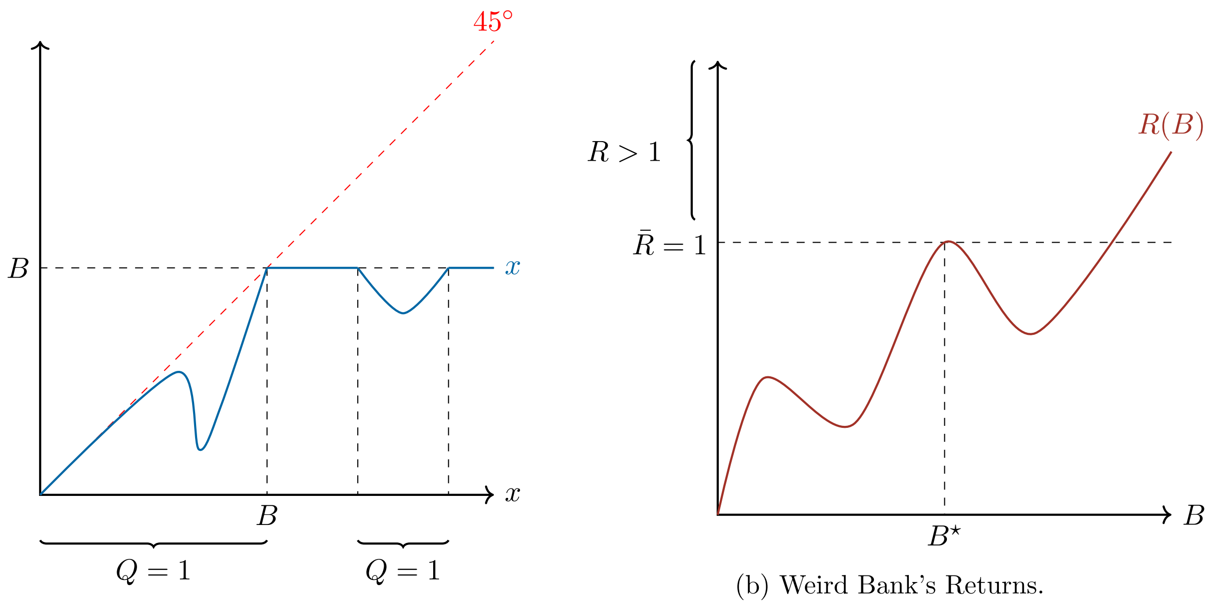
- entails no higher verification contract;
- satisfies (LL) and (IC) by Lemma 3.3; and
- satisfies (FC) since it pays the same as (Q, P) when $Q(x) = 0$.

There are two possibilities.

1. We have that:
$$\mathbb{E}\left[P'(x) - \phi(x)Q'(x)\right] = 1, \tag{3.36}$$
which means that (FC') is satisfied. This implies that (Q', p') is at least as good as (Q, P).

2. We have that:
$$\mathbb{E}\left[P'(x) - \phi(x)Q'(x)\right] > 1, \tag{3.37}$$
which means that $B' > B^\star$. This implies that the SDC with $B^\star$ (which satisfies (FC') and entails no larger verification costs) is at least as good as (Q, P).

This concludes the proof. $\square$



(a) Weird Cashflow & Verification.

(b) Weird Bank's Returns.

Figure 3.7: Analysing Complicated Cases, An Example.

3.2.5 Extensions & Limitations

There are multiple ways that this model could be **extended**.

1. **Maximum equity participation**: If the entrepreneur has some wealth at $t = 0$, investing all of it in the project relaxes (FC), allowing for a lower B and less "waste" of verification costs.

2. **Possibility of credit rationing (Williamson, 1987)**: This is due to the likely non-monotonicity of R(B). This is obtained, opposite to Stiglitz and Weiss (1981), with optimal contracts!

3. **Rationale for intermediation**: Who pays the verification costs? Avoiding cost duplication with multiple lenders may justify delegation to a single "monitor". But, who monitors the monitor? This moves us to the theory of financial intermediation of Diamond (1984).

4. **Rationale for risk management and indexation**: If there are verifiable variables correlated with x, hedging against poorer realisations of x would save on verification costs.

The framework has its **limitations**.

- ⚠ The **proposed SDC is not optimal**:

    1. if random verification is feasible; and
    2. if entrepreneurs are risk averse.

- ⚠ **Verification may be ex post inefficient**. That is, lenders' commitment to verify may be essential with large verification costs.

- ⚠ This is fine for entrepreneurial firms, but **how can we explain the presence of outside equity**? Is verifiability less of a problem? Is risk-sharing more important? Alternatively, could it be the case that bankruptcy is not only about verification?

3.3 Unverifiable Cash Flows and Strategic Default (Hart and Moore, 1998)

In this model, we have three dates ($t = 0, 1, 2$), risk-neutrality, and a zero riskless rate.

An entrepreneur has limited liability, no wealth, and an investment project which requires one unit of investment at $t = 0$, and yields:

1. a random cash flow, x_1, and assets with liquidation value L at $t = 1$; and

2. nothing if $t = 1$ liquidated at $t = 2$ or random cash flow x_2.

Lenders compete for financing the entrepreneur.

The contractual setup is as follows. Cash flows are observable but not verifiable. Assets and payments are verifiable. Negotiations are a la *Nash*, with bargaining powers: $(\beta, 1 - \beta)$ for lenders and entrepreneurs, respectively.

The contract (between the entrepreneur and a lender) is:

- set a repayment obligation B at $t = 1$;
- if the entrepreneur pays B to the lender, the project is not liquidated; and
- otherwise, the lender has the right to liquidate the project.

There are a few questions in helping to solve this problem.

1. For a given B, how much does the entrepreneur pay in each possible state of the world at $t = 1$?

2. Is it always feasible to finance the project?

3. If so, how is the equilibrium B determined? Does it depend on β?

4. Do we observe renegotiation in equilibrium?

5. Can we do better on the contracting side?

You need to identify the surplus for not changing the decision to liquidate. This would be $\mathbb{E}\, x_2 - L$ in the stochastic case. The lender gets $L + \beta\, (\mathbb{E}\, x_2 - L)$. The entrepreneur gets $x_1 + (1 - \beta)\, (\mathbb{E}\, x_2 - L)$. You effectively divide a larger cake when you decide not to liquidate. Remember that $L + \beta\, (\mathbb{E}\, x_2 - L) < x_1$ must be met.

Consider the simplifying assumptions. x_1 is X with probability π, and zero otherwise. x_2 is deterministic. We have:

$$x_2 > X > \pi X > I = 1 > L. \tag{3.38}$$

4 Banks as Monitors

4.1 Delegated Monitoring and Diversification (Diamond, 1984)

In this chapter, we seek to answer why financial intermediaries exist, outlining the role of delegated monitoring based on Diamond (1984).

4.1.1 Why Do Financial Intermediaries Exist?

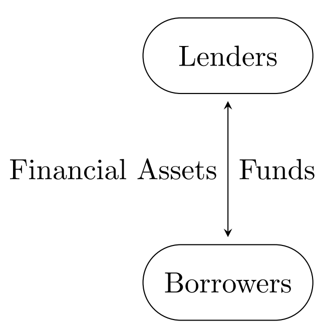
Definition 4.1 (Financial Intermediary)**.** A **financial intermediary** is an agent that channels funds from those who wish to save or lend to those who wish to invest or borrow.

This activity typically involves the trading of financial assets either on the agent's own account or on the account of its customers.

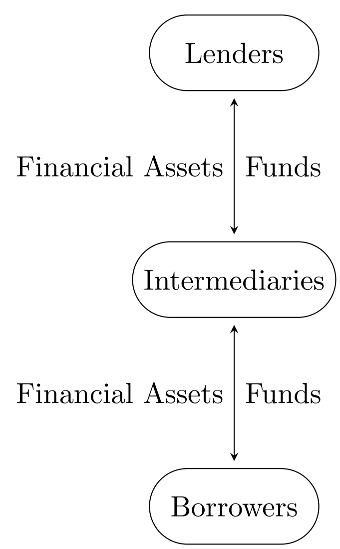
A (traditional) bank is a financial intermediary that performs its function by receiving deposits and making loans. This has great quantitative importance (now and in the past). Traditional banking business is well-defined, but the actual boundaries of the business model are blurred. Banking is heavily regulated (now and in the past) and intertwined with the transmission of monetary policy.

In the pure theory of financial intermediation and banking, we seek to understand why financial intermediaries /banks exist, their contribution to social welfare, what contracts they should use, and how they should be regulated.

In the standard Arrow-Debreu and Modigliani-Miller worlds banks are unneeded.



(a) Direct Financing.



(b) Indirect Financing.

Figure 4.1: Understanding Intermediated Financing.

Banks' Functions. They include:

1. providing payment services (and contributing to the working of the payment system);
2. transforming assets (size, term, risk, etc. of the assets offered to lenders differ from those of the assets taken from borrowers);

3. managing financial risks (credit risk, interest rate risk, exchange rate risk, liquidity risk, etc.); and

4. screening, monitoring, and disciplining potential and actual borrowers.

Theoretical Paradigms.

- **Transaction costs**: Gurley et al. (1979) ask why the asset transformation is a function performed by intermediaries. Why not entities created by a borrower & his lenders? Specialists can reduce unmodelled organisational frictions (e.g., search costs, contract writing, legal costs, etc.). Where do transaction costs come from?

- **Information and incentive problems**: Regarding types and actions correlated with borrowers' solvency, see Leland and Pyle (1977), Boyd and Prescott (1986), and Calomiris and Kahn (1991). Regarding the liquidity needs of lenders and/or borrowers, see Section 6.

4.1.2 Advantages of Delegated Monitoring

Diamond (1984)'s key idea is as follows:

- **intermediaries perform delegated monitoring**; and

- delegation to a **single agent allows for savings on monitoring costs**, associated with the imperfect observability of borrowers' cash flows (ex-post asymmetric information).

The **idea can be extended to any informational asymmetry that can be reduced at some cost**. Ex-ante asymmetries are reducible by prior assessment and screening. Interim asymmetries are reducible by supervision and corrective actions.

Originally Diamond (1984) considers a world with unverifiable cash-flows and non-pecuniary penalties for the entrepreneurs who do not repay. Instead, we developed his model in CSV setup.

The savings come from avoiding: duplicating monitoring, screening, and supervision costs, as well as free-riding among lenders.

However, who monitors the monitor? Perhaps, monitoring the monitor implies costs at least as high as the direct monitoring of the borrower by all lenders. In fact, it does NOT, because there is a natural scale economy. Diversification reduces the costs of monitoring the monitor (by washing away the impact of borrowers' idiosyncratic risk on the monitor's risk).

4.1.3 The Model

Set-Up. There are two dates ($t = 0, 1$), risk-neutrality, and a riskless rate r. We see many entrepreneurs and many savers.

Entrepreneurs, indexed by $i \in \mathbb{Z}_{++}$, are penniless and want to undertake a project:

$$(-1) \text{ (at } t = 0) \rightarrow \; x_i \in [0, \infty) \text{ (at } t = 1), \tag{4.1}$$

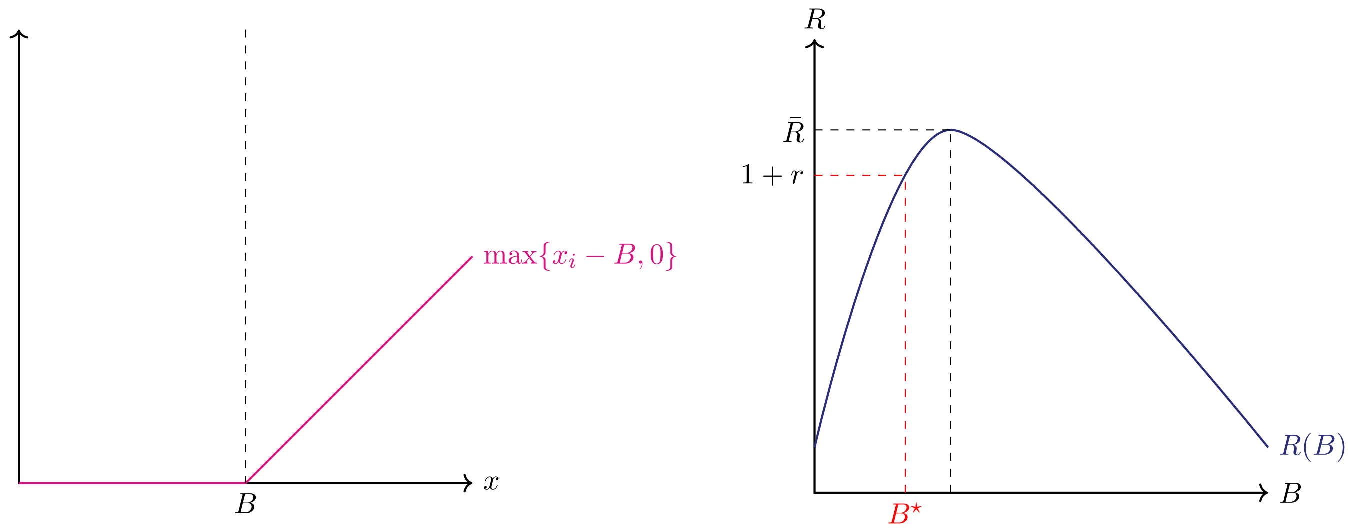
with:

$$x_i \overset{iid}{\sim} F(x), \tag{4.2}$$

where $\mathbb{E}\, x_i > 1 + r$.[4.1]

Savers, indexed by $j \in \mathbb{Z}_{++}$, have an initial wealth $\frac{1}{m}$ each, with $m \in \mathbb{Z}_{++,\geq 2}$, and can invest at the riskless rate. This effectively means that we want more than 2 people to finance each project.

The **information structure is as in Gale and Hellwig (1985)**. Each entrepreneur i costlessly observes the realisation of x_i. Any other agent has to incur a cost $\phi > 0$ to verify x_i. Note that this implies a constant verification costs, unlike in the previous model.



(a) Cash-Flow for the Entrepreneurs.

(b) Finding $B^\star$.

Figure 4.2: Two Figures Next to Each Other

4.1.4 Allocation Problem

Each project requires funds from $m \geq 2$ savers. For some realizations of x_i someone will have to incur ϕ. Consider two possible arrangements (modes of financing) with symmetric contracts:

1. direct financing: m savers directly fund one of the entrepreneurs, incurring $m\phi$ when verifying x_i; and

2. intermediated financing: nm savers delegate to a single "bank" the verification of the cash flows $\{x_i\}_{i=1}^n$ of n entrepreneurs.

We proceed in the following way: (i) we first characterize the optimal contracts for each financing model; and (ii) we then find which financing mode is cheaper/more efficient.

4.1.5 Direct Financing

Gale and Hellwig (1985) show that the **use of standard debt contracts is optimal in this setup**. There would be a contract per each saver-entrepreneur pair. For an entrepreneur as a whole, we would have:

- a total repayment B promised to his lenders: if B is paid, x_i is not verified, otherwise, it is verified with each of m lenders incurring ϕ cost; and

[4.1] In the corporate finance terms, this implies that the NPV is positive. That is, $NPV = -1 + \frac{\mathbb{E}\, x_i}{1+r} \geq 0$. This means that, in perfect condition, it's worth financing the project.

The m lenders as a whole obtain $\min\{B, x_i\}$ net of verification costs, on expectation:

$$R_m(B) = \int_0^B (x - m\phi)\, dF(x) + \int_B^\infty B dF(x) \implies \tag{4.3a}$$

$$R_m(B) = B - \int_0^B F(x)dx - m\phi F(B), \tag{4.3b}$$

which is possibly non-monotonic.

Thus, **direct financing is feasible if and only if**:

$$\max_{B \in \mathbb{R}_{++}} R_m(B) \geq 1 + r. \tag{4.4}$$

If feasible, competition between lenders implies:

$$B^\star = \min\{B : R_m(B) = 1 + r\}. \tag{4.5}$$

The associated information costs are:

$$c_m \equiv m\phi F(B^\star), \tag{4.6}$$

where $F(B^\star)$ denotes the proportion of entrepreneurs with less than $B^\star$ in cash-flow.

4.1.6 Intermediated Financing

Bank-entrepreneur and savers-bank relationships are subject to **informational problems of the same qualitative nature as those of savers-entrepreneur relationships under direct financing**.

It is **optimal to use debt contracts**! Bank-entrepreneur relationships are based on loans. Loan conditions are set by banks, but entrepreneurs only accept $B \leq B^\star$. Savers-bank relationships are based on deposits. Deposits impose on the bank an obligation to repay $D^\star$. Such a repayment must compensate the savers for he opportunity cost of their funds and the expected verification costs.

Let s_n **denote the marginal cost of funds for the bank** (the required rate of return per unit of deposits). Can the bank profit from offering loans with $B < B^\star$? To check this, it is sufficient to verify whether the bank would get positive profits by charging $B^\star$ on its loans. **Consider the profit function of the banks wishing to mimic the world without lenders**:[4.2]

$$\Pi_n(B^\star) = n \times \left[B^\star - \int_0^{B^\star} F(x)dx - \phi F(B^\star) - (1 + s_n)\right] \implies \tag{4.7a}$$

$$\Pi_n(B^\star) = n\left(r - s_n + (m-1)\phi F(B^\star)\right) \implies \tag{4.7b}$$

$$\Pi_n(B^\star) > 0 \iff \underbrace{s_n + \phi F(B^\star)}_{\text{Cost of Banking Financing.}} < \underbrace{r + m\phi F(B^\star).}_{\text{Costs of Direct Financing.}} \tag{4.7c}$$

We use equation (4.5) to leave equation (4.7a). Note that, in equation (4.7a), ϕ is not multiplied by m as the bank lends to the entrepreneur on its own!

[4.2] If this profit is positive, the bank can choose $B < B^\star$ to undercut its competitors. As a result, this is going to be the equilibrium result, meaning that we can establish that B can be used.

4.1.7 Diamond (1984)'s Main Result

Diamond (1984) shows that:

$$\lim_{n\to\infty} \{s_n\} = r, \tag{4.8}$$

which implies:

$$\lim_{n\to\infty} \left\{ \frac{\Pi_n(B^\star)}{n} \right\} > 0 \quad \forall m \geq 2. \tag{4.9}$$

This result is due to diversification. By the **strong law of larger numbers, for large n, the bank's loan portfolio becomes riskless** (due to the iid property). As a result, the **bank is almost surely solvent**!

The gross returns of a portfolio of n loans with $B^\star$ are:

$$Y_n = \sum_{i=1}^{n} y_i, \quad \text{where} \quad y_i = \min\{x_i, B^\star\} - \phi\mathbb{I}(x_i < B^\star). \tag{4.10}$$

By the strong law of large numbers:

$$\frac{Y_n}{n} \xrightarrow{\text{d}} \mathbb{E}\, y_i = B^\star - \int_0^{B^\star} F(x)dx - \phi F(B^\star) = 1 + r + (m-1)\phi F(B^\star). \tag{4.11}$$

The bank signs a **contract with each saver**. As a whole, this implies a promised repayment D such that if D is paid, Y_n is not verified. Otherwise, Y_n is verified, and each of the mn savers incur ϕ. Thus savers' net payments are $\min\{D, Y_n\}$ net of verification costs.

On expectations, savers receive:

$$P_{mn}(D) = \int_0^D (Y - mn\phi)\, dG_n(Y) + \int_D^\infty D dG_n(Y) \implies \tag{4.12a}$$

$$P_{mn}(D) = D - \int_0^D G_n(Y)dY - mn\phi G_n(D), \tag{4.12b}$$

where:

$$Y_n \sim G_n(x). \tag{4.13}$$

Assuming feasibility, the **optimal arrangement entails**:

$$D^\star = \min\{D : P_{mn}(D) = (1+r)n\}. \tag{4.14}$$

The implied information costs are $mn\phi G_n(D^\star)$. This means that:

$$1 + s_n = \frac{\mathbb{E}\min\{D^\star, Y_n\}}{n} \implies \tag{4.15a}$$

$$1 + s_n = 1 + r + m\phi G_n(D^\star). \tag{4.15b}$$

This allows us to conclude the central proof.

Proposition 4.1. *We have:*

$$\lim_{n\to\infty} \{s_n\} = r. \tag{4.16}$$

Proof.

$$G_n(D) = \mathbb{P}\left(Y_n < D\right) \implies \tag{4.17a}$$

$$G_n(D) = \mathbb{P}\left(\frac{Y_n}{n} < \frac{D}{n}\right). \tag{4.17b}$$

Suppose:

$$D^\star = (1+r)n. \tag{4.18}$$

This implies:

$$\lim_{n\to\infty}\{G_n(D)\} = \lim_{n\to\infty}\left\{\mathbb{P}\left(\frac{Y_n}{n} < 1+r\right)\right\} \implies \tag{4.19a}$$

$$\lim_{n\to\infty}\{G_n(D)\} = 0. \tag{4.19b}$$

This concludes the proof. □

4.1.8 Final Comments

Institutional Implementation. What is the objective function of the bank? Which agents manage this institution?

Scale Economy. There is a **natural scale economy coming from diversification gains**. Can a bank work as any other firm? Is there a natural monopoly situation? Is it compatible with perfect competition in banking? Does it need to be regulated?

Robustness. Will the presence of systematic risk change the conclusions? Will a profit-maximising bank diversify at a socially optimal level? Can the model be extended to explain the co-existence of direct and intermediated finance?

Empirical Predictions. Banks have a comparative advantage as lenders for:

- the risky ones with less correlated outcomes,
- but not for the fully safe, or
- the risk ones with highly correlated outcomes.

4.2 The Incentive Role of Bank Capital (Holmstrom and Tirole, 1997)

4.2.1 The Role of Bank Capital

The own funds of **financial intermediaries play a role similar to the well-known role of own funds in corporate finance**:

- in reducing the need for external financing; and/ or
- by providing a guarantee to more senior security holders.

Holmstrom and Tirole (1997) consider a setup in which the relationships of both firms and banks with their financiers are subject to moral hazard problems. Bank **monitoring and firms' own funds are substitutes**

in guaranteeing that firms behave properly. Bank capital guarantees that banks monitor properly. They also show that the implied incentive-based capital requirements decrease with the cost of bank capital (i.e., in recessions).

4.2.2 The Model

There are 2 dates ($t = 0, 1$) with 3 classes of **risk neutral agents**. The **riskless rate is R_d**.

1. There are **entrepreneurs**:
   (a) with **heterogenous wealth**, $w \sim F(w)$;
   (b) on a measure one continuum with a project each:
   $$(-1) \rightarrow \begin{cases} X & \text{if the project succeeds,} \\ 0 & \text{otheriwse; and} \end{cases} \tag{4.20}$$
   (c) with three **unobservable modes of managing the projects**:

Mode	Private Benefits	Success Probability
Efficient	0	p_H
Negligent	γ	p_L
Fraudulent	Γ	p_L

Table 4.1: Modes of Managing the Project.

where bank monitoring can prevent the fraudulent mode!

Assumption 4.1.
$$\Delta p = p_H - p_L > 0. \tag{4.21}$$

Assumption 4.2.
$$\Gamma > \gamma. \tag{4.22}$$

Assumption 4.3.
$$p_H X - R_d > 0 > p_L X + \Gamma - R_d. \tag{4.23}$$

2. There are **depositors/ unsophisticated investors**:
   (a) e.g., a large number of financiers without the ability to monitor the entrepreneurs; and
   (b) who have a **perfectly elastic supply of funds** at the expected gross rate of return R_d.

3. There are **banks**:
   (a) i.e., a large number of financiers with the ability to monitor the entrepreneurs at a cost, $\phi > 0$, per project (payable at $t = 1$);
   (b) whose **monitoring can prevent the fraudulent management mode**, but is not observable (leading to the potential moral hazard problem);
   (c) who have an aggregate amount of own funds, $\bar{K}$; and

(d) who have limited opportunities to diversify risk across projects.

Assumption 4.4. *Project cash flows are perfectly correlated.*

The above assumption is justified on the basis of systematic risk, specialisation, and incentives.

4.2.3 Direct Financing Version of the Model

An **optimal contract sets**:

1. **funding contributions of entrepreneur & depositors** $(w, 1-w)$;
2. **payment $B^\star$ to depositors** that solves:

$$\begin{aligned} \max_{B\in\mathbb{R}} \quad & p_H(X-B) \\ \text{s.t.} \quad & B \leq X && (LL) \\ & p_H(X-B) \geq p_L(X-B) + \Gamma && (IC) \\ & p_H B \geq R_d(1-w), && (PC) \end{aligned} \tag{4.24}$$

where the IC guarantees choice of efficient mode (the only viable one).

Note that the IC can be re-written in the following way:

$$B^\star \leq X - \frac{\Gamma}{\Delta p}, \tag{4.25}$$

which implies that the **LL is never binding**. We can combine it with the PC to get:

$$w \geq w^\star \equiv 1 - \frac{p_H}{R_d}\left(X - \frac{\Gamma}{\Delta p}\right). \tag{4.26}$$

Assumption 4.5. *We assume that $w^\star > 0$, which implies:*

$$p_H X - R_d < p_H \frac{\Gamma}{\Delta p}. \tag{4.27}$$

4.2.4 Intermediated Financing Version of the Model

In this mode of financing banks act as monitoring financiers, while depositors are passive financiers. Bank monitoring is subject to a moral hazard problem: they need "skin in the game" to monitor properly. The required **expected return on banks' own funds, R_b, is endogenous**! Banks are perfectly competitive and, hence, take R_b as given. **Equilibrium value of R_b on the relative scarcity of $\bar{K}$**. However, we will always have $R_b \geq R_d$.

An **optimal contract** will satisfy the following conditions.

1. **Contributions of entrepreneur, depositors & bank** (w, I_d, I_b) are:

$$I_d + I_b = 1 - w. \tag{4.28}$$

2. Payments to **depositors & bank upon success** (B_d, B_b) **satisfy**:

$$B_d, B_b \geq 0 \quad \wedge \quad B_d + B_b \leq X, \tag{4.29}$$

which solve:

$$\begin{aligned} \max_{B_b, B_d \in \mathbb{R}} \quad & \{p_H (X - B_d - B_b)\} \\ \text{s.t.} \quad & B_d + B_b \leq X - \frac{\gamma}{\Delta p} && (\text{IC}_e) \\ & B_b \geq \frac{\phi}{\Delta p} && (\text{IC}_b) \\ & p_H B_d \geq R_d I_d && (\text{PC}_d) \\ & p_H B_b - \phi \geq R_b I_b. && (\text{PC}_b) \end{aligned} \tag{4.30}$$

Let's start solving for the **optimal contract** (assuming w.l.o.g. $R_b > R_d$).

- Objective function is decreasing in B_d and B_b. Clearly (PC_d) will be binding. We can reduce B_b until (IC_b) or (PC_b) becomes binding.
- We can substitute I_d for I_b until both (IC_b) and (PC_b) become binding.

$$B_b = \frac{\phi}{\Delta p} \implies \tag{4.31a}$$

$$I_b = \frac{p_L}{R_b} \frac{\phi}{\Delta p} \implies \tag{4.31b}$$

$$B_d = \frac{R_d}{p_H} \left(1 - w - \frac{p_L}{R_b} \frac{\phi}{\Delta p} \right) \implies \tag{4.31c}$$

$$I_d = 1 - w - \frac{p_L}{R_b} \frac{\phi}{\Delta p}. \tag{4.31d}$$

- A necessary and sufficient condition for intermediated financing to be viable can be found from (IC_e) after substitution:

$$\frac{\phi}{\Delta p} + \frac{R_d}{p_H} \left(1 - w - \frac{p_L}{R_b} \frac{\phi}{\Delta p} \right) \leq X - \frac{\gamma}{\Delta p} \iff \tag{4.32a}$$

$$w \geq \widehat{w} \equiv 1 - \frac{p_H}{R_d} \left(X - \frac{\gamma}{\Delta p} \right) + \frac{\phi \left(p_H R_b - p_L R_d \right)}{\Delta p R_b R_d}. \tag{4.32b}$$

Equation (4.32b) reflects costs & benefits of intermediated financing.

- For solution to involve $I_d, B_d \geq 0$, we impose the sufficient condition.

Assumption 4.6.

$$X \geq \frac{\phi + \gamma}{\Delta p}. \tag{4.33}$$

4.2.5 Choosing between Two Models of Finance

We face two pending questions.

1. What is the preferred mode of financing for entrepreneur with:

$$w \geq \max\{w^\star, \hat{w}\}? \tag{4.34}$$

   Answer: Direct financing is the best option.

2. What are the relative positions of $w^\star$ and $\hat{w}$? Ambiguous. We will guarantee:

$$\lim_{R_b \to R_d} \hat{w} \leq w^\star, \tag{4.35}$$

   by assuming the following.

   Assumption 4.7. *Intermediation adds value:*

$$\phi \leq \frac{p_H}{\Delta p}(\Gamma - \gamma). \tag{4.36}$$

4.2.6 Comments

1. The bank's contribution to each project, I_b, does not depend on the entrepreneur's wealth.
2. We can analyse **interest rate effects**:
   (a) $w^\star$ increases in R_d and does not react to R_b (as it is not imposed by the banks); and
   (b) $\hat{w}$ increases in R_d and R_b.
3. There are **two possible interpretations of intermediated finance**:
   (a) **certification**: "depositors" contribute funds directly to projects, co-financing them with the banks; and
   (b) **strict intermediation**: the banks borrow funds from depositors and use these "deposits" (together with their funds) to lend to entrepreneurs.

4.2.7 Equilibrium Analysis

The **demand for bank capital is as follows**:

$$K(R_d, R_b) \equiv [F(w^\star) - F(\hat{w})] \frac{p_L}{R_b} \frac{\phi}{\Delta p}. \tag{4.37}$$

Note that $K_1 \lesseqgtr 0$ and $K_2 < 0$. Therefore, **the market for bank capital clears** when:

$$K(R_d, R_b) \leq \bar{K}. \tag{4.38}$$

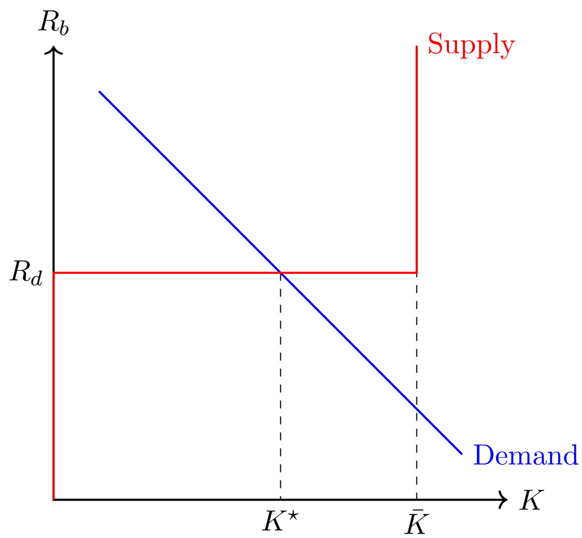


Figure 4.3: Finding Equilibrium.

For each exogenous value of R_d we have a unique solution in R_b.

The **aggregate investment** is given by:

$$I\left(R_d, R_b\right) = 1 - F\left(\hat{w}\right), \tag{4.39}$$

with $I_1, I_2 < 0$.

4.2.8 Comparative Statics

We want to analyse **the effects of three types of shocks on the financing capacity of the agents in this economy**:

- a **proportional reduction in entrepreneurs' wealth**, w;[4.3]
- a **reduction in banks' capital, $\bar{K}$**; and
- an **increase in the opportunity cost of depositors' funds**, R_d.

Proposition 4.2. *Any of the above-mentioned shocks to the financing capacity of the agents in the economy leads to:*

1. *a* ***fall in the measure of entrepreneurs whose wealth is sufficient to guarantee the financing of their investment projects****;*
2. *a* ***fall in aggregate investment****.*

[4.3] This entails shifting the distribution. Then, if $F_\mu(w) = G(\mu w)$, then this is equivalent to an increase in μ, where $\mu > 1$.

5 Banks as Suppliers of Liquidity

5.1 Introduction

Now, we take another view of banks' function:

- financing long-term investment projects with the funds of consumers with potentially short investment horizons; and
- by pooling the liquidity risk of many consumers, expand the capacity to invest in long-term projects.

The provision of liquidity explains in part the existence of banks, but it does shine much light on the vulnerability to runs. Bank runs reflect a **coordination problem**:

1. caused by a **shift in expectations**; and
2. producing **social deadweight losses** (inefficient liquidations and suboptimal risk-sharing).

Several mechanisms may help to prevent runs without fully neutralizing the liquidity provision function of banks: suspension of convertibility; deposit insurance; and lender of last resort (alternatively, these could be discount window facilities).

Diamond and Dybvig (1983) provide the **first rigorous rationale for deposit contracts and model of rational bank panics**. Doing so, they construct a useful framework for: modeling the demand for liquidity; and comparing different financial systems (bank- vs. market-based).

5.2 The Model of Diamond and Dybvig (1983)

There are three dates: $t = 0, 1, 2$.

There are **investment technologies with CRS**.

t	0	1	2
Long Term (LT)	-1	lL	$(1-l)R$
Short Term, Date $t=0$ (ST)	-1	1	0
Short Term, Date $t=1$ (ST)	0	-1	1

Table 5.1: Payoffs in Diamond and Dybvig (1983).

We seek to capture **the liquidation decision in $l \in [0,1]$** (at $t = 1$). We also have:

$$L \leq 1 < R. \tag{5.1}$$

This allows to capture the productive storage contrasted with **the costly short-term liquidation**.

There is a measure-one continuum of consumers who maximise their expected utility:

$$U(c_1, c_2, \theta) = (1-\theta)u(c_1) + \theta u(c_2), \tag{5.2}$$

which is **subject to a preference shock**:

$$\theta = \begin{cases} 0 & \text{w.p. } \gamma \\ 1 & \text{w.p. } 1-\gamma, \end{cases} \tag{5.3}$$

Note that they are iid, realised at $t = 1$.

You could think of this as of a model with "early" and "late" dyers. We also have that $u' > 0$ and $u'' < 0$, with the $RRA > 1$:

$$-c\frac{u''}{u'} > 1 \quad \forall c > 0. \tag{5.4}$$

Everyone gets an endowment of one unit at $t = 0$.

The Allocative Problem. How consumers' funds should be allocated across the available investment technologies:

1. autarky;
2. optimal allocation with perfect information;
3. optimal allocation with asymmetric information;
4. banks (operating at $t = 0, 1, 2$); and
5. financial markets (operating at $t = 1$).

5.2.1 Autarky

The **consumption plan** is:

$$C = \left\{c_t^{\theta}\right\}_{t=1,2}^{\theta=0,1} = \begin{pmatrix} c_1^0 & c_1^1 \\ c_2^0 & c_2^1 \end{pmatrix}. \tag{5.5}$$

A **consumer must decide**:

- how to allocate their endowment of one unit across technologies: LT ($\equiv I$) and ST ($1 - I$);
- how much to liquidate, l (fraction), of LT investment at $t = 1$:
  1. $\theta = 0 \implies l = 1$, and
  2. $\theta = 1 \implies l = 0$; and
- how much to reinvest, k (fraction) of ST funds at $t = 1$:
  1. $\theta = 0 \implies k = 0$, and
  2. $\theta = 1 \implies l = 1$.

This implies that:

$$C = \begin{pmatrix} c_1^a = LI + (1-I) & 0 \\ 0 & c_2^a = RI + (1-I) \end{pmatrix}. \tag{5.6}$$

We can plug this into the expected utility calculation:

$$\mathbb{E}\, U = \gamma u\left[LI + (1-I)\right] + (1-\gamma)u\left[RI + (1-I)\right]. \tag{5.7}$$

This leads us to the **first-order condition**:

$$-\gamma u'\left(c_1^a\right)(1-L) + (1-\gamma)u'\left(c_2^a\right)(R-1) = 0. \tag{5.8}$$

The LHS can be conveniently interpreted. Investing an extra unit in the $t = 2$ consumption, we lose a fraction of the $t = 1$ consumption's utility, discounted by the probabilities of survival. We can interpret it in terms of the**marginal rate of substitution and marginal rate of transformation**:

$$\mathrm{MRS}_1^2 = \frac{\gamma}{1-\gamma}\frac{u'\left(c_1^a\right)}{u'\left(c_2^a\right)} = \frac{R-1}{1-L} = \mathrm{MRT}_1^2. \tag{5.9}$$

5.2.2 Optimal Allocation with Perfect Information

Consumer types are publicly observable at $t = 1$ with fractions γ and $1-\gamma$ of types 0 and 1, respectively.

An **optimal insurance contract can be implemented**. It will set:

$$C = \begin{pmatrix} c_1^\star & 0 \\ 0 & c_2^\star \end{pmatrix}, \tag{5.10}$$

which maximises:

$$\mathbb{E}\, U = \gamma u\left(c_1^\star\right) + (1-\gamma)u\left(c_2^\star\right). \tag{5.11}$$

with the **restrictions**:

$$\gamma c_1^\star = lLI + (1-k)(1-I);\ \text{and} \tag{5.12a}$$

$$(1-\gamma)c_2^\star = (1-l)RI + k(1-I), \tag{5.12b}$$

where $I, k, l \in [0, 1]$.

Risk-pooling allows to set:

$$l = k = 0. \tag{5.13}$$

This means that:

- **ST consumption is provided through ST investments**; and
- **LT consumption is provided through LT investments**.

The resulting constraints are:

$$\gamma c_1^\star = 1 - I;\,;\text{and} \tag{5.14a}$$

$$\frac{1}{R}(1-\gamma)c_2^\star = I. \tag{5.14b}$$

I can add them up:

$$\gamma c_1^\star + \frac{1}{R}(1-\gamma)c_2^\star = 1. \tag{5.15}$$

This looks like a budget constraint.

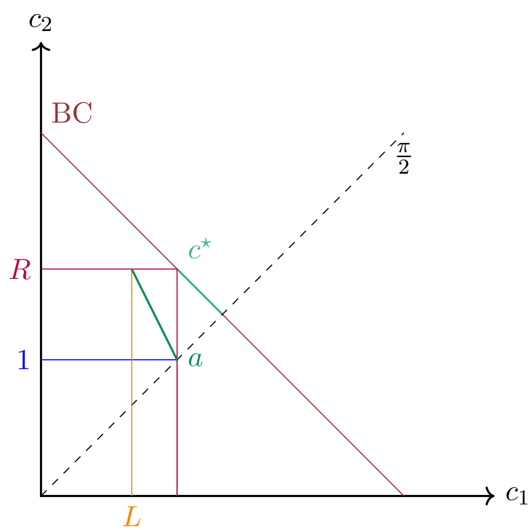


Figure 5.1: Comparing Perfect Allocation to Autarky.

Notice that the green line can be reached with the autarky. Irrespective of one's preferences, it's dominated by anything on the perfect information budget constraint. The sea green line is where the optimum lies for the perfect information case.

The Lagrangian is as follows:

$$\mathcal{L} = \gamma u\left(c_1^\star\right) + (1-\gamma)u\left(c_2^\star\right) + \lambda\left[1 - \gamma c_1^\star - \frac{1}{R}(1-\gamma)c_2^\star\right]. \tag{5.16}$$

The first-order condition yields:

$$\frac{u'\left(c_2^\star\right)}{u'\left(c_1^\star\right)} = \frac{1}{R}. \tag{5.17}$$

Under **the $RRA > 1$ assumption (equation (5.4))**, we have:

$$1 < c_1^\star < c_2^\star < R. \tag{5.18}$$

If **$RRA = 1$ (log-utility)**, we have:

$$c_1^\star = 1; \text{ and} \tag{5.19a}$$
$$c_2^\star = R. \tag{5.19b}$$

With **infinite risk aversion (*Leontief* preferences)**, we have:

$$c_1^\star \to c_2^\star. \tag{5.20}$$

5.2.3 Optimal Allocation with Asymmetric Information

At $t = 1$ each consumer observes their type, nobody else does. We should add IC or **self-selection constraints to the previous problem**! We need to introduce the following constraints (IC):

$$u\left(c_1^\star\right) \geq u\left(0\right); \text{ and} \tag{5.21a}$$
$$u\left(c_2^\star\right) \geq u\left(c_1^\star\right). \tag{5.21b}$$

The **first best solution is incentive compatible**!

Consumers do not hesitate to reveal their type, θ. The **investment policy** is:

$$1 - I^\star = \gamma c_1^\star; \text{ and} \tag{5.22a}$$
$$I^\star = \frac{\left(1-\gamma\right) c_2^\star}{R}. \tag{5.22b}$$

There must be an **institution that can implement the arrangement as a Nash equilibrium**! Diamond and Dybvig (1983) argue that banks offering demand deposit contracts can do it. We will also prove that a financial market which opens at t = 1 cannot do it.

5.2.4 Banks (Offering Demand Deposit Contracts)

Banks invest in ST and LT technologies but guarantee a reasonable return (larger than ST technology) to consumers who cash in early. To tackle this, we introduce a deposit contract with **sequential service constraint**:

- consumers have the right to withdraw $D > 1$ at $t = 1$ 1 per each unit of funds deposited at $t = 0$;
- withdrawal tenders are served sequentially (in random order) until the bank exhausts its assets; and
- mutually-owned bank: others obtain a pro-rata share in the assets remaining at $t = 2$.

This allows us to generate a bank run.

The **payoff to a depositor depends on position in the queue**:

- mass of depositors attempting to withdraw, $F \in [\gamma, 1]$; and
- mass of depositors served before on, $f \in [0, f]$.

The **withdrawal payoff is**:

$$P_1 = \begin{cases} D, & fD \leq 1 - I + LI = 1 - (1-L)I + 1 \\ 0, & fD > 1 - (1-L)I. \end{cases} \tag{5.23}$$

Hence, we have:

$$\exists\, \bar{F} \equiv \frac{1-(1-L)I}{D} \in (0,1) : P_1 = D\ f \leq F \iff F \leq \bar{F}. \tag{5.24}$$

This effectively implies that there exists the **maximum queue size for which the bank is not going insolvent**.

Provided that the queue's size satisfies: $FD \geq 1 - I$, the non-withdrawal payoff is:

$$P_2 = \max\left\{0, \frac{R}{1-F}\left[I - \frac{FD-(1-I)}{L}\right]\right\} \tag{5.25}$$

This implies:

$$P_2 > 0 \iff F < \bar{F}. \tag{5.26}$$

For $(D, I) = (c_1^\star, I^\star)$, there are two purse strategy equilibria.

Efficient Equilibrium. We consider only purse-strategy equilibria.[5.1] Only consumers with $\theta = 0$ attempt to withdraw.

$$F = \gamma \implies \tag{5.27a}$$
$$FD = \gamma c_1^\star = 1 - I^\star \implies \tag{5.27b}$$
$$P_1 = D = c_1^\star \; \forall \, f \leq F \implies \tag{5.27c}$$
$$P_2 = \frac{RI^\star}{1-\gamma} = c_2^\star > P_1. \tag{5.27d}$$

This is an **optimal allocation**! Note that this game has a strategic complementarity.[5.2]

Bank Run Equilibrium. Both consumer types attempt to withdraw:

$$F = 1 > \bar{F} \implies \tag{5.28a}$$
$$\underbrace{FD = c_1^\star > 1}_{\text{Optimal to run.}} \implies \tag{5.28b}$$
$$P_1 = \begin{cases} c_1^\star & f \leq \frac{1-(1-L)I^\star}{c_1^\star} \\ 0 & f \leq \frac{1-(1-L)I^\star}{c_1^\star} \end{cases} \implies \tag{5.28c}$$
$$P_2 = 0. \tag{5.28d}$$

5.2.5 Financial Markets Opening at $t = 1$

Suppose no bank is in place at $t = 0$. So, consumers have to directly invest in LT/ST. At $t = 1$, it is **possible to sell shares on future returns of LT investments** (shares $\simeq$ units of future consumption).

Denote by p the price at $t = 1$ of a unit of consumption at $t = 2$. Consider the equilibrium in the market for future consumption units at $t = 1$. Consumers with $\theta = 0$ definitely want to sell their shares. Consumers with $\theta = 1$ have the chance of selling their shares and investing ST until $t = 2$.

[5.1] Using the sub-perfect Nash equilibrium, we could rule out the inefficient equilibrium. However, bank runs still take place so it makes no sense to use such a strong equilibrium concept.

[5.2] The response function is positive. That is, if other people join the queue, you want to join it as well.

It can be shown that:

$$p = \frac{1}{R}; \tag{5.29a}$$

$$C = \begin{pmatrix} c_1^{FM} = 1 & 0 \\ 0 & c_2^{FM} = R \end{pmatrix}; \text{ and} \tag{5.29b}$$

$$\underbrace{\mathbb{E}\, U^A < \mathbb{E}\, U^{FM} < \mathbb{E}\, U^\star}_{RRA>1}. \tag{5.29c}$$

5.3 The Problem of Bank Runs

If a **bank run is anticipated**, the **bank run occurs** ("sun-spot equilibrium"): demand deposit contract fails to provide optimal risk-sharing. If anticipated, never started! If not, inefficient liquidation and very uneven distribution of losses!

The potential remedies are as follows.

1. **Suspension of Convertibility**: Once $F = \gamma$, the bank closes its windows. Those who do not withdraw have their returns guaranteed, so consumers with $\theta = 1$, do not withdraw. With uncertainty about γ, some liquidity needs would not be properly satisfied.

2. **Lender of Last Resort Facilities (LOLR)**: Bank with withdrawals in excess of $\gamma c_1^\star$ might borrow from other bank/agent holding liquidity at $t = 1$, using LT investments as collateral. It could pay up to $\frac{c_2^\star}{c_1^\star}$. There are possible problems: doubts about quality of LT investments and liquidity shocks (or lack of confidence) may be positively correlated across banks (due to panics, systemic risk, etc.). Monetary authorities may issue fiduciary money instead of accumulating "liquidity" in advance.

3. **Deposit Insurance**: Withdrawals beyond certain level are covered by government. Those who do not withdraw have their returns guaranteed, so consumers with $\theta = 1$ do not withdraw. With uncertainty about γ, arrangement may require tax funding. Its effectiveness requires the government itself to be considered solvent.

5.4 Dealing with Multiplicity (Vives, 2014)

[MISSING REFERENCES HERE]

We cover a stylised model of crises in games with strategic complementarities tradition, including the so-called **global games**. This provides a **compact summary of the lessons extracted from recent literature**.

There are **insightful discussion on uniqueness vs. multiplicity of equilibria** in symmetric binary action games of strategic complementarities with incomplete information.

Consequently, we analyse implications of a bank-run model for liquidity and solvency regulation.

5.4.1 The Vives (2014)'s Model in Nutshell

Again, we have $t = 0, 1, 2$.

Consider $t = 0$. The following table summarises it.

Assets	Liabilities
Cash: M Risky Assets: I	ST Debt: D_0 Stable Funding: E

Table 5.2

Consider $t = 1$. The **ST debt-holders observe private iid signals**:

$$s_i \sim \mathcal{N}\left(\theta, \tau_\epsilon^{-1}\right) \tag{5.30}$$

about θ:

$$\theta \sim \mathcal{N}\left(\mu_\theta, \tau_\theta^{-1}\right). \tag{5.31}$$

Fraction y of ST debt-holders decide to cancel their position. They are entitled to recover D. The bank accommodates using M liquidating I at value:

$$\frac{\theta}{1+\lambda} \tag{5.32}$$

per unit. The bank liquidates I at a loss which means that we need to introduce the liquidation cost, λ.[5.3]

Consider $t = 2$. Define M' and I' as what's left after the cancellations at $t = 1$.

Assets	Liabilities
M' $\theta I'$	$\min\{(1-y)D, M' + \theta I'\}$ $\max\{M' + \theta I' - 1 - y)D, 0\}$

Table 5.3

Whether the bank is "bankrupt" at $t = 2$ depends on fundamentals, θ, and cancelations, y. There are **two critical values** ($\bar{\theta}$ and $\underline{\theta}$).

- For $\theta \geq \bar{\theta}$, any y can be met and the bank is **supersolvent**.
- For $\theta < \underline{\theta}$, the bank is bankrupt even if $y = 0$ - it is **bankrupt**.
- For $\theta \in \left[\underline{\theta}, \bar{\theta}\right)$, there is a critical $h(\theta) \in \left(\frac{M}{D}, 1\right)$ such that the bank is **solvent whenever $y \leq h(\theta)$**. We say that the bank is **vulnerable**.

We can **find the above thresholds in the following way**:

$$\underline{\theta}: \quad \underline{\theta} I = D - M; \tag{5.33a}$$

$$\bar{\theta}: \quad \frac{\bar{\theta}}{1+\lambda} I = D - M; \text{ and} \tag{5.33b}$$

$$h(\theta): \quad D(1-h) = \theta I - (1-\lambda)(Dh - M). \tag{5.33c}$$

Cancellation decisions of ST debt-holders are made by managers with ad hoc differential reward to "cancelling":

$$\pi_1 - \pi_0 = \begin{cases} B & \text{if bankrupt;} \\ -C & \text{if not bankrupt,} \end{cases} \tag{5.34}$$

[5.3] If the run was costless, $\lambda = 0$.

where π_1 and π_0 are the profits of the manager contingent on cancelling and not cancelling, respectively. **Managers will cancel if**:

$$\mathbb{E}\left(\pi_1 - \pi_0 | s_i\right) > 0, \tag{5.35}$$

which implies:

$$\mathbb{P}\left(y > h | s_i\right) > \frac{C}{B+C} \equiv \gamma. \tag{5.36}$$

Relevant strategies are of the threshold type:

$$\text{Manager } i \text{ cancels} \iff s_i < s^\star = r\left(\hat{s}\right), \tag{5.37}$$

where $r(\cdot)$ is the best response to others' threshold $\hat{s}$. **Equilibria are the fixed points of $r(\hat{s})$** and can be described as $(\hat{s}^\star, \theta^\star)$.

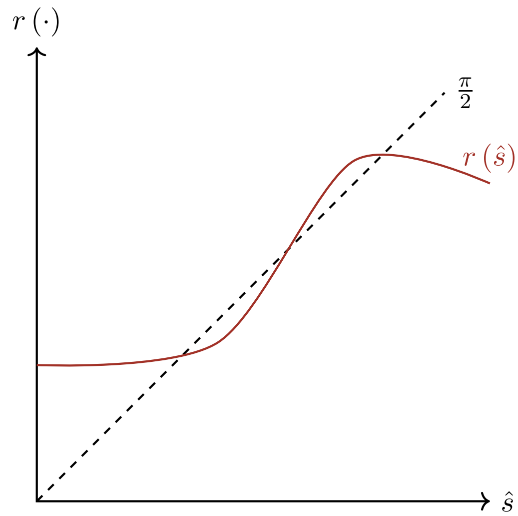


Figure 5.2: Expressing the Equilibrium as the Fixed Point of $r(\hat{s})$ - The Case of Three Equilibria.

5.4.2 Key Results, Regulatory Implications, & Limitations

Start by thinking about the number of equilibria.

- There might be **one or three equilibria**.
- The existence of **multiple equilibria depends on the slope and position of $r(\hat{s})$**.
- **If the maximum slope is less than 1, we have a unique equilibrium**.
- An equilibrium involves meaningful measures of:

$$\mathbb{P}\left(\text{run}\right) = p \tag{5.38a}$$

$$\mathbb{P}\left(\text{(pure) insolvency}\right) = q \tag{5.38b}$$

$$\mathbb{P}\left(\text{illiquidity}\right) = p - q. \tag{5.38c}$$

- Comparative statics can be analyzed for all relevant equilibria, especially if out-of-equilibrium adjustment is "adaptive".

- **Illquiditiy rises with $\downarrow \gamma$, $\downarrow \mu_\theta$, $\uparrow \lambda$ and perhaps $\downarrow \frac{M}{D}$.**
- There are multiplier effects of public signals, μ_θ.
- There are (sometimes) paradoxical effects of "transparency" (τ_θ).

Regulatory Implications. The analysis is made without explicit social welfare function, but rather some target probability of insolvency, q, and crisis, p. The analysis is conducted without explicit consideration of banks' decision problems, but assuming imposed constraints are binding:

1. maximum on leverage: $l \equiv \frac{D}{E}$; and
2. minimum on liquidity: $m \equiv \frac{M}{D}$.

For the limit case $\tau_\varepsilon \to 0$, the limits on l and m are **the only available instruments**. The key results **highlight the usefulness of using both liquidity and solvency requirements**.

Limitations. The limitations are related to the partial equilibrium approach:

1. given distribution of θ and signal structure;
2. given the "cost of cancelling" (C) and "value of success" of success $(B + C)$

6 The Prudential Regulation of Banks: Capital Regulation

6.1 Background, Static Analysis, & Repullo and Suarez (2004)

6.1.1 Raison D'être of Capital Requirements

Banks are vulnerable. On top of that, bank failures and financial instability produce sizable costs for the financial system, taxpayers, and the economy at large.

First, liabilities are held by small, dispersed, unsophisticated or short-term investors, often implicitly or explicitly insured. This calls for **regulation and supervision** ($\simeq$ delegated monitoring).

Bank failures produce **large negative externalities** (on the payment and financial systems, public finances, and macro economy).

Banks' **maturity transformation makes them vulnerable to runs**. Some failures may be triggered by **liquidity problems**. The safety-net type solutions to liquidity problems cause or aggravate moral hazard, calling for regulation.

Capital Adequacy Capital adequacy (that is, the definition of capital, capital requirements, and corrective action) is central to contemporaneous bank regulation:

- ***Basel I*** constituted the first agreements on credit risk (1988) and market risk (1996).
- ***Basel II*** (2004) improved treatment of credit risk. It also introduced the new operating risk, role for supervision (*Pillar 2*) and market discipline (*Pillar 3*).

The deficits detected in the **Global Financial Crisis led to *Basel III*** with its:

- better definition of capital and larger requirements;
- better treatment of off-balance sheet and complex exposures;
- new regulatory buffers: CCB, CCyB, *Systemic Risk Buffer*;
- new leverage ratio requirement; and
- new TLAC and MREL requirements.

6.1.2 Alternative Views on the Role of Capital Requirements

There are **alternative views on the role of capital requirements**.

1. **Buffer View** (regulators' favourite): Equity and other junior liabilities provide 1st line of defence for more senior creditors, especially the deposit insurance provider. The capital requirements make banks less likely to fail.
2. **Incentive View** (microeconomists' favourite): conflicts of interest between debtholders and shareholders increase with "leverage". For example, see risk-shifting incentives in Jensen and Meckling (1976) and debt-overhand problems in Myers (1977). There are also problems between insiders and outsiders that decrease with "inside equity participation", as with moral hazard in monitoring in Holmstrom and Tirole (1997).

3. **Pigovian View**: Banks regard equity financing as especially costly (perhaps due to distortions such as taxes or safety net guarantees (Admati et al., 2013) or due to deeper information or agency imperfections). In risk-based capital requirements, risk-weights establish an implicit system of "regulatory prices" for risk-taking /certain types of investments. These prices could be set so as to correct the underlying externalities (pecuniary or just direct "systemic risk" externalities).

6.1.3 Capital Requirements: Basel II (and III)

In 2004, Pillar 1 of Basel II introduced a substantial **reform of the capital requirements applicable to the banking book** (to deal with credit risk).

Banks had to adopt one of two approaches:

1. **standardised**: it is based on risk weights (like Basel I), but with a finer partition of credit exposures; and

2. **internal ratings-based** (IRB): it is determined by a formula fed with characteristics of each exposure provided by external agencies or each bank's internal rating system.

There are foundation IRB estimating default probabilities (PDs) and advanced IRB estimating PDs & losses given default (LGDs).

The **IRB formula is based on a VaR-type statistical criterion**:

- guaranteeing that the required capital (γ per unit of assets) suffices to absorb "estimated" one-year credit losses with a confidence level $c = 99.9\%$; and

- its quantitative foundations come from a single-factor model due to Vasicek (2002) and adapted by Gordy (2003) (the correlation between defaults implies a fluctuation in aggregate losses).

How does it work? For each asset, i, the IRB formula determines a risk weight, RW_i. In Basel I, II, and III, the **required total capital is 8% of**:

$$\sum_i RW_i A_i, \tag{6.1}$$

where A_i is the unweighted position of asset i (net of provisions). The total capital includes tiers 1 and 2. **In Basel II and III, one needs Tier 1 ≥ 0.5 and ≥ 0.75, respectively.**

The Single Risk Factor Model. Consider a loan portfolio with a unit mass of homogeneous loans indexed by $i \in [0, 1]$ and with payoffs:

$$\begin{cases} 1 + r & \text{if } d_i = 0 \\ 1 - \lambda & \text{if } d_i = 1. \end{cases} \tag{6.2}$$

where the default of loan i ($d_i = 1$) is determined as:

$$d_i = \begin{cases} 0 & \text{if } y_i \leq 0 \\ 1 & \text{if } y_i > 0. \end{cases} \tag{6.3}$$

where:

$$y_i = \mu + \sqrt{\rho} z + \sqrt{1 - \rho}\varepsilon_i. \tag{6.4}$$

We have $\mu \equiv$ financial vulnerability; $\rho \equiv$ exposure to systematic risk; $z \equiv$ systematic risk factor; and $\varepsilon_i \equiv$ idiosyncratic risk term. We also assume:

$$z \sim \mathcal{N}(0,1) \tag{6.5a}$$

$$\varepsilon_i \overset{iid}{\sim} \mathcal{N}(0,1) \tag{6.5b}$$

$$z \perp \varepsilon_i \tag{6.5c}$$

Then, **each loan has an LGD (loss given default) of λ and a PD**:

$$p = \mathbb{P}\left(y_i > 0\right) \implies \tag{6.6a}$$

$$p = \mathbb{P}\left(y_i - \mu > -\mu\right) \implies \tag{6.6b}$$

$$p = 1 - \Phi(-\mu) = \Phi(\mu) \implies \tag{6.6c}$$

$$y_i = \mu + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon_i \implies \tag{6.6d}$$

$$y_i = \mathcal{N}(\mu, 1). \tag{6.6e}$$

By the *Law of Large Numbers*, **ex-post default rate x** (proportion of defaulting loans) equals conditional-on-z probability of default $x(z)$:

$$x(z) = \mathbb{P}\left(y_i > 0 \mid z\right) \implies \tag{6.7a}$$

$$x(z) = \Phi\left(\frac{\mu + \sqrt{\rho}z}{\sqrt{1-\rho}}\right), \tag{6.7b}$$

since:

$$y_i \mid z \sim \mathcal{N}\left(\mu + \sqrt{\rho}z, 1-\rho\right). \tag{6.8}$$

The **default rate $x(z)$ is a random variable determined by the random variable z**, which is $\mathcal{N}(0,1)$, hence its CDF $F(x) \equiv \mathbb{P}[x(z) \le x]$ can be written in terms of $\Phi(\cdot)$. We have:

$$x(z) \le x \iff \tag{6.9a}$$

$$z \le \frac{\sqrt{1-\rho}\Phi^{-1}(x) - \mu}{\sqrt{\rho}} \implies \tag{6.9b}$$

$$z \le \frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}. \tag{6.9c}$$

We can write:

$$F(x) = \Phi\left[\frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right], \tag{6.10}$$

as:

$$\mathbb{P}[x(z) \le x] = \mathbb{P}\left[z \le \frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right]. \tag{6.11}$$

As an example, the following figure illustrates the density function of x for $p = 0.1, 0.2$.

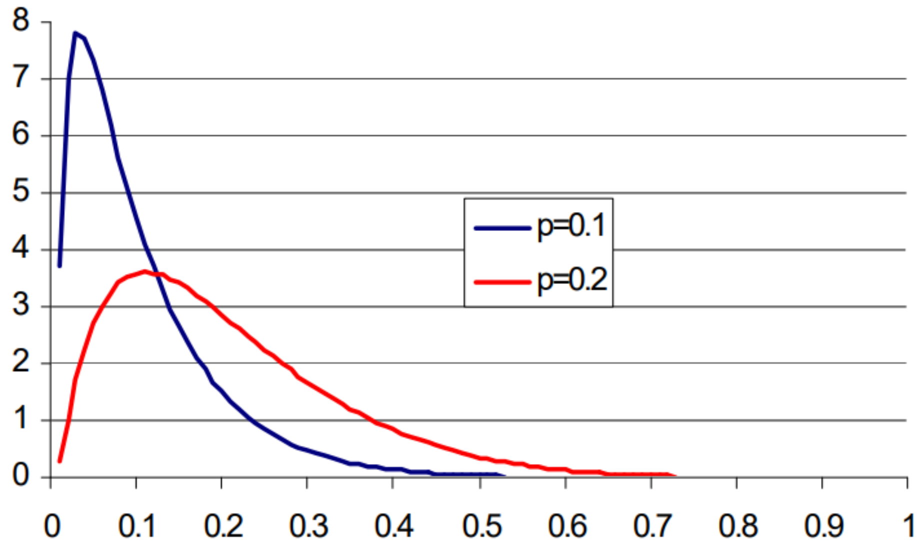


Figure 6.1: Illustrating the Density Functions for a Single Factor Model.

Consider the figure's shape when (substitute to the previous equations)

- $\rho \to 0$: we have a degenerate distribution at p;
- $\rho \to 1$: we have a mass of $1 - PD$ at 0 and mass PD at 1.

IRB Formula for Capital Requirements. Under the IRB approach, capital must cover potential losses with a confidence level c:

$$k \geq \gamma, \text{ where} \tag{6.12a}$$
$$\gamma \equiv \lambda F^{-1}(c) \implies \tag{6.12b}$$
$$\gamma = \lambda \Phi \left[z \leq \frac{\sqrt{1-\rho}\Phi^{-1}(p) - \Phi^{-1}(c)}{\sqrt{\rho}} \right]. \tag{6.12c}$$

It increases in in p, c, λ, and, typically, ρ.

For **corporate exposures with one-year maturity, the IRB prescribes a functional relationship between ρ and p**:

$$\rho = 12\% \times \left(2 - \frac{1 - \exp(-50p)}{1 - \exp(-50)} \right), \tag{6.13}$$

and fixes $c = 0.999$ (for a year horizon). The formula is subject to further adjustments for other types of exposures and maturities.

The **assumptions of the single risk factor model guarantee additivity** across sub-portfolios with different classes of loans.

1. If a loan portfolio is made of $j = 1, ...J$ classes of loans with different p_j & ρ_j parameters (& continuum of loans within each class), w_j is the weight of class-j exposures on total exposures and γ_j is class-j IRB requirement:

$$\gamma_j = \lambda_j \Phi \left[z \leq \frac{\sqrt{1-\rho_j}\Phi^{-1}(p_j) - \Phi^{-1}(c)}{\sqrt{\rho_j}} \right]. \tag{6.14}$$

2. Then, the IRB capital requirement can be expressed as:

$$\gamma = \sum_{j=1}^{J} w_j \gamma_j. \tag{6.15}$$

6.1.4 The Model of Repullo and Suarez (2004)

The **IRB capital requirements have obvious advantages**:

1. **impose higher capital charges** on riskier loan portfolios; and
2. should the underlying quantitative representation of credit risk be correct, **keep the risk of bank insolvency at very low levels**.

However, the whole construction is pretty ad hoc and its **positive and normative properties are a priori unclear**. What about the distribution of the effects across credit risk categories, impact on loan pricing (& volumes), probabilities of bank failure, and welfare trade-offs?

Repullo and Suarez (2004) **analyse some of these questions in a simple static model focused on loan pricing implications**:

- perfect competition & CRS, full deposit insurance, exogenous excess cost of bank capital (δ in the paper);
- credit losses are determined exactly as presumed in the IRB formula, giving the best chance to demonstrate/refute its internal consistency.

The technical details are as follows. The **single pricing formula** is based on the definition:

$$V_j = -k_j + \frac{1}{1+\delta} \int_0^{\hat{x}_j} \left[k_j + r_j - x_j \left(\lambda + r_j \right) \right] dF\left(x_j\right), \tag{6.16}$$

where condition $V_j = 0$ determines $r_j^\star$. Note that:

$$\hat{x}_j = \frac{k_j + r_j}{\lambda + r_j} < 1 \quad \text{if } \lambda > k_j. \tag{6.17}$$

The **social welfare** is defined as:

$$W_j = (1 - p_j)\, a - p_j \lambda - \delta k_j - \left[1 - F_j\left(\hat{x}\right)\right] C. \tag{6.18}$$

The model calibration stipulates:

- $\lambda = 0.45$;
- $\delta = 0.10$; and
- $c = 0.999$.

The value of a does not need to be calculated.

The **results identify**:

- **small impact on loan prices**;
- room for **regulatory arbitrage** (if there is a choice between the standardized and IRB approaches);
- **cross-sectional inconsistency in the capital charges**: high PDs are penalized as if bank failures provoked by portfolios of high PD loans were more costly than bank failures caused by portfolios of low PD loans.

	γ_j	Loan rates		Failure probabilities		Implicit C †
p_j	IRB	Basel I or Standard	IRB	Basel I or Standard	IRB	IRB
0.03	0.62	0.81	0.08	0.00	0.08	33.89
0.05	0.92	0.82	0.11	0.00	0.07	48.35
0.10	1.54	0.85	0.20	0.00	0.07	76.43
0.20	2.49	0.89	0.34	0.00	0.07	116.57
0.50	4.40	1.03	0.67	0.00	0.06	190.82
1.00	6.31	*1.26*	*1.09*	*0.02*	*0.05*	265.17
2.00	8.56	*1.73*	*1.79*	*0.06*	*0.04*	373.40
4.00	11.51	2.70	3.07	0.23	0.03	592.58
7.00	15.24	4.23	5.03	0.85	0.02	964.57
10.00	18.56	5.83	7.06	2.01	0.01	1346.49

Figure 6.2: The Results in Repullo and Suarez (2004).

Note that this implies that riskier loans can be associated with lower probabilities of bankruptcy. This has one reason: interest profits can offset the risk of going bankrupt. This implies that the regulators are not so conservative with risky loans and extremely conservative with safe loan banks.

6.2 Dynamics: Procyclical Effects of Capital Regulation (Repullo and Suarez, 2013)

6.2.1 Why Procyclicality?

For fixed K, **capital requirements impose an upper limit on a bank's risky investments (say, loans L)**:

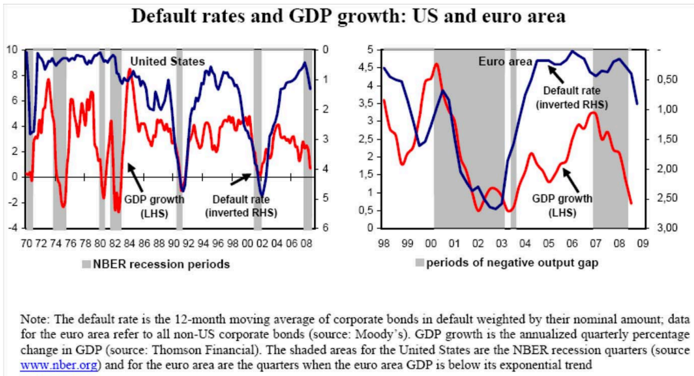
$$K \geq \gamma L \implies \tag{6.19a}$$

$$L \leq \frac{K}{\gamma}. \tag{6.19b}$$

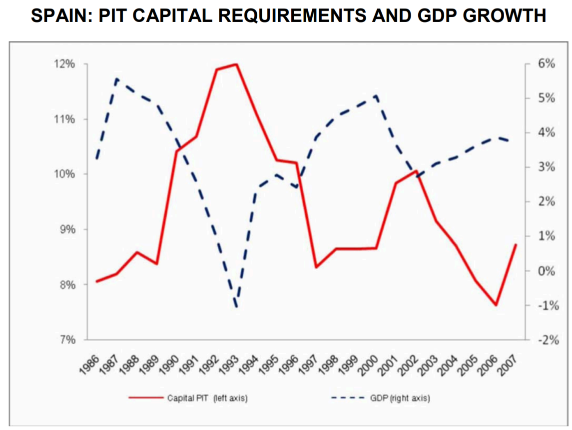
The **procyclicality of bank capital regulation is related to the effects of the business cycle on this upper limit**. For example, in the Basel world, $\gamma : 0.08 \to 0.12$ **when times are bad**. This means that banks can lend much less or need to raise very much capital.

In recessions, loan defaults and other losses may turn profits into losses. **Equity issues become even more expensive than usual**. Prospective PDs and LGDs increase!

1. In the Basel I framework, recessions imply $K \downarrow$.
2. In the Basel II framework, **recession imply $K \downarrow$ and $\gamma \uparrow$. This implies a stronger effect.**



(a) Visualising Procyclicality (BdI Report).



(b) Spain: Point in Time (PIT) Capital Requirements and GDP Growth (Repullo et al., 2010).

Figure 6.3: Visualising Problems with Procyclicality.

For capital requirements to have on significant impact on aggregate credit, two conditions must be met.

C1: Some **banks must find it difficult to issue equity when needed**.

C2: Some **borrowers must find it difficult to switch from a constrained bank to other financing sources**.

For example, see Blum and Hellwig (1995) and Kashyap and Stein (2004)

Additionally, banks' endogenously determined capital buffers must not be sufficient to neutralize the procyclicality of banks' lending capacity.

6.2.2 The Model of Repullo and Suarez (2013)

The **research questions** are as follows.

1. Can **endogenously determined capital buffers neutralize the inherent procyclicality of bank capital regulation**?
2. What are the **normative trade-offs involved in the attempt to correct this procyclicality**?

Their relationship banking model captures C1-C2 in a way that produces a tractable OLG structure. Borrowers need loans for **two consecutive periods** and become dependent on initial lenders. Banks with **ongoing relationships cannot issue equity** (they only access the equity market every other date). Both frictions might be due to lemon problems arising once banks get private information about their borrowers!

Other features of the model include:

- **perfect competition** (and **free entry**) in the market for first-period loans, which implies endogenous loan rates;

- banks may hold capital in excess of the minimum requirements, which implies **endogenous capital buffers**;
- loan losses are as in (recalibrated) model behind the IRB approach, which yields realistic quantitative predictions;
- **business cycle captured as a two-state Markov chain for loans' PDs**, making it tractable and easy to "calibrate".
- deposit insurance, risk free rate =0, and excess cost of equity $\delta > 0$.

Details . We consider a discrete-time, infinite horizon economy with OLG of entrepreneurs (with projects to be financed on two consecutive dates). Date t projects are as follows:

$$\text{At t: } (-1) \to \text{At t+1: } \begin{cases} 1+a & \text{w.p. } 1-p_s \\ 1-\lambda & \text{w.p. } p_s \end{cases} \tag{6.20}$$

where $s \in \{l, h\}$ is the state of the economy at date t, with $p_l < p_h$.

We observe the **correlation between failures results in aggregate default rate x_t with**:

$$F_s(x_t) = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}(x_t) - \Phi^{-1}(p_s)}{\sqrt{\rho}}\right]. \tag{6.21}$$

PDs follow a **2-state Markov chain** with:

$$q_{ss'} = \mathbb{P}\left(s_{t+1} = s' | s_t = s\right) \tag{6.22}$$

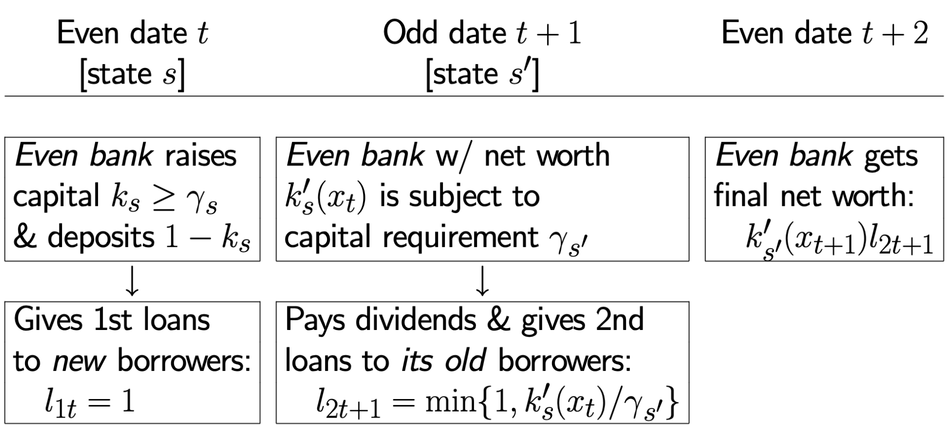


Figure 6.4: OLG Structure in Repullo and Suarez (2013).

We have the following **law of motion of capital**:

$$k_s'(x_t) = k_s - r_s - x_t(\lambda + r_s) - \mu, \tag{6.23}$$

where $\mu \equiv$ the loan origination cost.

The Representative Bank's Problem. At the interim date, the **bank's situation is crucially affected by the realised default rate**.

- If $x_t \leq \hat{x}_{ss'}$, the bank can lend as much as desired (l_{2t+1}).
- If $\hat{x}_{ss'} < x_t < \hat{x}_s$, the bank can only lend $\frac{k_s'(x_t)}{\gamma_{s'}}$.
- If $x_t > \hat{x}_s$, the bank is insolvent.

Let $v_s(k_s, r_s)$ denote the net present value of shareholders' expected payoffs at the initial date:

$$v_s(k_s, r_s) = \beta\,\mathbb{E}_t\left[v_{ss'}(x_t)\right] - k_s, \tag{6.24}$$

where $\beta = \frac{1}{1+\delta}$.

Equilibrium. There is a **sequence of state-contingent pairs $(k_s^\star, r_s^\star)_{s=l,h}$ which satisfy**:

1. the **bank optimisation**:
$$k_s^\star = \arg\left\{\max_{k_s\in[\gamma_s,1]} v_s(k_s, r_s^\star)\right\};\ \text{and} \tag{6.25}$$
2. **banks' zero net value condition**:
$$v(k_s^\star, r_s^\star) = 0. \tag{6.26}$$

Remember that $v(\cdot)$ represents the present value of expected shareholders' payoff.

To analyse the equilibrium, remember the following. **Banks' decision problems may have interior or corner solutions. One can obtain analytical comparative statics for $(k_s^\star, r_s^\star)_{s=l,h}$**. However, numerical evaluation is required for cross-state and cross-regime comparison, and measurement of expected credit rationing.

6.2.3 Main Results

Determination of equilibrium buffers involves a simple trade-of: the cost of excess capital vs. the probability of satisfying loan demand. The **impact of capital requirements on them is analytically ambiguous**. There are two effects at play.

1. **precaution effect**: larger future requirements call for greater buffers;
2. **profitability effect**: larger future requirements make future lending less profitable.

Under a realistic parameterisation, we have the following findings. Banks react to Basel II by increasing their buffers. Basel II is more procyclical than Basel I but makes banks safer and is generally superior in welfare terms. For a high social cost of bank failure, the socially optimal requirements are higher but less cyclically varying.

Parametrisation. The baseline parameter values are as follows.

a	λ	μ	δ	p_l	p_h	q_{ll}	q_{hh}	ρ
0.04	0.45	0.03	0.08	1.0%	3.6%	0.80	0.64	0.174

Their calibration is **based on US pre-crisis data** (with $\rho >$ regulatory model). The transition probabilities reflect observed default cycles (high and low PD states last 2.8y and 5y on average). PDs imply an average Tier 1 capital charge of 4 per cent in Basel II:

$$\gamma_l = 3.2\% \wedge \gamma_h = 5.5\%. \tag{6.27}$$

Capital and γs in the model refer to Tier 1 capital only, but the cost of the overall requirement is captured by imputed δ.

Numerical Results. Consider loan rates and capital buffers (%).

	Rates		Capital		Buffers	
	r_l	r_h	k_l	k_h	Δ_l	Δ_h
Basel I	1.3	3.2	6.7	6.3	2.7	2.3
Basel II	1.3	3.3	6.9	6.7	**3.8**	**1.2**
Laissez-Faire	0.8	2.5	4.2	3.4	4.2	3.4

Remember that:

$$\Delta_s = k_s - \gamma_s. \tag{6.28}$$

Capital regime has a small effect on loan rates. There are sizeable buffers: **noncyclical under Basel I; and higher in expansions under Basel II**.

Credit Crunches. Consider the **expected credit rationing**.

	$l \to l$	$l \to h$	$h \to h$	$h \to l$	Uncond.
Basel I	2.4	2.4	9.3	9.3	4.9
Basel II	0.9	**12.6**	**12.4**	5.3	5.6
Laissez-Faire	3.2	3.2	17.2	17.2	8.2

It is given by:

$$\text{Expected Credit Rationing } = 1 - \mathbb{E}_t\, l_{2t+1}. \tag{6.29}$$

Basel II has procyclical effects:

- we observe increases rationing in $s' = h$, especially after $s = l$; and
- we observe decreases rationing in $s' = l$, especially after $s = h$.

Unconditionally, Basel II increases expected credit rationing!

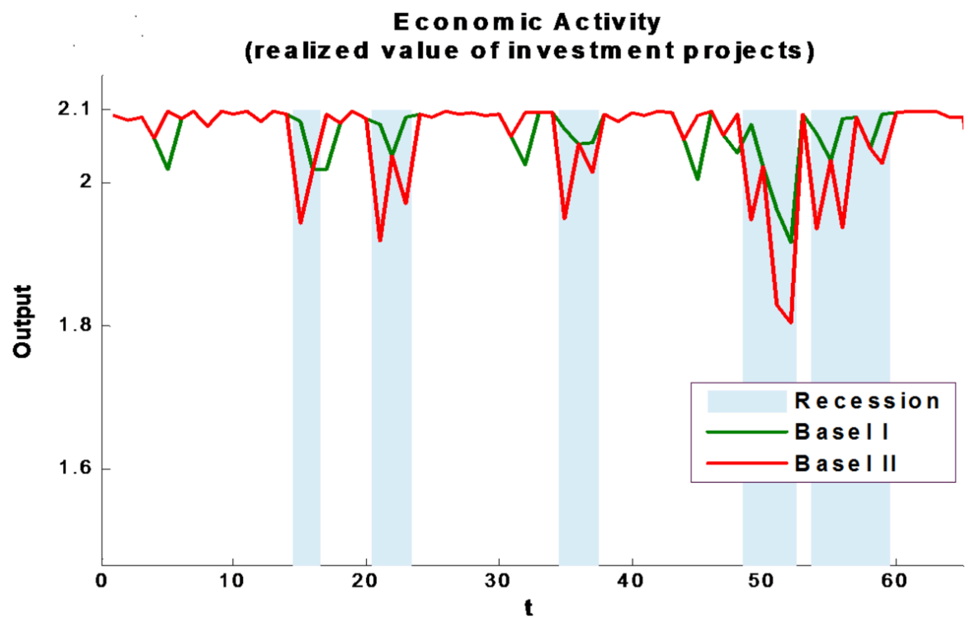


Figure 6.5: Visualisation Procyclical Nature of Basel II.

Probability of Bank Failure.

	1st Period Banks		2nd Period Banks		
	$s = l$	$s = h$	$s = l$	$s = h$	Uncond.
Basel I	0.20	**2.87**	0.03	**1.50**	0.86
Basel II	0.16	**2.25**	0.05	**0.76**	0.61
Laissez-Faire	3.7	17.15	0.55	10.21	**6.09**

γs have a clear impact on the risk of bank failure. **Basel II makes banks safer than Basel I, especially in state $s = h$.**

6.2.4 Welfare Analysis

Social welfare is measured as the **sum of the expected net present value of income extracted, directly or through banks, from investment projects**. Repullo and Suarez (2013) include two important additions:

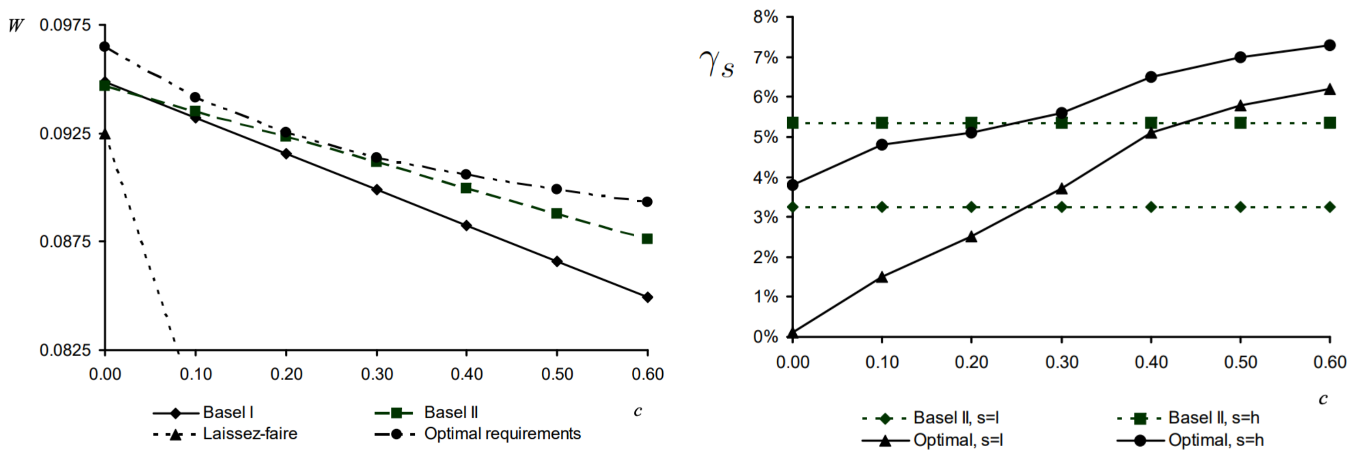
1. **social cost of bank failure** $\simeq$ proportion $c \in [0, 0.60]$ of failed banks' assets, accounting for negative externalities on payment and financial system, public finances & macroeconomic climate; and

2. **non-pledgeable success return** $b = 0.04$ appropriated by entrepreneurs whose projects are undertaken:

$$W_{ss'} = U_{ss'} + DI_{ss'} + BF_{ss'}, \tag{6.30}$$

where $U_{ss'} \equiv$ entrepreneurs' returns; $DI_{ss'} \equiv$ cost of deposit insurance to government; and $BF_{ss'} \equiv$ social cost of bank failure.

Social Welfare vs. Social Cost of Bank Failure c. Repullo and Suarez (2013) investigate these costs under different regulatory regimes.



(a) Social Welfare vs Social Cost of Bank Failure, c.

(b) Optimal γs vs Social Cost of Bank Failure, c.

Figure 6.6: Figures from Repullo and Suarez (2013).

We can see that **Basel 1 $\gtrsim$ Basel II for very low c**. Basel II requirements are socially optimal CRs for $c \simeq 0.25$. For **larger c, requirements should be larger and less cyclically varying**.

6.2.5 Further Research & Limitations

1. They could incorporate elastic loan demands. It is trivial, conditional on having estimates of them. It is interesting for the analysis of macroeconomic implications.

2. One could go for endogenising the excess cost of bank capital

3. One should capture the macroeconomic feedback!

7 The Prudential Regulation of Banks: Liquidity Regulation

7.1 Introduction

Assets	Liabilities
HQLA (m) Illiquid Assets (a)	ST Funding (x) Stable Funding (e)

Table 7.1: A Stylised Balance Sheet of a Bank. (HQLA ≡ high-quality liquid assets)

The Basel III regulatory regime specifies that:

- the LCR (liquidity coverage ratio) requirement:

$$m \geq \phi x; \text{ and} \tag{7.1}$$

- the NSFR (net stable funding ratio) requirement:

$$e \geq \lambda a. \tag{7.2}$$

Background. The banks perform **maturity transformation**. The provision of liquidity is one of banks' core functions. Maturity mismatches and asset illiquidity leave banks exposed to refinancing risk and self-fulfilling runs. Ordinary withdrawals and refinancing needs are covered with liquid reserves, interbank borrowing and discount window facilities Additional backstops include deposit insurance, lending of last resort (LLR) and suspension of convertibility/payments. Before the crisis, there was no consensus on the need or way to address liquidity regulation.

Liquidity problems witnessed during the crisis (in ABCP, repo and interbank markets) reopened debates on:

1. the need to regulate banks' liquidity;
2. the relative merits and demerits of regulatory alternatives; and
3. the relationship with other aspects of the prudential system: capital requirements, LLR, CCPs, etc.

As **part of the Basel III package**, the *Basel Committee for Bank Superivision* (2010) proposes two new requirements to deal with liquidity risk:

1. **liquidity coverage ratio** (LCR): HQLA to cover estimated net outflows in a one-month stress scenario; and
2. **net stable funding ratio** (NSFR): Stable funding to cover investment in illiquid assets

Table 1
Structure of Eurozone banks' outstanding debt in 2006

Debt category	Amount (b€)	Fraction (%)	Weight in overall δ	Assigned δ_i	Implied $1/\delta_i$
Retail deposits	5,821	27.4			
Wholesale debt	15,404	72.6	1.000	0.359	2.8
- Deposits & repos from banks	7,340	34.6	0.476	0.560	1.8
- Commercial paper & bonds	4,463	21.0	0.290	0.027	37.0
- Other deposits	2,906	13.7	0.189	0.336	3.0
- Other repos	245	1.2	0.016	0.693	1.4
- Eurosystem lending	451	2.1	0.000	–	–
Total outstanding debt	21,225	100.0			

This table describes the structure of Eurozone banks' outstanding debt in 2006 and assigns a maturity parameter δ_i to each of the wholesale debt categories based on existing breakdowns by maturity ranges. For details on the underlying data and the assigment of δ_i, see Appendix A. The value of δ_i assigned to Wholesale debt ("the overall δ") is a weighted average of the δ_i assigned to its components (excluding Eurosystem lending for which no maturity data is publicly available). One model period is one month.

Figure 7.1: Structure of the Eurozone Banks' Outstanding Debt (Segura and Suarez, 2017).

7.2 Literature Review

The **pre-crisis research on liquidity regulation** pointed to positive incentive effects coming from banks' vulnerability (Calomiris and Kahn, 1991; Diamond and Rajan, 2005; Chen and Hasan, 2006) The literature has shown a **potential redundancy of liquidity regulation in the presence of an active LLR** (Flannery, 1996; Freixas et al., 2000). Some of these views are not very popular right now.

However, there are inefficiencies in the private provision of liquidity (Bhattacharya and Gale, 1987; Holmström and Tirole, 1998). There are potential welfare-enhancing effects of liquidity regulation e.g. by reducing fire-sale effects (Allen and Gale, 2004) and panic risks (Rochet and Vives, 2004).

The **recent papers emphasize (negative) externalities**:

- contagion through interbank markets (Allen and Gale, 2004);
- generic systemic externalities (Perotti and Suarez, 2011);
- pecuniary externalities (Stein, 2012; Segura and Suarez, 2017);
- impact on the broad economy (Kroszner et al., 2007); and
- distortions due to the safety net (Farhi and Tirole, 2012; Santos and Suarez, 2019).

7.3 The Model of Perotti and Suarez (2011)

The paper studies the effectiveness of different approaches to the regulation of banks' refinancing risk. **Short-term (ST) funding helps banks expand their credit activity** but makes them more vulnerable to systemic liquidity problems. The reason is the fire sales or counterparty risk externalities. In the absence of regulation, **banks rely excessively on ST funding**.

Perotti and Suarez (2011) provide a theoretical assessment of the performance of:

- **Pigovian taxes**: levies on banks' short-term funding; and

- **quantity regulations**: ratios introduced by Basel II.

The analysis stresses bank heterogeneity and potential constraints to making regulation contingent on the relevant bank characteristics. Depending on the dominant source of heterogeneity, the socially efficient solution may be attained with Pigovian taxes, quantity regulations or a combination of both.

There are **two main sources of heterogeneity**:

1. **credit ability and quality of investment opportunities**: better banks want to expand more; and
2. **incentives to take risks**: overconfident managers and less capitalized banks want to "gamble" more.

The key findings:

1. a strong case for a simple Pigovian tax when banks differ in credit ability/quality of investment opportunities;
2. a strong case for quantity regulation (net stable funding ratio) if banks differ in risk-shifting incentives;
3. a scepticism about the effectiveness and efficiency of a liquidity coverage ratio (in both scenarios); and
4. the potential optimality of a mixed approach if the two sources of heterogeneity are important.

7.3.1 Baseline model: Heterogeneity in Credit Ability

Setting. Consider a **simple one-period model** in which agents are risk-neutral. There is a single round of ST funding decisions. Banks are characterized by type $\theta \in [0, 1]$, distributed with density $f(\theta)$ across banks.

Bank owners make an ST funding decision $x \in [0, \infty]$ to expand activities. They maximise bank value (NPV of their claims).

Other investors: (i) could invest at some exogenous market rates, and (ii) provide funding at competitive terms.

Without regulation, **bank value** is:

$$v\left(x, X, \theta\right) = \pi(x, \theta) - \varepsilon(x, \theta)c(X), \tag{7.3}$$

where:

- $\pi(x, \theta) \equiv$ value generated in the absence of systemic crisis risk with $\pi_x, \pi_\theta > 0$;
- $\varepsilon(x, \theta) \equiv$ **exposure to crisis** due to (x, θ) with $\varepsilon_x > 0$ and $\varepsilon_\theta \leq 0$; and
- $c(X) \equiv$ **crisis cost determined by systemic risk** with $C_X > 0$ and:

$$c(X) = \int_0^1 x(\theta)f(\theta)d\theta. \tag{7.4}$$

Other assumptions, including $\pi_{x\theta} > 0$, $\varepsilon_{x\theta} \leq 0$, $c'' \geq 0$, need to be made to obtain an interior solution.

Hence, **net marginal benefit from ST funding** x is:

1. decreasing in x and X; and

2. increasing in θ.

The social welfare is approached as follows. As **other investors obtain zero NPV from the banks**, a natural measure of social welfare is just:

$$W = \int_0^1 v\left[x(\theta), X, \theta\right] f(\theta) d\theta = \int_0^1 \left[\pi(x,\theta) - \varepsilon(x,\theta) c(X)\right] f(\theta) d\theta. \tag{7.5}$$

The total NPV of cash flows received by bank owners!

7.3.2 Equilibrium vs. Social Optimum

Consider **the unregulated equilibrium**:

$$x^e(\theta) = \arg\max_x \left\{\pi(x,\theta) - \varepsilon(x,\theta) c\left(X^e\right)\right\} \quad \forall\, \theta \in [0,1]; \text{ and} \tag{7.6a}$$

$$X^e = \int_0^1 x^e(\theta) f(\theta) d\theta. \tag{7.6b}$$

The FOCs for interior solution (X^e is treated as a constant):

$$\pi_x\left(x^e(\theta), \theta\right) - \varepsilon_x\left(x^e(\theta), \theta\right) c\left(X^e\right) = 0. \tag{7.7}$$

The **socially optimal allocation is given by**:

$$\begin{gathered} \max_{x(\theta), X^\star} \left\{\pi(x,\theta) - \varepsilon(x,\theta) c\left(X^\star\right)\right\} \\ \text{s.t.} X^\star = \int_0^1 x(\theta) f(\theta) d\theta. \end{gathered} \tag{7.8}$$

The FOCs for interior solution (each x affects everybody's X):

$$\pi_x\left(x^\star(\theta), \theta\right) - \varepsilon_x\left(x^\star(\theta), \theta\right) c\left(X^\star\right) - \underbrace{\mathbb{E}_z\left[\varepsilon\left(x^\star(z), z\right)\right] c'\left(X^\star\right)}_{\text{Marginal Cost of Each } x(\theta)} = 0. \tag{7.9}$$

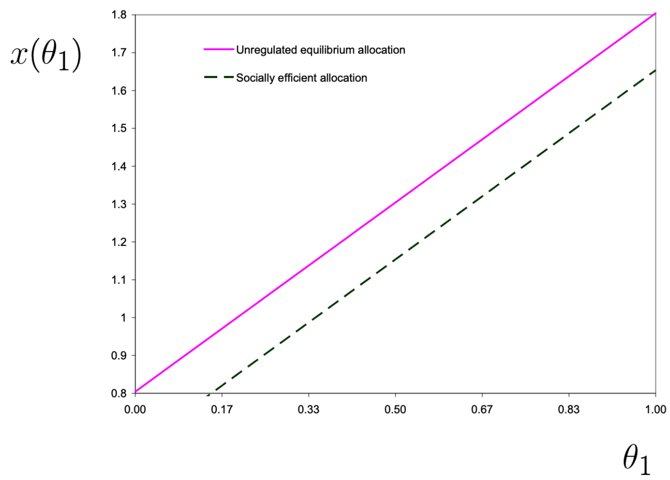


Figure 7.2: Socially Optimal vs. Laissez-Faire Equilibrium in Perotti and Suarez (2011).

Proposition 7.1. ***The equilibrium allocation is not socially efficient.*** *The systematic externalities imply that* $X^e > X^\star$.

7.3.3 The Simple Pigovian Solution

As in textbook discussions on negative production externalities, the **efficiency can be restored by imposing a Pigovian tax/charge**. The tax rate is the same as the difference between the social cost and private marginal cost.

Proposition 7.2. *With heterogeneity in investment opportunities, the* ***social efficiency of equilibrium can be restored by charging a flat tax, $\tau^\star$, on banks' ST funding.***

The equilibrium with taxes is defined by:

$$x^\tau(\theta) = \arg\max_x \{\pi(x,\theta) - \varepsilon(x,\theta)c(X^\tau) - \tau(\theta)x\} \quad \forall\, \theta \in [0,1]; \text{ and} \tag{7.10a}$$

$$X^\tau = \int_0^1 x^\tau(\theta) f(\theta) d\theta. \tag{7.10b}$$

The FOCs for interior solution are:

$$\pi_x\left(x^\tau(\theta),\theta\right) - \varepsilon_x\left(x^\tau(\theta),\theta\right) c\left(X^\tau\right) - \tau(\theta) = 0. \tag{7.11}$$

Then, if:

$$\tau^\star(\theta) = \mathbb{E}_z\left[\varepsilon\left(x^\star(z),z\right) c'\left(X^\star\right)\right], \tag{7.12}$$

the **efficiency is restored**. Importantly, $\tau^\star$ is independent of θ.

7.3.4 Quantity-Based Alternatives

Consider the **pure quantity regulation** (a la the USSR), prescribing $x^\star(\theta)$ to each θ. This would require bank-level knowledge of $\pi_x(x,\theta)$ and $\varepsilon_x(x,\theta)$. Given the strong informational requirements, this is not considered in practice. The **proposals considered in practice are ratio-based**.

The Net Stable Funding Ratio. Consider:

$$\frac{\text{Stable Funding}}{\text{Non-Liquid Assets}} \geq \text{Regulatory Minimum}, \tag{7.13}$$

where:

$$\text{Stable Funding} = \text{Equity} + \text{Customers Deposits} + \text{Other LT Debt}. \tag{7.14}$$

If the stable form of funding is taken as given, **the requirement is equivalent to the upper limit of $\bar{x}$ to ST funding**:

$$e \geq \lambda a \implies \tag{7.15a}$$
$$e \geq \lambda(x + e - m) \implies \tag{7.15b}$$
$$x \leq \frac{1-\lambda}{\lambda} e + m \equiv \bar{x}. \tag{7.15c}$$

Then, in an equilibrium with a stable funding requirement $\bar{x}$ is:

$$x^{\bar{x}}(\theta) = \arg\max_{x \leq \bar{x}} \left\{ \pi(x,\theta) - \varepsilon(x,\theta) c\left(X^{\bar{x}}\right) \right\}. \tag{7.16}$$

Then, we can **observe three cases**.

1. If $\bar{x} \geq x^e(1)$, the constraint is not binding for any θ. There is no effect!
2. If $\bar{x} \leq x^e(0)$, the constraint is binding for all θ.
3. If $\bar{x} \in [x^e(0), x^e(1)]$, we have **inefficiency and potential asymmetry**. The banks with the largest θ: $x^{\bar{x}}(\theta) = \bar{x} < x^e(\theta)$ - you constraint the largest-profit banks. Paradoxically, for unconstrained banks $x^{\bar{x}}(\theta) > x^e(\theta)$ - the worse banks put more quantity.

Proposition 7.3. *A net stable funding requirement may reduce X, but at* ***the cost of redistributing ST funding inefficiently across banks****.*

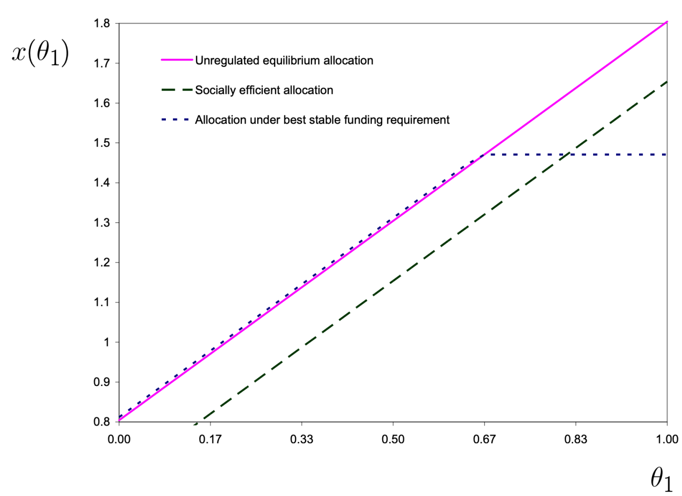


Figure 7.3: Inefficiency of Best Stable Funding Requirement (Perotti and Suarez, 2011).

The Liquidity Coverage Ratio. The **ST funding x must be backed with high-quality liquid assets** m (so as to confront a one-month disruption in markets). This can be captured like a fractional "reserve" requirement:

$$m \geq \theta x, \tag{7.17}$$

with $\theta \leq 1$. This kind of kills the banking business model of maturity transformation.

There are **two adaptations**. What matters for profitability and systemic risk are net refinancing needs:

$$\hat{x} = x - m; \text{ and} \tag{7.18a}$$
$$\hat{X} = X - M. \tag{7.18b}$$

However, holding liquidity may have an extra cost:

$$\delta = r_b - r_m \geq 0, \tag{7.19}$$

which is the **source of the deadweight loss**.

In an **equilibrium with liquidity requirement**, the following needs to be solved:

$$\begin{pmatrix} x^{\phi}(\theta) \\ m^{\phi}(\theta) \end{pmatrix} = \arg\max_{m \geq \phi x} \left\{ \pi(x-m,\theta) - \varepsilon(x-m,\theta)c\left(\hat{X}^{\phi}\right) - \delta m \right\}. \tag{7.20}$$

This is akin to the **equilibrium with tax**:

$$\tau(\theta) = \frac{\delta\phi}{1-\phi} \tag{7.21}$$

on **the ST funding**! $\delta > 0$ implies a social deadweight loss.

Proposition 7.4. ***With $\delta = 0$ ($\simeq$ normal times), ϕ is innocuous, except because it generates artificial demand for liquid assets.***

Proposition 7.5. *With $\delta > 0$, **ϕ can be set so as to seemingly replicate any flat-tax τ on the ST funding but at a deadweight cost** $(-\delta M)$, so its optimal level is lower than that.*

7.3.5 The Case for Quantity Regulation

What if some **"crazy," risk-inclined banks are willing to pay the tax and "abuse" ST funding**? Perotti and Suarez (2011) add a new dimension of heterogeneity: **heterogeneity in risk-taking/ gambling incentives**.

Assume bank owners do not internalize fraction θ_2 of crisis losses (due to, say, differences in governance, charter value, capitalisation). Fraction **θ_2 (uncompensatedly) passed to other stakeholders** (e.g., deposit insurers).

Bank owners' payoff function becomes:

$$v(x, X, \theta_1, \theta_2) = \pi(x, \theta_1) - (1-\theta_2)\,\varepsilon(x, \theta_1)\,c(X). \tag{7.22}$$

The social welfare W must now also account for the "missed" losses:

$$-\,\theta_2 \varepsilon\,(x, \theta_1)\, c(X). \tag{7.23}$$

Proposition 7.6. *If gambling incentives constitute the only source of heterogeneity, a flat tax on ST funding does not implement the first best. A stable funding requirement implying $\bar{x} = \bar{x}^{\star\star}$ can do it!*

For liquidity requirements, the same conclusions obtained above apply!

With two sources of heterogeneity, no clear-cut results: the 1st best is generally not attainable, with instruments non-contingent on θ_1 and θ_2.

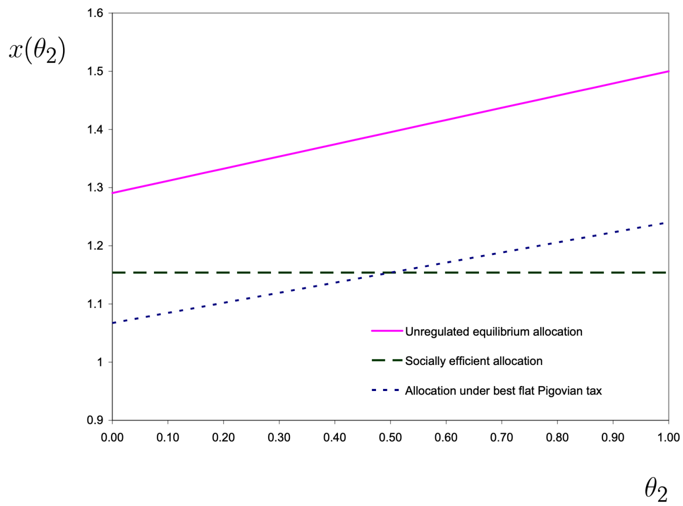


Figure 7.4: Inefficiency of Best Stable Funding Requirement (Perotti and Suarez, 2011).

There is the continuity argument, as follows.

- If θ_1 is the dominant source of heterogeneity:

$$\text{Flat tax on ST funding} \succ \text{Stable Funding Requirement}. \tag{7.24}$$

- The vice versa is the case if θ_2 is the dominant source of heterogeneity.

More generally, a **combination may be optimal**. If stronger capital regulation pushes θ_2 towards zero, the case for the Pigovian approach gets reinforced.

8 Banks in Dynamic General Equilibrium

8.1 Capital Requirements in a Small GE Model (Martinez-Miera and Suarez, 2014)

8.1.1 Introduction

The recent crisis has evidenced the need to better understand banks' contribution to **systemic risk**. One of the dimensions of this multifaceted phenomenon is the exposure to **common shocks**.

The model of Martinez-Miera and Suarez (2014) analyses:

1. the dynamic trade-offs behind banks' **voluntary exposure** to an infrequent and large common shock (attractive to them due to standard risk-shifting incentives);
2. the extent to which **capital requirements** (CRs) contribute to reducing the resulting systemic risk and increasing social welfare; and
3. the issues such as the optimal level of CRs, their gradual introduction and cyclical adjustment.

Martinez-Miera and Suarez (2014) build a simple dynamic equilibrium model where:

- **bank capital dynamics is formalized like in other papers in recent literature** (limited wealth of bankers who retain earnings and/or suffer losses from prior investments);
- however, the **role of bank capital is different**: Meh and Moran (2010)'s monitoring incentives a la Holmstrom and Tirole (1997) or Gertler and Kiyotaki (2010)'s preventing of fund diversion a la Hart and Moore (1998); and
- here, it reduces systemic gambling incentives through:
  1. the **leverage reduction effect** (standard) (see Van Den Heuvel (2008) and other micro-banking models), and
  2. the **last bank standing effect** (akin to Perotti and Suarez (2002)).

8.1.2 Related Literature

Ranciere et al. (2008) show that **myopic firms** adopt "risky growth strategies" due to lenders' expectation of a systemic bailout when a crisis occurs.

Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2019) highlight **similar capital dynamics** but include no time-varying systemic risk-taking and present no discussion on CRs.

There is **literature on risk-taking in banking**. Kareken and Wallace (1978) and many others cover the deposit risk insurance. Hellmann et al. (2000) and Repullo (2004) cover effects of the CRs. Acharya and Yorulmazer (2008) and Farhi and Tirole (2012) include equilibrium/dynamic considerations.

8.1.3 Modelling of Systemic Risk Taking in Martinez-Miera and Suarez (2014)

Set-Up. **Firms' production technology** is subject to failure risk and can be managed in two modes:

1. non-systemic ($x_i = 0$): the failure is purely iid; and

2. systemic ($x_i = 0$): if a rare shock occurs, all fail at once.

Firms need bank loans to pay inputs in advance:

$$l_i = k_i + wn_i. \tag{8.1}$$

Lending to systemic firms is socially inefficient, but highly levered banks may find it **privately profitable**. On top of that, systemic lending is not ex-ante detectable.

Regulation sets a common **capital requirement**:

$$e_i \geq \gamma l_i. \tag{8.2}$$

Bankers competitively allocate their wealth e as capital across banks.

Key Insights .

1. Systemic **risk-taking is maximum after several calm periods due to bankers' reaction to the lower shadow value of their wealth**.

2. Higher capital requirements:

   - reinforce the **last bank standing effect** (good); and
   - make bank **capital effectively scarcer at all times, which leads to less credit and lower economic activity** (bad).

3. The socially optimal capital requirements are quite high and should be gradually introduced. They should not be lowered in the crisis.

8.1.4 The Model of Martinez-Miera and Suarez (2014)

- The time is: $t \in \{0, 1, ...\}$.
- **Risk neutral agents**, including regulator, include

  1. **patient agents**: deep pockets, required expected rate of return ρ (the risk-free rate); and
  2. **impatient agents**: infinitely lived, discount factor $\beta < \frac{1}{1+\rho}$ measure one, supply unit of labour at wage w_t: **pure workers** just work, **bankers** run the banks (their wealth is the bank capital), and **entrepreneurs** run th firms just for one period.

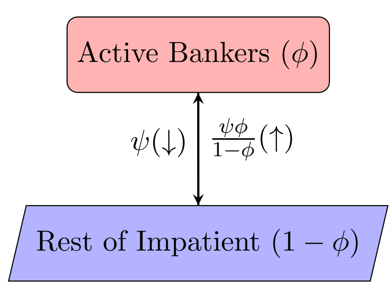


Figure 8.1: Conversion into Banker Is Learned In Advance.

Competitive firms $i \in [0, \mu]$ are run by penniless entrepreneurs who need loan:

$$l_{it} = k_{it} + w_t n_{it}, \tag{8.3}$$

used to pay for capital and labour in advance. You could think of entrepreneurs as one-period hand-to-mouth workers. They just "wake up" and become entrepreneurs for one period.

Random CRS technology subject to failure shock: $z_{it+1} \in \{0, 1\}$:

$$y_{it+1} = (1 - z_{it+1}) \left[AF\left(k_{it}, n_{it}\right) + (1 - \delta)k_{it}\right] + z_{it+1}(1 - \lambda)k_{it}, \tag{8.4}$$

where $\delta \equiv$ depreciation rate and $\lambda \equiv$ depreciation rate for a failing firm.

There are **two production modes**:

1. **non-systemic** ($x_{it} = 0$) involves iid failures, with failure rate π_0; and
2. **systemic** ($x_{it} = 1$) involves correlated failure in bad state:

$$\text{Failure Rate} = \begin{cases} 1 & \text{if systemic shock } (u_t = 1) \text{ w.p. } \epsilon \\ \pi_1 & \text{otherwise } (u_t = 0) \text{ w.p. } 1 - \epsilon. \end{cases} \tag{8.5}$$

Assumption 8.1. ***The systemic mode is attractive but inefficient!***

$$\pi_1 < \pi_0 < \underbrace{(1 - \epsilon)\pi_1 + \epsilon}_{\textit{Inefficiency}}. \tag{8.6}$$

Banks are owned by bankers who provide equity capital e_{jt}. They take insured one-period deposits d_{jt} from patient agents:

$$l_{jt} = d_{jt} + e_{jt}. \tag{8.7}$$

Convexities due to limited liability lead to **specialization in lending to non-systemic firms ($x_{it} = 0$) or systemic firms ($x_{it} = 1$)**. You can show it graphically!

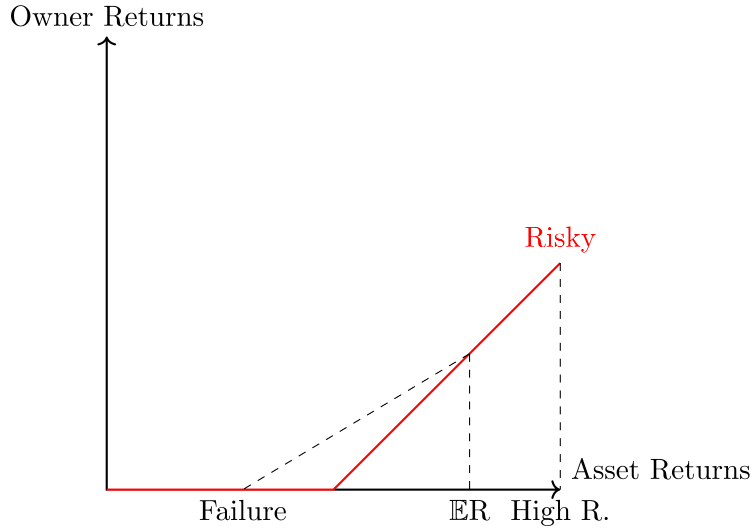


Figure 8.2: Reason behind Specialisation.

The specialisation is necessary for the indifference condition.
Loan contracts $(x_{it}, k_{it}, n_{it}, l_{it}, B_{it})$ are set by mutual agreement with each entrepreneur i. Banks take bankers' required return (in value terms) as given when fixing the terms of their supply of loans to firms!

Bankers can freely allocate their wealth e_t across banks! They **maximise their value**, taking returns as given:

$$v_t = \psi + (1-\psi)\beta \max \left\{ \mathbb{E}_t \left(v_{t+1} R_{0t+1} \right), \mathbb{E}_t \left(v_{t+1} R_{1t+1} \right) \right\}, \tag{8.8}$$

where $R_{xt+1} \equiv$ gross return on equity invested in bank with $x_{jt} \in \{0,1\}$ and $v_{t+1} \equiv$ marginal value of bankers' wealth at end of t. The **indifference requires**:

$$\mathbb{E}_t \left(v_{t+1} R_{0t+1} \right) = \mathbb{E}_t \left(v_{t+1} R_{1t+1} \right). \tag{8.9}$$

v_t can be considered as a shadow value of the resource. Alternatively, we could interpret it as Tobin's q of bankers' wealth.

The law of motion of total bank capital is:

$$e_{t+1} = \underbrace{(1+\rho)\phi w_t}_{\text{Future Bankers' Wages*}} + \underbrace{(1-\psi)\left[(1-x_t) R_{0t+1} + x_t R_{1t+1} \right] e_t}_{\text{Those Who Stay Bankers at } t+1}. \tag{8.10}$$

*: You only invest (behave patiently) if you know that you will be a banker next term.

Regulatory Environment. Banks are forced to hold non-granular loan portfolios. x_t is the private information of bank owners. In the pooling equilibrium, a systemic bank mimics a non-systemic bank. The **minimum capital requirement imposes**:

$$e_{jt} \geq \gamma l_{jt}. \tag{8.11}$$

Deposit insurance is funded with non-distortionary taxes (on impatient agents).

8.1.5 Equilibrium

Definition 8.1 (Equilibrium)**.** We need:

- a **stationary law of motion** for $e_t \in [\underline{e}, \bar{e}]$; and
- **tuple** $\left(v(e), x(e); k(e), w(e), R_0(e), R_1^0(e)\right)$ describing endogenous variables for each $e_t \in [\underline{e}, \bar{e}]$ such that $\{e_t\}$ and $\left\{v(e), x(e), k(e), w(e), R_0(e), R_1^0(e)\right\}$ are compatible with:
  1. individual optimisation, and
  2. market clearing.

List of Endogenous Variables:

- marginal value of bank capital: $v_t = v\left(e_t\right) \geq 1$;
- fraction of systemic banks: $x_t = x\left(e_t\right) \in [0, 1)$;
- physical capital used by firms: $k_t = k\left(e_t\right) \geq 0$;
- wage rate: $w_t = w\left(e_t\right) \geq 0$;
- ROE ($\equiv$ return on equity) $R_{0t+1} = R_0\left(e_t\right) \geq 1 + \rho$; and
- ROE at systemic bank without shock: $R_{1t+1}^0 = R_1^0\left(e_t\right) \geq 0$.

Intuition for Interior Systemic Risk-Taking Decisions. The indifference condition for $x_t \in (0, 1)$:

$$\left[(1-\epsilon)v\left(e_{t+1}^0\right) + \epsilon v\left(e_{t+1}^1\right)\right] R_{0t+1} = (1-\epsilon)v\left(e_{t+1}^0\right) R_{1t+1}^0, \tag{8.12}$$

where:

$$e_{t+1}^0 = (1+\rho)\phi w_t + (1-\psi)\left[\left(1-x_t\right) R_{0t+1} + x_t R_{1t+1}^0\right]; \tag{8.13a}$$

$$e_{t+1}^1 = (1+\rho)\phi w_t + (1-\psi)\left(1-x_t\right) R_{0t+1}; \text{ and} \tag{8.13b}$$

$$R_{1t+1} \geq R_{0t+1}. \tag{8.13c}$$

Note that $x_t < 1$ requires a large $v\left(e_{t+1}^1\right)$. The model is **characterised by the self-equilibrating mechanism for x_t**:

- Risk taking $\uparrow \Longrightarrow$

Social Welfare. Social welfare is defined as:

$$W_t \equiv \mathbb{E}\left\{\text{PV of Net Consumption Flows}\right\} \Longrightarrow \tag{8.14a}$$

$$W_t = \mathbb{E}_t\left(\sum_{s=0}^{\infty} \beta^s nc_{t+s}\right), \tag{8.14b}$$

where:

$$nc_t = -e_t + \left[1-(1+\psi)\phi\right] w_t + \beta\left\{y_{t+1} - (1-\rho)\left[d_t - (1+\psi)\phi w_t\right]\right\}; \text{ and} \tag{8.15a}$$

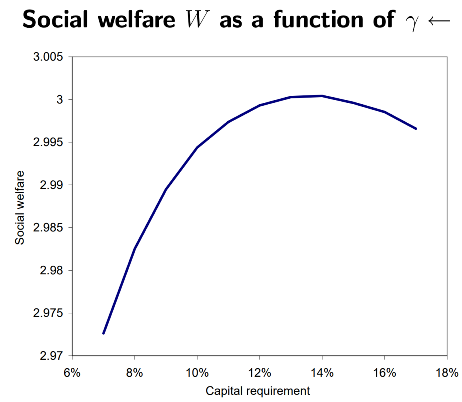
$$y_{t+1} = \text{GDP}_{t+1} + \left(1-\Delta_{t+1}\right) k_t. \tag{8.15b}$$

Calibration (1 period = 1 year)

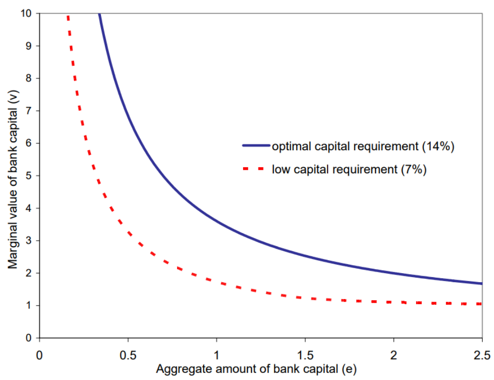
T1. Baseline parameter values

Patient agents' discount rate	ρ	0.02
Impatient agents' discount factor	β	0.96
Total factor productivity	A	2
Physical capital elasticity	α	0.3
Depreciation rate in successful firms	δ	0.05
Depreciation rate in failed firms	λ	0.35
Idiosyncratic default rate of non-systemic firms	π_0	0.03
Idiosyncratic default rate of systemic firms	π_1	0.018
Probability of a systemic shock	ε	0.03
Bankers' exit rate	ψ	0.20
Fraction of wage income earned by bankers	ϕ	0.05

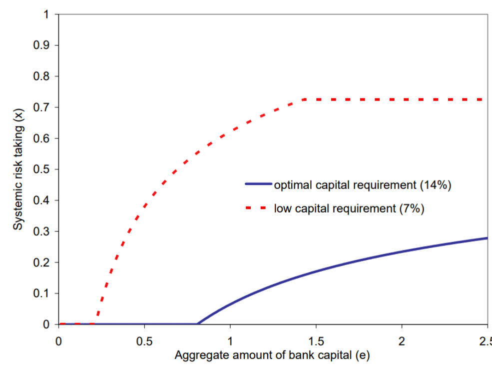
(a) Calibration.



(b) Social Welfare.



(c) $v(e)$.



(d) $x(e)$.

Figure 8.3: Figures in Martinez-Miera and Suarez (2014).

ADD REMAINING GRAPHS.

8.2 Banks in Dynamic General Equilibrium (Mendicino et al., 2018)

8.2.1 Introduction

Bank capital requirements (CRs) are still at the core of micro and macroprudential policies. The GFC pushed for the adoption of a macroprudential perspective on bank regulation. Developing GE models that help understand the channels of transmission of macroprudential policies is a top research priority now.

Within this research program, Mendicino et al. (2018) **focus on two issues**:

1. policy rules that mimic closely current Basel regulations (optimal level and default-sensitivity of sectoral CRs); and

2. agent heterogeneity and redistributive impact of prudential policies.

Related Literature

- There is a family of models that merge DSGE with banking (Cúrdia and Woodford, 2010; Gertler and Kiyotaki, 2010; Meh and Moran, 2010; Gertler et al., 2012).

- There are models that introduce bank fragility to general equilibrium models CONTINUE.

8.2.2 The Set-Up of Mendicino et al. (2018) & Model Overview

Bank fragility is key to bank-related **transmission channels**. The key distortions are:

- limited liability and absence of bank-risk pricing at the margin;
- net worth channel a la Bernanke et al. (1999), also for banks; and
- potential pecuniary externalities (Lorenzoni, 2008).

The main policy conclusions are as follows. CRs must keep the risk of bank failure low. Increasing CRs is Pareto-improving up to a point. CRs on corporate and mortgage loans should be higher, but less time-varying than implied by IRB formulas with PIT PDs.

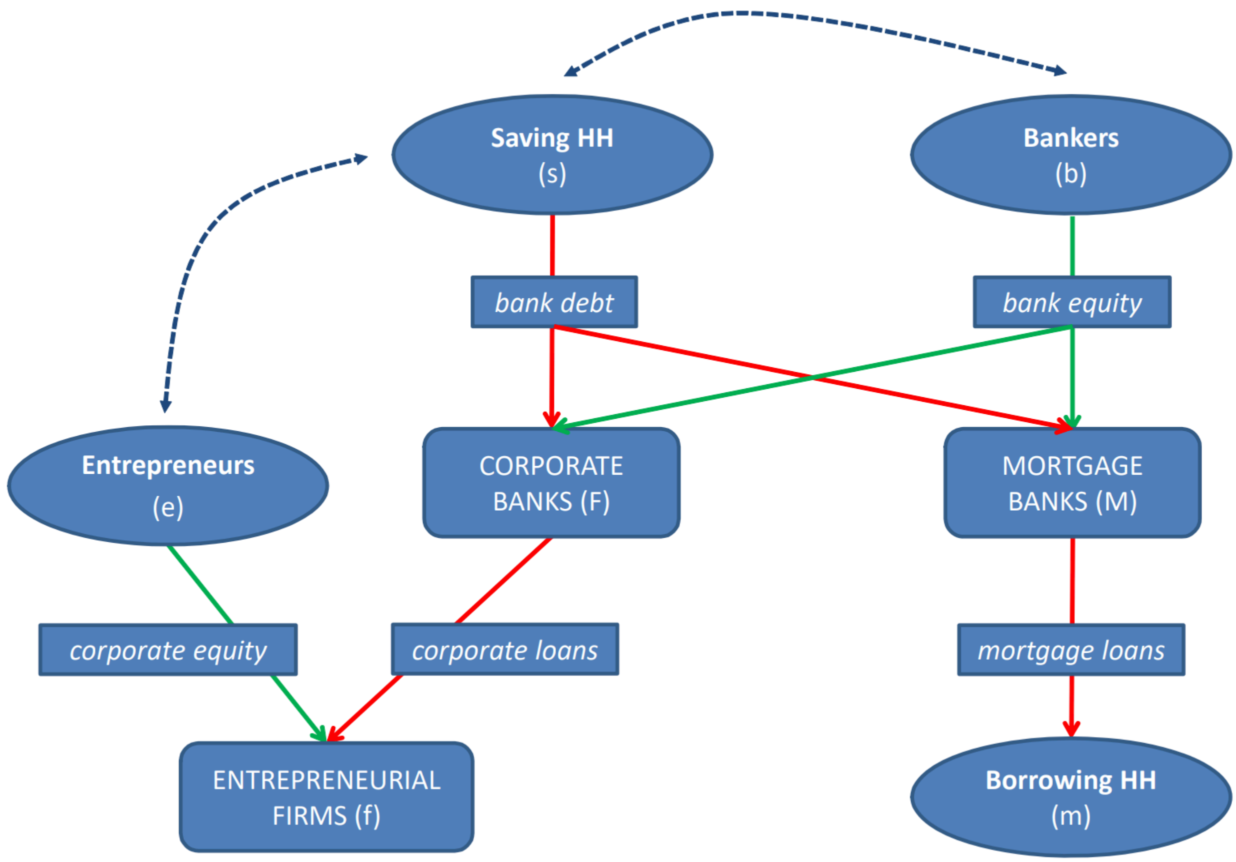


Figure 8.4: The Structure of Mendicino et al. (2018).

Dividing the banks into two classes avoids cross-subsidisation across client types. It's a simple trick. Saving households have as members entrepreneurs and bankers (this is what the two-sided arrow represents). In the tradition of the macro-finance models, entrepreneurial firms do not produce. They own the capital and rent it to the neoclassical firm that combines it with labour at zero profit.

- The model features **three interconnected net-worth channels** (m, e, b). The connection between leverage and default is as in Bernanke et al. (1999).
- SLIDES 7-9

8.2.3 Modelling Default

All borrowers' assets are subject to idiosyncratic shocks $\omega_{i,t+1}$. They are iid across borrowers of class i and **independent across classes**:

$$\omega_{i,t+1} \overset{iid(i)}{\sim} \log\mathcal{N}\left(1, \sigma_{i,t+1}\right), \tag{8.16}$$

where

$$\sigma_{i,t+1} \sim \mathrm{AR}(1). \tag{8.17}$$

Agent with assets $A_{i,t}$ and liabilities $B_{i,t}$ default at $t+1$ iff:

$$\omega_{i,t+1} R^A_{t+1} - R^B_{t+1} B_{i,t} < 0 \iff \tag{8.18a}$$

$$\omega_{i,t+1} < \bar{\omega}_{i,t+1} \equiv \frac{R^B_{t+1} B_{i,t}}{R^A_{t+1}}. \tag{8.18b}$$

The **share of final asset value owned by defaulting borrowers of class** i is:

$$G_{i,t+1}\left(\bar{\omega}_{i,t+1}\right) = \int_0^{\bar{\omega}_{i,t+1}} \omega_{i,t+1} dF_{i,t+1}\left(\omega_{i,t+1}\right). \tag{8.19}$$

The share of final asset value (gross of repossession costs) eventually going to lenders is:

$$\Gamma\left(\bar{\omega}_{i,t+1}\right) = G_{i,t+1}\left(\bar{\omega}_{i,t+1}\right) + \bar{\omega}_{i,t+1}\left[1 - F_{i,t+1}\left(\bar{\omega}_{i,t+1}\right)\right]. \tag{8.20}$$

8.2.4 Households, Savers, and Borrowers

Each dynasty, $\varkappa \in \{s, m\}$ maximises:

$$\mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \beta_{\varkappa}^{t+1} \left[\log\left(c_{\varkappa,t+i}\right) + \nu_{\varkappa} \lambda_{t+1} \log\left(h_{\varkappa,t+i}\right) - \frac{\phi_{\varkappa}}{1+\eta} l_{\varkappa,t+1}^{1-\eta} \right] \right\} \tag{8.21}$$

Slides 11-12

Some Detail on Borrowers. The budget constraint is:

$$c_{m,t} + q_{h,t} - b_{m,t} \leq w_m l_{m,t} + \left[1 - \Gamma_{m,t}\left(\bar{\omega}_{i,t+1}\right)\right] R^H_t q_{h,t-1} h_{m,t-1} - T_{m,t}. \tag{8.22}$$

SLIDE 13

Some Detail on Entrepeneurs. They are ∞-lived. The return net worth to patient dynasty at retirement. They solve:

$$v_{e,t} n_{e,t} = \max_{a_t, \mathrm{div}_{e,t} \geq 0} \left\{ \mathrm{div}_{e,t} + \mathbb{E}_t \Lambda_{s,t+1}\left[1 - \theta_e + \theta_e v_{e,t+1}\right] n_{e,t+1} \right\}, \tag{8.23}$$

where $\mathrm{div}_{e,t}$ is the dividend and $n_{e,t}$ is the number of units of net worth. θ_e is the probability that one loses the ability to be an entrepreneur. Their firms maximize:

$$\max_{k_t, R^F_t} \left\{ \mathbb{E}_t \Lambda_{e,t+1}\left(1 - \Gamma_{f,t+1}\left(\bar{\omega}_{f,t+1}\right)\right) R^K_{t+1} q_{k,t} k_{f,t} \right\}. \tag{8.24}$$

subject to the **participation constraint of their bank**:

$$\mathbb{E}_t \, \Lambda_{b,t+1} \left[1 - \Gamma_{F,t+1}\left(\bar{\omega}_{F,t+1}\right)\right] \widetilde{R}^{F}_{t+1} b_{f,t} \geq v_{b,t} \phi_{F,t} b_{f,t}, \tag{8.25}$$

where

- $k_{f,t} \equiv$ capital purchased with net worth a_t and loan $b_{f,t} = (q_{k,t} k_{f,t} - a_t)$;
- $b_{f,t} \equiv$ non-contingent debt charging agreed gross rate R^F_t;
- $\bar{\omega}_{F,t+1} \equiv$ F banks' idiosyncratic-shock default threshold; and
- $\phi_{F,t} b_{f,t} \equiv$ bankers' equity involved in funding the loan.

Further, we have:

$$\bar{\omega}_{f,t+1} \equiv \frac{x_{f,t}}{R^K_{t+1}}; \tag{8.26a}$$

$$x_{f,t} = \frac{R^{F_b}_t b_{f,t}}{q_{k,t} k_{f,t}}; \text{ and} \tag{8.26b}$$

$$R^K_{t+1} \equiv \frac{r_{k,t+1} + \left(1 - \delta_{k,t+1}\right) q_{k,t+1}}{q_{k,t}}. \tag{8.26c}$$

Some Detail on Bankers. They are ∞-lived. They return their net worth to the patient dynasty at retirement. They solve:

$$V_{b,t} = \max_{e^M_t, e^F_t, \mathrm{div}_{b,t}} \left\{ \mathrm{div}_{b,t} + \mathbb{E}_t \, \Lambda_{s,t+1} \left[\left(1 - \theta_b\right) n_{b,t+1} + \theta_b V_{b,t+1} \right] \right\} \tag{8.27a}$$

$$e_{M,t} + e_{F,t} + \mathrm{div}_{b,t} = n_{b,t} \tag{8.27b}$$

$$n_{b,t+1} = \int_0^\infty \rho_{M,t+1}(\omega) dF_{M,t+1}(\omega) e_{M,t} + \int_0^\infty \rho_{F,t+1}(\omega) dF_{F,t+1}(\omega) e_{F,t} \tag{8.27c}$$

$$\mathrm{div}_{b,t} \geq 0 \tag{8.27d}$$

The interior solution equilibrium requires "no arbitrage condition":

$$\mathbb{E}_t \left[\Lambda_{b,t+1} \rho_{M,t+1}\right] = \mathbb{E}_t \left[\Lambda_{b,t+1} \rho_{F,t+1}\right] = v_{b,t} \tag{8.28}$$

Resulting laws of motion of e&b net worth*

$$n_{e,t+1} = \theta_e \rho_{f,t+1} a_t + \iota_{e,t} \quad [\text{ with } \iota_{e,t} = \chi_e(\text{ exiting net worth})]$$

$$n_{b,t+1} = \theta_b \left(\rho_{F,t+1} e_{F,t} + \rho_{M,t+1} e_{M,t}\right) + \iota_{b,t} \quad [\text{with } \iota_{b,t} = \chi_b \text{ (exiting net worth)}]$$

15 Capital policy rules CRs applicable to each class of loans are determined by simple rules:

$$\phi_{M,t} = \phi_M + \tau_M \left(\mathbb{E}_t \, \Psi_{m,t+1} - \Psi_m\right)$$
$$\phi_{F,t} = \phi_F + \tau_F \left(\mathbb{E}_t \, \Psi_{f,t+1} - \Psi_f\right)$$

TTC=through the cycle.

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