NPRE 555
Computer Project 2

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1 Introduction

This computer project calculated the neutron flux in a moderator slab using the P_N approximation. Table 1 summarizes the input parameters. The following sections define the equations for each P_N approximation.

Table 1: Input parameters.

	Value	Units
Slab thickness (L)	8	cm
Σ_t	1.0	1/cm
Σ_{s0}	0.4	1/cm
Σ_{s1}	0.1	1/cm
Σ_{s2}	0.1	1/cm
Σ_{s3}	0.1	1/cm
q_0	1	n/cm^3/s

2 P_1 approximation

$$\frac{d}{dx}\phi_1 + \Sigma_0\phi_0 = q_0 \tag{1}$$

$$\frac{d}{dx}\phi_0 + 3\Sigma_1\phi_1 = 0 \tag{2}$$

where

$$\Sigma_n = \Sigma_t - \Sigma_{sn}$$

2.a P_1 approximation Marshak boundary condition

$$\frac{1}{4}\phi_0(x=0) + \frac{1}{2}\phi_1(x=0) = 0 \tag{3}$$

$$-\frac{1}{4}\phi_0(x=L) + \frac{1}{2}\phi_1(x=L) = 0 \tag{4}$$

2.b P_1 approximation Mark boundary condition

$$\frac{1}{2}\phi_0(x=0) + \frac{3}{2}\phi_1(x=0)\mu_0 = 0 \tag{5}$$

$$\frac{1}{2}\phi_0(x=L) + \frac{3}{2}\phi_1(x=L)\mu_1 = 0 \tag{6}$$

where

$$\mu_{0,1} = \pm 0.57735$$

2.c Numerical method

Through some algebraic manipulation of Equations 1 and 2, the method obtains the following equation. The solver discretizes the equation with the finite difference method. Equation 2 combined with the boundary condition equations (Sections 2.a and 2.b) allow to impose the boundary conditions on the numerical solution.

$$-\frac{d}{dx}\left(\frac{1}{3\Sigma_1}\frac{d}{dx}\phi_0\right) + \Sigma_0\phi_0 = q_0 \tag{7}$$

3 P_3 approximation

$$\frac{d}{dx}\phi_1 + \Sigma_0\phi_0 = q_0 \tag{8}$$

$$2\frac{d}{dx}\phi_2 + \frac{d}{dx}\phi_0 + 3\Sigma_1\phi_1 = 0 \tag{9}$$

$$3\frac{d}{dx}\phi_2 + 2\frac{d}{dx}\phi_1 + 5\Sigma_2\phi_2 = 0 \tag{10}$$

$$3\frac{d}{dx}\phi_2 + 7\Sigma_3\phi_3 = 0 \tag{11}$$

where

$$\Sigma_n = \Sigma_t - \Sigma_{sn}$$

3.a P_3 approximation Marshak boundary condition

$$\frac{1}{2}\phi_0(x=0) + \phi_1(x=0) + \frac{5}{8}\phi_2(x=0) = 0 \tag{12}$$

$$-\frac{1}{8}\phi_0(x=0) + \frac{5}{8}\phi_2(x=0) + \phi_3(x=0) = 0 \tag{13}$$

$$-\frac{1}{2}\phi_0(x=L) + \phi_1(x=L) - \frac{5}{8}\phi_2(x=L) = 0 \tag{14}$$

$$\frac{1}{8}\phi_0(x=L) - \frac{5}{8}\phi_2(x=L) + \phi_3(x=L) = 0 \tag{15}$$

3.b P_3 approximation Mark boundary condition

$$\frac{1}{2}\phi_0(x=0)P_0(\mu_0) + \frac{3}{2}\phi_1(x=0)P_1(\mu_0) + \frac{5}{2}\phi_2(x=0)P_2(\mu_0) + \frac{7}{2}\phi_3(x=0)P_3(\mu_0) = 0 \tag{16}$$

$$\frac{1}{2}\phi_0(x=0)P_0(\mu_1) + \frac{3}{2}\phi_1(x=0)P_1(\mu_1) + \frac{5}{2}\phi_2(x=0)P_2(\mu_1) + \frac{7}{2}\phi_3(x=0)P_3(\mu_1) = 0 \tag{17}$$

$$\frac{1}{2}\phi_0(x=L)P_0(\mu_2) + \frac{3}{2}\phi_1(x=L)P_1(\mu_2) + \frac{5}{2}\phi_2(x=L)P_2(\mu_2) + \frac{7}{2}\phi_3(x=L)P_3(\mu_2) = 0 \tag{18}$$

$$\frac{1}{2}\phi_0(x=L)P_0(\mu_3) + \frac{3}{2}\phi_1(x=L)P_1(\mu_3) + \frac{5}{2}\phi_2(x=L)P_2(\mu_3) + \frac{7}{2}\phi_3(x=L)P_3(\mu_3) = 0 \tag{19}$$

where

$$P_0(\mu) = 1$$
$$P_1(\mu) = \mu$$
$$P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$$
$$P_3(\mu) = \frac{1}{2}(5\mu^3 - \mu)$$
$$\mu_{0,1,2,3} = [0.86114, 0.33998, -0.33998, -0.86114]$$

3.c Numerical method

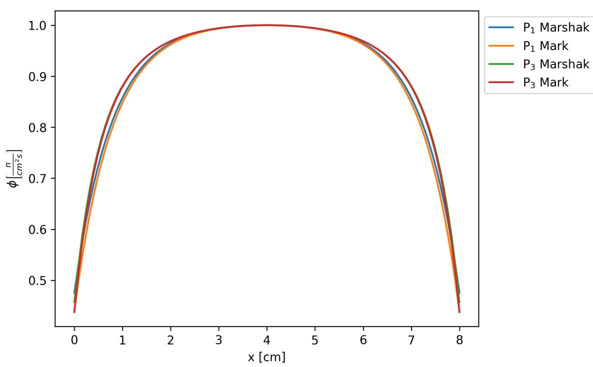
Through some algebraic manipulation of Equations 8 to 11, the method obtains the following equations. The solver discretizes the equations with the finite difference method. To solution of the coupled system used an explicit solver based on the previous iteration solution, requiring an iterative solver. The convergence criteria was an L_2-norm of the relative difference between fluxes smaller than 1×10^{-6}. Equations 9 and 11 combined with the boundary condition equations (Sections 3.a and 3.b) allow to impose the boundary conditions on the numerical solution.

$$-\frac{d}{dx}\left(\frac{1}{3\Sigma_1}\frac{d}{dx}\phi_0 + \frac{2}{3\Sigma_1}\frac{d}{dx}\phi_2\right) + \Sigma_0\phi_0 = q_0 \tag{20}$$

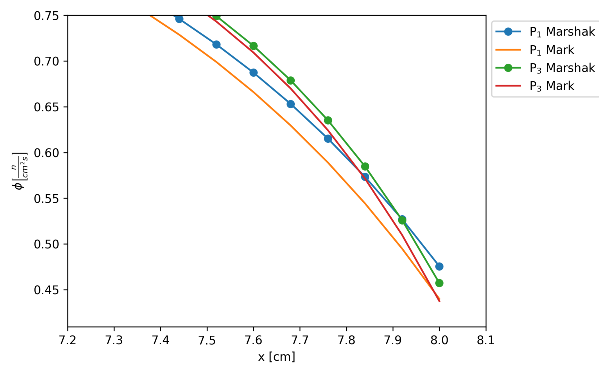
$$-\frac{d}{dx}\left(\frac{2}{3\Sigma_1}\frac{d}{dx}\phi_0 + \left(\frac{4}{3\Sigma_1} + \frac{9}{7\Sigma_3}\right)\frac{d}{dx}\phi_2\right) + 5\Sigma_2\phi_2 = 0 \tag{21}$$

4 Results

Figure 1 displays the scalar flux for all the approximations with the different boundary conditions. Figure 1(a) shows a full range plot of all the approximations normalized to their maximum value. Figure 1(b) exhibits a reduced range plot close to the right boundary for a better appreciation of the boundary condition effects. P_3 flux has a larger magnitude than P_1 flux away from the boundaries, which indicates that lower P_N approximation fluxes have rounder shapes. A rounder shape is an indicator of poorer neutron conservation. Additionally, imposing a Mark boundary condition causes the scalar flux to have a smaller magnitude than the Marshak boundary condition case. A smaller flux on the boundary is a sign of 'more neutrons' leaving the domain, which indicates that the Marshak boundary condition better preserves the neutrons in the domain.



(a) Full range plot.



(b) Reduced range plot.

Figure 1: Scalar flux normalized to maximum value.

5 Conclusions

In order to conduct the computer project, I had to understand the basics of the P_N method. The results help visualize some characteristics of the approximations. Using a higher-order P_N approximation better preserves the neutrons inside the domain. Using a higher-order P_N approximation also requires a more elaborate solver. Choosing a Marshak boundary condition better conserves the neutrons inside the domain. This type of boundary condition also requires a more elaborate formulation.