

MULTI GROUP SP3

CP3-1

$$a. \frac{d}{dx} \phi_{1,g} + \sum_{t,g} \phi_{0,g} = \sum_{g'=1}^G \sum_{0,gg'} \phi_{0,g'} + \frac{x_g}{k_{eff}} \sum_{g'=1}^G \sum_{f,g'} \phi_{0,g'}$$

$$b. \frac{2}{3} \frac{d}{dx} \phi_{0,g} + \frac{2}{3} \frac{d}{dx} \phi_{2,g} + \sum_{t,g} \phi_{1,g} = \sum_{g'=1}^G \sum_{1,gg'} \phi_{1,g'}$$

$$c. \frac{2}{5} \frac{d}{dx} \phi_{1,g} + \frac{3}{5} \frac{d}{dx} \phi_{3,g} + \sum_{t,g} \phi_{2,g} = \sum_{g'=1}^G \sum_{2,gg'} \phi_{2,g'}$$

$$d. \frac{3}{7} \frac{d}{dx} \phi_{2,g} + \sum_{t,g} \phi_{3,g} = \sum_{g'=1}^G \sum_{3,gg'} \phi_{3,g'}$$

Following Beckert and Grundmann, 2007:

$$\sum_{n,gg'} = 0 \quad \wedge \quad g' \neq g \quad \wedge \quad n = 1, 2, 3$$

$$\sum_{r,n,g} = \sum_{t,g} - \sum_{n,gg} \quad n = 0, 1, 2, 3$$

$$b) \phi_{1,g} = -\frac{1}{3 \sum_{r,1,g}} \frac{d}{dx} [\phi_{0,g} + 2 \phi_{2,g}]$$

$$d) \phi_{3,g} = -\frac{3}{7 \sum_{r,3,g}} \frac{d}{dx} \phi_{2,g}$$

into

$$1) -\frac{1}{3 \sum_{r,1,g}} \frac{d^2}{dx^2}$$

$$\tilde{\phi}_0 = \phi_0 + 2\phi_2, \quad \tilde{\phi}_2 = \phi_2$$

$$\frac{d}{dx} \phi_{1,g} + \sum_{r,0,g} \phi_{0,g} = \frac{x_g}{k_{eff}} \sum_{j'=1}^G \nu \Sigma_{fg'} \phi_{0,g'}$$
~~$$\frac{d}{dx} \phi_{2,g} + \sum_{r,0,g} \phi_{0,g} = \frac{x_g}{k_{eff}} \sum_{j'=1}^G \nu \Sigma_{fg'} \phi_{0,g'}$$~~

$$+ \sum_{\substack{j'=1 \\ j' \neq g}}^G \sum_{r,0,g'} \phi_{0,g'}$$

$$\frac{2}{5} \frac{d}{dx} \phi_{1,g} + \frac{3}{5} \frac{d}{dx} \phi_{2,g} + \sum_{r,2,g} \phi_{2,g} = 0$$

$$\phi_{1,g} = -\frac{1}{3 \sum_{r,1,g}} \frac{d}{dx} [\phi_{0,g} + 2\phi_{2,g}]$$

$$\phi_{2,g} = -\frac{3}{7 \sum_{r,3,g}} \frac{d}{dx} \phi_{2,g}$$

$$A) \frac{d}{dx} \left[-\frac{1}{3 \sum_{r,1,g}} \left(\frac{d}{dx} \phi_{0,g} + 2 \frac{d}{dx} \phi_{2,g} \right) \right] + \sum_{r,0,g} \phi_{0,g} =$$

$$= \frac{x_g}{k_{eff}} \sum_{j'=1}^G \nu \Sigma_{fg'} \phi_{0,g'}$$

$$B) \frac{2}{5} \frac{d}{dx} \left[-\frac{1}{3 \sum_{r,1,g}} \left(\frac{d}{dx} \phi_{0,g} + 2 \frac{d}{dx} \phi_{2,g} \right) \right] +$$

$$+ \frac{3}{5} \frac{d}{dx} \left[-\frac{3}{7 \sum_{r,3,g}} \frac{d}{dx} \phi_{2,g} \right] + \sum_{r,2,g} \phi_{2,g} = 0$$

$$A) -\frac{1}{3 \sum_{r,1,g}} \left[\frac{d^2}{dx^2} \phi_{0,g} + 2 \frac{d^2}{dx^2} \phi_{2,g} \right] +$$

$$+ \sum_{r,0,g} \phi_{0,g} = \frac{x_g}{k_{eff}} \sum_{j'=1}^G \nu \Sigma_{fg'} \phi_{0,g'}$$

$$\sum_{\substack{j'=1 \\ j' \neq g}}^G \sum_{r,0,g'} \phi_{0,g'}$$

$$B) -\frac{2}{15} \frac{d^2}{dx^2} \phi_{0,g} - \frac{4}{15} \frac{d^2}{dx^2} \phi_{2,g} - \frac{9}{35} \sum_{r,3,g} \frac{d^2}{dx^2} \phi_{2,g} + \sum_{r,2,g} \phi_{2,g} = 0$$

$$\uparrow \sum_{r,1,g} \quad \uparrow \sum_{r,1,g}$$

$$A) -\frac{1}{3\bar{\Sigma}_{r,1,g}} \left[\frac{d^2}{dx^2} \phi_{0,g} + 2 \frac{d^2}{dx^2} \phi_{2,g} \right] + \bar{\Sigma}_{r,0,g} \phi_{0,g} =$$

$$= \sum_{\substack{j'=1 \\ j' \neq g}}^G \bar{\Sigma}_{0,j',g} \phi_{0,j'} + \frac{x_g}{k_{eff}} \sum_{j'=1}^G \nu \bar{\Sigma}_{f,j'} \phi_{0,j'}$$

$$B) -\frac{2}{15\bar{\Sigma}_{r,1,g}} \frac{d^2}{dx^2} \phi_{0,g} - \frac{4}{15\bar{\Sigma}_{r,1,g}} \frac{d^2}{dx^2} \phi_{2,g} - \frac{9}{35\bar{\Sigma}_{r,3,g}} \frac{d^2}{dx^2} \phi_{2,g} + \bar{\Sigma}_{r,2,g} \phi_{2,g} = 0$$

$$\downarrow \tilde{\phi}_0 = \phi_0 + 2\phi_2, \quad \tilde{\phi}_2 = \phi_2$$

$$\textcircled{A} -\frac{1}{3\bar{\Sigma}_{r,1,g}} \frac{d^2}{dx^2} \tilde{\phi}_{0,g} + \bar{\Sigma}_{r,0,g} \tilde{\phi}_{0,g} - 2\bar{\Sigma}_{r,0,g} \tilde{\phi}_{2,g} = \sum_{\substack{j'=1 \\ j' \neq g}}^G \bar{\Sigma}_{0,j',g} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'}) + \frac{x_g}{k_{eff}} \sum_{j'=1}^G \nu \bar{\Sigma}_{f,j'} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'})$$

$$\textcircled{B} -\frac{2}{15\bar{\Sigma}_{r,1,g}} \frac{d^2}{dx^2} \tilde{\phi}_{0,g} - \frac{9}{35\bar{\Sigma}_{r,3,g}} \frac{d^2}{dx^2} \tilde{\phi}_{2,g} + \bar{\Sigma}_{r,2,g} \tilde{\phi}_{2,g} = 0$$

$$\textcircled{A} \rightarrow -\frac{1}{3\bar{\Sigma}_{r,1,g}} \frac{d^2}{dx^2} \tilde{\phi}_{0,g} = -\bar{\Sigma}_{r,0,g} (\tilde{\phi}_{0,g} - 2\bar{\Sigma}_{r,0,g} \tilde{\phi}_{2,g}) + \sum_{\substack{j'=1 \\ j' \neq g}}^G \bar{\Sigma}_{0,j',g} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'}) +$$

$$\downarrow \times \frac{2}{5} \text{ into } \textcircled{B} \quad + \frac{x_g}{k_f} \sum_{j'=1}^G \nu \bar{\Sigma}_{f,j'} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'})$$

$$-\frac{9}{35\bar{\Sigma}_{r,3,g}} \frac{d^2}{dx^2} \tilde{\phi}_{2,g} + \bar{\Sigma}_{r,2,g} \tilde{\phi}_{2,g} - \frac{2}{5} \bar{\Sigma}_{r,0,g} \tilde{\phi}_{0,g} + \frac{4}{5} \bar{\Sigma}_{r,0,g} \tilde{\phi}_{2,g} + \frac{2}{5} \sum_{\substack{j'=1 \\ j' \neq g}}^G \bar{\Sigma}_{0,j',g} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'}) + \frac{2}{5} \frac{x_g}{k_f} \sum_{j'=1}^G \nu \bar{\Sigma}_{f,j'} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'}) = 0$$

$$\textcircled{B} -\frac{9}{35\bar{\Sigma}_{r,3,g}} \frac{d^2}{dx^2} \tilde{\phi}_{2,g} + (\bar{\Sigma}_{r,2,g} + \frac{4}{5} \bar{\Sigma}_{r,0,g}) \tilde{\phi}_{2,g} - \frac{2}{5} \bar{\Sigma}_{r,0,g} \tilde{\phi}_{0,g} = -\frac{2}{5} \sum_{\substack{j'=1 \\ j' \neq g}}^G \bar{\Sigma}_{0,j',g} (\tilde{\phi}_{0,j'} - 2\tilde{\phi}_{2,j'}) - \frac{2}{5} \frac{x_g}{k_f} \dots$$

$\rightarrow \textcircled{A} \text{ \& } \textcircled{B} \text{ equal (13)}$

$$\psi(x, \mu) = \frac{1}{4\pi} \phi_0 + \frac{3}{4\pi} \phi_1 \mu + \frac{5}{4\pi} \phi_2 \mu^2 + \frac{7}{4\pi} \phi_3 \mu^3 \quad \left(\frac{1}{2}(3\mu^2 - 1) \right)$$

$$\int \psi(x, \mu) P_1(\mu) d\mu = 0$$

$$\int \left[\phi_0 + 3\phi_1 \mu + 5\phi_2 \frac{1}{2}(3\mu^2 - 1) + 7\phi_3 \frac{1}{2}(5\mu^3 - 3\mu) \right] \mu d\mu =$$

$$= \int \phi_0 \mu + 3\phi_1 \mu^2 + \frac{5}{2}\phi_2 (3\mu^3 - \mu) + \frac{7}{2}\phi_3 (5\mu^4 - 3\mu^2) d\mu$$

$$\rightarrow \mu > 0 : \int_0^1 d\mu : \phi_0 \cdot \frac{1}{2} + 3\phi_1 \frac{1}{3} + \frac{5}{2}\phi_2 \left(\frac{3}{4} - \frac{1}{2} \right) + \frac{7}{2}\phi_3 \left(\frac{5}{5} - \frac{3}{3} \right) = 0$$

$$\cdot \frac{1}{2}\phi_0 + \phi_1 + \frac{5}{8}\phi_2 = 0$$

$$\rightarrow x=a : \int_{-1}^0 d\mu : -\frac{1}{2}\phi_0 + \phi_1 - \frac{5}{8}\phi_2 = 0$$

$$\rightarrow \frac{1}{2}\phi_0 + \phi_1 + \frac{5}{8}\phi_2 = 0$$

$$\frac{1}{2}\tilde{\phi}_0 - \tilde{\phi}_2 + \frac{1}{8}\hat{J}_0 + \frac{5}{8}\tilde{\phi}_2 = \frac{1}{2}\tilde{\phi}_0 + \hat{J}_0 - \frac{3}{8}\tilde{\phi}_2 = 0 \quad = \pi(18)$$

$$\boxed{\frac{1}{4}\tilde{\phi}_0 + \frac{1}{2}\hat{J}_0 - \frac{3}{8}\tilde{\phi}_2 = 0}$$

$$\int \psi(x, \mu) P_3(\mu) d\mu = 0$$

$$\int \left(\phi_0 \frac{1}{2}(5\mu^3 - 3\mu) + 3\phi_1 \mu \frac{1}{2}(5\mu^3 - 3\mu) + 5\phi_2 \frac{1}{2}(3\mu^2 - 1) \frac{1}{2}(5\mu^3 - 3\mu) + 7\phi_3 \left[\frac{1}{2}(5\mu^3 - 3\mu) \right]^2 \right) d\mu$$

$$\Rightarrow x=0 : -\frac{1}{8}\phi_0 + 0 + \frac{5}{8}\phi_2 + \phi_3 = 0$$

$$\Rightarrow x=a : +\frac{1}{8}\phi_0 + 0 - \frac{5}{8}\phi_2 + \phi_3 = 0$$

$$-\frac{1}{8}\phi_0 + \frac{5}{8}\phi_2 + \frac{-3}{7\sum} \frac{1}{dx} \phi_2 = 0 \rightarrow -\frac{1}{8}\tilde{\phi}_0 + \frac{21}{8}\tilde{\phi}_2 + \frac{5}{8}\tilde{\phi}_2 + \hat{J}_2 \frac{5}{3} = 0$$

$$-\frac{1}{8}\tilde{\phi}_0 + \frac{7}{8}\tilde{\phi}_2 + \hat{J}_2 \frac{5}{3} = 0$$

$$\boxed{-\frac{3}{80}\tilde{\phi}_0 + \frac{21}{80}\tilde{\phi}_2 + \frac{1}{2}\hat{J}_2 = 0}$$

$$= \pi(18)$$

$$\tilde{J}_2 = -\frac{9}{35\sum} \frac{1}{dx} \tilde{\phi}_0 = \frac{3}{5} \cdot \phi_3$$

VARIATIONAL FORM

CP3-4

$$\textcircled{A} \quad -D_0 \frac{d^2}{dx^2} \tilde{\phi}_{0g} + \sum_{r,0,g} \tilde{\phi}_{0g} - 2 \sum_{r,0,g} \tilde{\phi}_{2g} - \sum_{\substack{s'=1 \\ s' \neq g}}^G \sum_{o,s,g} (\tilde{\phi}_{0s'} - 2\tilde{\phi}_{2s'}) \frac{\chi_{gs}}{k_{eff}} \sum_{s'=1}^G \sum_{o,s,g} (\tilde{\phi}_{0s'} - 2\tilde{\phi}_{2s'}) = 0$$

$$\downarrow$$

$$-D_0 \frac{d^2}{dx^2} \tilde{\phi}_{0g} + D_0 \frac{d}{dx} \tilde{\phi}_{0g} \frac{d^4}{dx^4} + \sum_{r,0,g} \tilde{\phi}_{0g} - \dots = 0$$

$$\textcircled{B} \quad -D_{2g} \frac{d^2}{dx^2} \tilde{\phi}_{2g} + \left(\sum_{r,2g} + \frac{4}{5} \sum_{r,0,g} \right) \tilde{\phi}_{2g} - \frac{2}{5} \sum_{r,0,g} \tilde{\phi}_{0g} + \frac{2}{5} \left(\sum_{\substack{s'=1 \\ s' \neq g}}^G \sum_{o,s,g} () + \frac{\chi_g}{k_{eff}} \sum_{s'=1}^G \sum_{o,s,g} \dots \right) = 0$$

$$\downarrow$$

$$-D_{2g} \frac{d^2}{dx^2} \tilde{\phi}_{2g} + D_{2g} \frac{d}{dx} \tilde{\phi}_{2g} + \dots = 0$$

$$\text{BCs} \quad \textcircled{A} \quad \underbrace{-D_{0g} \frac{d}{dx} \tilde{\phi}_{0g}}_{=\hat{\Gamma}_0} = -\frac{1}{2} \tilde{\phi}_0 + \frac{3}{4} \tilde{\phi}_2 \xRightarrow{\text{in molecules}} \frac{1}{2} \tilde{\phi}_0 - \frac{3}{4} \tilde{\phi}_2$$

$$\textcircled{B} \quad -D_{2g} \frac{d}{dx} \tilde{\phi}_{2g} = \hat{\Gamma}_2 = \frac{3}{40} \tilde{\phi}_0 - \frac{21}{40} \tilde{\phi}_2 \Rightarrow -\frac{3}{40} \tilde{\phi}_0 + \frac{21}{40} \tilde{\phi}_2$$

1G, Fixed Source

CP3-S

$$\textcircled{A} -D_0 \frac{d^2}{dx^2} \tilde{\phi}_0 + \underbrace{\sum_{r,0} \tilde{\phi}_0}_{\text{P3 Diff}} - 2 \underbrace{\sum_{r,0} \tilde{\phi}_2}_{\text{P3 Sigma R}} - Q_0 = 0$$

$$\textcircled{B} -D_2 \frac{d^2}{dx^2} \tilde{\phi}_{2,2} + \left(\underbrace{\sum_{r,2}}_{\text{P3 Diff}} + \frac{4}{5} \sum_{r,0} \right) \tilde{\phi}_2 - \frac{2}{5} \sum_{r,0} \tilde{\phi}_0 + \frac{2}{5} Q_0 = 0$$

\textcircled{A} Group Diffusion \rightarrow P3 Diff
~~Source~~
 Sigma R \rightarrow P3 Sigma R
 P3 Sigma Coupled \rightarrow
 Source

\textcircled{B} P3 Diff
 P3 Sigma R
 P3 Sigma Coupled
 Source

$$D_0 = \frac{1}{3 \sum_{r,1,g}} \rightarrow \sum_{r,1,g} = \sum_{t,g} - \sum_{1,gg}$$

$$\sum_{r,1} = \sum_t - \sum_{\text{abs}} \text{ (or } \sum_{1,gg} \text{)}$$

$$\sum_{r,0} = \sum_t - \sum_{\text{abs}} = \sum_{\text{abs}}(?)$$

$$D_2 = \frac{9}{35 \sum_{r,3,g}} \rightarrow \sum_{r,3} = \sum_t - \sum_{s,3}$$

$$\sum_{r,2} = \sum_t - \sum_{s,2}$$

1G, Criticality Source

CP3-6

$$\textcircled{A} - \Delta_0 \frac{d^2}{dx^2} \tilde{\phi}_0 + \Sigma_{r0} \tilde{\phi}_0 - 2 \Sigma_{r0} \tilde{\phi}_2 - \frac{\chi^T}{k_{eff}} \nu \Sigma_f (\tilde{\phi}_0 - 2\tilde{\phi}_2) = 0$$

$$\textcircled{B} - \Delta_2 \frac{d^2}{dx^2} \tilde{\phi}_2 + \left(\Sigma_{r2} + \frac{4}{5} \Sigma_{r0} \right) \tilde{\phi}_2 - \frac{2}{5} \Sigma_{r0} \tilde{\phi}_0 + \frac{2}{5} \frac{\chi^T}{k_{eff}} \nu \Sigma_f (\tilde{\phi}_0 - 2\tilde{\phi}_2) = 0$$

fission
P3 Eigen kernel

$$+ \frac{2}{5} \frac{1}{k_{eff}} \nu \Sigma_f \left(\tilde{\phi}_0 - 2\tilde{\phi}_2 \right)$$

$\frac{2}{5} \tilde{\phi}_0 - \frac{4}{5} \tilde{\phi}_2$

$$= \frac{\chi^T}{k_{eff}} \nu \Sigma_f \left[\frac{4}{5} \tilde{\phi}_2 - \frac{2}{5} \tilde{\phi}_0 \right]$$

MULTI GROUP, FIXED SOURCE

CP3-7

$$\bullet - D_{0,g} \frac{d^2}{dx^2} \tilde{\phi}_{0g} + \sum_{r,0,g} \tilde{\phi}_{0g} - 2 \sum_{r,0,g} \tilde{\phi}_{2g} -$$

$$- \sum_{\substack{g'=1 \\ g' \neq g}}^G \sum_{0,g,g'} (\tilde{\phi}_{0,g'} - 2\tilde{\phi}_{2,g'}) - \frac{\chi_{0g}}{k_{eff}} \sum_{g'=1}^G \nu \sum_{r,g'} (\tilde{\phi}_{0,g'} - 2\tilde{\phi}_{2,g'}) - Q_{0,g} = 0$$

$$\bullet - D_{2,g} \frac{d^2}{dx^2} \tilde{\phi}_{2g} + \left(\sum_{r,2,g} \tilde{\phi}_{2g} + \frac{4}{5} \sum_{r,0,g} \right) \tilde{\phi}_{2g} - \frac{2}{5} \sum_{r,0,g} \tilde{\phi}_{0g} +$$

$$+ \frac{2}{5} \left[\sum_{\substack{g'=1 \\ g' \neq g}}^G \sum_{0,g,g'} (\tilde{\phi}_{0,g'} - 2\tilde{\phi}_{2,g'}) + \frac{\chi_{0g}}{k_{eff}} \sum_{g'=1}^G \nu \sum_{r,g'} (\tilde{\phi}_{0,g'} - 2\tilde{\phi}_{2,g'}) + Q_{0g} \right] = 0$$

$$Q_{0g} \Rightarrow Q_{0,1} = 1$$

$$G=2 \quad Q_{0,2} = 0$$

Diffusion $\rightarrow D_{01} \frac{d^2}{dx^2} \tilde{\phi}_{01}$

$D_{02} \frac{d^2}{dx^2} \tilde{\phi}_{02}$

Coupling

$$- 2 \sum_{r,0,1} \tilde{\phi}_{21}$$

$$- 2 \sum_{r,0,2} \tilde{\phi}_{22}$$

Removal $\rightarrow \sum_{r,0,1} \tilde{\phi}_{01}$

$$\sum_{r,0,2} \tilde{\phi}_{02}$$

Insertion

$$g=1 \quad - \sum_{50,11} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{2,1}) -$$

$$- \sum_{50,21} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{2,2})$$

$$g=2 \quad - \sum_{50,12} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{2,1})$$

$$- \sum_{50,22} (\phi_2$$

Fixed Source 3G

Eq A

CP3-8

Diffusion:

$$D_{01} \frac{d^2}{dx^2} \tilde{\phi}_{0,1}$$

$$D_{02} \frac{d^2}{dx^2} \tilde{\phi}_{0,2}$$

$$D_{03} \frac{d^2}{dx^2} \tilde{\phi}_{0,3}$$

Removal:

$$\sum_{r,0,1} \tilde{\phi}_{0,1}$$

$$\sum_{r,0,2} \tilde{\phi}_{0,2}$$

$$\sum_{r,0,3} \tilde{\phi}_{0,3}$$

Coupled:

$$-2 \sum_{r,0,1} \tilde{\phi}_{2,1}$$

$$-2 \sum_{r,0,2} \tilde{\phi}_{2,2}$$

$$-2 \sum_{r,0,3} \tilde{\phi}_{2,3}$$

In scatter:

$$\begin{aligned} g=1 & \left| \begin{aligned} & -\cancel{\sum_{s0,11} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{2,1})} - \sum_{s0,21} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{2,2}) - \sum_{s0,31} (\tilde{\phi}_{0,3} - 2\tilde{\phi}_{2,3}) \end{aligned} \right. \\ g=2 & \left| \begin{aligned} & -\sum_{s0,12} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{2,1}) - \cancel{\sum_{s0,22} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{2,2})} - \sum_{s0,32} (\tilde{\phi}_{0,3} - 2\tilde{\phi}_{2,3}) \end{aligned} \right. \\ g=3 & \left| \begin{aligned} & -\sum_{s0,13} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{2,1}) - \sum_{s0,23} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{2,2}) - \cancel{\sum_{s0,33} (\tilde{\phi}_{0,3} - 2\tilde{\phi}_{2,3})} \end{aligned} \right. \end{aligned}$$

Eq B
Renormal

Diffusion

$$\begin{aligned}
 -D_{z,01} \frac{d^2}{dx^2} \tilde{\phi}_{z,1} &+ \left(\sum r_{z,1} + \frac{4}{5} \sum r_{01} \right) \tilde{\phi}_{z,1} \\
 -D_{z,2} \frac{d^2}{dx^2} \tilde{\phi}_{z,2} &+ \left(\sum r_{z,2} + \frac{4}{5} \sum r_{02} \right) \tilde{\phi}_{z,2} \\
 -D_{z,3} \frac{d^2}{dx^2} \tilde{\phi}_{z,3} &+ \left(\sum r_{z,3} + \frac{4}{5} \sum r_{0,3} \right) \tilde{\phi}_{z,3}
 \end{aligned}$$

Coupling :

$$\begin{aligned}
 -\frac{2}{5} \sum r_{01} \tilde{\phi}_{0,1} \\
 -\frac{2}{5} \sum r_{02} \tilde{\phi}_{0,2} \\
 -\frac{2}{5} \sum r_{03} \tilde{\phi}_{0,3}
 \end{aligned}$$

Interactions :

$$\begin{aligned}
 g=1 \quad & \frac{2}{5} \left[\cancel{\sum_{s0,11} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{z,1})} + \sum_{s0,21} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{z,2}) - \right. \\
 & \left. + \sum_{s0,31} (\tilde{\phi}_{0,3} - 2\tilde{\phi}_{z,3}) \right] \\
 g=2 \quad & \frac{2}{5} \left[\sum_{s0,12} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{z,1}) + \cancel{\sum_{s0,22} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{z,2})} \right. \\
 & \left. + \sum_{s0,32} (\tilde{\phi}_{0,3} - 2\tilde{\phi}_{z,3}) \right] \\
 g=3 \quad & \frac{2}{5} \left[\sum_{s0,13} (\tilde{\phi}_{0,1} - 2\tilde{\phi}_{z,1}) + \sum_{s0,22} (\tilde{\phi}_{0,2} - 2\tilde{\phi}_{z,2}) \right. \\
 & \left. + \cancel{\sum_{s0,33} (\tilde{\phi}_{0,3} - 2\tilde{\phi}_{z,3})} \right]
 \end{aligned}$$

MULTIGROUP, CRIMINALITY SOURCES

CP3-10

$$\textcircled{A} \quad \underbrace{\text{Fixed source}}_{w/o - Q_{0,j}} + - \frac{x_j}{k_{eff}} \sum_{j'=1}^G v_{fj'} (\tilde{\phi}_{0j'} - 2\tilde{\phi}_{2j'}) = 0$$

$$\textcircled{B} \quad \underbrace{\text{Fixed source}}_{w/o \frac{2}{5} + Q_{0,j}} + \frac{2}{5} \frac{x_j}{k_{eff}} \sum_{j'=1}^G v_{fj'} (\tilde{\phi}_{0j'} - 2\tilde{\phi}_{2j'}) = 0$$

Eg A

$$\begin{aligned} & - \frac{x_1}{k_{eff}} \sum_{j'=1}^G \left[v_{f1} (\tilde{\phi}_{01} - 2\tilde{\phi}_{21}) + v_{f2} (\tilde{\phi}_{02} - 2\tilde{\phi}_{22}) + v_{f3} (\tilde{\phi}_{03} - 2\tilde{\phi}_{23}) \right] \\ & - \frac{x_2}{k_{eff}} \left[\begin{array}{c} || \\ || \\ || \end{array} \right] \\ & - \frac{x_3}{k_{eff}} \left[\begin{array}{c} || \\ || \\ || \end{array} \right] \end{aligned}$$

Eg B

$$\begin{aligned} & + \frac{2}{5} \frac{x_1}{k_{eff}} \left[v_{f1} (\tilde{\phi}_{01} - 2\tilde{\phi}_{21}) + v_{f2} (\tilde{\phi}_{02} - 2\tilde{\phi}_{22}) + v_{f3} (\tilde{\phi}_{03} - 2\tilde{\phi}_{23}) \right] \\ & + \frac{2}{5} \frac{x_2}{k_{eff}} \left[v_{f1} (\tilde{\phi}_{01} - 2\tilde{\phi}_{21}) + v_{f2} (\tilde{\phi}_{02} - 2\tilde{\phi}_{22}) + v_{f3} (\tilde{\phi}_{03} - 2\tilde{\phi}_{23}) \right] \\ & + \frac{2}{5} \frac{x_3}{k_{eff}} \left[v_{f1} (\tilde{\phi}_{01} - 2\tilde{\phi}_{21}) + v_{f2} (\tilde{\phi}_{02} - 2\tilde{\phi}_{22}) + v_{f3} (\tilde{\phi}_{03} - 2\tilde{\phi}_{23}) \right] \end{aligned}$$

$$D_{0,g} = \frac{1}{3(\bar{z}_{t,g} - \bar{z}_{s1,g})}$$

Diffcoef A

$$D_{2,g} = \frac{9}{35(\bar{z}_{t,g} - \bar{z}_{s3,g})}$$

Diffcoef B

$$\overset{\text{REMXSA}}{\bar{z}_{r0,g}} = \bar{z}_{t,g} - \bar{z}_{s0,g}$$

REMXSA

$$2\bar{z}_{r0}$$

~~REMXB~~
COUPLEXSA

$$-\bar{z}_{r2,g} + \frac{4}{5}\bar{z}_{r0,g} =$$

REMXB

$$= \bar{z}_{t,g} - \bar{z}_{s2,g} + \frac{4}{5}(\bar{z}_{t,g} - \bar{z}_{s0,g})$$

$$+ \frac{2}{5}\bar{z}_{r0,g} = \frac{2}{5}(\bar{z}_{t,g} - \bar{z}_{s0,g}) \quad \text{COUPLEXB}$$