Numerical Analysis

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Secant Method

Secant method mimics Newton's method but avoids the calculation of derivatives.

Newton's method

Let x_0 be an initial approximation of the root.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1 \ldots .$$

The definition of derivative

$$f'(x) = \lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}.$$

For small h

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

In particular, if $x = x_n$ and $h = x_{n-1} - x_n$

$$f'(x_n) \approx \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}.$$

$$x_{n+1} = x_n - \left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \right) f(x_n), \quad n = 1, 2, \ldots .$$

Let's note that we can start calculations if we have initial approximations x_0 and x_1.

At each iteration we have only one function evaluation.

Stopping conditions

- $|x_n - x_{n-1}| < \varepsilon$
- $\frac{|x_n - x_{n-1}|}{|x_n|} < \varepsilon, \quad |x_n| \neq 0$
- $|f(x_n)| < \varepsilon$

Example

Assume we want to approximate roots of the following equation

$$e^{x^2} - 2 = 0.$$

a. Use Secant Method to calculate the third approximation x_2 with $x_0 = 0$ and $x_1 = 1$;

b. (MATLAB) Calculate x_7.

Some examples of bad behavior of Secant Method:

- Next iteration is not defined;
- Method is divergent.

Convergence Analysis

Let's assume f, f' and f'' are continuous in a neighborhood of a zero r of f and $f'(r) \neq 0$. If initial approximations x_0 and x_1 are sufficiently close to the zero r then it can be shown that the rate of convergence for Secant Method is $\alpha = \frac{1+\sqrt{5}}{2} \approx 1.62$

$$|r - x_{n+1}| \leq C|r - x_n|^{\alpha},$$

where $C = const > 0$.

Pros. and Cons.

The Secant Method is nearly as fast as Newton's Method, but doesn't require knowledge of derivative.

The Secant Method requires two good starting points.

The Secant Method can fail to find a zero of nonlinear function which has a small slope near root.