Optimization

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Theorem

If $\Omega \subset \mathbb{R}^n$ *is an open convex set and* $f \in \mathbb{C}^2(\Omega)$, *then* f *is convex if and only if*

$$\nabla^2 f(x) \succeq 0, \quad \forall x \in \Omega.$$

Theorem

If $\Omega \subset \mathbb{R}^n$ *is an open convex set and* $f \in \mathbb{C}^2(\Omega)$ *such that* $\nabla^2 f(x) \succ 0$, $\forall x \in \Omega$, *then* f *is strictly convex.*

Theorem

If $\Omega \subset \mathbb{R}^n$ is an open convex set and $f \in \mathbb{C}^2(\Omega)$, then f is concave if and only if

$$\nabla^2 f(x) \preceq 0, \quad \forall x \in \Omega.$$

Theorem

If $\Omega \subset \mathbb{R}^n$ is an open convex set and $f \in \mathbb{C}^2(\Omega)$ such that $\nabla^2 f(x) \prec 0$, $\forall x \in \Omega$, then f is strictly concave.

Example

Check whether f is convex (strictly convex), concave (strictly concave) on Ω if

a. $f(x_1, x_2, x_3) = x_1^2 + 3x_1x_2 + 4x_2^2 + x_3^2 - x_1x_3$, $\Omega = \mathbb{R}^3$;

b. $f(x_1, x_2) = -x_1^4 + 2x_1x_2 - x_2^4 - x_1^2 - x_2^2$, $\Omega = \mathbb{R}^2$;

c. $f(x_1, x_2) = e^{x_1x_2}$, $\Omega = \mathbb{R}^2$;

d. $f(x_1, x_2) = x_1^3 + x_2^3$, $\Omega = \mathbb{R}^2$.