Classification with Logistic Regression and Linear Discriminant Analysis

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When customers leave a company, this is often referred to as 'churn'. Churn is a big concern for many service-oriented businesses as there is a cost of acquiring new customers, so keeping existing customers is important to keep costs down. Let's look into predicting whether a customer will churn for a telecommunications company.

Let's modify the objects somewhat to get a consistent naming convention

```
Xtrain = select(trainingData, -Churn)
Xtest  = select(testingData, -Churn)
Ytrain = select(trainingData, Churn) %>% unlist()
Ytest  = select(testingData, Churn) %>% unlist()
```

Let's look at the data structures for the training data:

```
str(Xtrain)
```

```
## tibble [6,340 x 19] (S3: tbl_df/tbl/data.frame)
##  $ customerID      : chr [1:6340] "5575-GNVDE" "3668-QPYBK" "7795-CFOCW" "9237-HQITU" ...
##  $ gender          : chr [1:6340] "Male" "Male" "Male" "Female" ...
##  $ SeniorCitizen   : num [1:6340] 0 0 0 0 0 0 0 0 0 0 ...
##  $ Partner         : chr [1:6340] "No" "No" "No" "No" ...
##  $ Dependents      : chr [1:6340] "No" "No" "No" "No" ...
##  $ tenure          : num [1:6340] 34 2 45 2 8 22 10 62 13 16 ...
##  $ MultipleLines   : chr [1:6340] "No" "No" "No phone service" "No" ...
##  $ InternetService : chr [1:6340] "DSL" "DSL" "DSL" "Fiber optic" ...
##  $ OnlineSecurity  : chr [1:6340] "Yes" "Yes" "Yes" "No" ...
##  $ OnlineBackup    : chr [1:6340] "No" "Yes" "No" "No" ...
##  $ DeviceProtection: chr [1:6340] "Yes" "No" "Yes" "No" ...
##  $ TechSupport     : chr [1:6340] "No" "No" "Yes" "No" ...
##  $ StreamingTV     : chr [1:6340] "No" "No" "No" "No" ...
##  $ StreamingMovies : chr [1:6340] "No" "No" "No" "No" ...
##  $ Contract        : chr [1:6340] "One year" "Month-to-month" "One year" "Month-to-month" ...
##  $ PaperlessBilling: chr [1:6340] "No" "Yes" "No" "Yes" ...
##  $ PaymentMethod   : chr [1:6340] "Mailed check" "Mailed check" "Bank transfer (automatic)"
"Electronic check" ...
##  $ MonthlyCharges  : num [1:6340] 57 53.9 42.3 70.7 99.7 ...
##  $ TotalCharges    : num [1:6340] 1890 108 1841 152 820 ...
```

```
#Note: we can put the number of unique values with the data structure:
rbind(sapply(Xtrain,function(x){ length(unique(x))}),
      sapply(Xtrain,class))
```

```
##      customerID  gender      SeniorCitizen Partner     Dependents  tenure
## [1,] "6340"      "2"         "2"           "2"         "2"         "73"
## [2,] "character" "character" "numeric"     "character" "character" "numeric"
##      MultipleLines InternetService OnlineSecurity OnlineBackup DeviceProtection
## [1,] "3"           "3"             "3"            "3"          "3"
## [2,] "character"   "character"     "character"    "character"  "character"
##      TechSupport StreamingTV StreamingMovies Contract    PaperlessBilling
## [1,] "3"         "3"         "3"             "3"         "2"
## [2,] "character" "character" "character"     "character" "character"
##      PaymentMethod MonthlyCharges TotalCharges
## [1,] "4"           "1546"         "5918"
## [2,] "character"   "numeric"      "numeric"
```

```
str(Ytrain)
```

```
##  Named chr [1:6340] "No" "Yes" "No" "Yes" "Yes" "No" "No" "No" "No" "No" ...
##  - attr(*, "names")= chr [1:6340] "Churn1" "Churn2" "Churn3" "Churn4" ...
```

```
table(Ytrain)
```

```
## Ytrain
##   No  Yes
## 4657 1683
```

Problem 0

Remember the items that usually need to be checked:

- data structures
- checking for missing data
- Converting qualitative features to dummy variables
- extreme observations
- transformations
- correlations

For this assignment, in the interest of keeping it short(er), let's just look at data structures and missing data

0.1 Missing data

```
anyNA(Xtrain)  # Check to see if there are any missing values in the training data
```

```
## [1] TRUE
```

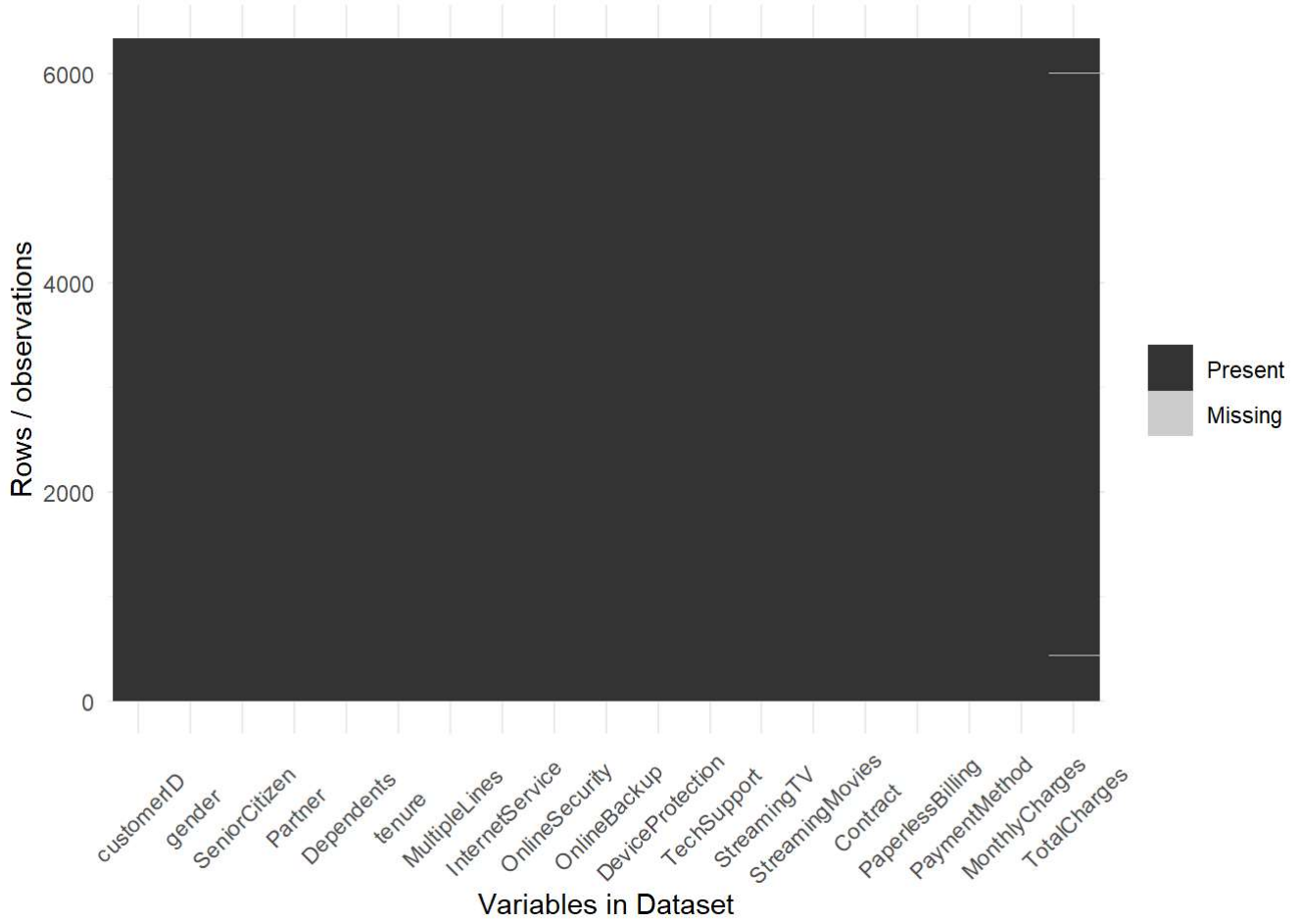
```
anyNA(Xtest)   # Check to see if there are any missing values in the test data
```

```
## [1] TRUE
```

```
ggplot_missing <- function(x){
    if(!require(reshape2)){warning('you need to install reshape2')}
    require(reshape2)
    require(ggplot2)

  x %>%
    is.na %>%
    melt %>%
    ggplot(data = .,
           aes(x = Var2,
               y = Var1)) +
    geom_raster(aes(fill = value)) +
    scale_fill_grey(name = "",
                    labels = c("Present","Missing")) +
    theme_minimal() +
    theme(axis.text.x  = element_text(angle=45, vjust=0.5)) +
    labs(x = "Variables in Dataset",
         y = "Rows / observations")
}
ggplot_missing(Xtrain) # Make a visualization with the function ggplot_missing of the missing da
ta in Xtrain
```

```
## Loading required package: reshape2
```



It looks like there are just missing values for TotalCharges in both Xtrain and Xtest. Let's use a linear regression model to impute TotalCharges with MonthlyCharges. Here is how that would work using the training data

```
trControl = trainControl(method='none')

imputationScheme = train(TotalCharges~MonthlyCharges,
                         data = select(Xtrain,MonthlyCharges, TotalCharges) %>% na.omit,
                         method = 'lm', trControl = trControl)
XtrainImp                 = Xtrain
M                         = is.na(XtrainImp$TotalCharges)
XtrainImp$TotalCharges[M] = predict(imputationScheme,
                                    select(Xtrain, MonthlyCharges, TotalCharges) %>% filter(M))
```

Now, we want to impute the test features using the imputation scheme learned using the training features.

```
XtestImp = Xtest
# Using 'imputationScheme' from 0.1.3, impute the missing value(s) in Xtest
#Don't retrain a new imputation scheme on the test data, just use the training
M                         = is.na(XtestImp$TotalCharges) # imputation scheme
XtestImp$TotalCharges[M] = predict(imputationScheme,
                                    select(Xtest, MonthlyCharges, TotalCharges) %>% filter(M))
```

0.2 Data structures

Now, convert the qualitative features and supervisors to factors.

```
XtrainImpFact = select(XtrainImp, -customerID,
                       -tenure, -MonthlyCharges, -TotalCharges) %>%
  mutate_all(factor)

XtestImpFact = select(XtestImp, -customerID,
                       -tenure, -MonthlyCharges, -TotalCharges) %>%
  mutate_all(factor) # Convert the character data structures in Xtest to factor

Ytrain   = factor(Ytrain)

Ytest    = factor(Ytest) # Convert the character data structures in Ytest to factor
```

0.3 Dummy variables

```
dummyModel = dummyVars(~ ., data = XtrainImpFact, fullRank = TRUE)

XtrainQualDummy = predict(dummyModel, XtrainImpFact)
XtestQualDummy  = predict(dummyModel, XtestImpFact)
```

Let's create the full feature matrices for the training and test set

```
XtrainQuan = select(XtrainImp, tenure, MonthlyCharges, TotalCharges)
XtrainFull = cbind(XtrainQualDummy, XtrainQuan)

XtestQuan  = select(XtestImp, tenure, MonthlyCharges, TotalCharges)
XtestFull  = cbind(XtestQualDummy, XtestQuan)
```

Some additional processing

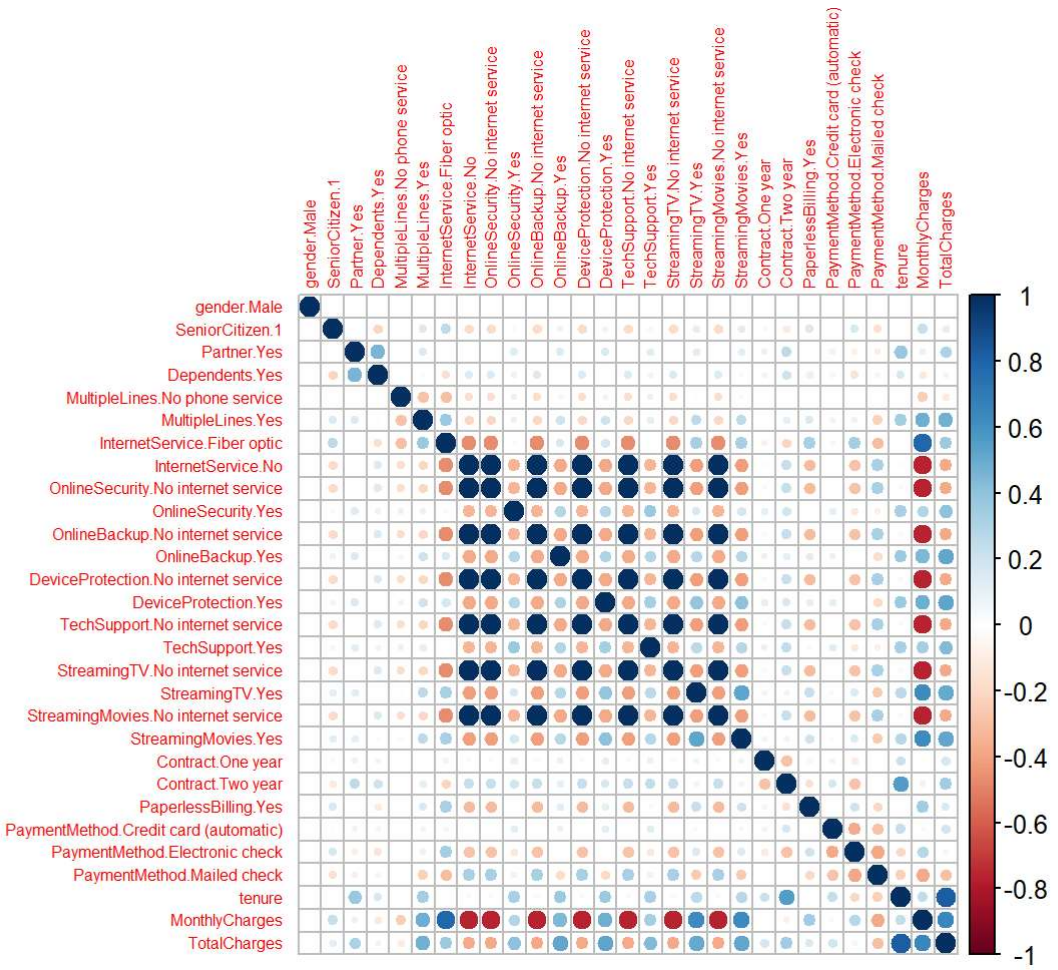
Note that there is some redundant information in the qualitative features. Though it isn't really "correlation" (because we are referring to indicator features), we can still use a corrplot to visualize the problem

```
require(corrplot)
```

```
## Loading required package: corrplot
```

```
## corrplot 0.91 loaded
```

```
corrplot(cor(XtrainFull), tl.cex = 0.5)
```



There are qualitative features like “InternetService” and then other qualitative features like “OnlineBackup” which have a level “no internet service”. These new features overlap exactly and hence we need to remove some of them

```
XtrainFull = select(XtrainFull,-contains('.No internet'))
XtestFull  = select(XtestFull,-contains('.No internet'))
```

Problem 1: Logistic Regression

Let’s go through and make a predictive model out of logistic regression

Train the logistic regression model on the training data

Make sure you are using the trainControl to only train the model by setting method = ‘none’. ‘train’ treats the first level as the event of interest, which is in alphabetical order. So, ‘no’ would be the event. However, we usually want to code results so that the outcome of interest is the event. We can make this adjustment in R via ‘relevel’ on the supervisor

```
YtrainRelevel = relevel(Ytrain, ref = 'Yes')
YtestRelevel  = relevel(Ytest, ref = 'Yes')

trControl    = trainControl(method = 'none')
outLogistic  = train(x = XtrainFull, y = YtrainRelevel,
                     method = 'glm', trControl = trControl)
```

Problem 1.2: Get the test predicted probabilities

Get the predicted probabilities for the test data and print out the first few predictions with the ‘head’ function

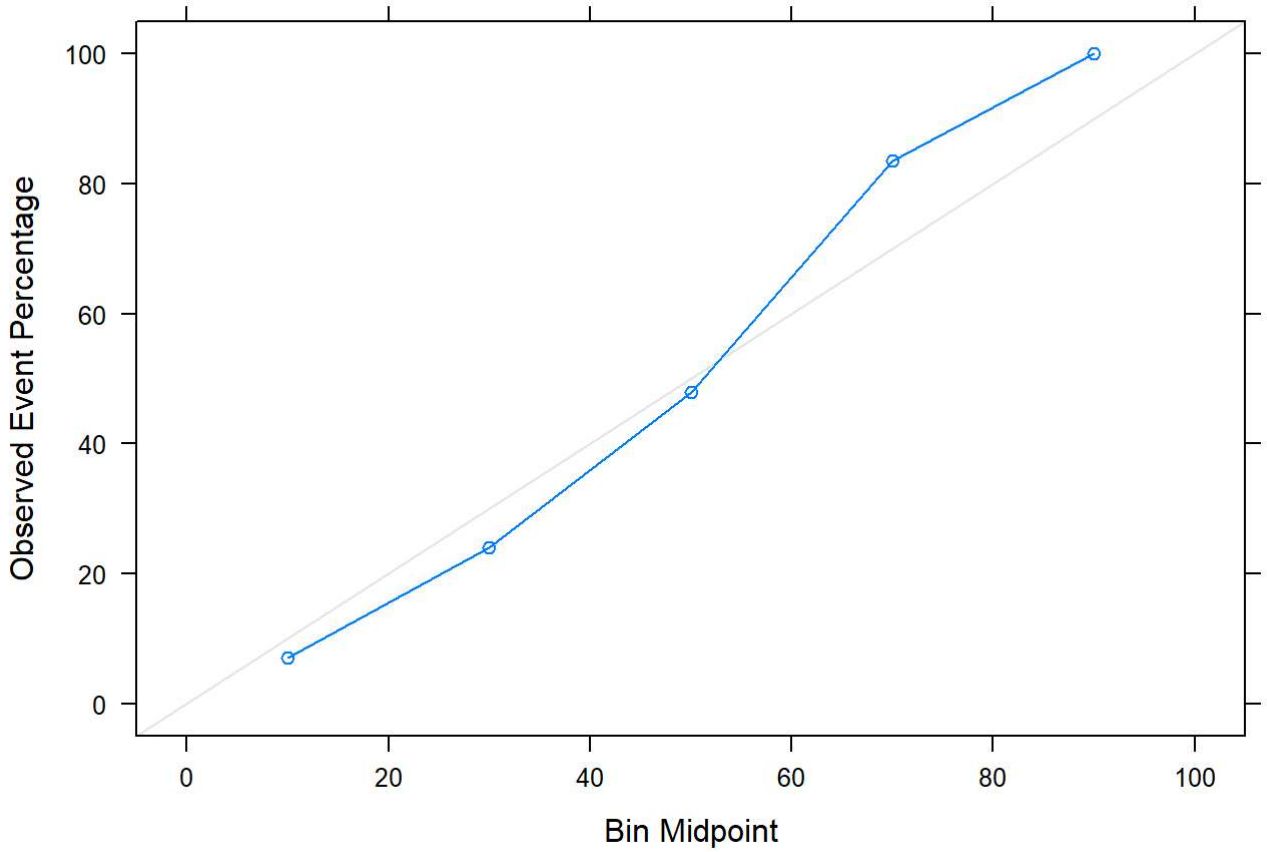
```
YhatTestProb = predict(outLogistic, XtestFull, type = 'prob')
head(YhatTestProb)
```

```
##          Yes        No
## 1 0.62959261 0.3704074
## 2 0.60063123 0.3993688
## 3 0.02781752 0.9721825
## 4 0.50092343 0.4990766
## 5 0.16875929 0.8312407
## 6 0.36771916 0.6322808
```

Problem 1.3: Well-calibrated probabilities

Produce a calibration plot using your predictions of the test Data

```
calibProbs = calibration(YtestRelevel ~ YhatTestProb$Yes, cuts = 5)
xyplot(calibProbs)
```



Are these well calibrated probabilities?

Yes, these probabilities are fairly well calibrated, at least for bin midpoints 60 and below. Above 60 and we start to see stronger deviation with higher observed events than we would expect for these bins.

Problem 1.4: The Confusion matrix

Get the classifications now using the default threshold.

```
YhatTest = predict(outLogistic, XtestFull, type = 'raw')
```

Look at the help file for the function 'confusionMatrix' function. Instead of producing all the output as in the lectures, produce only the confusion matrix, the accuracy rate, kappa, the sensitivity, and the specificity.

```
confusionMatrixOut = confusionMatrix(reference = YtestRelevel, data = YhatTest)

print(confusionMatrixOut$table)
```

```
##            Reference
## Prediction Yes  No
##        Yes 114  35
##        No   72 482
```

```
print(confusionMatrixOut$overall[1:2])
```

```
##  Accuracy     Kappa
## 0.8477952 0.5822842
```

```
print(confusionMatrixOut$byClass[1:2])
```

```
## Sensitivity Specificity
##   0.6129032   0.9323017
```

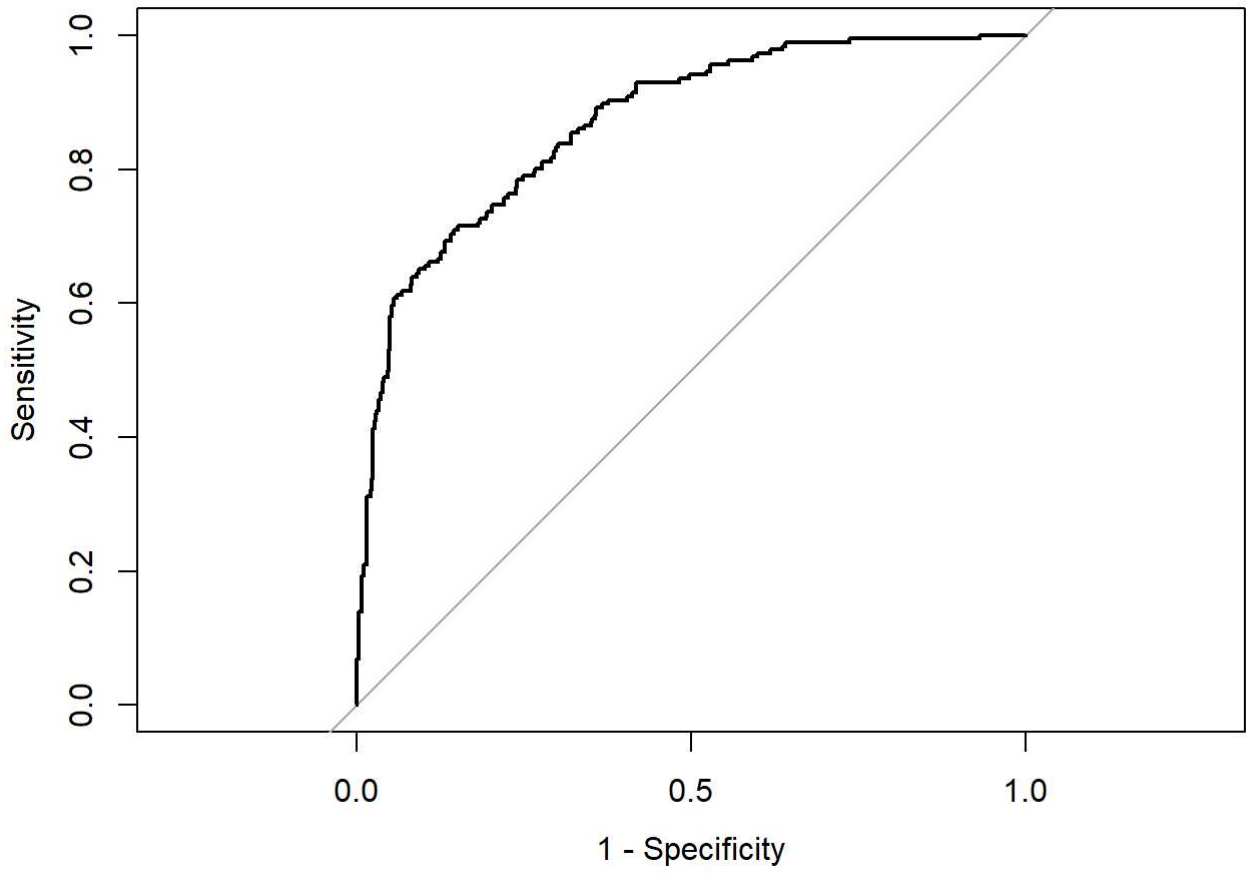
Problem 1.5: Produce the ROC curve using the test data

```
rocCurve = roc(Ytest, YhatTestProb$Yes)
```

```
## Setting levels: control = No, case = Yes
```

```
## Setting direction: controls < cases
```

```
plot(rocCurve, legacy.axes=TRUE)
```



Briefly describe this curve. What the objects are being plotted? What value for the threshold would put a classifier at the point (0,0)?

The ROC curve plots the sensitivity (# true positives / total # positives), 1-specificity (1-#true negatives/total # negatives) and implicitly plots threshold values. The point (0,0) corresponds to a threshold value of 1.

Problem 1.6: AUC

We can get a one number summary of the ROC from the 'rocCurve' object: auc

```
rocCurve$auc
```

```
## Area under the curve: 0.8689
```

What does the AUC describe? What is a realistic minimum value for the AUC? Maximum value?

The AUC is a single value quantitative summary of the quality of a classifier averaged across all possible threshold values. The maximum AUC is 1 and classifiers with AUC's closer to 1 are generally considered to be better, but the process of averaging across all thresholds diminishes its utility. A minimum value would be 0.5 (or no better than a coin flip method).

Problem 1.7: Achieving a particular sensitivity

What is the specificity for a model that has a sensitivity of at least 0.8? What threshold does it occur at?

```
thresholds = rocCurve$thresholds

pt8        = which.min(rocCurve$sensitivities >= 0.8)

threshold   = thresholds[pt8]
specificity = rocCurve$specificities[pt8]
```

The specificity is 0.7330754.

Problem 2: Linear Discriminant Analysis (LDA)

Let's do the same thing for LDA.

Train the LDA model on the training data

Make sure you are using the trainControl to only train the model by setting method = 'none'. 'train' treats the first level as the event of interest, which is in alphabetical order. So, 'no' would be the event. However, we usually want to code results so that the outcome of interest is the event. We can make this adjustment in R via 'relevel' on the supervisor.

```
YtrainRelevel = relevel(Ytrain, ref = 'Yes')
YtestRelevel  = relevel(Ytest, ref = 'Yes')

trControl = trainControl(method = 'none')
outLDA    = train(x = XtrainFull, y = YtrainRelevel,
                  method = 'lda', trControl = trControl)
```

Get the test predicted probabilities

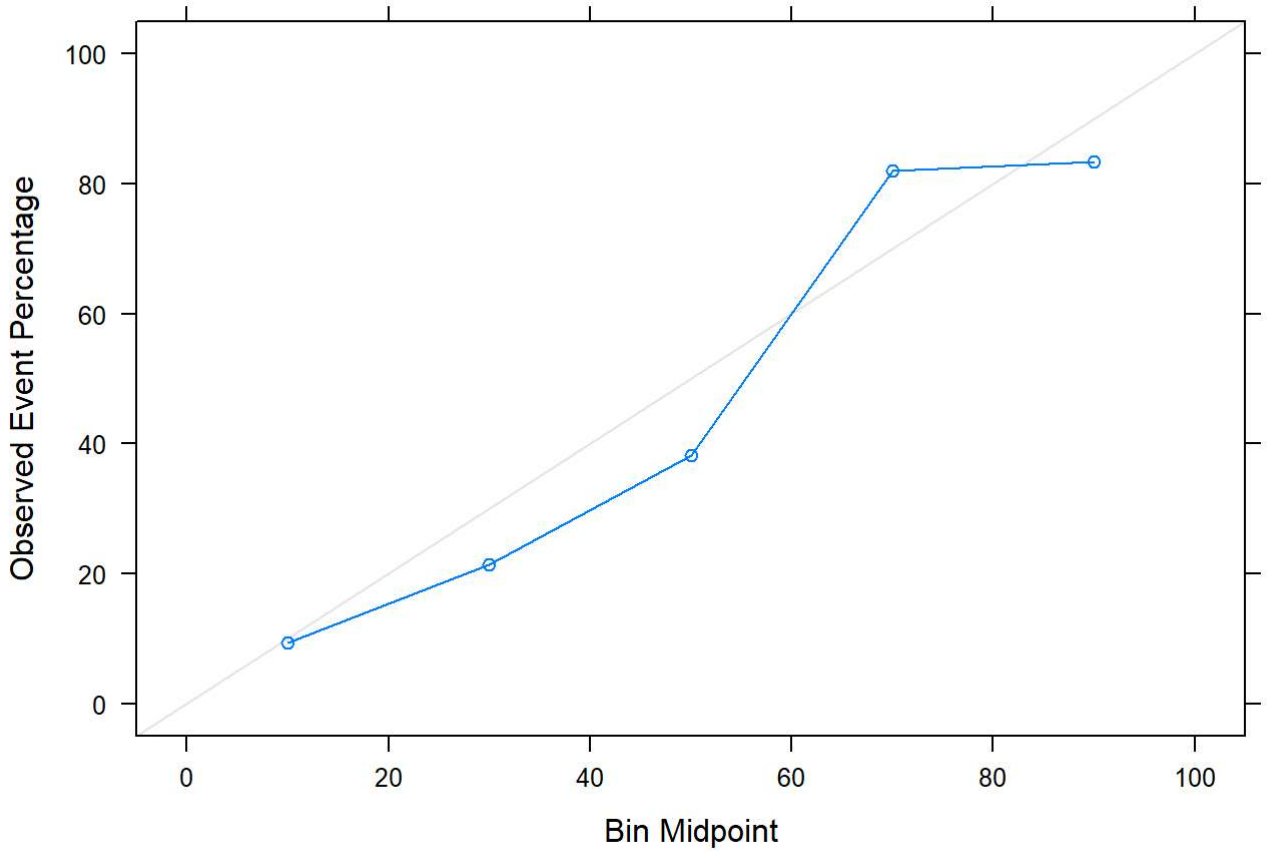
```
YhatTestProb2 = predict(outLDA, XtestFull, type = 'prob')
head(YhatTestProb2)
```

```
##          Yes        No
## 1 0.48308863 0.5169114
## 2 0.62943861 0.3705614
## 3 0.05668856 0.9433114
## 4 0.53500672 0.4649933
## 5 0.13409761 0.8659024
## 6 0.29653606 0.7034639
```

Problem 2.1: Well-calibrated probabilities

Produce a calibration plot using your predictions of the test Data

```
calibProbs = calibration(YtestRelevel ~ YhatTestProb2$Yes, cuts = 5)
xyplot(calibProbs)
```



Are these well calibrated probabilities?

These probabilities are fairly well calibrated. But it's not as accurate in the lower range bins as in logisitc regression.

Problem 2.2: The Confusion matrix

Get the classifications now using the default threshold.

```
YhatTest2 = predict(outLDA, XtestFull, type = 'raw')
```

Look at the help file for the function 'confusionMatrix' function. Instead of producing all the output as in the lectures, produce only the confusion matrix, the accuracy rate, kappa, the sensitivity, and the specificity.

```
confusionMatrixOut2 = confusionMatrix(reference = YtestRelevel, data = YhatTest2)

print(confusionMatrixOut2$table)
```

```
##            Reference
## Prediction Yes  No
##        Yes 111  40
##        No   75 477
```

```
print(confusionMatrixOut2$overall[1:2])
```

```
##  Accuracy     Kappa
## 0.8364154 0.5526975
```

```
print(confusionMatrixOut2$byClass[1:2])
```

```
## Sensitivity Specificity
##   0.5967742   0.9226306
```

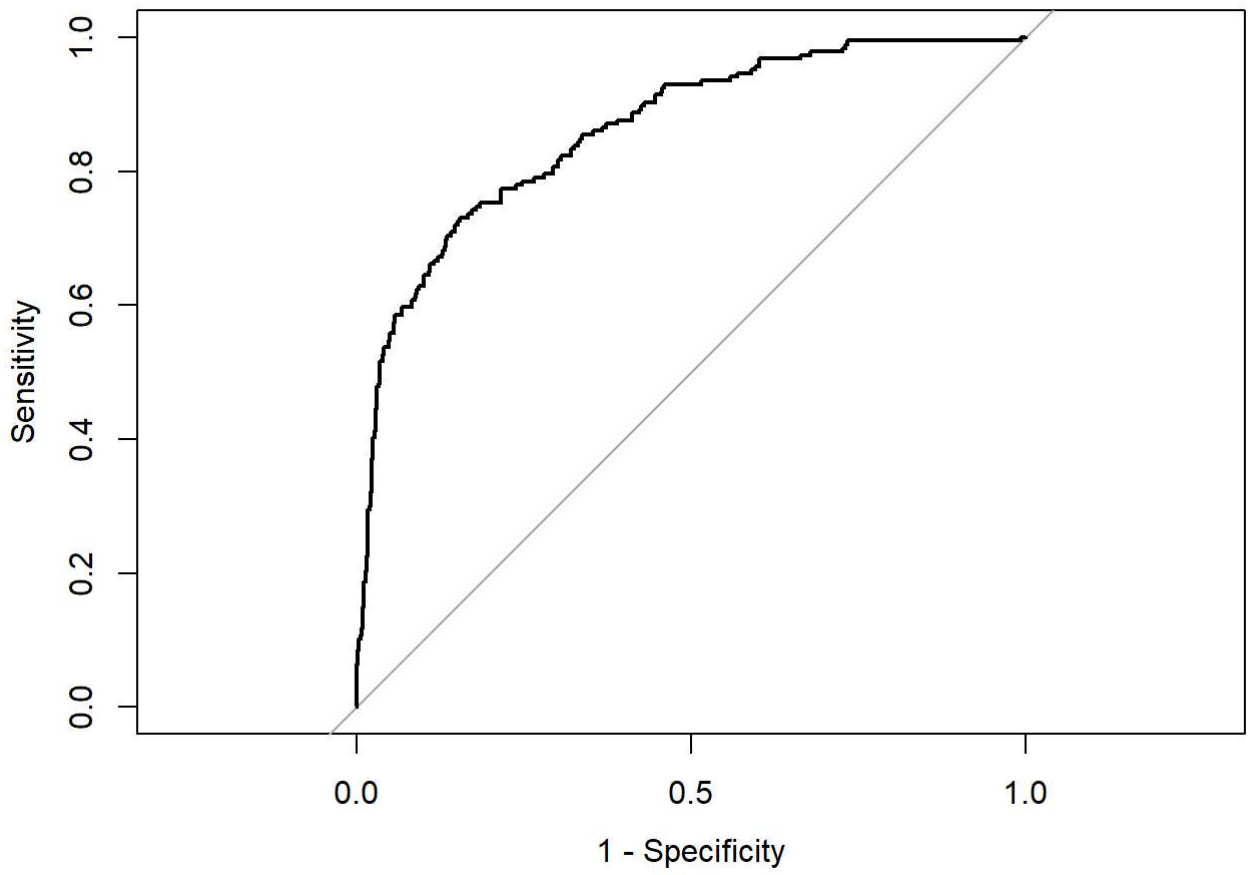
Produce the ROC curve using the test data

```
rocCurve2 = roc(Ytest, YhatTestProb2$Yes)
```

```
## Setting levels: control = No, case = Yes
```

```
## Setting direction: controls < cases
```

```
plot(rocCurve2, legacy.axes=TRUE)
```



Problem 2.3: AUC

Get the AUC for LDA

```
rocCurve2$auc
```

```
## Area under the curve: 0.8613
```

Problem 2.4: Achieving a particular sensitivity

What is the specificity for a model that has a sensitivity of at least 0.8? What threshold does it occur at?

```
thresholds2 = rocCurve2$thresholds

pt8        = which.min(rocCurve2$sensitivities >= 0.8)

threshold   = thresholds2[pt8]
specificity2 = rocCurve2$specificities[pt8]
```

The specificity is 0.7059961.

Problem 3: Comparison

Report your answers to at least 4 digits after the decimal.

What are the models's accuracies? Which model would you prefer based on accuracy?

The accuracy of the logistic regression model is 0.8478. The accuracy of the LDA model is 0.8364. Since the logistic regression model has higher accuracy, that model is preferable.

What are the models's AUCs? Which model would you prefer based on AUC?

The AUC of the logistic regression model is 0.8689. The AUC of the LDA model is 0.8613. Again, the AUC of the logistic regression model is higher and closer to 1, so the logistic regression model is preferable.

What are the models's maximum specificity for a sensitivity at least 0.8? Which model would you prefer based on this?

The max specificity for the logistic regression model is 0.7331. The max specificity for the LDA model is 0.706. Since the specificity is higher for the logistic regression model, that model is preferable.