1. R-Lab§5.19 Problem 2.

2. In this problem, we will fit distributions to the weekly returns of S&P 500 in the same time frame as the last homework,

```
> getSymbols("^GSPC", from = "1991-01-01", to = "2021-02-01")
> rt = weeklyReturn(Ad(GSPC), type = "log")
```

   (a) Fit standardized t, skewed t, GED, skewed GED to the weekly returns of S&P 500 `rt` using `fitdist()` of R's package `rugarch`.

   Tips: The `fitdist()` of `rugarch` is convenient for fitting several distributions.

```
> dists = c("std", "sstd", "ged", "sged")
> fits = vector("list", 4)
> for(i in 1:4) fits[[i]] = fitdist(dists[i], rt)
```

   The `std` fit is stored in `fits[[1]]`, etc. You can compute/plot parts (b) and (c) easily with a loop. The object `dists` can be used for labelling plots.

   (b) For each fitted distribution, plot the density curve of fitted distribution with estimated parameter values from part (a) overlaying the kernel density estimate curve (`density()`). Please refer to Problem 6 of §R-Lab 4.10 in the last homework. Use different colors for the two curves. Make the 4 plots on a 2×2 layout. Clearly label them. Examine the plots, which distribution is the best fit?

   If necessary, zoom in the plot by setting `xlim` to be shorter. Also mind the limits of y-axis, your plots should not be cut off.

   Tips: Store the kernel estimates, `den = density(rt)`. The return values are `den$x` and `den$y`, both are of the default length 512. The vector `den$y` consists of kernel estimates evaluated at `den$x`. You can plot the kernel density curve with them. For the parametric density curve, use `ddist()` evaluate density at `den$x` with the MLE estimates for the arguments. For example, the `std` density sequence can be obtained by

```
> est = fits[[1]]$pars
> ddist(dists[1], den$x, mu = est["mu"], sigma = est["sigma"],
            skew = est["skew"], shape = est["shape"])
```

   The four plots can be done with a loop in `R` code.

   (c) Compute the AIC and BIC criteria for all 4 models. Which model is selected by the AIC? The BIC? Which model would you choose? Why?

   (d) For the two skew distributions, construct the 95% confidence intervals for the skewness parameter. Based on these confidence intervals, would you reject the null hypothesis that the distribution is symmetric?

   (e) Consider the skew t distribution. We would like to test for the distribution symmetry using the likelihood ratio test based on the estimated skew t model. Compute the likelihood ratio statistic and give the test. Your answer should include the null, alternative hypotheses (in terms of parameter), test statistic, p-value and conclusion.

I would suggest using `nlminb()` instead of `optim()` for optimization. Just like `optim()`, the first and second arguments are starting values and the negative log likelihood. In addition, set the lower bound (`lower =`) . For the returned values, `par` is the vector of MLE and `objective` is the the negative log likelihood evaluated at MLE.

3. R-Lab§7.13.2, Problems 3-6. For Problem 6, part (b), set random seed, `set.seed(200128)` and use `rt()` to generate. **Note:** This is not a suggestion, you are required to use R's basic function `rt()` to simulate.