Please load the data "HW06.RData" for this assignment.

There are 3 objects, rt, yt and wt, use command ls() to see the list of object names. Important: Both rt and yt are daily returns in percentage (%).

1. The object rt contains the daily log returns of 8 stocks, Advanced Micro Devices (AMD), Johnson & Johnson (JNJ), Coca-Cola (KO), Mcdonald's (MCD), Oracle (ORCL), Procter & Gamble (PG), Tesla (TSLA), UnitedHealth (UNH) between January 4, 2011 and December 31, 2020.

```
> cat("Starting dates:\n"); head(rt,2)
```

Starting dates:

```
AMD JNJ KO MCD ORCL PG TSLA UNH
2011-01-04 3.4806 0.840134 -2.09166 -3.03517 -0.44371 0.27752 0.18765 0.91148
2011-01-05 1.5837 -0.063165 -0.59672 0.46992 -1.40762 -0.23122 0.59813 0.31976
> cat("Ending dates:\n"); tail(rt,2)
```

Ending dates:

```
AMD JNJ KO MCD ORCL PG TSLA UNH
2020-12-30 1.82608 1.23152 0.57106 -0.54211 -0.093121 -0.47069 4.2321 -0.68176
2020-12-31 -0.63044 0.84868 0.73207 1.41740 0.449300 0.98950 1.5552 1.63587
```

- (a) Fit a multivariate-t model to the returns in rt. Show all the MLE's.
- (b) The annual risk free rate is 3%, there are 253 trading days in a year (the daily rate is 3/253). The object wt is a portfolio that has 0.10 proportion of risk free asset and 0.90 of risky assets of AMD, JNJ, KO, MCD, ORCL, PG, TSLA, UNH stocks.

> wt

```
UNH
                                                                                     PG
risk_free
                ORCL
                             MCD
                                      TSLA
                                                    ΚO
                                                                         JNJ
                                                                                              AMD
   0.1000
             -0.0437
                         0.4367
                                    0.1128
                                              -0.0768
                                                           0.3305
                                                                      0.0764
                                                                                 0.0865
                                                                                          -0.0224
```

What is the distribution of the return of this portfolio? Please include all the parameter estimates.

Note: In R, a matrix product operation will return a value in a matrix form including a scalar. E.g. For 2 vectors a, b and a matrix A, the product t(a)%*%A%*%b returns a 1 × 1 matrix. Remove the matrix form of a scalar by applying as.vector().

(c) For an investment of \$500,000, what are estimates of the one-day Value at Risk (VaR) and Expected Shortfall (ES) at $\alpha = 0.01, 0.02, 0.03, 0.04, 0.05$ (alpha = seq(.01,.05, .01)) based on the estimated distribution in part (b)?

Note: Since the return data are in %, please convert the investment S = 500000/100. Please do NOT change the return data.

2. The object yt is the daily log returns of Goldman Sachs from January 4, 2011 and December 31, 2020.

```
> cat("Starting dates:\n"); head(yt,2)
```

Starting dates:

```
GS
2011-01-04 0.01734
2011-01-05 0.53014
> cat("Ending dates:\n"); tail(yt,2)
Ending dates:

GS
2020-12-30 0.55656
2020-12-31 1.62860
```

For an investment of \$200,000, compute the estimated one-day VaR and ES at $\alpha = 0.01, 0.02, 0.03, 0.04, 0.05$ using the following procedures.

- (a) Nonparametric estimation.
- (b) Parametric estimation with a t distribution.
- (c) Semiparametric regression estimation. Use the following candidate bandwidths m = 50, 100, 200, 300, 400.

R commands for superimposing a fitted line on a scatter plot of \boldsymbol{x} and \boldsymbol{y}

```
> plot(x,y)
> abline(lsfit(x,y)$coef)
```