The data `HW07.RData` contains 4 objects, `Rt`, `RM` and `syb`:

`RM`: Weekly returns of S&P 500, Jan 1, 2008 to Feb 28, 2021.

`Rf`: Weekly rate of US 3 month Treasure Bill for the same period of RM.

`Rt`: Weekly returns of 8 stocks, Ambev S.A. (ABEV), Hyatt Hotels Co. (HD), IBM Co. (IBM), Johnson & Johnson (JNJ), McDonald's Co. (MCD), Micron Technology, Inc. (MU), Koninklijke Philips N.V. (PHG) and Polaris Industries Inc. (PII).

`syb`: the ticker symbols of Rt.

To study the CAPM, we will use RM, the S&P 500, as a proxy for Market portfolio and Rf, the US 3-Month Treasury Bill, as a proxy for risky-free asset returns.

Important: Both `RM` and `Rf` are matrices of dimension $n \times 1$. When apply R's `var()`, the return values is a 1×1 matrix. Please remove the matrix with `as.vector()` or `as.numeric()`. Eg. `YM = RM-Rf;` `as.vector(var(YM))`. Whenever you copy my code with `var(YM)`, always replace it with `as.vector(var(YM))`.

1. Let Y_{jt} be the excess return of stock j and $\boldsymbol{Y}_t = (Y_{1t} \ldots, Y_{Nt})^T$, where $N = 8$. Fit the excess return model of (8.9) on page 163 of Handout 8.

   (a) Give the estimates of betas for all 8 assets. Which two assets have the highest betas? Which two assets have the lowest betas?

   (b) Give the proportion of the square risk that is due to the systematic risk for each asset.

   (c) Suppose each Y_{it}, $i = 1, \ldots, 8$ follows the CAPM, give the estimates for the excess return based on the model.

   (d) Test if the CAPM holds for each individual asset. What are the hypotheses of the tests? State your conclusion.

   (e) Compute the sample mean $\overline{Y_i}$, $i, = 1, \ldots, 8$. Are they close to the estimates of part (b)? Are the highest few sample means consistent with the highest betas? Give a possible explanation if they are not consistent.

2. In Question 1, we consider the CAPM for each individual asset. In this question, we will consider $\boldsymbol{Y}_t = (Y_{1t}, \ldots, Y_{Nt})^T$ as a whole.

   (a) Test if the CAPM holds for the 8 assets as a whole using both the Wald and likelihood ratio tests. Are the test results the same as what you expected?

   (b) What are the estimated systematic component and unique component of the risk of $\boldsymbol{Y}_t$? (Please note: both are matrices).

(c) For a CAPM, the systematic component is unavoidable risk and the unique component can be diversified out. Find the portfolio that minimizes the unique risk of $\boldsymbol{Y}_t$ allowing short selling. (this was a midterm question in Spring 2020).

(d) Consider 2 portfolios, one is from part(b) and the other is an equal weight portfolio (i.e., $w_i = 1/8, i = 1, \ldots, 8$, in R: rep(1/8, 8)). Give a Wald test to check if the CAPM holds for the 2 portfolios.