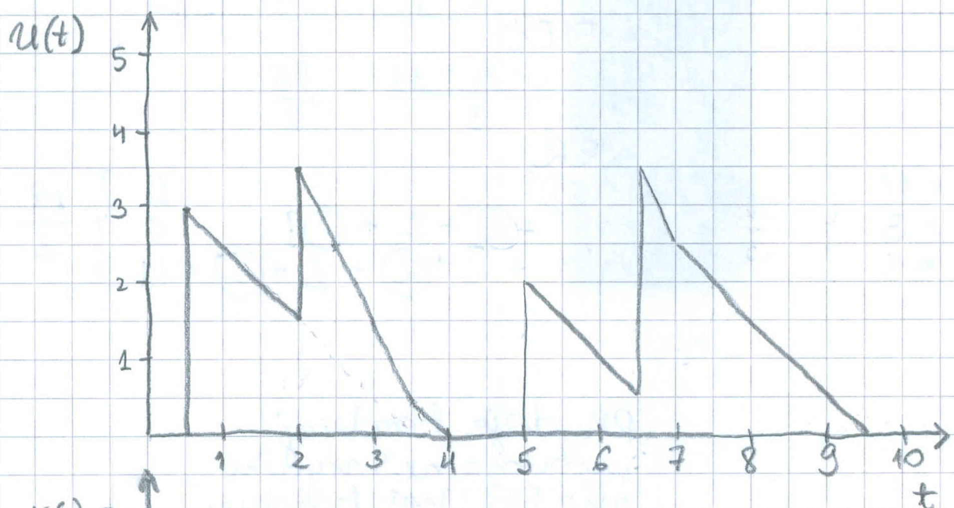
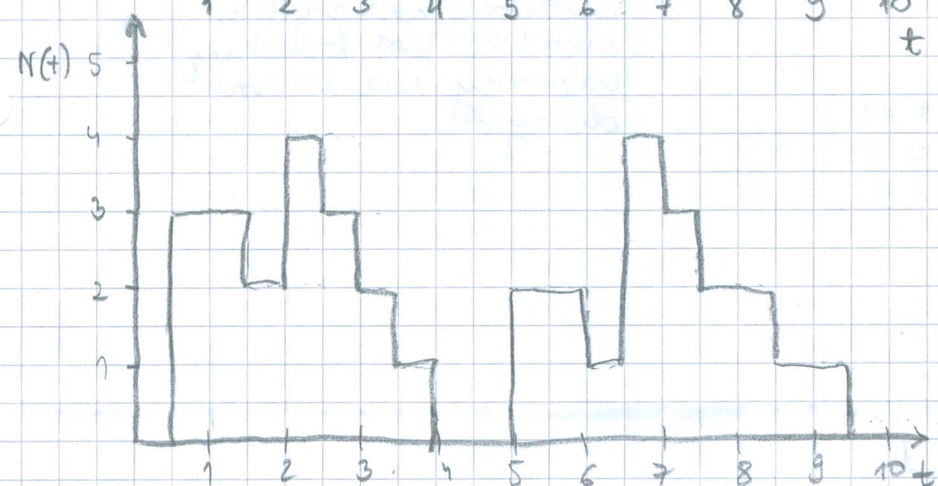
Problem 1, Practice set 1

$N(t)$ - queue length $\qquad (t_n^+) = (0.5, 2, 5, 6.5)\ s$

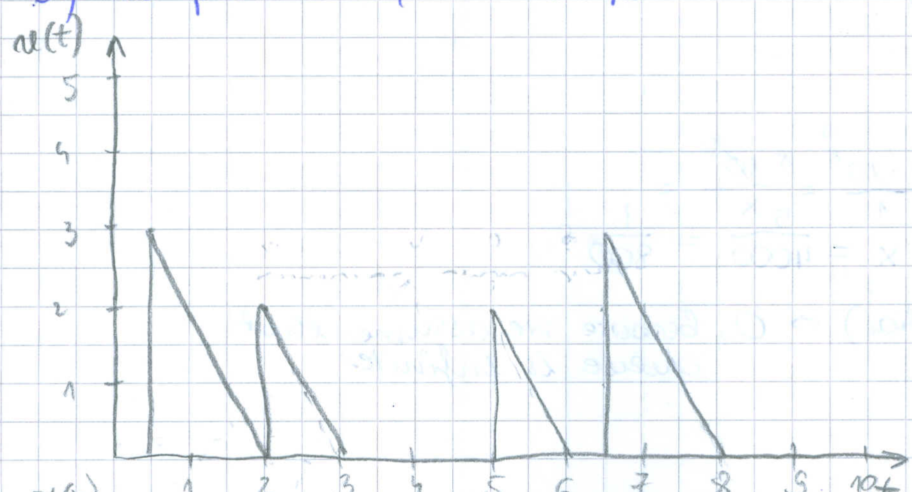
$U(t)$ - unfinished work $\qquad (b_n) = (3, 2, 2, 3)\ s.u$

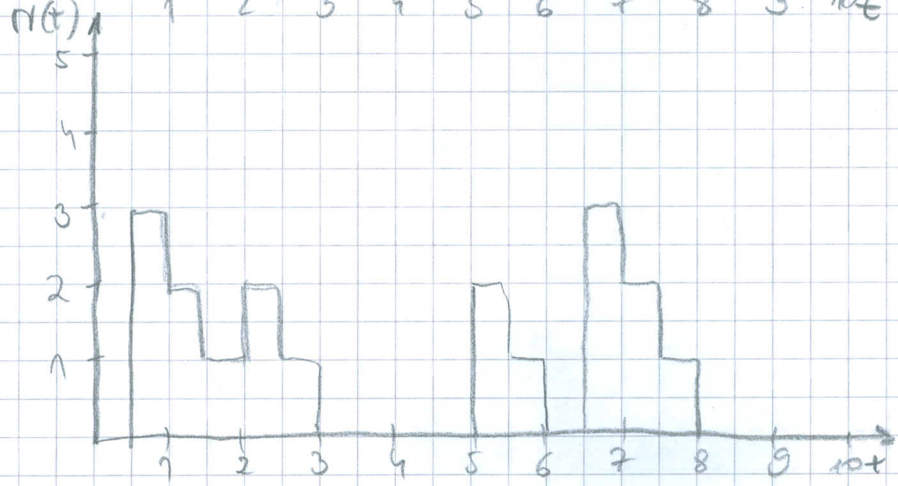
a.) two processors, $v = 1$ s.u/s each





b.) one processor, $v = 2$ s.u/s





Problem 2, Practice set 1

X - 7 s.u
Y - 1 s.u
Z - 3 s.u

a.) XYZ

FIFO

$$\frac{7+8+11}{3} = \frac{26}{3}$$

RR

$$\left.\begin{array}{ll} X & 2+1+2+2+1+3 = 11 \\ Y & 2+1] = 3 \\ Z & 2+1+2+2+1] = 8 \end{array}\right\} \frac{22}{3}$$

b.) YZX

FIFO

$$\frac{1+4+11}{3} = \frac{16}{3}$$

RR

$$\left.\begin{array}{ll} Y & 1] = 1 \\ Z & 1+2+2+1] = 6 \\ X & 1+2+2+1+5 = 11 \end{array}\right\} \frac{18}{3}$$

c.) XZY

FIFO

$$\frac{7+10+11}{3} = \frac{28}{3}$$

RR

$$\left.\begin{array}{ll} X & 2+2+1+2+1+3 = 11 \\ Z & 2+2+1+2+1 = 8 \\ Y & 2+2+1] = 5 \end{array}\right\} \frac{24}{3}$$

RR daje bardziej „zrównane" wyniki niż FIFO. Jest bardziej odporny na losowość danych

Problem 3, practice set 1

Little's law (slide 30)

$$N_m = \frac{1-L}{a_m} d_m$$

a_m = interval → $\frac{1}{800}$ s

d_m = processing time → $\frac{4 \cdot 10^{8}}{1} = \frac{5 \cdot 10^{8}}{5x} =$

$x = 4000 = \frac{1}{800}$ s.

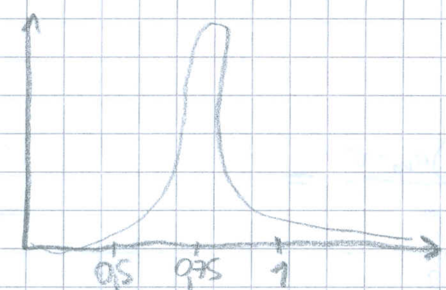
L = requests rejected (fraction) → 0, because we assume that queue is infinite

$$N_m = \frac{1-0}{\frac{1}{800}} \cdot \frac{1}{800} = 1$$

Problem 1, practice set 2 (check Gauss distribution)

$v = 1$ s.u/s
„dense" arrival stream $\rightarrow r = 75\%$ offered load
$\sigma = 0.1$ s. $\rightarrow$ standard deviation
jakie szanse, że procesor będzie mógł wykorzystać 0,5 s bez robienia backlogu?

Service demand



$b_1 = r = 0{,}75\ \%$
$b_2 \le \cancel{\ } 50\% \rightarrow$ gdy 50% jest „wolne"

$P(a \le X_{(r,\sigma)} \le b) = P(0 \le X_{(0{,}75,\,0{,}1)} \le 0{,}5) =$
$= \Phi\left(\frac{b-r}{\sigma}\right) - \Phi\left(\frac{a-r}{\sigma}\right) = \Phi\left(\frac{0{,}5-0{,}75}{0{,}1}\right) - \Phi\left(\frac{0-0{,}75}{0{,}1}\right)$
$= \Phi(-2{,}5) - \Phi(-7{,}5) = \Phi(7{,}5) - \Phi(2{,}5) =$
$\approx 0{,}5 - 0{,}4938 = 0{,}0062 = 0{,}62\% = 6{,}2‰$

Szanse wynoszą 0,62 %

Problem 2, practice set 2

finite buffer of capacity Q
offered load $r > 1$
L – ?

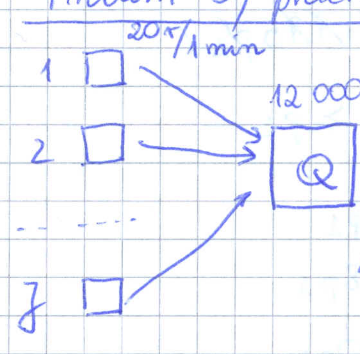
$1 - p_0 = (1-L)r$

if $Q \to \infty$ then $p_0 \to 0$ (processor idle time)

$1 - 0 = (1-L)r$
$1 = r - Lr$
$Lr = r - 1$
$L = \frac{r-1}{r}$

$\frac{1}{1-L} = r > 1$
$\frac{1}{1-L} > 1$
$1 > 1 - L$
$L > 0$

Problem 3, practice set 2



time needed for 1 report:
$b_m = \frac{\cancel{1800}\,3}{\cancel{12000}\,20} = \frac{3}{20} = 0{,}15$ s

boldface $\le \frac{d_m}{b_m} = \frac{1{,}8}{0{,}15} = 12 \Rightarrow Q = 22$ lub 23

$J \cdot r_1 \le 1$

J – ?
$d_m = 1{,}8$ s

dla jednej końcówki
$r_1 = \frac{b}{a \cdot v} = \frac{\cancel{1800}}{\cancel{3} \cdot \cancel{12000}\,20} = \frac{1}{20} = 5\%$

$J \cdot 0.05 \le 1$
$J \le 20$

ilość źródeł: 20 kolejka: $Q = 23$

Problem 1, practice set 3

$J = 50$
think time $= t_t = 2/3$ s
length $=$ 80% $\to$ 1000 bytes, 20% $\to$ 160 bytes $\Big\}$ $b_m = 1000 \cdot 0.8 + 160 \cdot 0.2 = 800 + 32 = 832$ bytes $=$
$= 832 \cdot 8 = 6656$ bits
$v = 1 \cdot 10^6$ b/s
~~a~~ 25% of time available for terminals
$L = ?$

$$r = \frac{b_m}{a_m \cdot v} \qquad 1 - p_0 = J \cdot (1-L) r$$

$$L = 1 - \frac{(1-p_0)}{J \cdot r} = 1 - \frac{(1-p_0) a_m \cdot v}{J \cdot b_m} =$$

$$= 1 - \frac{0.25 \cdot \frac{2}{3} \cdot 10^6}{50 \cdot 6656} = 0.4992$$

Frakcja utraconych zgłoszeń to prawie 50%

problem 2, practice set 3

$Q = 2$ $\qquad p_0 \geq p_1 \geq p_2$
$r = 0{,}75$

p_x – frakcja czasu jaką processor spędził w danym „stanie kolejki". Ponieważ $Q=2$ to $p_0 + p_1 + p_2 = 1$

$L = p_Q$ w tym wypadku $L = p_2$ $\qquad p_1 = ?$

$$1 - p_0 = (1-L) r$$
$$1 - p_0 = (1 - p_2) r$$
$$1 - p_0 = 0.75 - 0.75 p_2$$
$$0.25 + 0.75 p_2 = p_0$$

$$0.25 + 0.75 p_2 + p_1 + p_2 = 1$$

$$p_1 = 0.75 - 1.75 p_2$$
$$4 p_1 = 3 - 7 p_2$$
$$7 p_2 = 3 - 4 p_1$$
$$p_2 = \frac{3 - 4p_1}{7}$$
$$p_0 = 0.75 \cdot \frac{3-4p_1}{7} + 0.25$$
$$p_1 \leq \frac{3-4p_1}{7}$$
$$7p_1 \leq 3 - 4p_1$$
$$11 p_1 \leq 3$$
$$\boxed{p_1 \leq \frac{3}{11}}$$

~~$0.25 + 0.75 p_2 \geq 0.75 - 1.75 p_2$~~

$$0.75 \cdot \frac{3-4p_1}{7} + 0.25 \geq p_1$$
$$\frac{9 - 12p_1}{7} + 1 \geq 4p_1$$
$$9 - 12p_1 + 7 \geq 28 p_1$$
$$16 \geq 40 p_1$$
$$\frac{\cancel{16}\,4}{\cancel{40}\,10} \geq p_1$$
$$\boxed{0{,}4 \geq p_1}$$

$$\boxed{\frac{3}{11} \leq p_1 \leq \frac{4}{10}}$$

Problem 1, practice set 4

population = circulation × lifetime

$\frac{\text{population}}{\text{circulation}} = \frac{J = 10}{\frac{1}{a_m}} = \frac{(1-p_0)v}{b_m}$ lifetime $= h_m + \frac{b_m}{v} + w_m$

flow conserv: $1-p_0 = (1-L)r$
$L=0$ here (we don't know anything about Q)
$1-p_0 = r$
$1-p_0 = \frac{b_m}{v \cdot a_m}$
$\frac{(1-p_0)v}{b_m} = \frac{1}{a_m}$

$$J = \frac{(1-p_0)v}{b_m} \cdot \left(h_m + \frac{b_m}{v} + w_m\right)$$

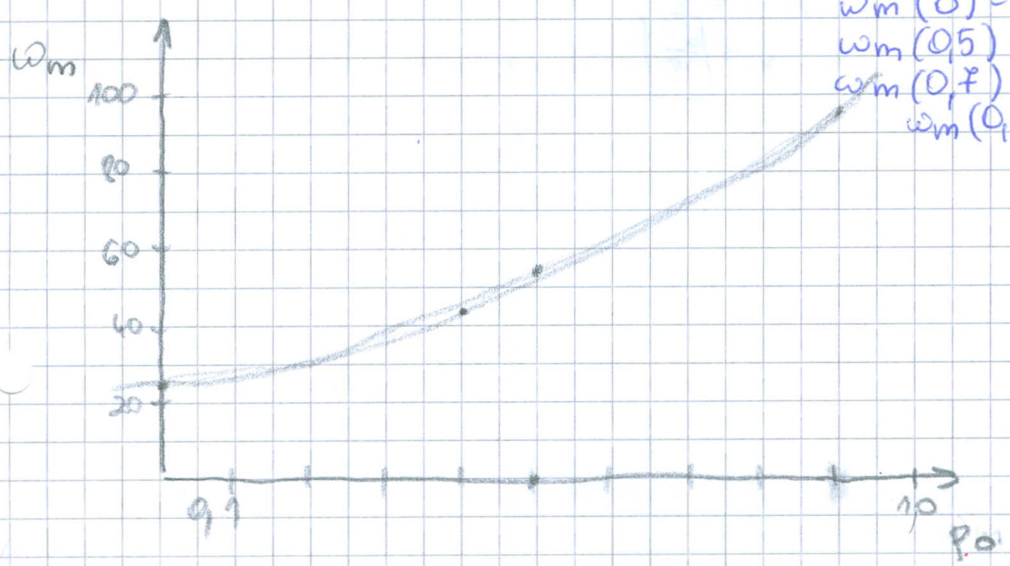
$$10 = \frac{(1-p_0)\cancel{5000}}{\cancel{15000}\ 3} \cdot \left(4 + \frac{\cancel{15000}\ 3}{\cancel{5000}} + w_m\right)$$

$$30 = (1-p_0) \cdot (7 + w_m)$$

$$\frac{30}{1-p_0} - 7 = w_m$$

$w_m(p_0) = \frac{30}{1-p_0} - 7$

$w_m(0) = 30 - 7 = 23$
$w_m(0{,}5) = 60 - 7 = 53$
$w_m(0{,}7) = \cancel{60}\ 100 - 7 = 93$
$w_m(0{,}4) = 50 - 7 = 43$



Problem 2, practice set 4

$J = 30$

$l_p = 21 \quad \tau_p = 0.05\,s \quad \frac{bm}{V} \quad 1.05\,s$

$l_{sd} = 12 \quad \tau_{sd} = 0.07\,s \quad 0.84\,s$

$l_{fd} = 8 \quad \tau_{fd} = 0.02\,s \quad 0.16\,s$

a.) wąskie gardło → procesor
gdy $\tau_p = 0.03$ wtedy wtedy dysk będzie ~~~~ gardłem wąskim

$$v_x = \frac{\tilde{\tau}_x}{a_m / l_x} = \frac{1}{a_m} \cdot \tilde{\tau}_x \cdot l_x$$

takie same

$$\tau_m = \frac{bm}{V}$$

$$r = \frac{bm}{V \cdot a_m}$$

populacja = cyrkulacja × czas życia

$$J = \frac{1}{a_m} \times (h_m + d_m) \Rightarrow a_m = \frac{h_m + d_m}{J}$$

dla $d^* = 12s$

$$a_m = \frac{15+12}{30} = 0.9$$

$$\frac{1}{a_m} = 1.11 \text{ cyrkulacje}$$

$$v_p = 1.11 \cdot \tau_p \cdot \cancel{21}$$

$$v_p' = 0.56 \cdot 0.05 \cdot 21 =$$

$$v_p' = 0.5967$$

$$21 \cdot x + 1 = 12$$

$$21 \cdot x = 11$$

$$x = \boxed{\frac{11}{21}}$$

dla $d_m^* = 9$

$$a_m = \frac{15+9}{30} = 0.8$$

$$\frac{1}{a_m} = 1.25$$

$$v_p = 1.25 \cdot \tau_p \cdot \cancel{21}$$

$$21 \cdot x + 1 = 9$$

$$21 \cdot x = 8$$

$$x = \boxed{\frac{21}{8}}$$

$d_m = 2.05\,s$

$$a_m = \frac{15+2.05}{30} = 0.5\cancel{7}$$

$$\frac{1}{a_m} = \cancel{0.56}\ 1.75438$$

$$v_p = 1.75 \cdot \tau_p \cdot \cancel{21}$$

$$v = 1.75 \cdot \frac{bm}{v} \cdot 21$$

$$30 = \frac{1}{a_m} \cdot d_m$$

Problem 1, practice set 5

a.) 40 req/s 20ms

M/M/1/2$_Q$

$a_m = \frac{1}{40} = 0.025\ s$

$b_m = 20ms = 0.02\ s$

$r = \frac{b_m}{a_m \cdot v} = \frac{0.02}{0.025 \cdot 1} = 0.8$

$r \neq 1 \qquad L = p_Q = \frac{1-r}{1-r^{Q+1}} \cdot r^Q = \frac{0.2}{1-(0.8)^3} \cdot (0.8)^2 = \frac{16}{61}$

$40 \cdot doba \cdot \frac{16}{61} = 40 \cdot \frac{16}{61} \cdot 24 \cdot 3600 \approx 907492$ lost

b.) M/M/1/5$_Q$

$r = \frac{b_m}{a_m \cdot v} = \frac{b_m}{\frac{b_m}{2} \cdot v} = \frac{2}{1} = 2$

$r \neq 1 \qquad L = p_Q = \frac{1-2}{1-2^6} \cdot 2^5 = \frac{-1}{1-64} \cdot 32 = \frac{1}{63} \cdot 32 \approx 50{,}8\%$

c.) M/M/1/Q

$r = \frac{b_m}{a_m \cdot v}$

$p_k = p_0 r^k$

$p_0 = \frac{p_k}{r^k}$

$1 - p_0 = (1-L) r$

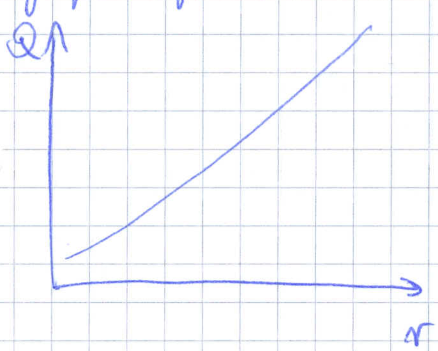
$L = 1 - \frac{1-p_0}{r} = 1 - \frac{(1-p) \cdot a_m \cdot v}{b_m}$

$r \neq 1$

$p_0 = \frac{1-r}{1-r^{Q+1}}$

$L = 1 - \frac{1-p}{r}$

aby p_0 było stałe



$\Rightarrow$

Problem 2, practice set 5

M/M/5/5
150 users ~~8000el.~~ transaction → 800 el op.
10 trans/s ~~→ 28000 el/s~~
$L \leq 3\%$ rej.

$$\rho_1 = bm / (am \cdot v) = \frac{800}{\frac{1}{1500} \cdot 5 \cdot 10^{7}} = \frac{8 \cdot \cancel{15}^{3}}{\cancel{5} \cdot 10} = 2.4 \Rightarrow 6$$

$$\rho_2 = bm / (am \cdot v) = \frac{800}{\frac{1}{1500} \cdot 4 \cdot 5 \cdot 10^{5}} = \frac{8 \cdot \cancel{15}^{3}}{\cancel{5} \cdot 10 \cdot 4} = 0.6 \Rightarrow 3$$

$bm = 800$

$am = \frac{1}{1500}$

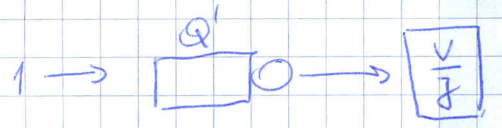
lepsze

$koszt\ 1 = 6 \cdot f$

$koszt\ 2 = 3 \cdot f \cdot 2.5 = 7.5 f$

Problem 3, practice set 3

a.) system możemy określić jako:



internal → $a_m = 6s$

size → $b_m = 600$ bytes

$v = 24000/J = v_1/J$

$J = ?$ $Q = ?$

$d_m \le c \cdot \frac{b_m}{v}$

$L \le 0.1\%$

$r = \frac{b_m \cdot J}{a_m \cdot v_1}$

$r = \frac{b_m}{a_m \cdot v} = \frac{600 \cdot J}{6 \cdot 24000} = \frac{J}{240}$

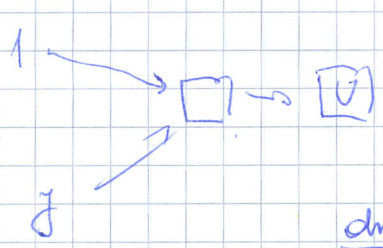
$J \cdot r \le 1 \Rightarrow \frac{J^2}{240} \le 1$

$J^2 \le 240$

$\boxed{15 \le J < 16}$ $J = 15$

$\frac{d_m}{b_m} = \frac{\frac{b_m}{v} \cdot c}{b_m} = \frac{c}{v} = \frac{5}{24000/15 \; 16000} = \frac{1}{3200}$

b.)



$r = \frac{b_m}{a_m \cdot v} = \frac{600}{6 \cdot 24000} = \frac{1}{240}$

$J \cdot r \le 1 \Rightarrow J \le 240$ $J = 240$

$\frac{d_m}{b_m} = \frac{5 \cdot J}{24000} = \frac{5 \cdot 240}{24000} = \frac{5}{100} = 5\%$

Problem 1, practice set 4

$J = 10$
$h_m = 4\,s$ (think time)
$b_m = 15\,000$
$v = 5000$ op/s

population = circulation × lifetime

J - population
$\frac{1}{a_m}$ - circulation $\qquad \frac{1}{a_m} = \frac{(1-p_0)\, v}{b_m}$

lifetime - $h_m + \frac{b_m}{v} + w_m$ (think time + processing time + waiting)

$$J = \frac{(1-p_0)\,\cancel{b_m}}{a_m \cdot v \cdot \cancel{b_m}} \cdot \left(h_m + \frac{b_m}{v} + w_m\right)$$

$$10 = \frac{1-p_0}{\quad}$$