Linear programming

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Part I

Zad. 1.   Using Gasussian elimination method solve systems of equations.

1.
$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ 2x_1 + 3x_2 + 4x_3 &= 20 \\ -x_1 + 1x_2 - x_3 &= -2 \end{aligned}$$

2.
$$\begin{aligned} 3x_1 + 0x_2 - x_3 + x_4 + 0x_5 &= 6 \\ x_1 + 3x_2 + x_3 + 0x_4 + 0x_5 &= 6 \\ 0x_1 - 2x_2 + 0x_3 - x_4 + x_5 &= -4 \end{aligned}$$

3.
$$\begin{aligned} x_1 + x_2 - x_3 &= -5 \\ 2x_1 + 3x_2 + x_3 &= 4 \\ 4x_1 + 5x_2 - x_3 &= 1 \end{aligned}$$

4.
$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 4 \\ 2x_1 + x_2 + 3x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

Zad. 2.   Draw a graph of the following constraints. Show the feasible region of them and nonnegativity constraints.

$$\begin{aligned} x_1 + 3x_2 &\le 6 \\ 4x_1 + 3x_2 &\le 12 \\ 4x_1 + x_2 &\le 8 \end{aligned}$$

Discus how the optimal solution depends on the objective function?

Zad. 3.   Let us consider a manufacturer who produces two types of goods W_1 and W_2. Four resources that are used are limited : energy, steel, wood and labor force. Table shows the consumption.

	Consumption of resources			
	Energy	Steel	Wood	Labor force
W_1	5	5	6	10
W_2	25	10	0	10
Limits	1200	600	420	900

Determine the optimal production sine profits are:
- Unit of W_1 brings 10 PLN,
- Unit of W_2 brings 20 PLN.

Zad. 4. Ancient village accumulated four means of production: wood, lines, grease and iron. Town in the neighborhood is interested in buying carriages and boats. The table bellow determines the consumption the limits of the means of production.

	Boats	Carriages	Limits
Wood	15	15	525
Lines	2	1	50
Grease	3	6	180
Iron	0	5	110

Price of both products is the same, 17 golden coins. Derive the optimal solution i.e. one that maximizes income.

Zad. 5. (Hillier) The following table summarizes the key facts about two products A and B, and resources Q, R and S. Find the maximal profit.

Resource	A	B	Availability
Q	2	1	2
R	1	2	2
S	3	3	4
profit	3	2	

Zad. 6. Let us consider two coal types: A and B. Both are polluted with ash and phosphorus. Production process requires 90 t of fuel containing at most 0.03% of phosphorus and 4% of ash. The table shows the prices of different types of coal and the percentage rate of pollution.

	Pollution		
	phosphorus	ash	price
A	0.02	3	100
B	0.05	5	80

1 How much coal of different types should be mixed to minimize the cost and satisfy the restrictions?
2 What should be changed if:
   - price of B increases to 100 zł?
   - new type C appears on the market that costs 80 zł is contaminated with 0.03% of phosphorus and 5% ash?

Zad. 7. Stud farm uses tow kinds of feed which consist of three major ingredients (nutrients) S_1, S_2 i S_3 important for the breed. It is widely known that horse should be provided with 18 kg of S_1 per day and 2 kg of S_2. S_3 is unhealthy and the mixture shouldn't consist of more then 5% of it. Percentage amounts of nutrients if feeds are shown in the table. Cost of one

kg of feed I is 6 zł and feed II 15 zł.

	Amounts on nutrients %		
	S_1	S_2	S_3
Feed I	50	4	6
Feed II	60	10	2

Use the graphic method to:

- determine the minimal cost of maintenance one horse?

- what are the optimal proportions of feed I and II?

Zad. 8. Sawmill was placed an order for 300 sets of desks. Each set consists of 7 desks of length 0.7 m and 4 of length 2.5 m. How should the standard desks be cut to minimize waste? The length of standard desk is 5,2 m.