Stability and Dynamics of Ships and Offshore Structures

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Room **331A**, (3rd floor, in a wing of the building)
Consultations: Wednesday 12:00 – 14:00

Complete the course:
qualifying test, at the last lecture
(attendance at lectures is obligatory)

Bibliography

- **James F. Wilson: „Dynamics of Offshore Structures”. WILEY 2003**
- Targut Sarpkaya: „Wave Forces on Offshore Structures”. Cambridge University Press 2010
- O.M. Faltinsen: „Sea Loads on Ships and Offshore Structures”. Cambridge University Press 1990
- „Principles of Naval Architecture”, vol. 1,3. SNAME 1988

Some „extra” handbooks:

- S.K. Chakrabarti: „Offshore Structure Modeling”
- S.K. Chakrabarti: „Handbook of Offshore Engineering”

What we need for exercises?

- Calculator (stability computations)
- Computer (simple notebook, netbook, etc.)
- Appropriate software:
  - C/C++ (if you know it)
  - or Mathlab (do you have a license ???)
  - or Octave/QOctave (something free, very similar to Mathlab, under Linux)

Why?

- We need to solve ordinary differential equations ODE
- Sometimes we have to solve nonlinear ODE (do not worry!)

Introduction

Knowlage about wave induced loads and motions of ships and offshore structures is important both in design and operational studies. [O.M. Faltinsen]

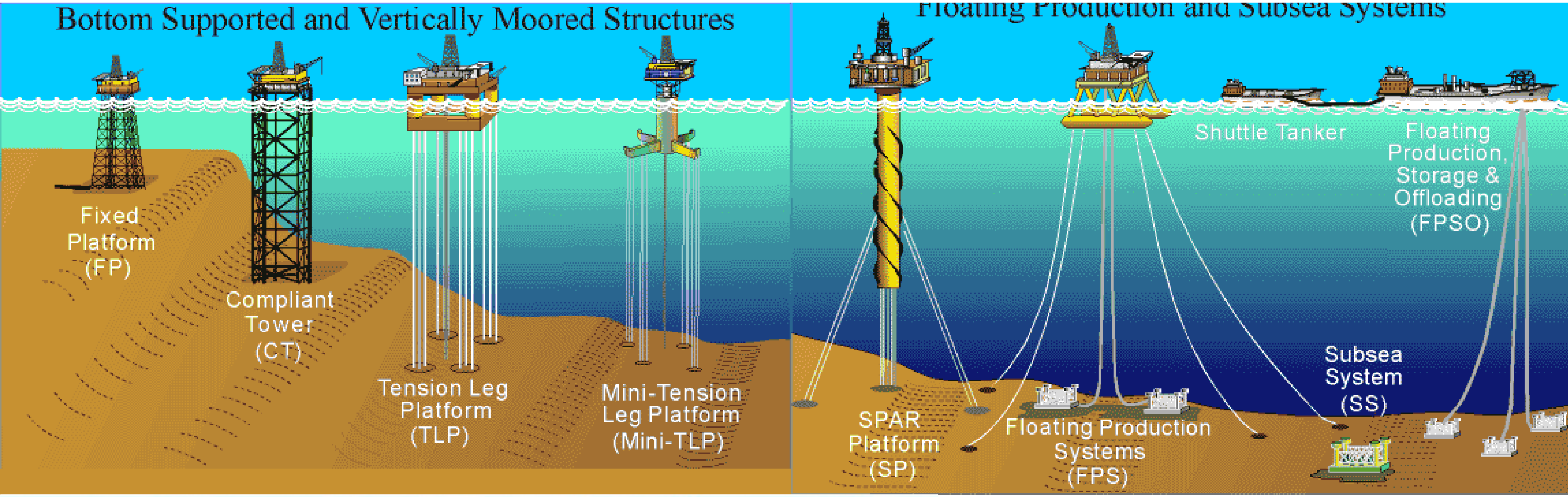
Why?

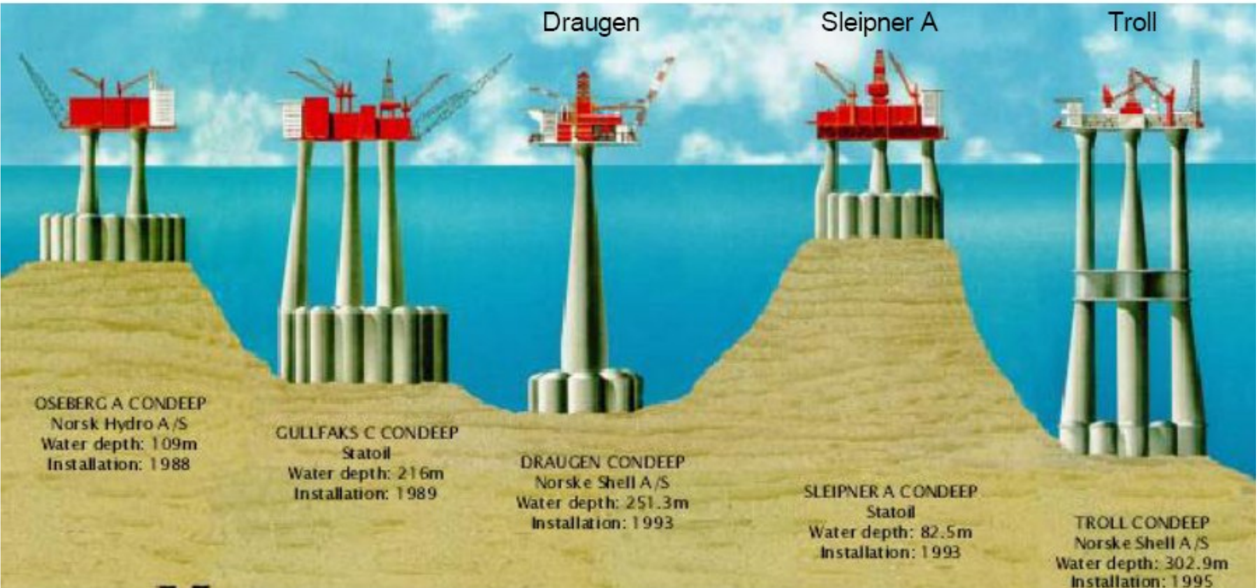
The significant wave height can be larger than 2 m for 60% of the time in hostile areas like the North Sea.

Maximum wave heights higher than 15 m can occur even on Baltic Sea.

Maximum ocean's wave can be higher than 30 m

Types of offshore platforms





Types of offshore platforms

Offshore platform can be broadly categorised into two types:

[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

1. ***Fixed platform structures*** - fixed platform is a platform in which its substructure is attached or piled to the seabed. This type of platform cannot be moved by virtue of its fixity. This include:

- Steel jacket
- Compliant tower
- Concrete gravity structure or concrete base structure (CBS)

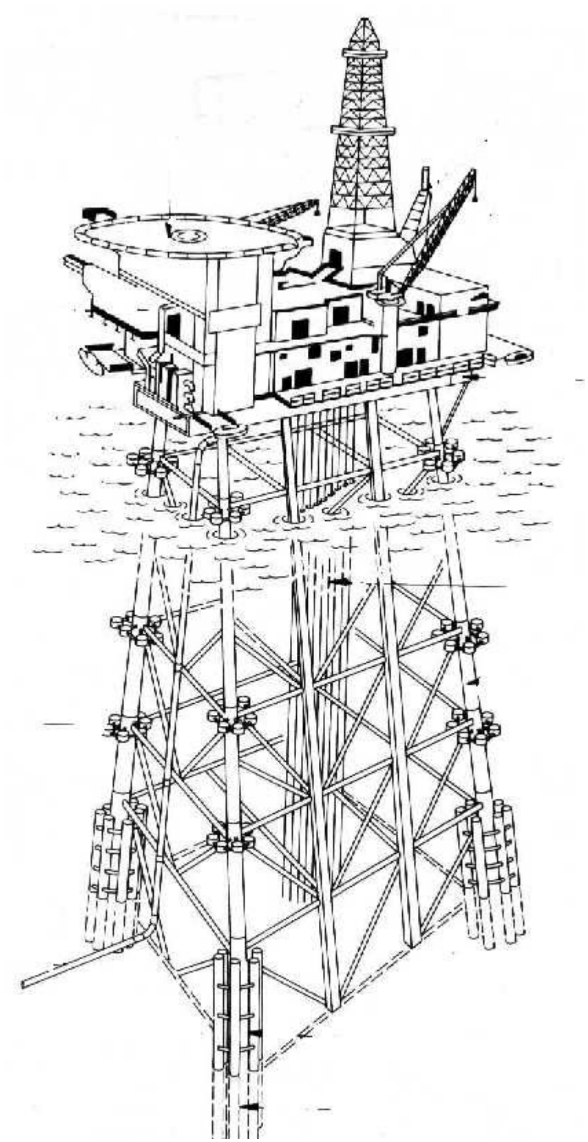
Types of offshore platforms

2. ***Floating platform structures*** - these are platforms that flow near the water surface. This include:

- Tension Leg Platform
- Semi submersible
- Spar
- Ship shaped vessel (FPSO)

Types of offshore platforms

Fixed platform structures: ***Steel jacket platform***



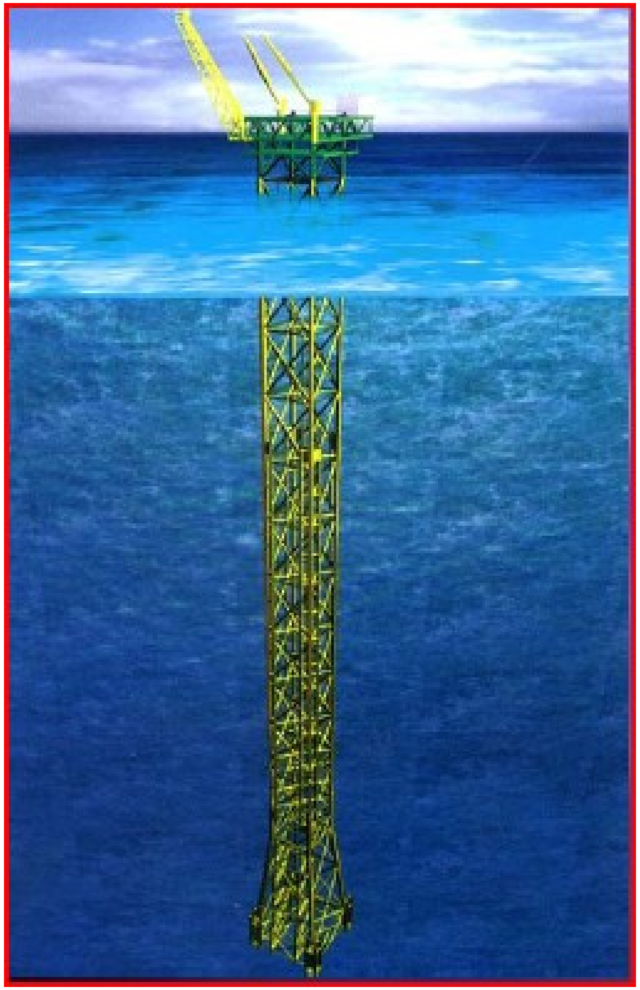
Jacket platform consist of topside (deck) and substruckture (jacket). The jacket provides support to the deck and transfer the load to pile. Jacket is usually made of steel with tubular members supported on piled foundation.

For water depths up to ~350m
[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

Steel jacked
http://www.esru.strath.ac.uk/EandE/Web_sites/98-9/offshore/steel.htm

Types of offshore platforms

Fixed platform structures: ***Compliant tower CT***



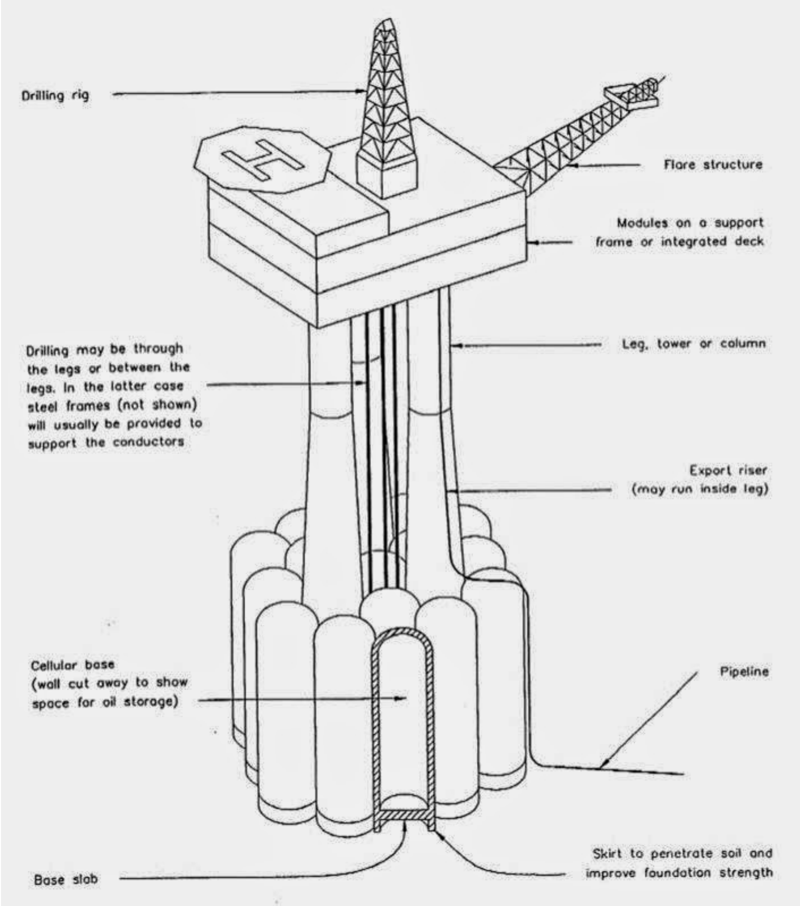
Compliant tower consist of topside plus flexible tower. That is, the topside structure is supported by flexible tower. Unlike steel jacket, this is narrow, flexible frame structures but also supported by piled foundations. By virtue of its flexibility, it can withstand large lateral deflections (up to 3m) under wave loading than jacket. It is used for water depths from 300 to 600 (800m).

[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

Types of offshore platforms

Fixed platform structures:

Concrete base structure CBS



Concrete base structure platform consist of topside and concrete base. That's, the topside (deck) is usually supported by concrete base. This type of structure is commonly used in the severe or harsh environment such as North Sea and Scandinavian.

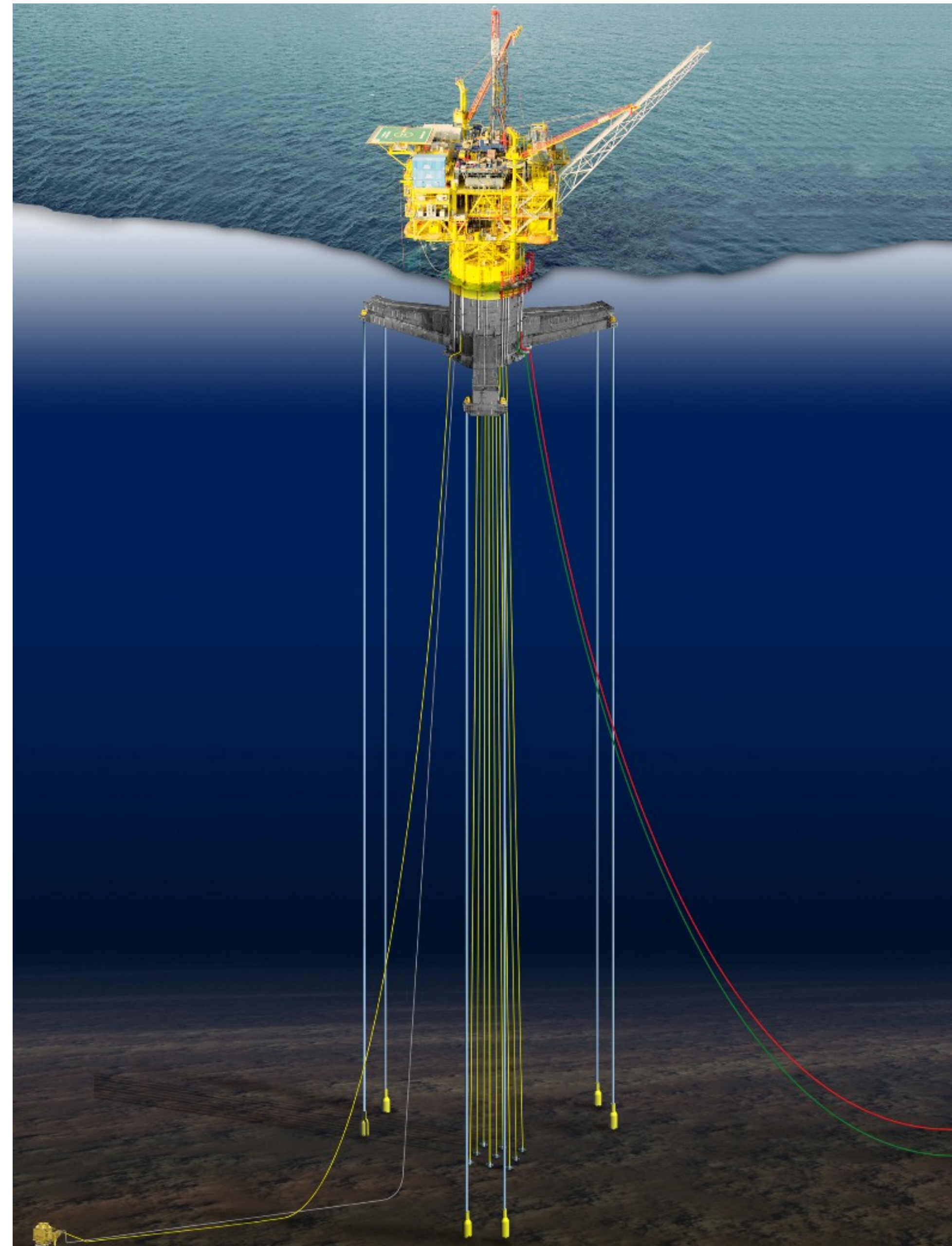
The topside (deck) relies on the gravity/weight of the base and the structure is normally made of concrete (occasionally steel may be used). The bottom of the structure is fixed, heavy and remains in place on the seabed without the need for piles to transfer the load to a suitable layer of foundation or soil; unlike the jacket platform in which the load from jacket is being transferred to the pile that carries it to a suitable layer of soil.

[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

Types of offshore platforms

Floating platform structures:

Tension Leg Platform TLP



Tension – Leg platform like others floating platforms consist of topside and hull; but the hull's mooring (anchoring) system are different. TLP system used a set of tension legs or tendons attached to the platform (deck) and connected to a foundation on the seabed using what is called tethers (tethers are anchoring device). The tension leg is moored (anchored) in a way that it allows horizontal movement to accommodate wave disturbances, but prevent vertical or bobbing movement to keep the platform (deck) in place. These characteristics makes TLPs a popular choice for stability especially in regions where there is high possibility of large waves or prone to hurricane such as Gulf of Mexico. TLPs are normally used for water depths up to 1200m.

[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

Types of offshore platforms

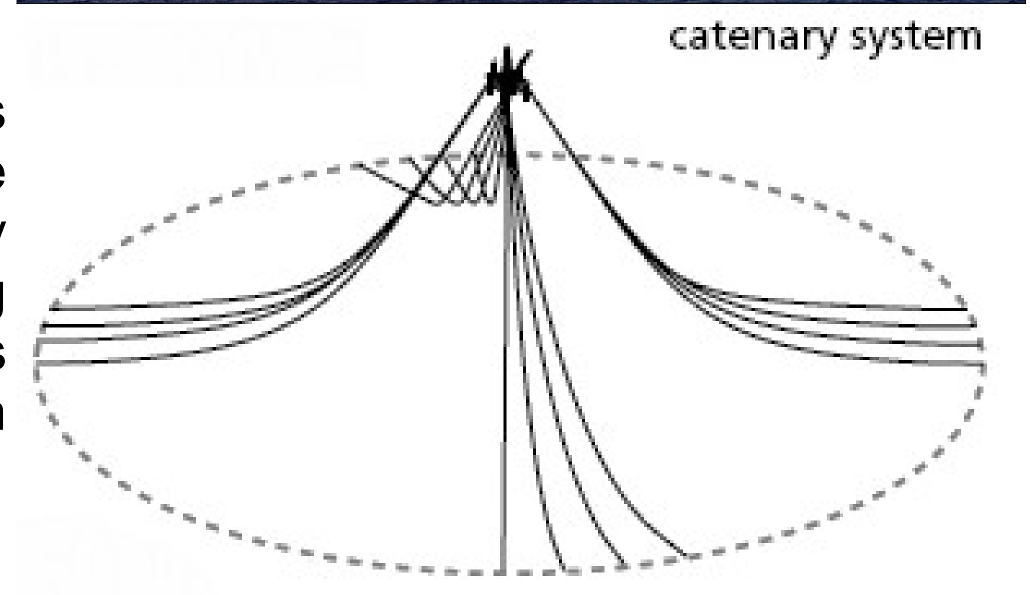
Floating platform structures:

Semi submersible

Semisubmersible platform like other platforms consist of topside (deck) but supported by hulls which can either be in box or cylindrical form. Hull consists of columns and pontoons (pontoons - cause the structure to float). Pontoons (or air-filled structure) are floating devices with sufficient buoyancy to cause it to float as well as heavy load. Hull is also used for storage and ballasting operations.

The advantage of this platform system is that it can be moved or towed from one location to another. It is held in place by anchors connected to a catenary mooring system. Semisubmersible platform is normally used for deep water production which is water depth is above 2000 metres.

[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

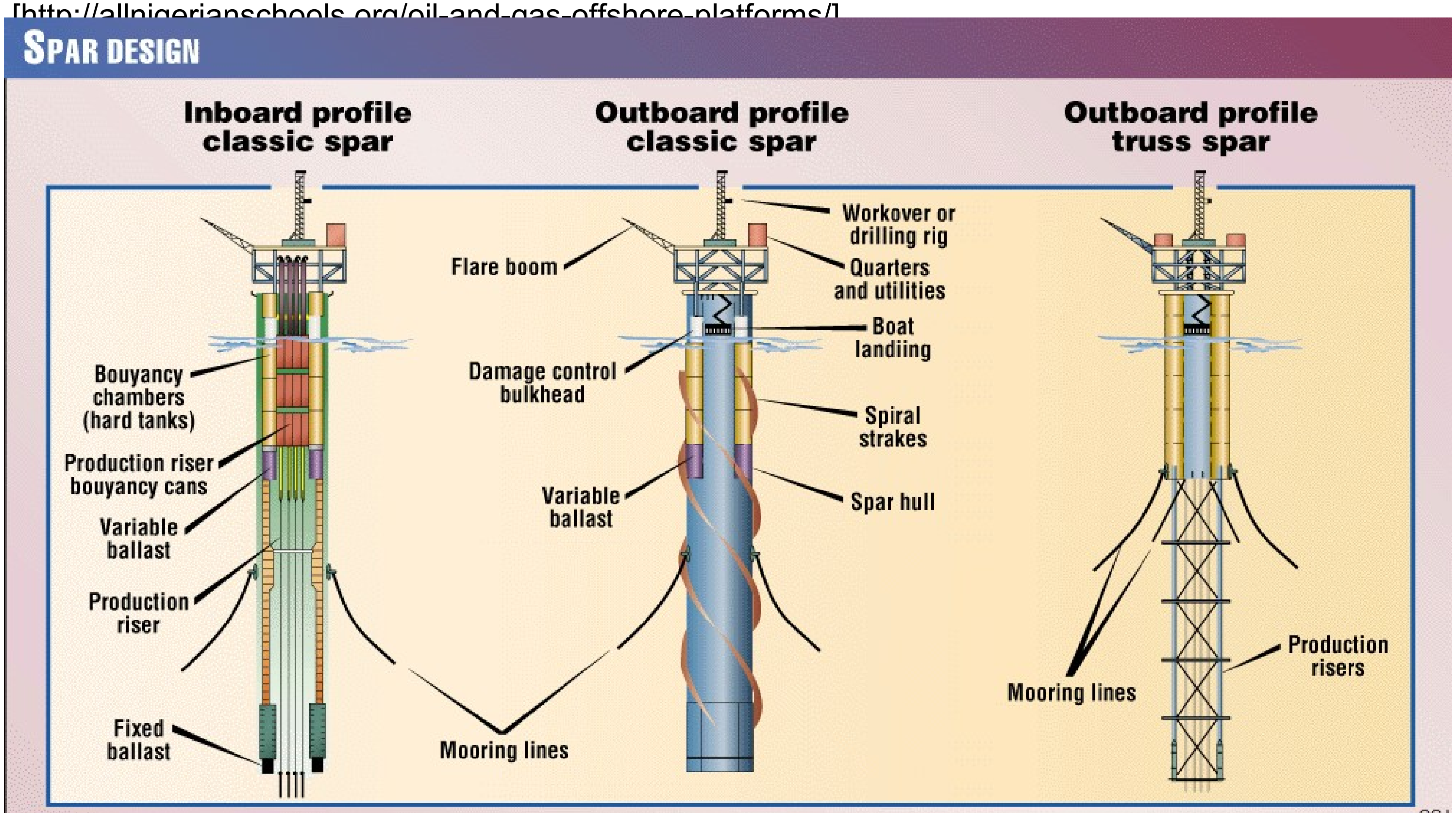


Types of offshore platforms

Floating platform structures: ***Spar***

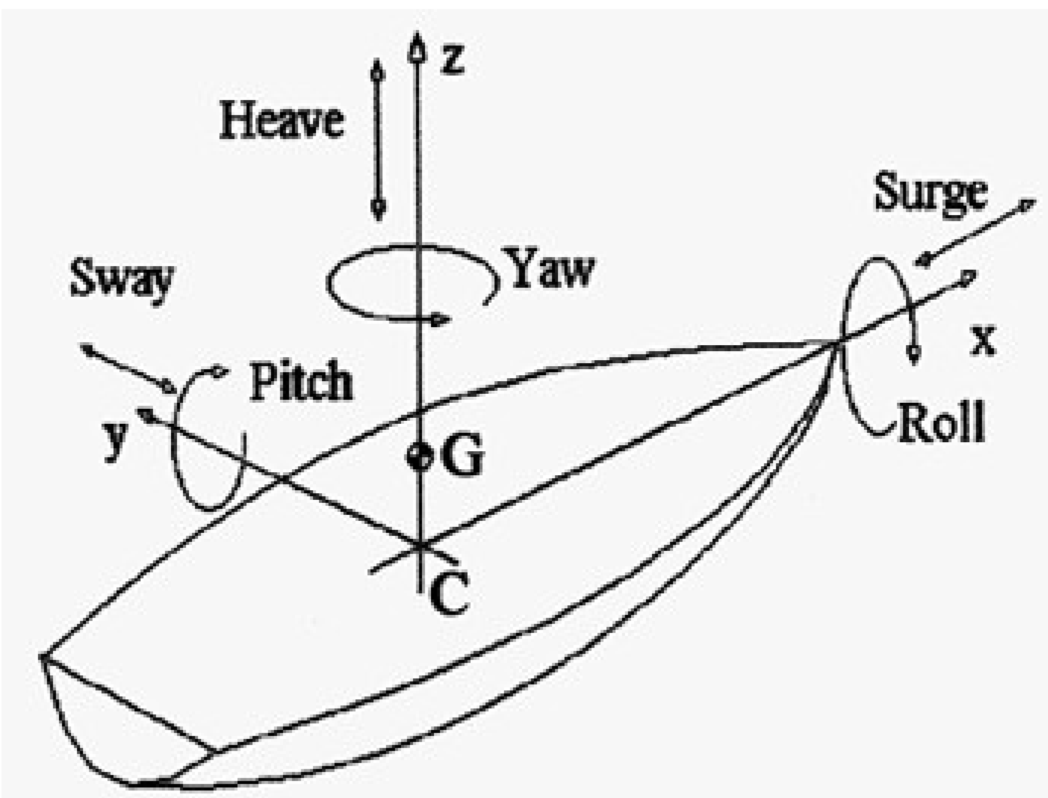
Spar platform consist of large cylindrical steel hull which support the platform (deck). Spars are moored to the seabed like TLPs. Unlike the TLP which has vertical tension tethers, a spar has more conventional mooring lines. The spar has more inherent stability than a TLP due to its large counterweight at the bottom and does not depend on the mooring to hold it upright. It also has the ability, by adjusting the mooring line tensions (using chain-jacks attached to the mooring lines), to move horizontally and to position itself over wells at some distance from the main platform location.

[http://allnigerianschools.org/oil-and-gas-offshore-platforms/]

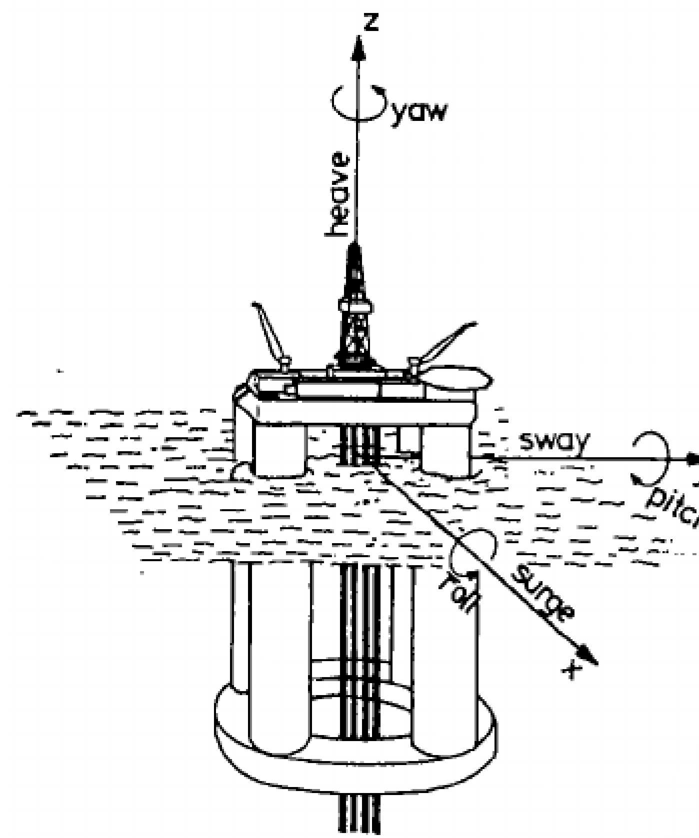


Definition of rigid-body motion modes

The oscillatory rigid body motions are reffered to as ***surge***, ***sway*** and ***heave***, with heave being the vertical motion and (for a ship) surge is the longitudal motion. The oscillatory angular motions are referred to as ***roll***, ***pitch*** and ***yaw***, with yaw being rotation about a vertical axis and roll being the rotation about the longitudinal axis (for a ship).



Ship



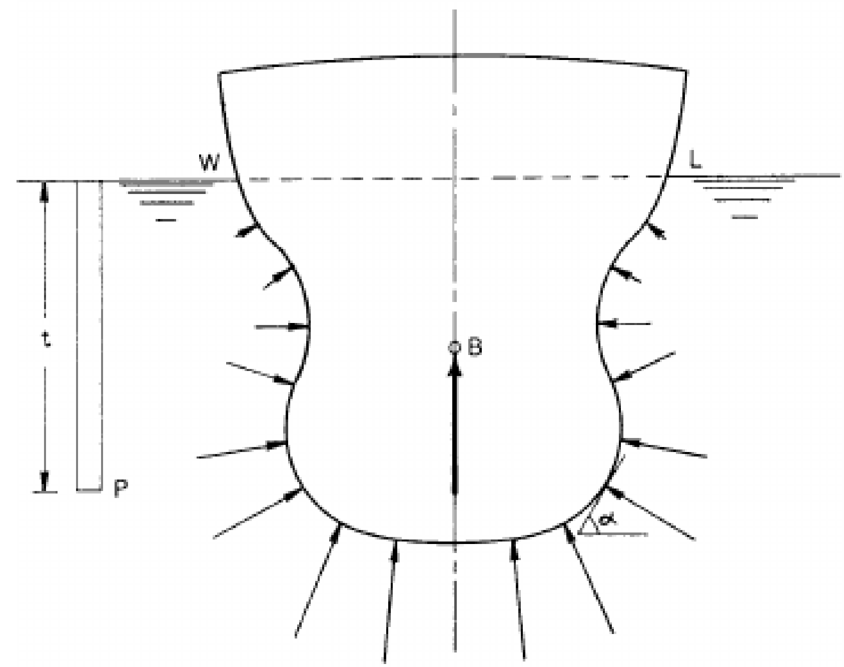
Platform

Why do ships float? Displacement and Weight Relationships

Archimedes principle.

Body immersed in a fluid is buoyed up by a force that equals the weight of the displaced fluid.

For body floating on a water surface, we have:



$$W - \rho g \nabla = 0$$

W – weight;
ρg▼ – displacement (force);

Stability of Ships and Offshore Structures. **Elementary Principles**

Gravitational Stability.

Not only must the designer provide adequate buoyancy to give support support for the ship and its contents, but it must be assured that it will float in the proper attitude, or trim, and remain upright when loaded with passengers and cargo.

This involves the problems of gravitational stability and trim.

[„Principles of Naval Architecture”, vol.1]

<u>Stability</u> of Ships and Offshore Structures. **Elementary Principles**

Equilibrium.

In general, a rigid body is considered to be in a state of equilibrium when the resultants of all forces and moments acting on the body are zero. In dealing with static floating body stability, we are interested in that state of equilibrium associated with the floating body upright and at rest in a still liquid.

[„Principles of Naval Architecture", vol.1]

<u>Stability</u> of Ships and Offshore Structures.
Elementary Principles

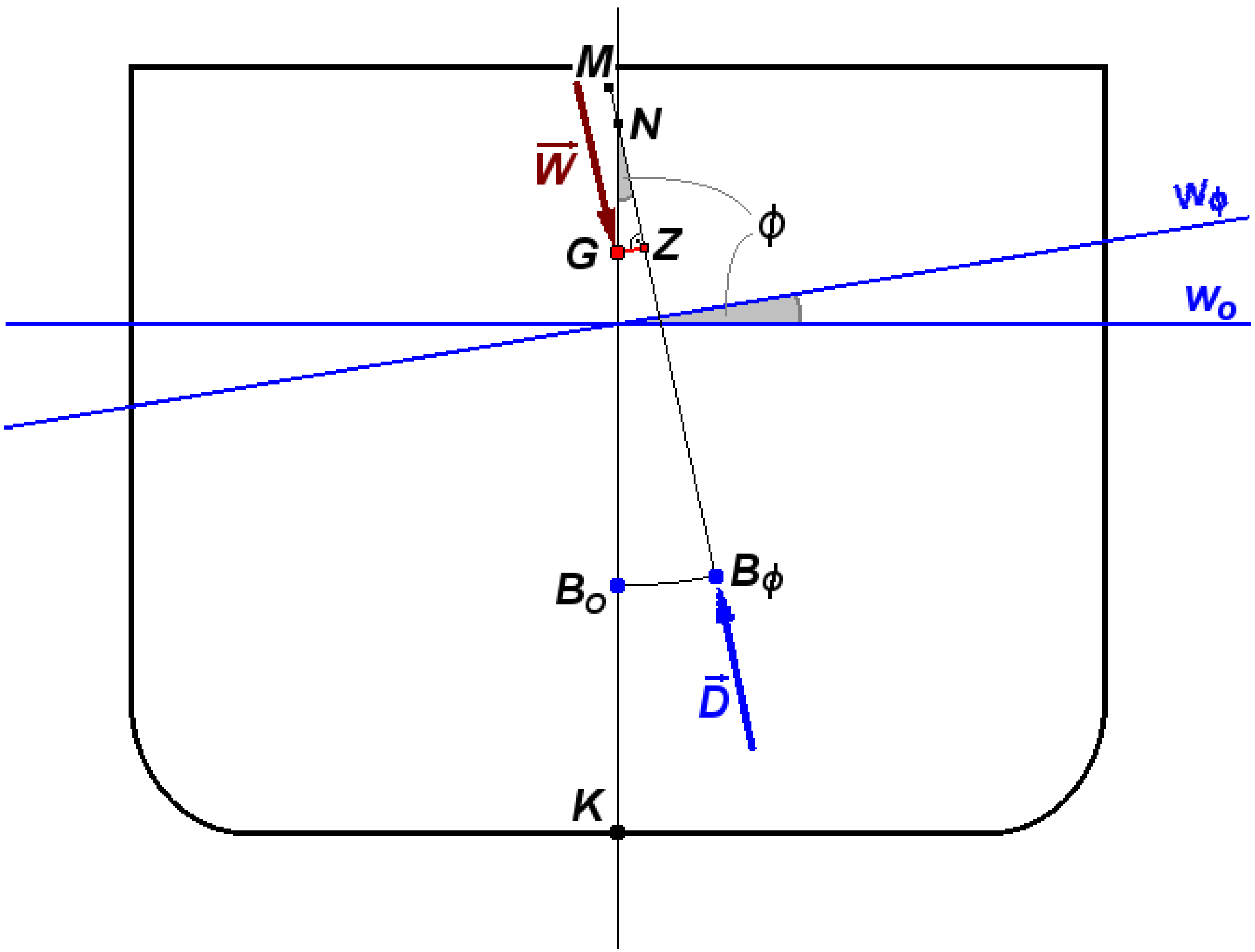
Stable equilibrium*.** If a floating body, initially at equilibrium, is disturbed by an external moment, there will be a change in its angular attitude. If upon removal of the external moment, the body returns to its original position, it is said to have been in ***stable equilibrium and to have <u>positive stability</u>.

Neutral equilibrium. If a floating body that assumes a displaced inclination because of an external moment remains in that displaced position when the external moment is removed, the body is said to have been in ***neutral equilibrium*** and has <u>neutral stability</u>.

Unstable equilibrium. If a floating body, displaced from its oryginal angular attitude by an external force, continues to move in the same direction after the force is removed, it is said ti have been in ***unstable equilibrium*** and was <u>initially unstable</u>.

[„Principles of Naval Architecture”, vol.1]

Stability of Ships and Offshore Structures. **Transverse stability** of floatig structures



***B**$_o$* , ***B**$_\Phi$* – centre of buoyancy;

G – centre of gravity;

Φ – heel angle;

M – metacentre;

N – intersection of the ***B**$_\Phi$**M*** line with the symmetry plane;

Z – a projection of ***G*** point on the ***B**$_\Phi$**M*** line;

K – keel pint;

W – weight;

D – displacement (force);

Stability of Ships and Offshore Structures. **Transverse stability** of floatig structures

Equilibrium conditions can be formulated in the form of the following equation:

$$M_H + M_R = 0 \qquad (1)$$

M_H , M_R – are heeling and righting moment. Righting moment can be calculated from the formula:
Moment = ***Arm*** x ***Force***

Based on the figure, we can write:

$$M_R = -D\,\overline{\boldsymbol{GZ}} \qquad (2)$$

Let's consider ***GZN*** trinagle:

$$\overline{\boldsymbol{GZ}} = \overline{\boldsymbol{GN}} \sin\Phi \qquad (3)$$

When Φ is small we can assume, that: ***M*** → ***N***, hence, from (3):

$$\overline{\boldsymbol{GZ}} = \overline{\boldsymbol{GM}} \sin\Phi \qquad (4)$$

From the equations (2) and (4), we obtain:

$$M_R = -D\,\overline{\boldsymbol{GM}} \sin\Phi \qquad (5)$$

The above eqution is called metacentric formula and ***GM*** is called ***metacentric height***. From eq. (1) and (2) one can calculate heeling angle for given heeling moment M_H:

$$\Phi = \arcsin\left(\frac{M_H}{D\,\overline{\boldsymbol{GM}}}\right) \qquad (6)$$

Stability of Ships and Offshore Structures.
Transverse stability of floatig structures

Important note 1: The „minus” sign in the metacentric formula is the result of the assumed conventions (see figure). Sign of the righting moment should be opposite to sign of heeling angle for stable ship.

Important note 2: The metacentric formula has been derived based on assumption, that the point ***M*** coincides with the point ***N*** (see figure). In practice it means, that the equation is true for heel angle less than 10 degrees (if the geometry of waterplane is not changed rapidly)

For small heeling angles, we have:

$$\sin\Phi \approx \Phi \qquad (7)$$

therefore, the metacentric formula may be replaced by a following linear expression:

$$M_R = -D\,\overline{\mathbf{GM}}\,\Phi \qquad (8)$$

The above linear form allows to solve simpler issues with the use of analytical methods.

Important note 3: Φ angle is expressed in radians.

Stability of Ships and Offshore Structures. **Transverse stability** of floatig structures

How to get the value of ***GM***?

On the basis of the figure, we can write:

$$\overline{GM} = \overline{KM} - \overline{KG} \qquad (9)$$

KM can be expressed as:

$$\overline{KM} = \overline{KB} + \overline{BM} \qquad (10)$$

where ***BM*** is so-called metacentric radius, which is calculated from the formula:

$$\overline{BM} = \frac{I_x}{\nabla}$$

I_x is a surface moment of inertia of waterplane:

$$I_x = \frac{1}{12}\int_0^{L_w} (2y)^3\, dx$$

The volume of the submerged part of the hull can be calculated from:

$$\nabla = \int_0^T A_W\, dz$$

Ordinate of the center of buoyancy is expressed by the following formula:

$$\overline{KB} = \frac{\int_0^T A_W\, z\, dz}{\nabla}$$

Stability of Ships and Offshore Structures. **Transverse stability** of floatig structures

The transverse stability of a floating body is **positive** when:

$$\overline{GM} > 0$$

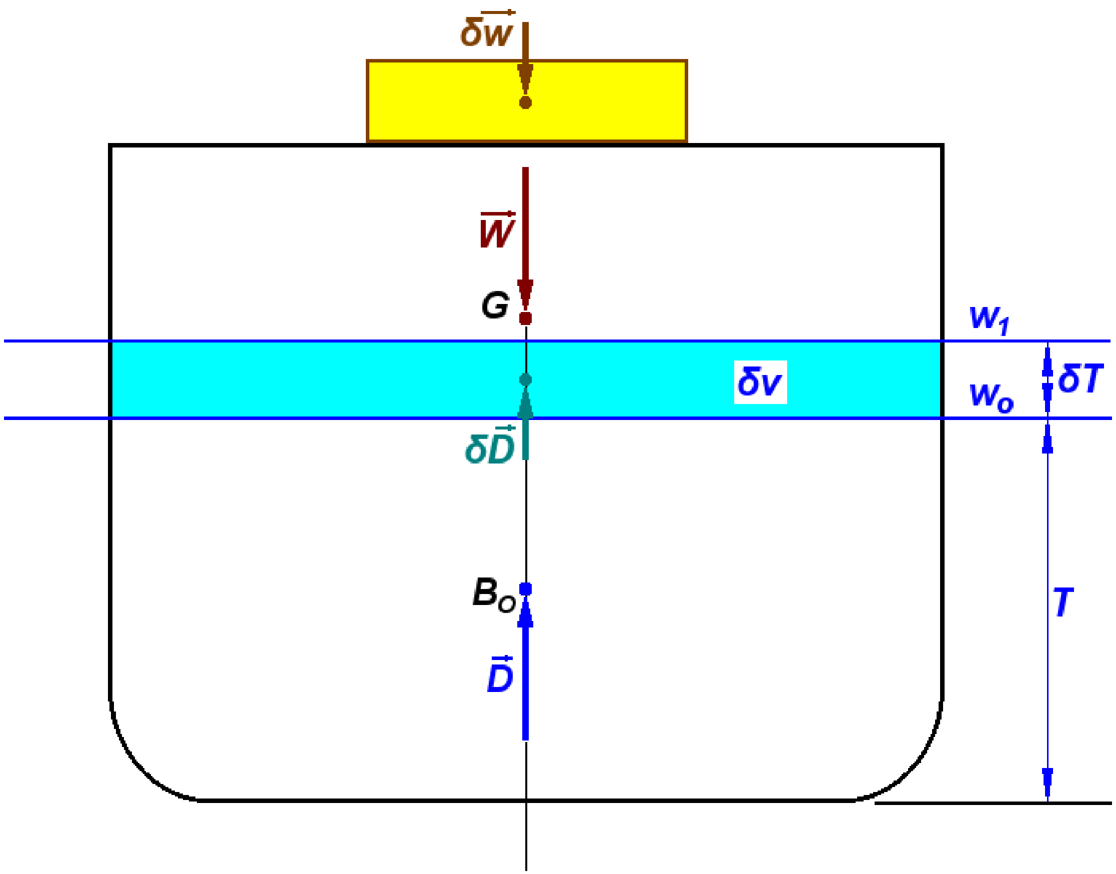
The transverse stability of a floating body is **neutral** when:

$$\overline{GM} = 0$$

The floating body is **initially unstable** when:

$$\overline{GM} < 0$$

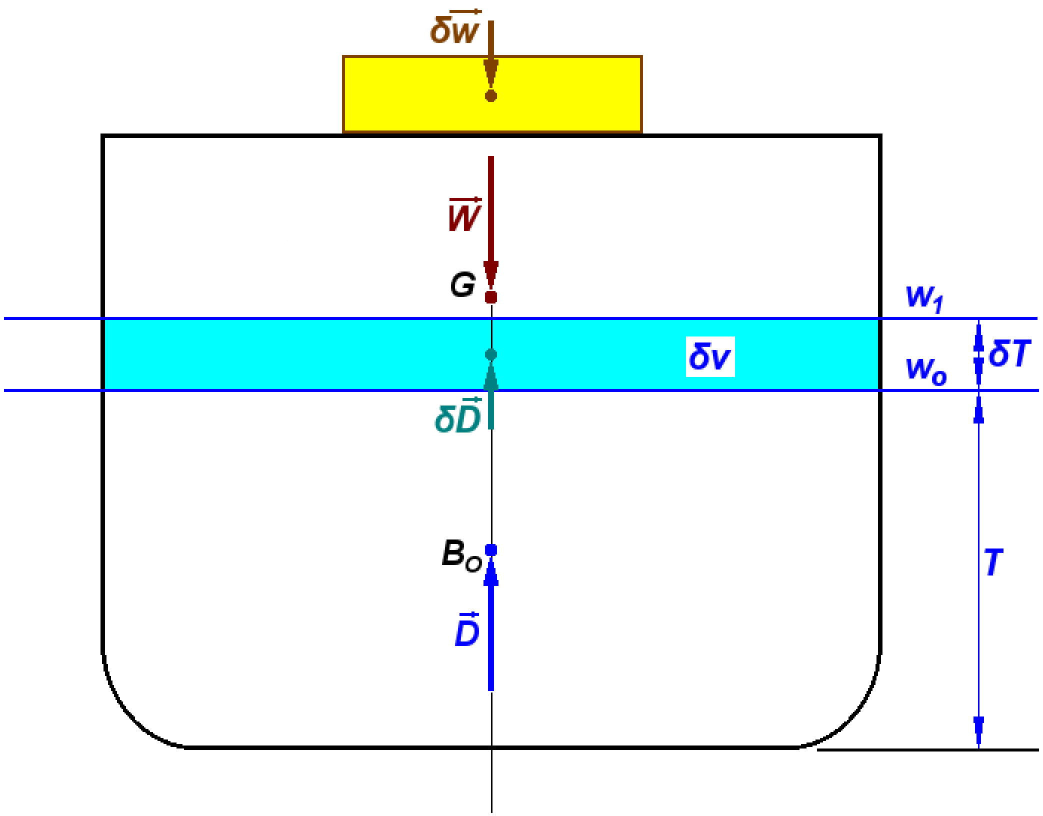
Analysis of the equilibrium in the vertical direction (heave)



Problem description:

- In the initial state the draft of the ship was T (waterline $\mathbf{w_0}$)
- Under the action of additional force $\boldsymbol{\delta w}$ (eg. weight) immersion of the vessel increased by δT (waterline $\mathbf{w_1}$)
- What is the relationship between $\boldsymbol{\delta w}$ and δT ?
- In what type of equilibrium is the ship?

Analysis of the equilibrium in the vertical direction (heave)



Equilibrium equation:

For body floating at draft *T* (waterline ***w₀***), the displacement force ***D*** is equal to the weight of the body ***W***:

$$W - D = 0 \qquad (1)$$

or:

$$W - \rho g \nabla = 0 \qquad (1')$$

After loading the additional weight ***δw,*** additional volume of the hull *δv* will be underwater (immersion of the hull increase of δ*T),* and the equilibrium equation takes the form:

$$W + \delta w - \rho g \left(\nabla + \delta v \right) = 0 \qquad (2)$$

On the basis of eq. (1') and (2) we get:

$$\delta w - \rho g \delta v = 0 \qquad (3)$$

where $\rho g \delta v$ is δD

Analysis of the equilibrium in the vertical direction (heave)

$$\delta w - \rho g \delta v = 0 \qquad (3)$$

The additional volume of the submerged hull *δv*, for small δ*T*, can be calculated using the formula for the volume of a cylinder:

$$\delta v = A_W \delta T \qquad (4)$$

This formula remains true even for large values of δ*T*, if the waterplane area is constant (structure with vertical sidewalls).

after substituting the expression (4) to (3), we obtain:

$$\delta w - \rho g A_W \delta T = 0 \qquad (5)$$

or: $$\delta T = \frac{\delta w}{\rho g A_W} \qquad (6)$$

after removing the load ***δw***, the force ***δD*** (additional displacement) will restore the body to its original position.
Let's look at the expression, which describes the restoring force:

$$\delta D = \rho g A_W \delta T \qquad (7)$$

Until the waterplane area A_W is greater than zero, as long ***δD*** balance the ***δw*** - the system is in **stable equilibrium**.
When A_w=0, the system is in **neutral equilibrium**

Analysis of the equilibrium on the other degrees of freedom

The **longitudinal stability** of a floating body is **positive** when:

$$\overline{GM}_L > 0$$

is **neutral** when:

$$\overline{GM}_L = 0$$

the **equilibrium is unstable**, when:

$$\overline{GM}_L < 0$$

where $\boldsymbol{GM_L}$ is a longitudinal metacentric height.

If there are no additional constraints, the ship is **in neutral equilibrium** for degrees of freedom along the axis *Ox* (**surge**), along axis *Oy* (**sway**), and the rotation around the vertical axis *Oz* (**yaw**)

Note:
During the movement in the horizontal plane, restoring forces do not appear.

Note 2:
If we want the movements in the horizontal plane appeared restoring forces, we need to impose additional constraints on the system

Dynamics of floating body structures. Introduction

Single degree of freedom structures

The simples matematical model has just one equation in terms of only one time dependent scalar coordinate (here it is *x*), which uniquely describes the structure's position.

The dynamics of body motion describes the Newton's second law:

$$m\ddot{x}=\sum_i F_{x,i}$$

Where:
m – is a mass of the structure (or mass of system)
x – is the scala coordinate describing the position of the structure

$\sum_i F_{x,i}$ - sum of the forces acting on the object (or the entire system)

We can distinguish the following types of forces:
(1) – self-weight;
(2) – externally applied environmental forces;
(3) – reaction forces, due to system restrains, that tend to restore the structure to its static equilibrium position;
(4) – damping forces that mitagate motion.

Dynamics of floating body structures. Introduction

Single degree of freedom structures

Some types of forces are almost always during the movement of the body in a liquid (water).

Here we can distinguish:

- additional inertia forces related to the presence of added mass (of water), which moves together with the submerged body

- damping forces: drag and forces due to the wave radiation (the wave arises as a result of body movement)

- restoring forces due to hydrostatics and/or mooring system.

The above-mentioned forces are induced due to movement of the body. These forces are also present in calm water.

Therefore, the equation of motion of a body immersed in a fluid (for one degree of freedom) can be written in the „general” form:

$$\left(m_0 + m_A\right)\ddot{x} + f\left(\dot{x}\right) + q\left(x\right) = p_1\left(t\right)$$

m_0 – mass of the body;
m_A – added (virtual) mass;
$f\left(\dot{x}\right)$ – damping forces
$q\left(x\right)$ – restoring forces
$p_1\left(t\right)$ – time dependent environmental forces

Dynamics of floating body structures.

Added (or virtual) mass

Added mass or **virtual mass** is the inertia added to a system because an accelerating or decelerating body must move (or deflect) some volume of surrounding fluid as it moves through it. Added mass is a common issue because the object and surrounding fluid cannot occupy the same physical space simultaneously. For simplicity this can be modeled as some volume of fluid moving with the object, though in reality "all" the fluid will be accelerated, to various degrees

In contrast to mass of a body, the added mass will be (in general) different for each of the degrees of freedom. In the case of angular motion, the added mass has a unit of a moment of inertia

The added mass is usually denoted by m_A or - at a certain degree of freedom *ij* – m_{ij} or A_{ij} .

In order to facilitate the use of data from the literature, introduced the dimensionless ratio C_A:

$$m_A = C_A m_b$$

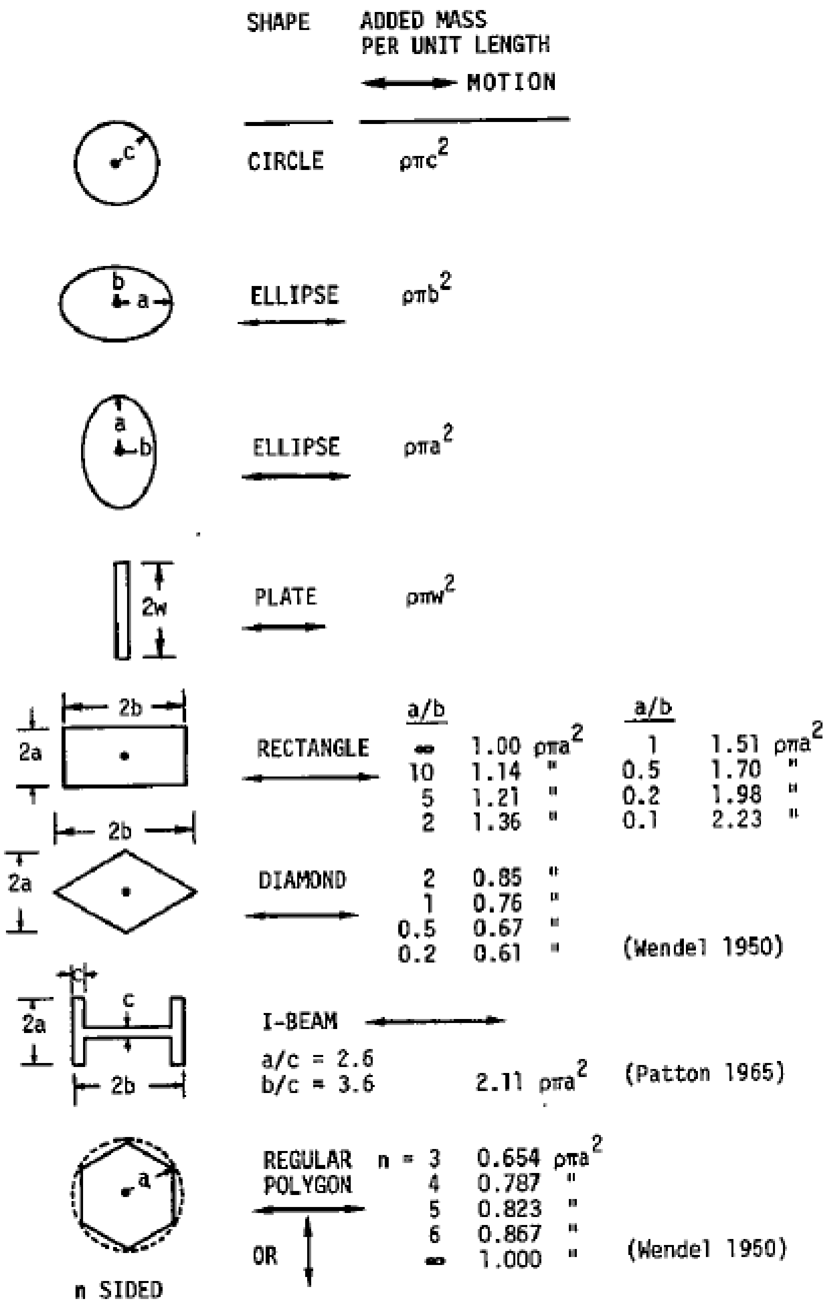
where: m_b – is a mass of a displaced water:

$$m_b = \rho V_b$$

Dimensionless coefficients make it easy to use the data of 2D cross sections for computatons of 3D (longitudinal) elements.

Dynamics of floating body structures.

Added (or virtual) mass – examples for various bodies



SHAPE		ADDED MASS PER UNIT LENGTH ←→ MOTION			
CIRCLE		$\rho\pi c^2$			
ELLIPSE		$\rho\pi b^2$			
ELLIPSE		$\rho\pi a^2$			
PLATE		$\rho\pi w^2$			
RECTANGLE	a/b		a/b		
	∞	1.00 $\rho\pi a^2$	1	1.51 $\rho\pi a^2$	
	10	1.14 "	0.5	1.70 "	
	5	1.21 "	0.2	1.98 "	
	2	1.36 "	0.1	2.23 "	
DIAMOND	2	0.85 "			
	1	0.76 "			
	0.5	0.67 "			
	0.2	0.61 "			(Wendel 1950)
I-BEAM	a/c = 2.6, b/c = 3.6	2.11 $\rho\pi a^2$			(Patton 1965)
REGULAR POLYGON	n = 3	0.654 $\rho\pi a^2$			
	4	0.787 "			
	5	0.823 "			
	6	0.867 "			
	∞	1.000 "			(Wendel 1950)

In order to obtain dimensionless ratio C_A, from 2D data, we can use the following formula:

$$C_A = \frac{\overline{m}_A}{\rho A}$$

where: A – is a surface area of the section.

Added mass for 3D longitudinal body, can be calculated with the use of formula:

$$m_A = \rho V_b C_A$$

Note: m_A (or C_A) value applies only to a particular direction of motion (degree of freedom)

[Sarpkaya T.: „Wave forces on offshore structures”]

Dynamics of floating body structures.

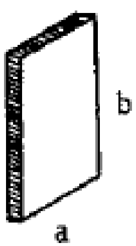
Added (or virtual) mass – examples for various bodies

CIRCULAR DISK * $\frac{8}{3}\rho c^3$

ELLIPTICAL DISK

(Munk 1934)

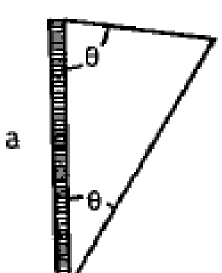
b/a		
∞	1.000	$(\pi/6)\rho b a^2$
3	0.900	
2	0.826	"
1.5	0.758	"
1.0	0.637	"



RECTANGULAR PLATE

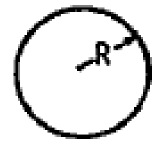
(Meyerhoff 1970)

b/a		
1.0	0.579	$(\rho/4)\pi a^2 b$
1.25	0.642	"
1.59	0.704	
2.0	0.757	"
2.5	0.801	"
4.0	0.872	"
5.0	0.897	"
8.0	0.934	"
10.0	0.947	"
∞	1.000	"

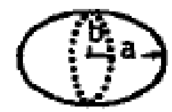


TRIANGULAR PLATE $\frac{\rho a^3(\tan\theta)^{3/2}}{3\pi}$

(Nielsen 1960)

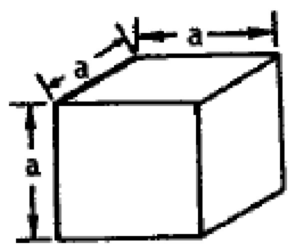


SPHERE $\frac{2}{3}\rho\pi R^3$



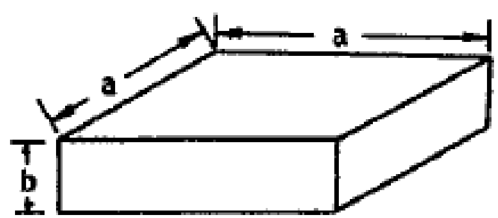
ELLIPSOID OF REVOLUTION $k\,\frac{4}{3}\rho\pi b^3$

a/b	k
0.01	0.6348
0.10	0.6148
0.20	0.6016
0.40	0.5712
0.60	0.5447
0.80	0.5211
1.00	0.5000
2.00	0.4200
3.00	0.3660
5.00	0.2956
7.00	0.2511
10.00	0.2071



CUBE $0.7\rho a^3$

(Sarpkaya 1960, Yu 1945)



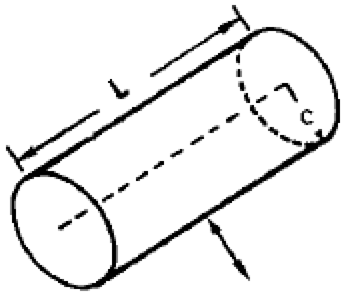
RECTANGULAR BLOCK $k\,\rho a^2 b$

(Sarpkaya 1960)

b/a	k
0.5	1.32
0.6	1.15
0.8	0.86
1.0	0.70
1.2	0.57
1.6	0.45
2.0	0.35
2.4	0.30
2.8	0.26
3.6	0.22

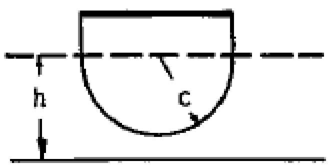
Dynamics of floating body structures.

Added (or virtual) mass – examples for various bodies



$L/(2c)$		
1.2	0.62	$\pi\rho a^2 b$
2.5	0.78	"
5.0	0.90	"
9.0	0.96	"
∞	1.00	"

(Wendel 1950)

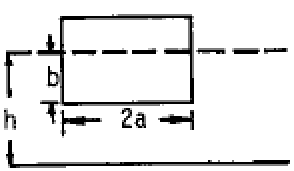


FLOATING CYLINDER

$k \frac{\pi}{2} c^2$

h/c	k
1.2	1.83
1.5	1.45
2.0	1.22
3.0	1.09
5.0	1.03
∞	1.00

(Bai 1977)



FLOATING RECTANGLE

$2k_1\rho ab$

	k_1			
	a/b			
b/h	0.2	0.5	1.0	2.0
0.0	4.75	2.14	1.18	0.67
0.2	4.92	2.25	1.29	0.78
0.4	5.43	2.63	1.66	1.16
0.6	6.63	3.56	2.53	2.02
0.8	10.15	6.46	5.23	4.62

$2k_2\rho ab$

h/b	k_2
1.1	5.52
1.2	3.49
1.5	2.11
3.0	1.35
8.0	1.21
∞	1.19

(Bai 1977, Flagg and Newman 1971)

Note: C_A coefficient is not a constant value for a given shape. In general, it depends on the Reynolds number *Rn*, (circular) frequency ω and amplitude of movement x_a, surface roughness k_r and other...

$$C_A = C_A(Rn, \omega, x_a, k_r)$$

In order to better recognition of the problem, two dimensionless coefficients have been introduced:
K_C – Keulegan-Carpenter number;
β – so-called „beta" number.

$$K_C = u_a T / D \quad ; \qquad \beta = \frac{Rn}{K_C} = \frac{D^2}{\nu T}$$

where: u_a – amplitude of velocity;
T – velocity variation period;
D – diameter; ν - kinematic viscosity

Dynamics of floating body structures.

Linear damping coefficient, drag coefficient

Linear damping coefficient. Definition (formula) of force due to linear damping is as follows:

$$F_{damp}(\dot{x}) = -b_1 \dot{x}$$

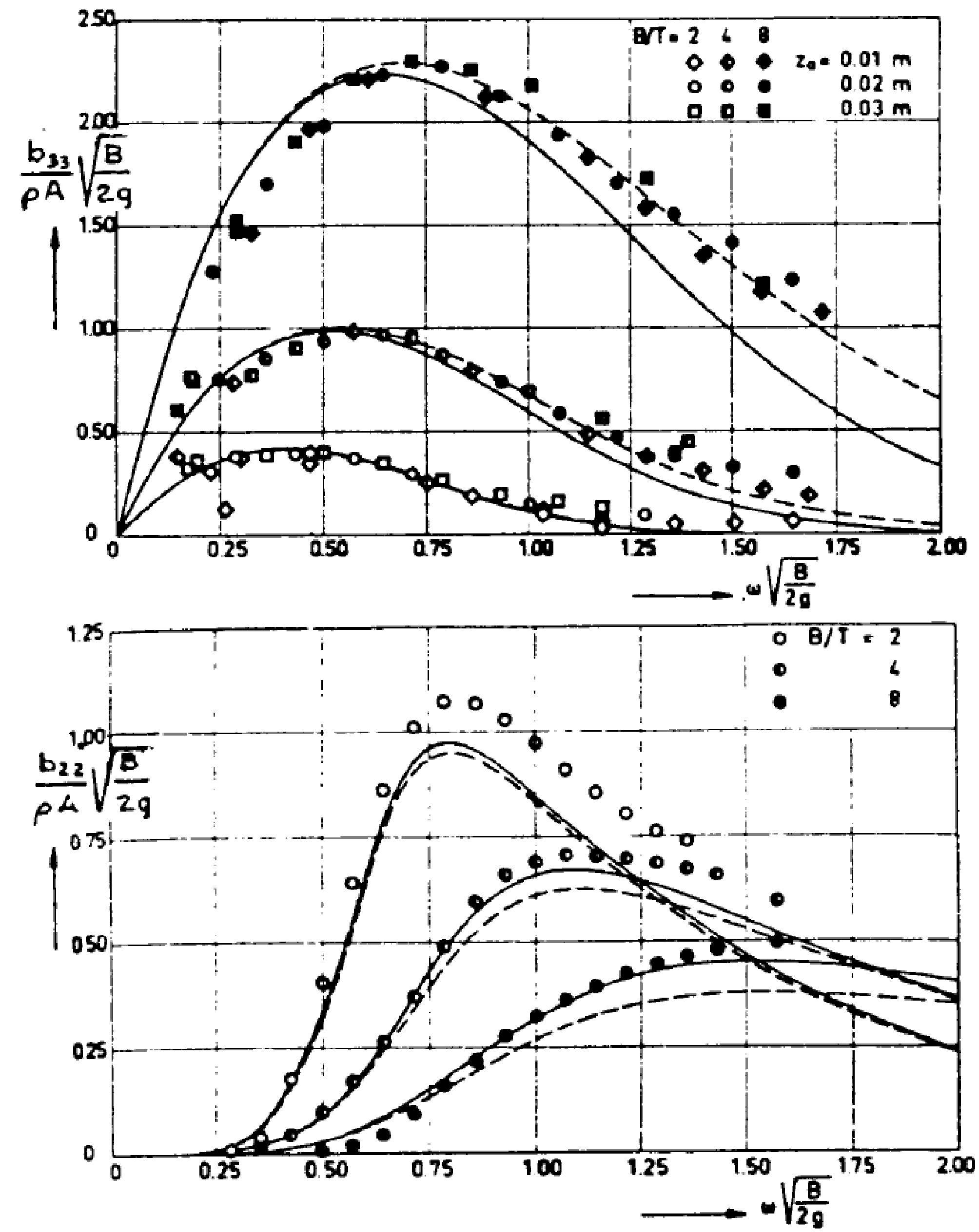
where: b_1 – is a (linear) damping coefficient.

The main reason of the presence of a linear damping is the wave radiation due to the movement of the body in the liquid, near (or at) the free surface.

The value of the damping coefficient is a function of the circular frequency and the Froude number:

$b_1 = b_1\,(\omega,\mathrm{Fn})$

The plots beside show the variations of the sectional damping coefficients in heaving b_{33} and swaying b_{22} for rectangular sections



[Source: *Principles of Naval Architcture*]

Dynamics of floating body structures.
Linear damping coefficient, drag coefficient

Linear damping coefficient can be also used for modelling of viscous forces during flow at (very) low Rn.

Note, however, that in the shipbuilding/offshore industry, the objects are rather large. So the value of the Reynolds number is in the range of (strongly) turbulent flow.

Despite the above, the linear damping coefficient is also used for turbulent flows in the case of oscillating motions/flow.

Application of linearization of damping forces allows the use of analytical methods for the integration of differential equations, as well as to obtain quickly an approximation of the transfer function, etc.

Viscous drag force is calculated with rhe use of following formula:

$$F_{drag} = -C_D \frac{1}{2} \rho A_p |\dot{x}| \dot{x}$$

where: C_D is a drag coefficient;

A_p - projection area of the body to the direction of motion.

In the case of stationary flow, the drag coefficient is a function of the *Rn* and the roughness of the surface k_r.

The above formula, can be written in more general form:

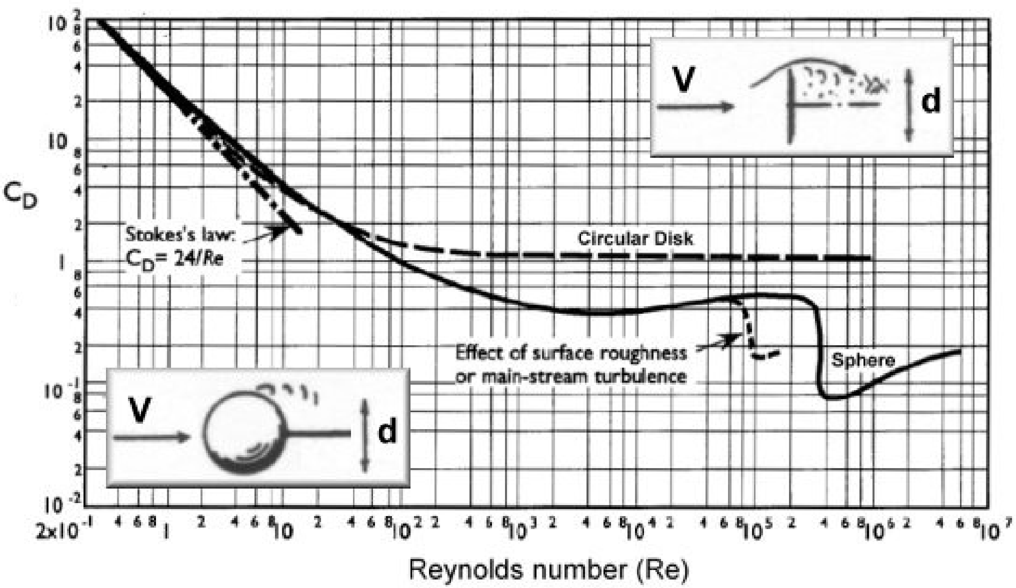
$$F_{drag} = C_D \frac{1}{2} \rho A_p |u - \dot{x}| (u - \dot{x})$$

which allows to take into account the velocity *u* of the surrounding liquid

Dynamics of floating body structures.

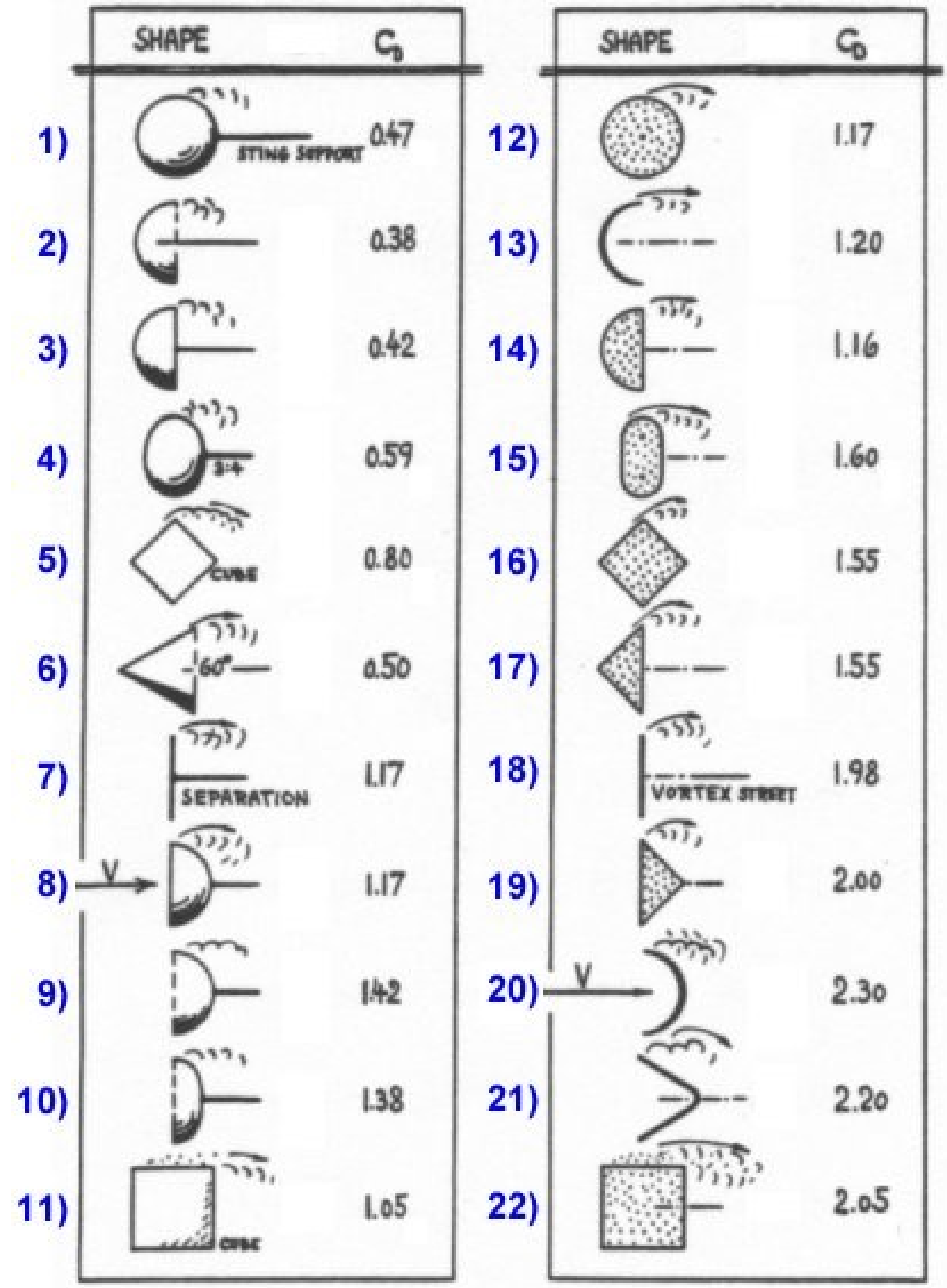
Linear damping coefficient, drag coefficient

Viscous drag coefficient C_D in a function of Reynolds number *Rn* for sphere (rounded body) and for circular disk (blunt body).



Viscous drag coefficient C_D of severial simple 3D and 2D shapes, figure on the right.

[http://www.aerospaceweb.org]



Dynamics of floating body structures.

Added mass, linear damping coefficient, drag coefficient

Remarks and summary

Dynamics of floating body structures.

Single degree of freedom (1DoF) example.

Exercise 1:

Calculate the movements of cylindrical buoy, floating in calm water, which was displaced from the equilibrium position of the δz = 1m.

D = 2m - diameter of the buoy,

T_0 = 4m - initial draught (in equilibrium),

H = 6m - depth (or height),

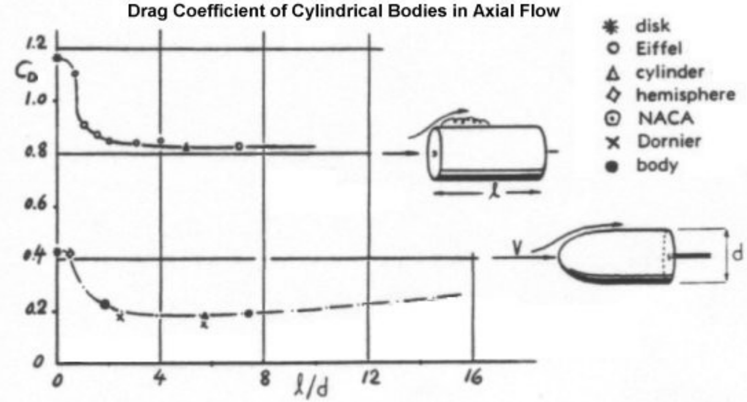
ρ = 1025 kg/m3 - water density,

C_A = 0.2 - added mass coefficient,

C_{D2} = 0.82 - drag coef. for fully submerged cylinder (both bases submerged).

b_{33} = ? - neglected due to lack of data

Drag Coefficient of Cylindrical Bodies in Axial Flow



Solution:

According to the second law of Newton (for z - motions):

$$m_0 \ddot{z} = \sum_i F_{z,i}$$

Added mass force:

$$F_{added} = -C_A m_b \ddot{z}$$

$$m_b = m_0 = \rho \frac{\pi D^2}{4} T_0$$

Damping force: $F_{damp} = -b_{33} \dot{z}$

Drag force:

$$F_{drag} = -C_D \frac{1}{2} \rho A_p |\dot{z}| \dot{z}$$; where:

$$A_p = \pi D^2/4 \qquad C_D = \frac{1}{2} C_{D2}$$

Restoring force:

$$F_{rest} = -\rho g A_W z$$; where:

$$A_W = A_p = \pi D^2/4$$

Dynamics of floating body structures.
Single degree of freedom (1DoF) example.

Solution (continuation):

The sum of the forces will be expressed by the formula:

$$\sum_i F_{z,i} = F_{added} + F_{dump} + F_{drag} + F_{rest}$$

or:

$$\sum_i F_{z,i} = -C_A m_b \ddot{z} - b_{33} \dot{z} - C_D \frac{1}{2} \rho A_p |\dot{z}| \dot{z} - \rho g A_W z$$

After substituting the sum of the forces to the main equation:

$$m_0 \ddot{z} = -C_A m_b \ddot{z} - b_{33} \dot{z} - C_D \frac{1}{2} \rho A_p |\dot{z}| \dot{z} - \rho g A_W z$$

Taking into account that $m_0 = m_b$, after ordering we can write:

$$(1 + C_A) m_0 \ddot{z} + b_{33} \dot{z} + C_D \frac{1}{2} \rho A_p |\dot{z}| \dot{z} + \rho g A_W z = 0$$

The equation is consistent with the general form: $(m_0 + m_A) \ddot{z} + f(\dot{z}) + q(z) = p_1(t)$, while at this case: $p_1(t) = 0$

With the assumption of free oscillations, time dependent environmental forces are equal to zero

Dynamics of floating body structures.
Single degree of freedom (1DoF) example.

Solution (continuation 2):

Formulation of initial conditions:

$$t=0 \quad ; \quad z(0)=\delta T=1\,m \quad ; \quad \dot{z}(0)=0$$

What is the solution of the differential equation?

$\ddot{z}=\ddot{z}(t)$ - acceleration as a function of time;

$\dot{z}=\dot{z}(t)$ - velocity;

$z=z(t)$ - position;

But also, as a solution, we can get:

$$F_{added}=F_{added}(t)$$
$$F_{damp}=F_{damp}(t)$$
$$F_{drag}=F_{drag}(t)$$
$$F_{rest}=F_{rest}(t)$$

How to solve the equation?:

1) using analytical methods (?),
2) using numerical methods for ODEs

Ad 1) Analytical methods are very useful, but they can be used only for certain types of equations (most often use is limited to linear equations).
Form of solution is sometimes very complex, and thus difficult to use. It increases the probability of making an error.
Ad 2) Numerical methods are usually easy to use. However, this solution is not accurate. Numerical error depends mainly on the method used (the 1st order, 2nd order, 3rd order methods), also depends on the time step. There is also an additional problem: the solution can be unstable

Dynamics of floating body structures.
Basic numerical methods for sloving ODEs.

Two level methods:

Let's consider the first order ODE with an initial condition:

$$\frac{d\varphi(t)}{dt} = f(t,\varphi(t)) \quad ; \quad \varphi(t_0) = \varphi^0 \qquad (1)$$

The basic problem is to find the solution φ a short time Δ*t* after the initial point. The solution φ^1 at

$$t_1 = t_0 + \Delta t$$

Can be regarded as a new initial condition and the solution can be advanced to:

$$t_2 = t_1 + \Delta t \quad ; \quad t_3 = t_2 + \Delta t$$, etc.

The simplest methods can be constructed by integrating eq. (1)

from t_n to $t_{n+1} = t_n + \Delta t$

$$\int_{t_n}^{t_{n+1}} \frac{d\varphi(t)}{dt} dt = \varphi^{n+1} - \varphi^n = \int_{t_n}^{t_{n+1}} f(t,\varphi(t))\, dt \qquad (2)$$

The above formula is an accurate expression. However, to calculate the integral on the right side of the equation, it is necessary to approximate the function $f(t,\varphi(t))$ for the time range from t_n to t_{n+1}.

There are many numerical methods for solving initial value problems, There are many methods for solving initial value problems. The most basic methods will be discussed below.

Dynamics of floating body structures.
Basic numerical methods for sloving ODEs.

Two level methods (continuation):

- ***explicit*** (or *forward*) ***Euler*** method:

$$\varphi^{n+1} = \varphi^n + f\left(t_n, \varphi^n\right)\Delta t$$

- ***implicit*** (or *backward*) ***Euler*** method:

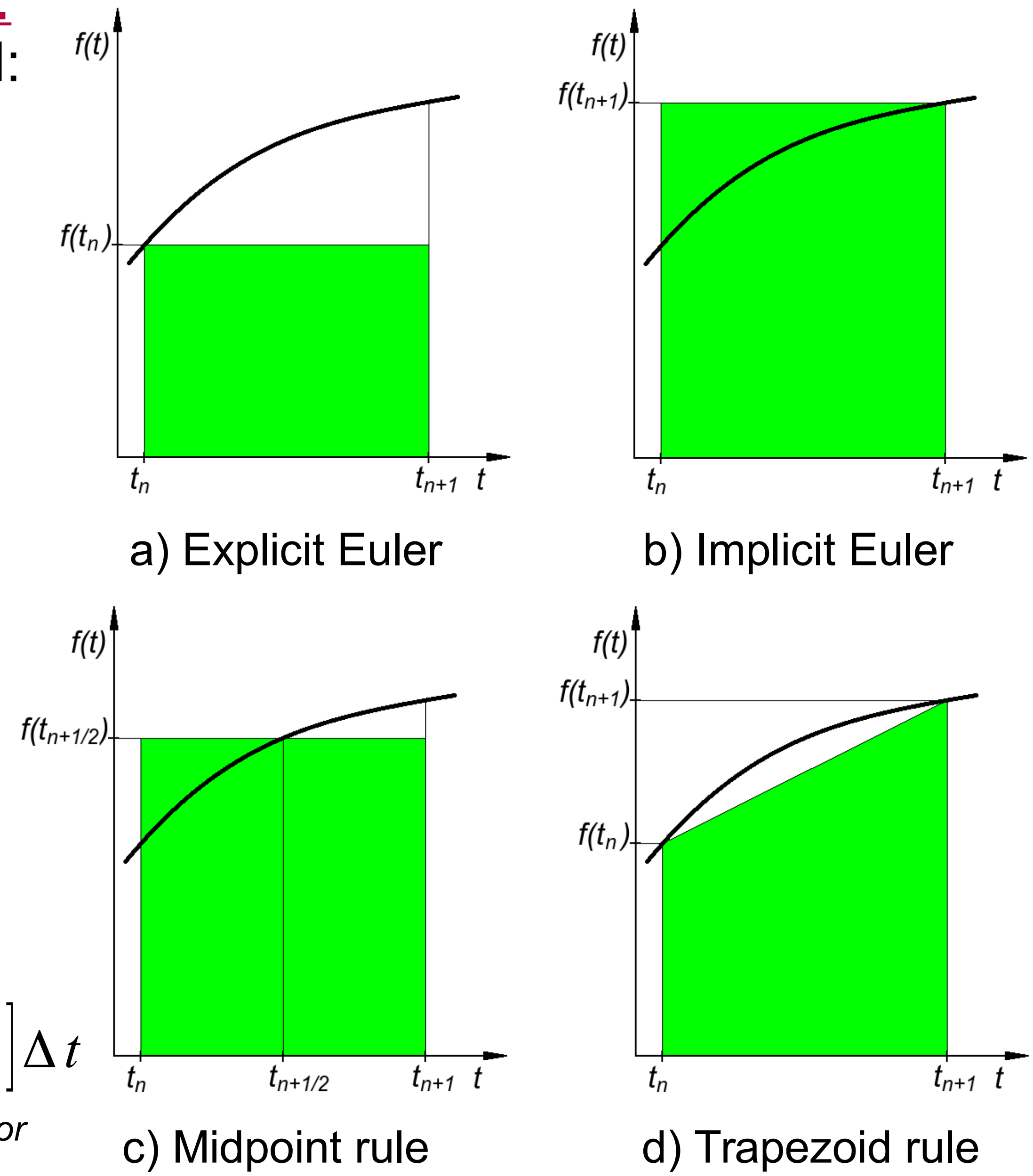
$$\varphi^{n+1} = \varphi^n + f\left(t_{n+1}, \varphi^{n+1}\right)\Delta t$$

- ***midpoint rule***:

$$\varphi^{n+1} = \varphi^n + f\left(t_{n+\frac{1}{2}}, \varphi^{n+\frac{1}{2}}\right)\Delta t$$

- ***trapezoid rule***:

$$\varphi^{n+1} = \varphi^n + \frac{1}{2}\left[f\left(t_n, \varphi^n\right) + f\left(t_{n+1}, \varphi^{n+1}\right)\right]\Delta t$$



[source: Ferziger J.H, Peric M.: *Computational Methods for Fluid Dynamics*]

Dynamics of floating body structures.
Single degree of freedom (1DoF) example.

Solution (continuation 3):

Based on previously derived 2nd order ordinary differential equation describing the dynamics of the buoy:

$$\left(1+C_A\right) m_0 \ddot{z} + b_{33} \dot{z} + C_D \frac{1}{2} \rho A_p \left|\dot{z}\right| \dot{z} + \rho g A_W z = 0$$

We can write:

$$\ddot{z} = -\frac{b_{33} \dot{z} + C_D \frac{1}{2} \rho A_p \left|\dot{z}\right| \dot{z} + \rho g A_W z}{\left(1+C_A\right) m_0}$$

The ***solution algorithm*** of the problem may be as follows:

Step 0: „rewriting" the initial conditions

$n=0$; - setting the iteration counter

$t_0=0$; $z^0 = z(0) = 1\,m$; $\dot{z}^0(0) = 0$

Dynamics of floating body structures.

Single degree of freedom (1DoF) example.

Solution (continuation 4):

Step 1: calc. of the the acceleration of the body for t_n:

$$\ddot{z}^n = -\frac{b_{33}\,\dot{z}^n + C_D \frac{1}{2}\rho A_p \left|\dot{z}^n\right| \dot{z}^n + \rho g A_W z^n}{\left(1 + C_A\right) m_0}$$

Step 2: calc. of the the velocity of the body for t_{n+1}:

$$\dot{z}^{n+1} = \dot{z}^n + \ddot{z}^n \Delta t$$ - explicit Euler method

Step 3: calc. of the the position of the body for t_{n+1}:

$$z^{n+1} = z^n + \frac{1}{2}\left(\dot{z}^n + \dot{z}^{n+1}\right)\Delta t$$ - trapezoid rule

Step 4: increasing the time and iteration counter:

Save results of iteration No. *n*

$$t_{n+1} = t_n + \Delta t \quad ; \quad n := n+1$$

Go to Step 1: (to start another iteration)

Dynamics of floating body structures.

Single degree of freedom (1DoF) example.

Solution (continuation 5):

Task: use the above solution algorithm in Spreadsheet (Excel/OpenOffice.Org Calc) in order to find plots of forces, acceleration, velocity and position in a function of time (solve the Exercise 1).

Check the impact of changes in the coefficients on the obtained results (acceleration, speed, position, frequency / period of oscillation, motion damping rate, etc.).

Try to show the impact of variability of main coefficents on the properties of the object motions on charts
No effect of a parameter changes on a specific characteristic of the object motion is also important.

Note, that the above algorithm is the one of the simplest. More complex methods of solution of the problem can be derived.

Dynamics of the environment.

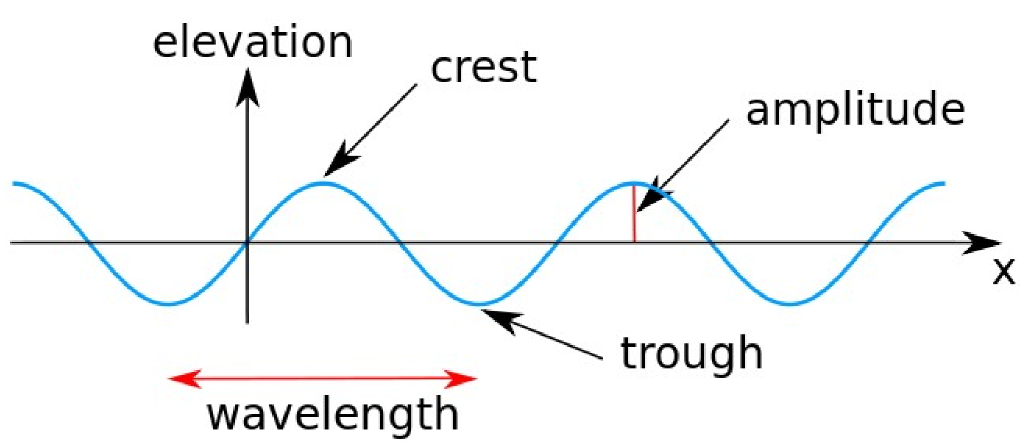
Inroduction to ocean wave modelling. Definition

Wind waves, or wind-generated waves, are surface waves that occur on the free surface of oceans, seas, lakes, rivers, and canals. They result from the wind blowing over an area of fluid surface. Waves in the oceans can travel thousands of miles before reaching land. Wind waves range in size from small ripples, to waves over 30 m high.

Wind waves have a certain amount of ***randomness***: <u>subsequent waves differ in height, duration, and shape with limited predictability</u>. They can be described as a stochastic process, in combination with the physics governing their generation, growth, propagation and decay - as well as governing the interdependence between flow quantities such as: the water surface movements, flow velocities and water pressure. The key statistics of wind waves (both seas and swells) in evolving sea states can be predicted with wind wave models. [source: en.wikipedia.org]

The size of wind waves and the structures of the flows are determined by:

- Wave height (from high trough to crest)
- Wave length (from crest to crest)
- Wave period (time interval between arrival of consecutive crests at a stationary point)
- Wave propagation direction



Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Airy wave theory (or linear wave theory) gives a linearised description of the propagation of gravity waves on the surface of a homogeneous fluid layer. The theory assumes that the fluid layer has a uniform mean depth, and that the fluid flow is inviscid, incompressible and irrotational.

Airy wave theory is often applied in ocean engineering and coastal engineering for the modelling of random sea states – giving a description of the wave kinematics and dynamics of high-enough accuracy for many purposes.

This approximation is accurate for small ratios of the wave height to water depth (for waves in shallow water), and wave height to wavelength (for waves in deep water).

[source: en.wikipedia.org]

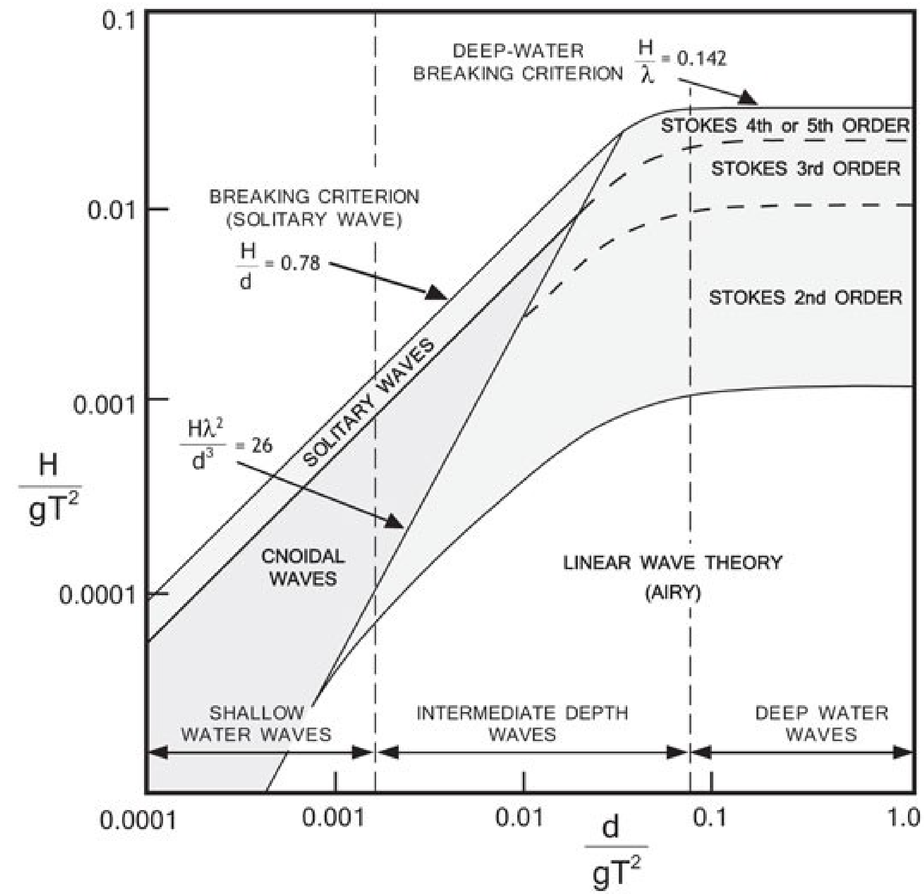


Fig. Limit for the application of the Airy wave theory and the other wave models.

Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Airy wave theory is a linear theory for the propagation of waves on the surface of a potential flow and above a horizontal bottom. The free surface elevation η(*x*,*t*) of one wave component is sinusoidal, as a function of horizontal position *x* and time *t:*

$$\eta(x,t)=a\cos(kx-\omega t)$$

where:

- *a* is the wave amplitude in [m],
- *k* is the angular wavenumber in [rad/m], related to the wavelength λ as:

$$k=\frac{2\pi}{\lambda}$$

- ω is the angular frequency in [rad/s], related to the period *T* and frequency *f* by:

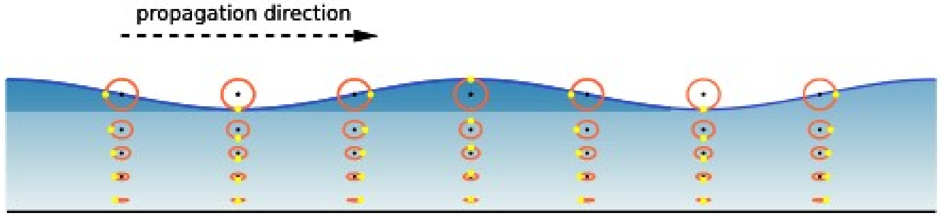
$$\omega=\frac{2\pi}{T}=2\pi f$$

The waves propagate along the water surface with the phase speed c_p:

$$c_p=\frac{\omega}{k}=\frac{\lambda}{T}$$

The angular wavenumber *k* and frequency ω are not independent parameters (and thus also wavelength λ and period *T* are not independent), but are coupled. Surface gravity waves on a fluid are dispersive waves – exhibiting frequency dispersion – meaning that each wavenumber has its own frequency and phase speed.

While the surface elevation shows a propagating wave, the fluid particles are in an orbital motion. Within the framework of Airy wave theory, the orbits are closed curves: circles in deep water, and ellipses in finite depth—with the ellipses becoming flatter near the bottom of the fluid layer.



Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Flow problem formulation.

When the flow is assumed to be incompressible and irrotational the potential theory can be used.

The velocity potential Φ(x,z,t) is related to the flow velocity components u_x and u_z in the horizontal *x* and vertical *z* directions by:

$$u_x = \frac{\partial \Phi}{\partial x}; \quad u_z = \frac{\partial \Phi}{\partial z}$$

Then, due to the continuity equation for an incompressible flow, the potential Φ has to satisfy the Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

Boundary conditions are needed at the bed and the free surface in order to close the system of equations.
The bed being impermeable, leads to the kinematic bed boundary condition:

$$\frac{\partial \Phi}{\partial z} = 0; \text{ at } z = -d,$$

or, in case of deep water, the above b. c. has to be satisfy at: z → ∞
At the free surface, for „small" waves, the vertical motion of the flow has to be equal to the vertical velocity of the free surface:

$$\frac{\partial \eta}{\partial t} = \frac{\partial \Phi}{\partial z}; \text{ at } z = \eta(x, t)$$

The surface elevation η is an extra unknown, for which an additional b. c. is needed. This is provided by Bernoulli's equation for an unsteady potential flow. The pressure above the free surface is assumed to be constant (and equal to zero). After linearisation, this gives the dynamic free-surface b. c.:

$$\frac{\partial \Phi}{\partial t} + g\eta = 0; \text{ at } z = \eta(x, t).$$

Note: Because this is a linear theory, in both free-surface b. c. – the kinematic and the dynamic one, the value of Φ and ∂Φ/∂z at the fixed mean level z = 0 is used.

Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Solution for a progressive regular (monochromatic) wave.

For a propagating wave of a single frequency – a regular wave – the surface elevation is of the form:

$$\eta(x, t) = a \cos(kx - \omega t)$$

The associated velocity potential, satisfying the Laplace equation in the fluid interior, as well as the kinematic boundary conditions at the free surface, and bed, is:

$$\Phi = \frac{\omega}{k} a \frac{\cosh(k(z+d))}{\cosh(kd)} \cos(kx - \omega t)$$

But η and Φ also have to satisfy the dynamic boundary condition, which results in non-trivial (non-zero) values for the wave amplitude *a* only if the linear ***dispersion relation*** is satisfied:

$$\omega^2 = gk \tanh(kd)$$

So angular frequency ω and wavenumber *k* – or equivalently period *T* and wavelength λ – cannot be chosen independently, but are related. When ω and *k* satisfy the dispersion relation, the wave amplitude *a* can be chosen freely (but small enough for Airy wave theory).

For more general situation, the waves may propagate in an arbitrary horizontal direction in the ***x*** = (*x*,*y*) plane. The wavenumber vector is ***k***, and is perpendicular to the cams of the wave crests:

$$\eta(x, t) = a \cos(\theta(\boldsymbol{x}, t))$$, where:

$$\theta(\boldsymbol{x}, t) = \boldsymbol{k} \cdot \boldsymbol{x} - \omega t$$

Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Solution for a progressive regular (monochromatic) wave.

Water depth is classified into three regimes:

- deep water – for a water depth larger than half the wavelength, $d > \frac{1}{2} \lambda$, the phase speed of the waves is hardly influenced by depth (this is the case for most wind waves on the sea and ocean surface),
- shallow water – for a water depth smaller than the wavelength divided by 20, $d < \frac{1}{20} \lambda$, the phase speed of the waves is only dependent on water depth, and no longer a function of period or wavelength,
- intermediate depth – all other cases, $\frac{1}{20} \lambda < d < \frac{1}{2} \lambda$, where both water depth and period (or wavelength) have a significant influence on the solution of Airy wave theory.

In the limiting cases of deep and shallow water, simplifying approximations to the solution can be made. While for intermediate depth, the full formulations have to be used.

Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Properties of gravity waves on the surface of deep water and at intermediate depth, according to Airy wave theory:

quantity	symbol	units	deep water	intermediate depth
surface elevation	$\eta(\boldsymbol{x}, t)$	[m]	$a\cos\theta(\boldsymbol{x}, t)$	
wave phase	$\theta(\boldsymbol{x}, t)$	[rad]	$\theta(\boldsymbol{x}, t) = \boldsymbol{k}\cdot\boldsymbol{x} - \omega t$	
wave propagation direction	$\boldsymbol{e}_k$	[-]	$\dfrac{\boldsymbol{k}}{k}$	
phase speed	c_p	[m/s]	$\sqrt{\dfrac{g}{k}\tanh(kd)}$	
horizontal velocity	$\boldsymbol{u}_x(\boldsymbol{x}, z, t)$	[m/s]	$\boldsymbol{e}_k \omega a e^{kz} \cos\theta$	$\boldsymbol{e}_k \omega a \dfrac{\cosh(k(z+d))}{\sinh(kd)}\cos\theta$
vertical velocity	$u_z(\boldsymbol{x}, z, t)$	[m/s]	$\omega a e^{kz} \sin\theta$	$\omega a \dfrac{\sinh(k(z+d))}{\sinh(kd)}\sin\theta$

Dynamics of the environment.

Inroduction to ocean wave modelling. Airy wave theory

Properties of gravity waves on the surface of deep water and at intermediate depth, according to Airy wave theory:

quantity	symbol	units	deep water	intermediate depth
horizontal particle excursion	$\xi_x(\boldsymbol{x}, z, t)$	[m]	$\boldsymbol{e}_k \omega a e^{kz} \cos\theta$	$-\boldsymbol{e}_k a \dfrac{\cosh(k(z+d))}{\sinh(kd)} \sin\theta$
vertical particle excursion	$\xi_z(\boldsymbol{x}, z, t)$	[m]	$\omega a e^{kz} \sin\theta$	$a \dfrac{\sinh(k(z+d))}{\sinh(kd)} \cos\theta$
horizontal acceleration	$\boldsymbol{a}_x(\boldsymbol{x}, z, t)$	[m/s^2]	$\boldsymbol{e}_k \omega^2 a e^{kz} \sin\theta$	$\boldsymbol{e}_k \omega^2 a \dfrac{\cosh(k(z+d))}{\sinh(kd)} \sin\theta$
vertical acceleration	$a_z(\boldsymbol{x}, z, t)$	[m/s^2]	$-\omega^2 a e^{kz} \cos\theta$	$-\omega^2 a \dfrac{\sinh(k(z+d))}{\sinh(kd)} \cos\theta$
pressure oscillation	$p(\boldsymbol{x}, z, t)$	[N/m^2]	$\rho g a e^{kz} \cos\theta$	$\rho g a \dfrac{\cosh(k(z+d))}{\cosh(kd)} \cos\theta$

Dynamics of floating body structures.
Single degree of freedom (1DoF) example.

Exercise 2:

Calculate the movements of cylindrical buoy, floating in (deep) water, which was treated with regular wave:

a = 0,5 m - wave amplitude,
T = 3 s - wave period,
D = 2m - diameter of the buoy,
T_0 = 4m - initial draught (in equilibrium),
H = 6m - depth (or height),
ρ = 1025 kg/m3 - water density,
C_A = 0.2 - added mass coefficient,
C_{D2} = 0.82 - drag coef. for fully submerged cylinder (both bases submerged).
b_{33} = ? - neglected due to lack of data

Solution:

According to the second law of Newton (for z - motions):

$$m_0 \ddot{z} = \sum_i F_{z,i}$$

Added mass force:

$$F_{added} = -C_A m_b \ddot{z}$$

$$m_b = m_0 = \rho \frac{\pi D^2}{4} T_0$$

Damping force: $F_{damp} = -b_{33} \dot{z}$

Drag force:

$$F_{drag} = C_D \frac{1}{2} \rho A_p |u_z - \dot{z}| (u_z - \dot{z})$$

$$A_p = \pi D^2 / 4 \qquad C_D = \frac{1}{2} C_{D2}$$

Restoring force:

$$F_{rest} = -\rho g A_W z$$; where:

$$A_W = A_p = \pi D^2 / 4$$

Driving force due to wave:

$$F_{wave} = \rho g A_W \eta$$; where:

$$\eta(x,t) = a \cos(kx - \omega t)$$

$$k = \omega^2 / g$$ - deep water

Dynamics of floating body structures.
Single degree of freedom (1DoF) example.

Solution (continuation):

The sum of the forces will be expressed by the formula:

$$\sum_i F_{z,i} = F_{added} + F_{dump} + F_{drag} + F_{rest} + F_{wave}$$

or:

$$\sum_i F_{z,i} = -C_A m_b \ddot{z} - b_{33}\dot{z} + C_D \frac{1}{2}\rho A_p \left|u_z - \dot{z}\right|\left(u_z - \dot{z}\right) - \rho g A_W z + \rho g A_W \eta$$

After substituting the sum of the forces to the main equation:

$$m_0 \ddot{z} = -C_A m_b \ddot{z} - b_{33}\dot{z} + C_D \frac{1}{2}\rho A_p \left|u_z - \dot{z}\right|\left(u_z - \dot{z}\right) + \rho g A_W (\eta - z)$$

Taking into account that $m_0 = m_b$, after ordering we can also write:

$$\left(1 + C_A\right) m_0 \ddot{z} + b_{33}\dot{z} - C_D \frac{1}{2}\rho A_p \left|u_z - \dot{z}\right|\left(u_z - \dot{z}\right) + \rho g A_W z = \rho g A_W a \cos\left(kx - \omega t\right)$$

The above equation is „close to” the general form: $\left(m_0 + m_A\right)\ddot{z} + f\left(\dot{z}\right) + q\left(z\right) = p_1\left(t\right)$,

while at this case: $p_1\left(t\right) = \rho g A_W a \cos\left(kx - \omega t\right)$

Dynamics of floating body structures.

Single degree of freedom (1DoF) example.

Solution (continuation 2):

Formulation of initial conditions:

$$t=0 \quad ; \quad z(0)=0 \; ; \; \dot{z}(0)=0 \; ; \; x(t)=0$$

What is the solution of the differential equation?

$\ddot{z}=\ddot{z}(t)$ - acceleration as a function of time;

$\dot{z}=\dot{z}(t)$ - velocity;

$z=z(t)$ - position;

But also, as a solution, we can get:

$$F_{added}=F_{added}(t)$$

$$F_{damp}=F_{damp}(t)$$

$$F_{drag}=F_{drag}(t)$$

$$F_{rest}=F_{rest}(t)$$

$$F_{wave}=F_{wave}(t)$$

How to solve the equation?:

1) using analytical methods (only after linearization - approximate solution)

2) **using numerical methods for ODEs** (as before, see Ex. 1)

Dynamics of floating body structures.

Single degree of freedom (1DoF) example.

Solution (continuation 3):

Based on previously derived 2nd order ordinary differential equation describing the dynamics of the buoy:

$$\left(1+C_A\right)m_0\ddot{z}+b_{33}\dot{z}-C_D\frac{1}{2}\rho A_p\left|u_z-\dot{z}\right|\left(u_z-\dot{z}\right)+\rho g A_W z=\rho g A_W a\cos\left(kx-\omega t\right)$$

We can write:

$$\ddot{z}=\frac{-b_{33}\dot{z}+C_D\frac{1}{2}\rho A_p\left|u_z-\dot{z}\right|\left(u_z-\dot{z}\right)-\rho g A_W z+\rho g A_W a\cos\left(kx-\omega t\right)}{\left(1+C_A\right)m_0}$$

The ***solution algorithm*** of the problem may be as follows:

Step 0: „rewriting" the initial conditions

$n=0$; - setting the iteration counter

$t_0=0$; $z^0=z\left(0\right)=0$; $\dot{z}^0\left(0\right)=0$

Dynamics of floating body structures.
Single degree of freedom (1DoF) example.

Solution (continuation 4):

Step 1: calc. of the the acceleration of the body for t_n:

$$\ddot{z}^n = \frac{-b_{33}\dot{z}^n + C_D \frac{1}{2}\rho A_p \left|u_z^n - \dot{z}^n\right|\left(u_z^n - \dot{z}^n\right) + \rho g A_W\left(\eta^n - z^n\right)}{\left(1 + C_A\right) m_0}$$

Step 2: calc. of the the velocity of the body for t_{n+1}:

$$\dot{z}^{n+1} = \dot{z}^n + \ddot{z}^n \Delta t$$ - explicit Euler method

Step 3: calc. of the the position of the body for t_{n+1}:

$$z^{n+1} = z^n + \frac{1}{2}\left(\dot{z}^n + \dot{z}^{n+1}\right)\Delta t$$ - trapezoid rule

Step 4: increasing the time and iteration counter:

Save results of iteration No. *n*

$$t_{n+1} = t_n + \Delta t \quad ; \quad n := n+1$$

Go to Step 1: (to start another iteration)

Structure-environmental force interactions.
Fluid-induced structural forces

Forces on bodies in separated unsteady flow.

In general, a body has a mass m_b of its own and experiences a resistance (in viscous fluids) that is primarily due to separation of the flow or the resulting pressure forces (*form drag*). Assuming that the instantaneous values of the drag and inertial forces can be added, for a body at rest in a unidirectional time-dependent flow $u(t)$, the force exerted on the body becomes:

$$F = C_D \frac{1}{2} \rho A_p |u| u + \rho V_b (1 + C_A) \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \qquad (1)$$

in which A_p is the projected area of the body on a plane normal to the flow and V_b is the volume of the body. If the convective acceleration terms:

$\left(u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right)$ are small enough relative to the local acceleration $\frac{\partial u}{\partial t}$,

the term $\frac{du}{dt}$ Is replaced with $\frac{\partial u}{\partial t}$.

[Sarpkaya T.: „*Wave forces on offshore structures*”]

Structure-environmental force interactions.
Fluid-induced structural forces

Forces on bodies in separated unsteady flow (continuation)

In general, C_A and C_D depend on time, bodyshape, representative Reynolds number, relative displacement ofthe fluid, and the parameters characterizing the history of the motion. In practice, they are time averaged and plotted as functions of Re (or β), K_C and structure roughness:

$$C_A = C_A\left(\beta, K_C, k_r/D\right) \;;\quad C_D = C_D\left(\beta, K_C, k_r/D\right)\text{ , where: } K_C = \frac{u_a T}{D} \;;\quad \beta = \frac{Rn}{K_C} = \frac{D^2}{\nu T}$$

Let us cosider the relative motion of the body in a fluid stream whose velocity may vary with both time and distance in the direction of cylinder motion, i.e., $u=u(x,t)$. The body is subjected to a displacement x, velocity dx/dt, and acceleration d^2x/dt^2 in the direction of the incident stream and the diffraction effects are negligible. Then the force acting on the cylinder becomes:

$$F = C_D \frac{1}{2} \rho A_p \left|u - \dot{x}\right| \left(u - \dot{x}\right) + \rho V_b \left(1 + C_A\right) \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}\right) - \rho V_b C_A \ddot{x} \qquad (2)$$

The first term on the right-hand side of (2) represents the form drag; the second term, the inertial force that is due to the local and convective accelerations of the fluid about the body; and the third term, the inertial force that is due to the motion of the body, as it would be in a fluid otherwise at rest.

Structure-environmental force interactions. **Fluid-induced structural forces**

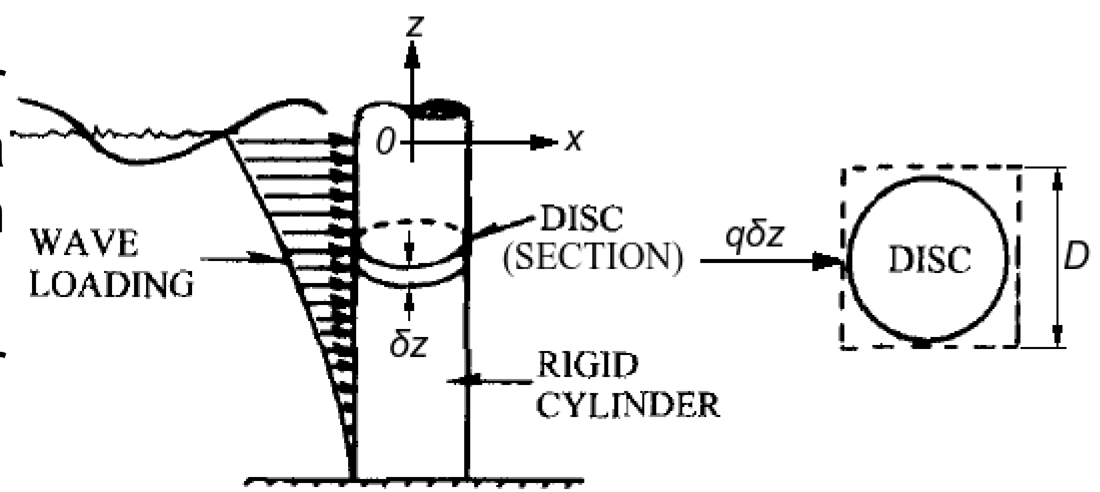
Forces on bodies in separated unsteady flow (continuation).

Nonuniform velocity field.

For stationary cylinder in a plane flow field with the free stream velocity $u=u(z,t)$, the total time-varying load acting on a thin section of the cylinder may be expressed (based on eq. (1)) as:

$$\delta F=\left[C_D\frac{1}{2}\rho D|u|u+\rho A_s(1+C_A)\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)\right]\delta z \quad , \qquad (1^*)$$

where: D is a width of the cylinder (for example diameter, in the case of a circular cylinder), A_s is the section area, δz is the section thickness.
Load per unit lenght on the cylinder may be expressed as $q=\delta F/\delta z$:



$$q=C_D\frac{1}{2}\rho D|u|u+\rho A_s(1+C_A)\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right) \qquad (1^{**})$$

Structure-environmental force interactions.
Fluid-induced structural forces

Forces on bodies in separated unsteady flow (continuation).

Total force and moment acting on fixed cylinder.

The total force acting on stationary cylinder in a plane flow field with the free stream velocity $u=u(z,t)$, may be calculated based differential form on eq. (1*)

$$d\ F=\left[C_D\frac{1}{2}\rho D|u|u+\rho A_s\left(1+C_A\right)\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)\right]d\ z\ , \qquad (1^{***})$$

as the integral determined in the range of z_0 (bottom level) to z_1 (water surface level):

$$F=\int_{z_0}^{z_1} q\ dz=\int_{z_0}^{z_1}\left[C_D\frac{1}{2}\rho D|u|u+\rho A_s\left(1+C_A\right)\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)\right]d\ z \qquad (3)$$

The total bending moment acting on the cylinder, calculated relative to the seabed plane (level z0) can be calculated as follows:

$$M=\int_{z_0}^{z_1} q\left(z-z_0\right)dz=\int_{z_0}^{z_1}\left[C_D\frac{1}{2}\rho D|u|u+\rho A_s\left(1+C_A\right)\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)\right]\left(z-z_0\right)d\ z \qquad (4)$$

Structure-environmental force interactions.
Fluid-induced structural forces

Forces on bodies in separated unsteady flow (continuation).

Morison equation.

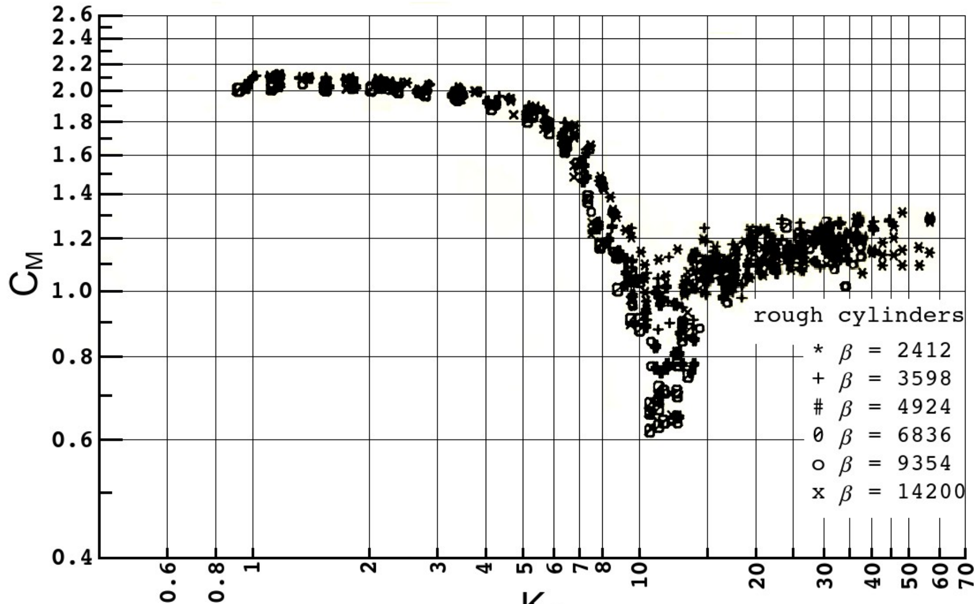
Equation (1*), can be formulate for the circular cylinder. If, in addition, we assume that the effect of convection member is negligibly small, we obtain the Morison equation:

$$q = C_D \frac{1}{2} \rho D |u| u + C_M \rho \pi \frac{D^2}{4} \dot{u} \quad , (5)$$

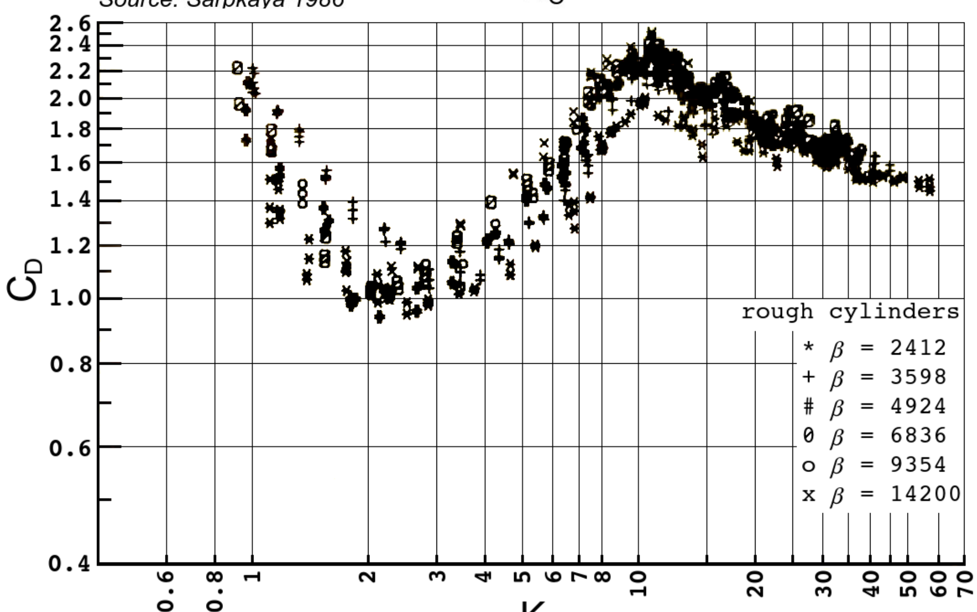
where C_M is so-called iertial force coefficient (due to the accelerations of the fluid), and:

$$C_M = 1 + C_A \quad , \quad (6)$$

The values of C_M and the C_D coefficients are determined experimentally. Sample graphs (based on [*Sarpkaya*]) are shown beside:



Source: Sarpkaya 1986



Source: Sarpkaya 1986

Structure-environmental force interactions.
Fluid-induced structural forces

Forces on bodies in separated unsteady flow (continuation).

Exercise 3:

Calculate the forces and moment on cylindrical, vertical monopile stuck in the seabed which is subjected to a regular wave during extreme storm:

a = 4.5 m - wave amplitude,
T = 11.3 s - wave period,
D = 6.5m - diameter of the buoy,
d = 40 m - water depth,
ρ = 1025 kg/m3 - water density,
C_M = ? - inertia coef. (rough surface),
C_D = ? - drag coef.

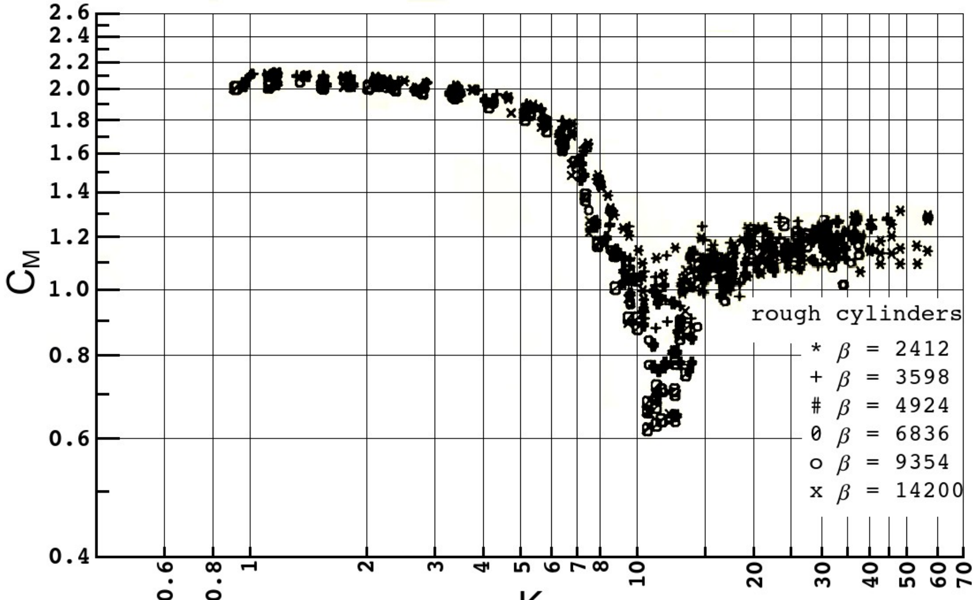
Structure-environmental force interactions.
Fluid-induced structural forces

Exercise 3 - solution alghorithm

Step 0. Data preparation:

0.1. Calculation of the KC (Keulegan-Carpenter number) to determine the C_M and C_D coefficients, as well as β number (if necessary)

$$K_C = \frac{u_a T}{D} \qquad \beta = \frac{D^2}{\nu T}$$

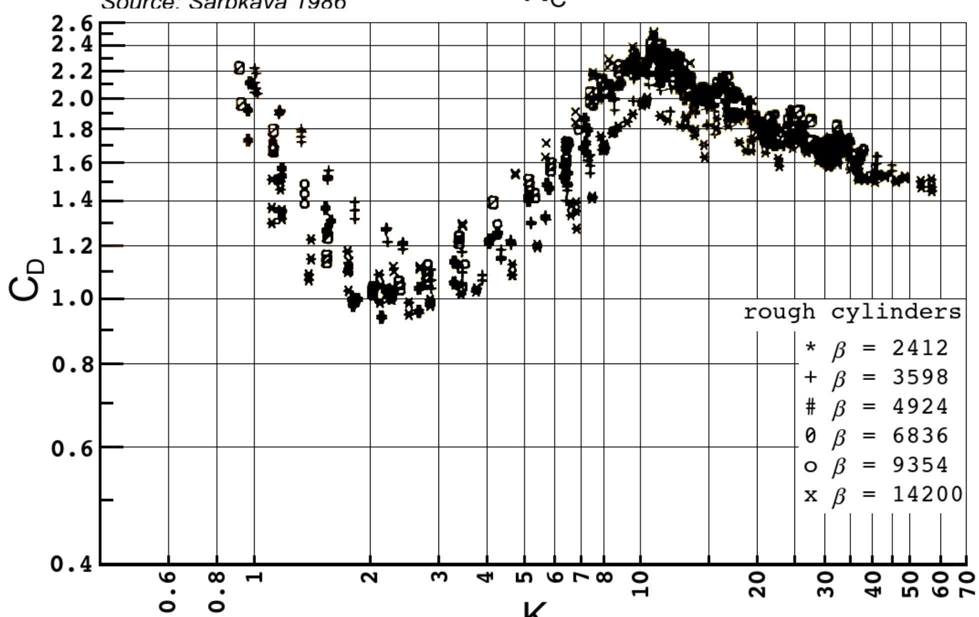


Source: Sarpkaya 1986

0.2. Split the pile into sections in order to calculate integrals:

$$F = \int_{z_0}^{z_1} q \, dz \quad \text{and} \quad M = \int_{z_0}^{z_1} q\left(z - z_0\right) dz$$

We obtain: array of $z_{c,i}$ and Δz_i



Source: Sarpkaya 1986

0.3. Calculate the wave number: k

$$\omega^2 = gk \tanh\left(kd\right) \qquad \omega = 2\pi / T$$

Structure-environmental force interactions.
Fluid-induced structural forces

Exercise 3 - solution alghorithm (continuation)

Step 0: Data preparation (continuation):

0.4. initial values: $t = 0$, $n = 0$

Step 1: Calculation of load acting on each section

For i=1 to i=n_{sec}

$$u_{x,i}=\omega a\frac{\cosh\left(k\left(z_{c,i}+d\right)\right)}{\sinh\left(kd\right)}\cos\left(kx-\omega t_n\right) \quad ; \quad \dot{u}_{x,i}=\omega^2 a\frac{\cosh\left(k\left(z_{c,i}+d\right)\right)}{\sinh\left(kd\right)}\sin\left(kx-\omega t_n\right)$$

$$q_i=C_D\frac{1}{2}\rho D\left|u_{x,i}\right|u_{x,i}+C_M\rho\pi\frac{D^2}{4}\dot{u}_{x,i}$$

Next i

Step 2: Calculation of the total force *F* and total moment *M*:

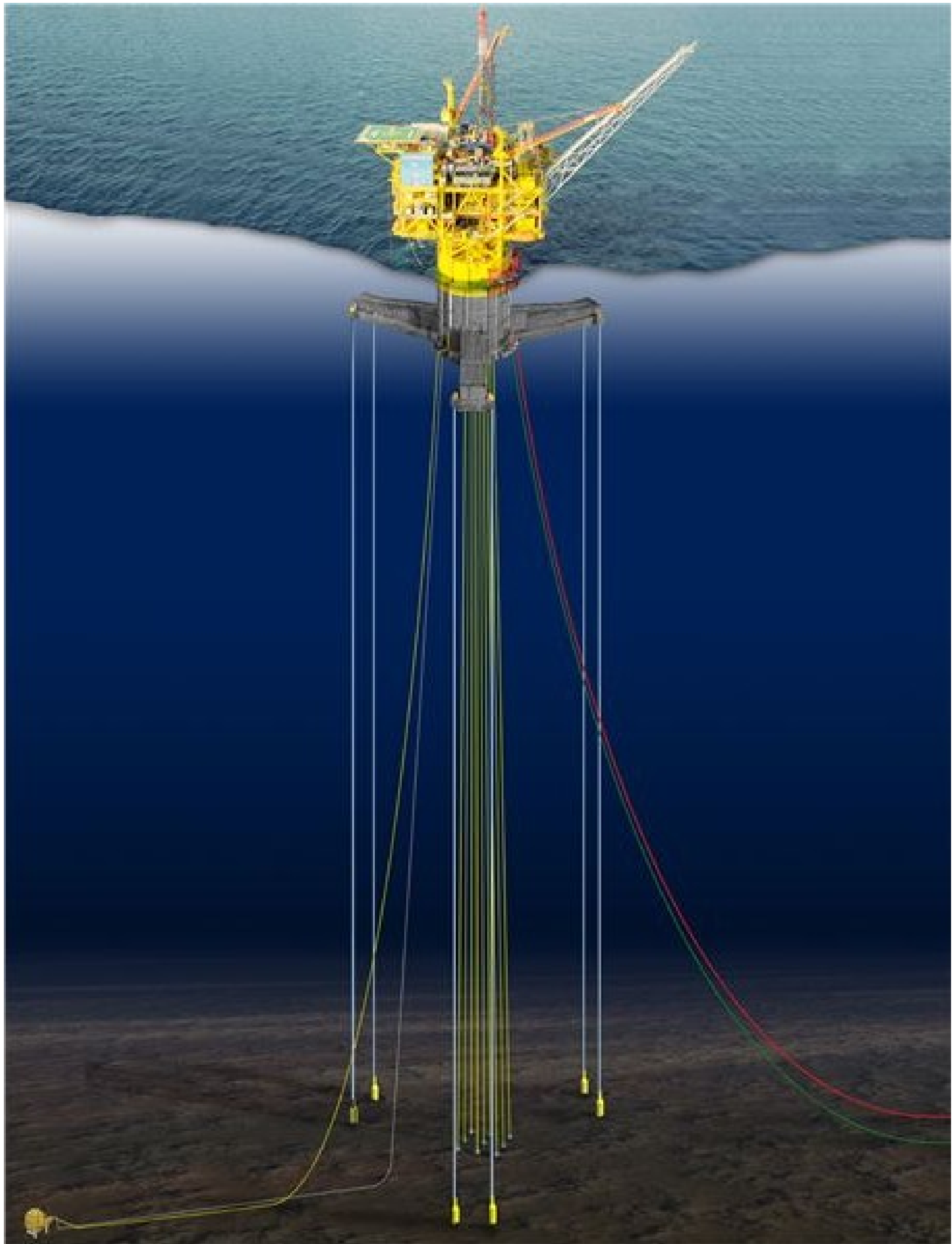
$$F=\sum_{i=1}^{n_{sec}} q_i\,\Delta z_i \qquad M=\sum_{i=0}^{i=n_{sec}} q_i\left(z_{c,i}-z_0\right)\Delta z_i$$

Step 3: *t=t+del_t ; n=n+1 ; go to* **step 1**

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Introduction

TLP anchoring system use a set of tension legs or tendons attached to the platform and connected to a foundation on the seabed using what is called tethers (tethers are anchoring device). The tension leg is anchored in a way that it allows horizontal movement (surge and sway) to accommodate wave disturbances, but prevent vertical (heave) or angular movement (pitch and roll) to keep the platform in place.



Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Equilibrium of forces equations

The platform is in static equilibrium, when the following conditions are satisfied:

- The vector sum of the forces is zero:

$$\sum_i \boldsymbol{F}_i = \boldsymbol{0} \qquad (1)$$

- The vector sum of the moments is zero:

$$\sum_i \boldsymbol{M}_i = \boldsymbol{0} \qquad (2)$$

For 2D case, the above two vector equations can be written in the scalar form as a system of three equations:

Sum of the horizontal forces equals zero:

$$\sum_i F_{x,i} = 0 \qquad (3)$$

Sum of the vertical forces equals zero:

$$\sum_i F_{z,i} = 0 \qquad (4)$$

Sum of the moments in the plane *Oxz* is zero:

$$\sum_i M_{y,i} = 0 \qquad (5)$$

Or, if we assume that the moment is triggered by forces at a certain arm:

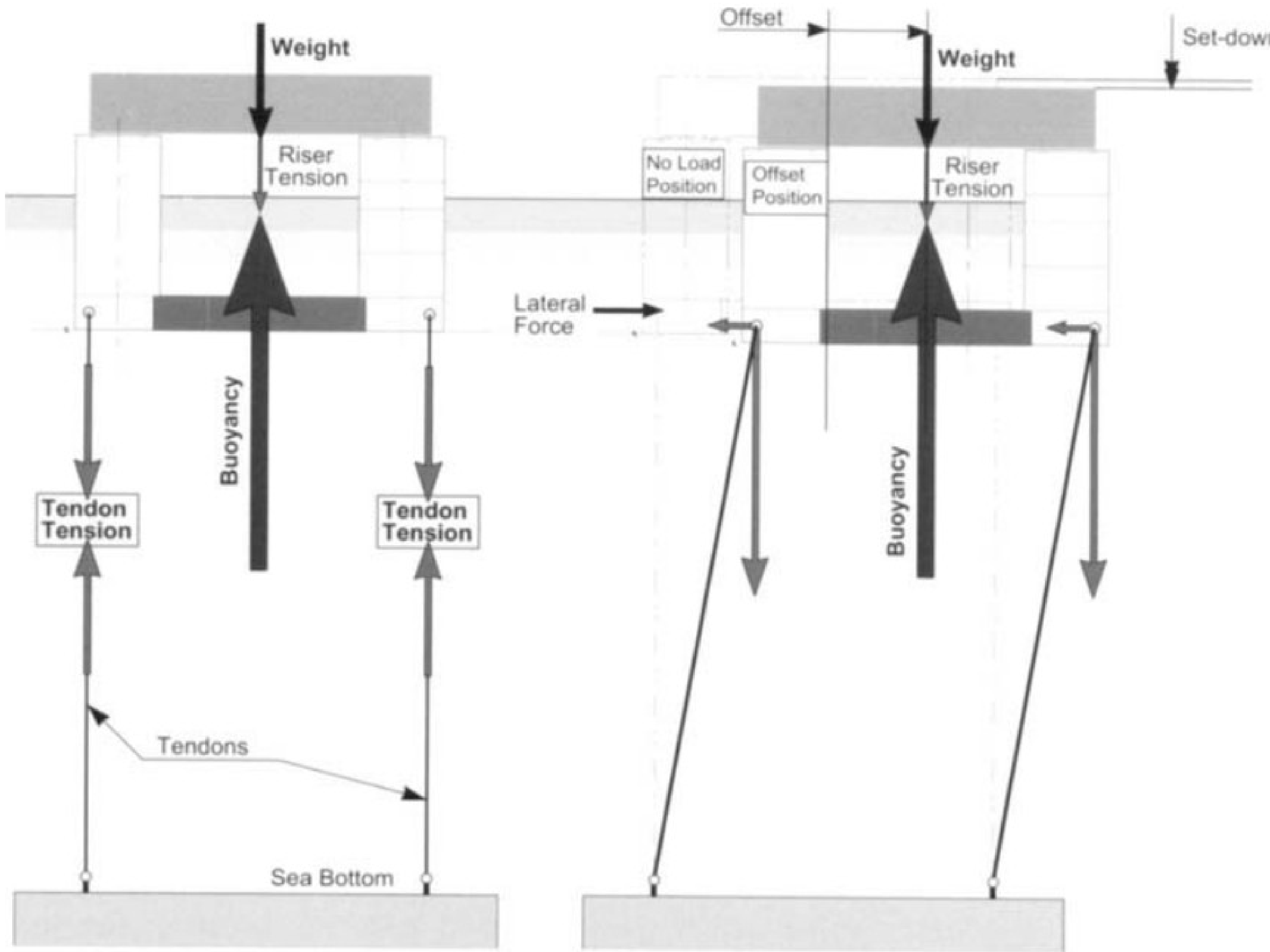
$$\sum_i F_{z,i}\left(x_i - x_O\right) - \sum_j F_{x,j}\left(z_j - z_O\right) = 0 \qquad (5^*)$$

where *O* is the pole, with respect to which the moment is calculated.

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Equilibrium of forces equations (continuation)

An example of the distribution of the forces acting on the platform shown in the following sketch [Subrata K. Chakrabarti: *Handbook Of Offshore Engineering*]:



Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Equilibrium of forces equations (continuation)

If we assume, that on the platform acts: hydrodynamic forces, aerodynamic forces, anchoring system respose forces (tendons tensions), raiser tension, weight and buoyancy, system of equations will be as follows:

$$F_{aero,x} + F_{hydro,x} + \sum_i T_{t,x,i} = 0 \qquad (3^*)$$

$$D - W + \sum_i T_{t,z,i} + T_{r,z} = 0 \qquad (4^*)$$

$$F_{areo,x}\left(z_{c,areo} - z_O\right) + F_{hydro,x}\left(z_{c,hydro} - z_O\right) - \sum_i T_{t,x,i}\left(z_{t,i} - z_O\right) - \sum_i T_{t,z,i}\left(x_{t,i} - x_O\right)$$
$$- T_{r,z}\left(x_r - x_O\right) - D\left(x_B - x_O\right) + W\left(x_G - x_O\right) = 0 \qquad (5^{**}),$$ where:

$F_{aero,x}$ - aerodynamic forces;

$F_{hydro,x}$ - hydrodynamic forces;

$T_{t,x,i}$; $T_{t,z,i}$ - tendons tensions;

$T_{r,z}$ - raiser tension;

D - buoyancy force; W - weight;

$x_{c,aero}$ - abscissa of aerodynamic centre;

$x_{c,hydro}$ - abscissa of hydrodynamic centre;

$x_{t,i}, z_{t,i}$ - coordinates of *i*-th tendon attachment point;

x_r - abscissa of raiser attachment point;

x_B, x_G - abscissa of ***B*** - and ***G*** - point;

x_O, z_O - coordinates of the pole point;

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Equilibrium of forces equations (continuation)

Further, we can assume, that the masses are distributed symmetrically to the center of buoyancy and, that the raiser is situated on the axis of symmetry:

$$x_B = x_G = x_r$$

The position of pole point can be chosen as follows:

$$x_O = x_B = x_G = x_r \quad \text{and:} \quad z_O = z_{t,i}$$

Then, momentum equilibrium equation takes the following form:

$$F_{areo,x}\left(z_{c,areo} - z_O\right) + F_{hydro,x}\left(z_{c,areo} - z_O\right) - \sum_i T_{t,z,i}\left(x_{t,i} - x_O\right) = 0 \quad (5^{***})$$

Note 1: The system of equations (3*), (4*) and (5***) can be used to determine the forces in the tendons of the TLP for the specified system of loads.

However, this requires the introduction of additional dependences

Note 2: You should very carefully check the signs (directions) of forces

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Determining of the restoring force due ro anchoring system

Horizontal component of tendons tension can be calculated as a function of lateral displacement and the vertical component of the tension:

$$\sum_i T_{t,x,i} = \tan\alpha \sum_i T_{t,z,i}$$, where:

$$\tan\alpha = \frac{x}{L_{t,z}} \quad ; \quad L_{t,z} = \sqrt{L_t^2 - x^2} \quad \text{or} \quad \alpha = \arcsin\frac{x}{L_t}$$

From the equilibrium equations of vertical forces, results:

$$\sum_i T_{t,z,i} = -D + W - T_{r,z}$$

Hence, the restoring force can be calculated by the formula:

$$\sum_i T_{t,x,i} = -\frac{x}{\sqrt{L_t^2 - x^2}}\left(D - W + T_{r,z}\right) \qquad (6)$$

In general, ***D*** depends on the movement *x*, which causes a change of platform draft *δz.*

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Determining of the restoring force due ro anchoring system

The buoyancy force is given by:

$$D = D_0 + \delta D \quad (7), \text{ where:}$$

D_0 - The buoyancy force (displacement) in the initial (undisturbed) state

δD - The buoyancy change as a result of changes of draft

The equation we can transform using the volume of the submerged part of the hull:

$$D = \rho g \left(\nabla_0 + \delta \nabla \right) \qquad (7^*)$$

Change of the volume of the submerged part of the hull (assuming a constant value of waterplane area A_W) can be written as follows:

$\delta \nabla = A_W \delta z$, where δz is a change of platform draft.

$\delta z = L_t - L_{t,z} = L_t - \sqrt{L_t^2 - x^2}$, or after conversion: (7**)

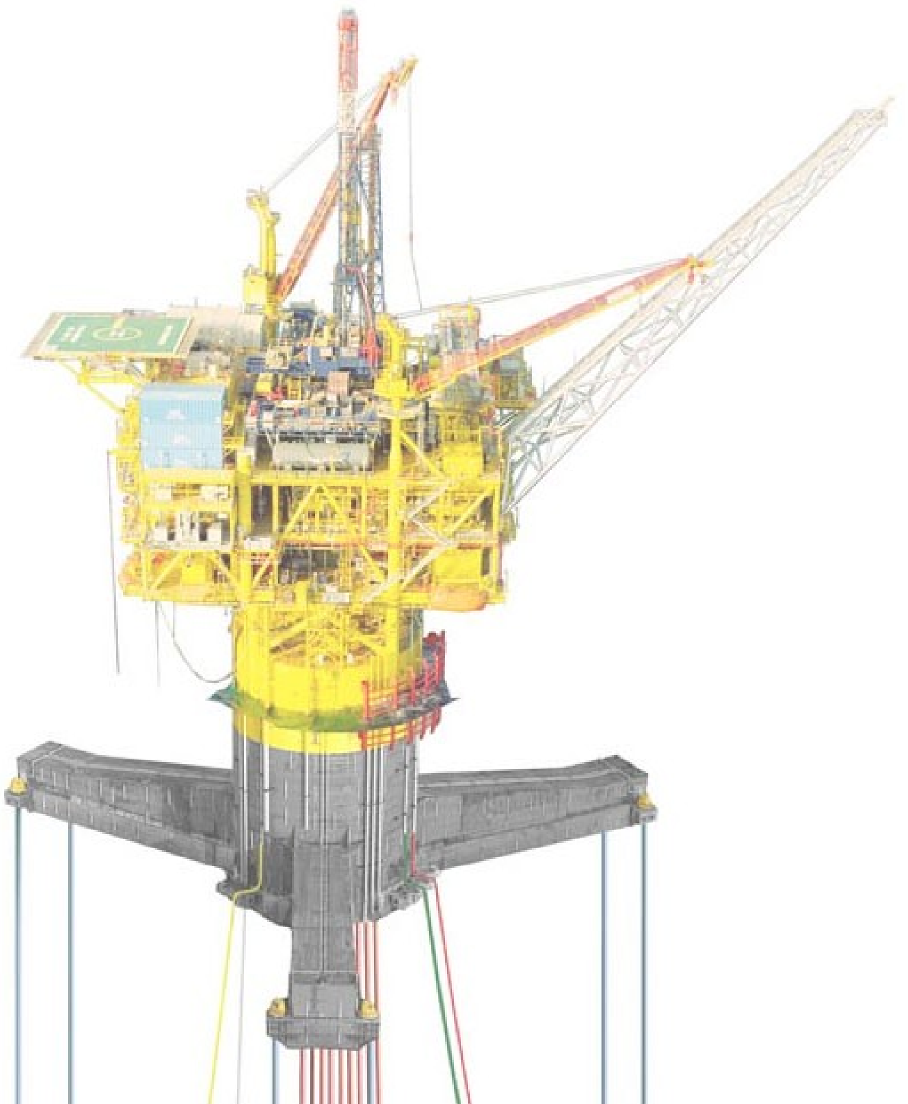
$$\delta z = L_t \left[1 - \sqrt{1 - \left(\frac{x}{L_t} \right)^2} \right]$$; hence: $$D = \rho g \left(\nabla_0 + A_W L_t \left[1 - \sqrt{1 - \left(\frac{x}{L_t} \right)^2} \right] \right)$$

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4:

Determine the static displacement of TLP platform, if the strength of the wind is:

u_w=48 m/s and the speed of the sea current u_c=2.37 m/s [DNV-OS-E301, October 2010]



Location Mississippi Canyon, Block 243
Water Depth: d=860 m

SeaStar® TLP Specifications:
Payload (deck/facilities/risers): 8 425 tons
Main column dimensions: D_c=17.8 m ; h_c=38.1 m
Pontoon dimensions: r_p=54.7 m; $h_{p,max}$=12.8 m
$l_p = r_p - 0.5D_c$=45.8 m
Draft: T=31.7 m
Deck Dimensions: B_d=42.7m x L_d=42.7m
(3 levels)

[http://www.zerohedge.com/article/possible-new-oil-spill-100-10-miles-reported-gulf-mexico]

Additional assumptions [by the teacher]:
Level Hight: h_l=8m;
Total Weight: W=0.75D_0; (including risers)
Pontoon width, mean height: w_p=6m; h_p=9.4 m;
Deck Freeboard: 17.5 m

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4: Solution

Calculation of the main parameters of the platform:

Displacement (buoyancy):

$D_0 = \rho g \nabla_0$, where: $\nabla_0 = \nabla_p + \nabla_{c0}$

Volume of pontoons (arms): $\nabla_p = n_p l_p w_p h_p$

Where: n_p - the number of pontoons

l_p, w_p, h_p - the length, width and height of a single pontoon

Volume of column (submerged part):

$$\nabla_{c0} = \frac{1}{4} \pi D_c^2 T_0$$

Length of tendons:

$$L_t = d - T_0$$

Stablity of Offshore Structures.

Stability of Tension Leg Platforms

Exercise 4: Solution (continuation)

In order to calculate the drag forces, it is necessary to determine the surface area of the submerged part of hull (projection onto the plane perpendicular to the direction Ox)

$$A_{p,hull} \approx A_{p,c} + A_{p,p}$$

Projection of a surface of the column is:

$$A_{p,c} = D_c T_0$$

Projection of a surface of pontoons is:

$A_{p,p} = 2 l_p' h_p$ where, for three pontoons:

$$A_{p,p} = 2\left[l_p \sin\left(60^o\right)\right] h_p$$

Averaged drag coefficient can be determined by a weighted average:

$$C_{D,hull} = \frac{C_{D,c} A_{p,c} + C_{D,p} A_{p,p}}{A_{p,hull}}$$

$C_{D,c}, C_{D,p}$ - drag coefficient of column and pontoons

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4: Solution (continuation)

In order to calculate the aerodynamic forces acting on the decks (and equipment), we also have determine the porojection of surface area of above water part:

$$A_{p,decks} = B_d h_d$$

h_d - height of equipment (including buildings) on all decks

$$h_d = n_{lev} h_{lev}$$

Based on the data, following geometric parameters and weights were calculated:

$$\nabla_c = 7888.4\, m^3$$
$$\nabla_p = 7749.4\, m^3$$
$$\nabla_0 = 15637.8\, m^3$$
$$D_0 = 157.2\, MN$$
$$W = 117.9\, MN$$
$$\rho = 1025\, kg/m^3$$

$$A_{p,c} = 564.3\, m^2$$
$$A_{p,p} = 745.7\, m^2$$
$$A_{p,hull} = 1310.0\, m^2$$
$$A_{p,decks} = 1024.8\, m^2$$
$$A_W = 248.8\, m^2$$
$$L_t = 828.3\, m$$

$$C_{D,c} = 1.17$$
$$C_{D,p} = 2.05$$
$$C_{D,hull} = 1.67$$
$$C_{D,decks} = 2.05$$

Stablity of Offshore Structures.

Stability of Tension Leg Platforms

Exercise 4: Solution (continuation)

In order to obtain displacement of the platform we have to solve a system of equations:

$$F_{aero,x} + F_{hydro,x} + \sum_i T_{t,x,i} = 0 \qquad (1)$$

(sum of the horizontal forces equals zero)

$$D - W + \sum_i T_{t,z,i} = 0 \qquad (2)$$

(sum of the vertical forces equals zero)

$$\sum_i T_{t,x,i} = \tan\alpha \sum_i T_{t,z,i} \qquad (3)$$

(Tendons tensions are parallel to the orientation of the tendons)

$$\tan\alpha = \frac{x}{L_{t,z}}$$, where: $$L_{t,z} = \sqrt{L_t^2 - x^2} \qquad (4)$$

From the equations (2), (3) and (4) after replacing the tan α and the vertical tensions, we obtain:

$$\sum_i T_{t,x,i} = -\frac{x}{\sqrt{L_t^2 - x^2}}\left(D - W\right) \qquad (3^*)$$

after substituting equation (3*) to (1) receive

$$F_{aero,x} + F_{hydro,x} - \frac{x}{\sqrt{L_t^2 - x^2}}\left(D - W\right) = 0 \qquad (1^*)$$

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4: Solution (continuation)

Hydro- and aerodynamic forces are calculated from formulas:

$$F_{hydro,x} = C_{D,hull} \frac{1}{2} \rho A_{p,hull} |u_{c,x}| (u_{c,x}) \qquad F_{hydro,x} = 6.298\, MN$$

$$F_{aero,x} = C_{D,decks} \frac{1}{2} \rho_{air} A_{p,decks} |u_{w,x}| (u_{w,x}) \qquad F_{aero,x} = 2.904\, MN$$

As a first approximation we can assume that: $\sqrt{L_t^2 - x^2} \approx L_t$ and $D \approx D_0$,
then we get a regular linear equation.

More accurate solution can be obtain considering the effect of additional buoyancy:

$$D = \rho g (\nabla_0 + \delta \nabla) \quad ; \quad \delta \nabla = A_W \delta z \quad ; \quad \delta z = L_t - \sqrt{L_t^2 - x^2} \quad ;$$

hence:

$$D = \rho g \left(\nabla_0 + A_W L_t \left[1 - \sqrt{1 - \left(\frac{x}{L_t} \right)^2} \right] \right)$$

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4: Solution (continuation)

Use a spreadsheet to calculate the displacement *x* and α angle.

Results:

$$F_{aero,x} = 2.904\,MN \qquad x = 122\,m$$

$$F_{hydro,x} = 6.298\,MN \qquad \alpha = 8{,}46^{o}$$

Note 1: The data in the task are approximate and are defined on the basis of (incomplete) data from Internet.
Note 2: In general, aerodynamic and hydrodynamic force also depends on the δz.
Note 3: Sea current speed u_c is generally a function of immersion *z*. Wind speed u_w is a function of the height *z* and time *t*

Stablity of Offshore Structures.

Stability of Tension Leg Platforms

Exercise 4.1: Additional task

Determine the tension of each of the tendons.

$$T_{t,1}=?;T_{t,2}=?;T_{t,3}=?$$

Solution: In order to determine the forces in tendons we need to introduce additional equation - the balance equation of the moments: sum of the moments in the plane *Oxz* is zero:

$$\sum_i M_{y,i}=0$$, hence:

$$F_{areo,x}\left(z_{c,areo}-z_O\right)+F_{hydro,x}\left(z_{c,hydro}-z_O\right)-\sum_i T_{t,x,i}\left(z_{t,i}-z_O\right)-\sum_i T_{t,z,i}\left(x_{t,i}-x_O\right)$$
$$-D\left(x_B-x_O\right)+W\left(x_G-x_O\right)=0$$

We assume, that the masses are distributed symmetrically: $x_B=x_G$

The position of pole point *O* we chose as: $x_O=x_B=x_G$ and: $z_O=z_{t,i}$, then:

$$F_{areo,x}\left(z_{c,areo}-z_O\right)+F_{hydro,x}\left(z_{c,hydro}-z_O\right)-\sum_i T_{t,z,i}\left(x_{t,i}-x_O\right)=0 \qquad (5)$$

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4.1: Additional task (solution)

Due to the symmetry:

$$T_{t,z,1} = T_{t,z,2} \quad (6)$$

Considering the equation (6) and geometrical relationships:

$$x_{t,1} = x_{t,2} = -r_p \cos\left(60^o\right) = -\frac{1}{2} r_p \quad ; \quad x_{t,3} = r_p \quad ; \quad x_{t,3} = r_p \quad ; \quad x_O = 0 \quad ;$$

we obtain:

$$\sum_i T_{t,z,i}\left(x_{t,i} - x_O\right) = 2\,T_{t,z,1}\left(-\frac{1}{2} r_p - 0\right) + T_{t,z,3}\left(r_p - 0\right) = -T_{t,z,1}\, r_p + T_{t,z,3}\, r_p$$

Then, the equation (5) can be written as follows:

$$F_{areo,x}\left(z_{c,areo} - z_O\right) + F_{hydro,x}\left(z_{c,hydro} - z_O\right) + T_{t,z,1}\, r_p - T_{t,z,3}\, r_p = 0 \quad (5^*)$$

Equation (5) must be supplemented by the equation (2) from the previous part of the task:

$$D - W + 2\,T_{t,z,1} + T_{t,z,3} = 0 \qquad (2^*)$$

We have to solve a system of equations (5 *) and (2 *). As a solution we obtain $T_{t,z,1}$ and $T_{t,z,3}$. The value of $T_{t,z,2}$ get from equation (6).

Use the α angle trigonometric functions to calculate $T_{t,x,1}$; $T_{t,x,2}$ and $T_{t,x,3}$

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4.1: Additional task (solution)

As a center of aerodynamic forces we can take the centre of $A_{p,decks}$ surface.

$$z_{c,areo} = T_0 + f_{B,d} + \frac{1}{2} h_d$$

Center of hydrodynamic forces can be taken as an weighted average of centres of submerged projected surfces $A_{p,c}$ and $A_{p,p}$, taking into account the different values of drag coefficient of each part of the hull.

$$z_{c,hydro} = \frac{z_{c,c} C_{D,c} A_{p,c} + z_{c,p} C_{D,p} A_{p,p}}{C_{D,c} A_{p,c} + C_{D,p} A_{p,p}}$$

, where: $z_{c,c} = \frac{1}{2} T_0$; $z_{c,p} = \frac{1}{2} h_p$

$$z_{c,areo} = xxx\ m$$

$$z_{c,hydro} = xxx\ m$$

Stablity of Offshore Structures.
Stability of Tension Leg Platforms

Exercise 4.1: Additional task (solution)

Equation (2*) can be transformed as follows:

$$T_{t,z,3} = -D + W - 2T_{t,z,1} \qquad (2^{**})$$

After substituting the expression for $T_{t,z,3}$ into the formula (5*), we obtain:

$$F_{areo,x}(z_{c,areo} - z_O) + F_{hydro,x}(z_{c,hydro} - z_O) + T_{t,z,1} r_p - (-D + W - 2T_{t,z,1}) r_p = 0 \qquad (5^{**})$$

After ordering the equation:

$$T_{t,z,1} = \frac{-F_{areo,x}(z_{c,areo} - z_O) - F_{hydro,x}(z_{c,hydro} - z_O) - (D - W) r_p}{3 r_p} \qquad (5^{***})$$

After substituting the calculated value of $T_{t,z,1}$ into (2**) we obtain $T_{t,z,3}$, and then from equation (6) we get $T_{t,z,2}$. The tension inside the tendons can be calculated using trigonometric functions:

$$T_{t,i} = -\frac{T_{t,z,i}}{\cos\alpha}$$, hence: $$T_{t,1} = T_{t,2} = 22.13\, MN; \qquad T_{t,3} = 18.29\, MN$$

Stablity of Offshore Structures.

Stability of Tension Leg Platforms

Exercise 4.1: Additional task (solution)

Conclusions:

Due to established convention of signs, positive values of $T_{t,i}$ mean stretching the tendons, while negative values indicate compression of tendons, which is unacceptable because the platform starts to rotate.

Maximum values of forces (designated for extreme load) will be used for dimensioning of tendons (determination of cross-sectional area, material selection, etc.)

Dynamics of the environment.
Simple models of wind and sea currents

<u>Wind</u> is the flow of air on a large scale.

Wind is caused by differences in the atmospheric pressure. When a difference in atmospheric pressure exists, air moves from the higher to the lower pressure area, resulting in winds of various speeds. On a rotating planet, air will also be deflected by the Coriolis effect, except exactly on the equator. Globally, the two major driving factors of large-scale wind patterns (the atmospheric circulation) are the differential heating between the equator and the poles (difference in absorption of solar energy leading to buoyancy forces) and the rotation of the planet. [*http://en.wikipedia.org/wiki/Wind*]

The results of measurements (or numerical simulations) of wind speed $\overline{u}$ are given for a specified height *h* above the surface of the sea.

Most often, this height is *h*=**10 m** (or 30ft).

Once $\overline{u}(h)$ is established for a particular offshore site, the mean horizontal wind velocity at height *z* above the sea surface is [Wilson F.: *Dynamics of Offshore Structures*]:

$$\overline{u}(z)=\left(\frac{z}{h}\right)^{1/n}\overline{u}(h)$$

The exponent *n* depends on many factors. For instance, *n*=3 fits data for rough coastal aeras; <u>*n*=7 to 8</u> for sustained winds over an unobstructed sea; and n=12 to 13 for gusts.

More wind models and coefficients can be found at [*DNV-RP-C205*] and other recommendations.

Dynamics of the environment.

Simple models of wind and sea currents

An ***ocean current*** is a continuous, directed movement of seawater generated by forces acting upon this mean flow, such as breaking waves, wind, the Coriolis effect, cabbeling, and temperature and salinity differences, while tides are caused by the gravitational pull of the Sun and Moon. Depth contours, shoreline configurations, and interactions with other currents influence a current's direction and strength. [*http://en.wikipedia.org/wiki/Ocean_current*]

Two types of ocean currents has a significant impact on the dynamics of offshore structures: ***tidal currents*** and ***wind-stress currents***.

Tidal currents often attain a maximum velocity of 1 to 2 knots and may even reach 10 knots in some locations. Data for the tidal current $u_t(z)$ as a function of water depth z generally follows a *power law* (similar to that for wind) [Wilson F.: *Dynamics of Offshore Structures*]:

$$u_t(z) = \left(1 + \frac{z}{d}\right)^{1/7} u_t(0)$$

, where: d – (total) water depth; $u_t(0)$ – is the tidal velocity at sea surface z=0.

The velocity profile of ***wind-stress current*** is approximated as linear with depth, with a maximum value $u_w(0)$ at sea surface, where $u_w(-d)$=0 at seafloor:

$$u_w(z) = \left(1 + \frac{z}{d}\right) u_w(0)$$

, the magnitude of u_w(z) is generally about 1 to 5 percent of the sustained wind speed.

More about sea currents can be found at [*DNV-RP-C205*] and other recommendations.

Dynamics of the environment.

Ocean waves – a short-term model

***Propagation of ocean wave*s**. The oustanding visible characteristic of waves in the open ocean is their irregularity. Study of wave records confirms this irregularity of the sea, both in time and space. However, one is equally impresed by the fact that over a fairly wide area and often for a period of a half-hour or more the sea may maintain a characteristic appearance, because record analyses indicate it is very nearly steady or stationary. Hence, for most problems of behaviour of ships and floating structures at sea, attention can be focused on describing mathematically the surface waves as random, or stochastic, process under short-term statistically stationary conditions.

The wave system in the neighborhood of a particular place and time is assumend to be the sum of many regular waves, each progressing in its own direction and speed.

For simplicity, we assume, however, that the direction of propagation is the same for each of the waves. The total wave system is then assumed to be a sumation of many (theoretically an infinite number) of independent components [*Principles...* vol. 3]:

$$\eta(x,t)=\sum_i a_i \cos\left(k_i x-\omega_i t+\epsilon_i\right)$$

Where, a_i is a component amplitude corresponding to wave frequency ω_i , wave number k_i and ε_i random phase angle x – location.

Dynamics of the environment.
Ocean waves – a short-term model

Wave components are defined in terms of function known as *variance spectrum* $S(\omega)$.

At any particular wave frequency, ω_i , the variance of all the wave components within a small finitefrequency band, $\delta\omega$, centered upon ω_i is given by:

$\langle \eta_i(t)^2 \rangle \equiv S(\omega_i)\,\delta\omega$, where: $\eta_i(t) = a_i \cos(k_i x - \omega_i t + \epsilon_i)$, for given *x.*

In general, the variance of a continuous function with zero mean is geven by:

$$\langle \eta(t)^2 \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \eta^2(t)\,dt$$

It can be proved that for a simple regular wave [*Principles of naval architecture* , vol.3]:

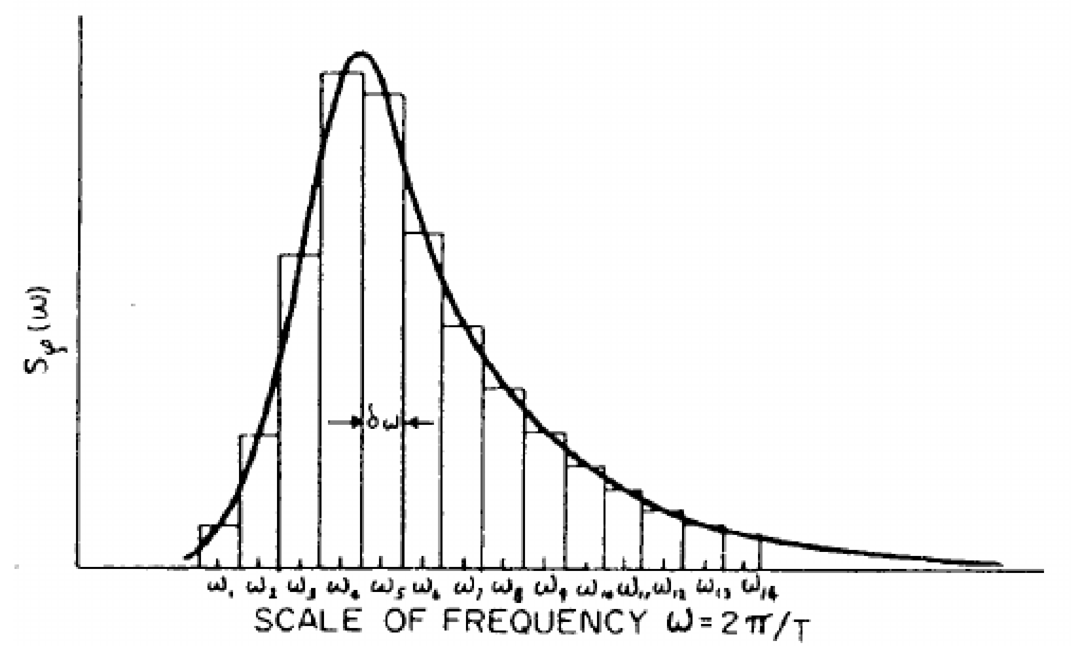
$\langle \eta_i(t)^2 \rangle = \frac{1}{2} a_i^2$, hence: $\frac{1}{2} a_i^2 = S(\omega_i)\,\delta\omega$,

then the component amplitude is:

$$a_i = \sqrt{2\,S(\omega_i)\,\delta\omega}$$

More important than the component amplitudes is the total variance of the wave system *E*, which is a good measure of severity of the sea:

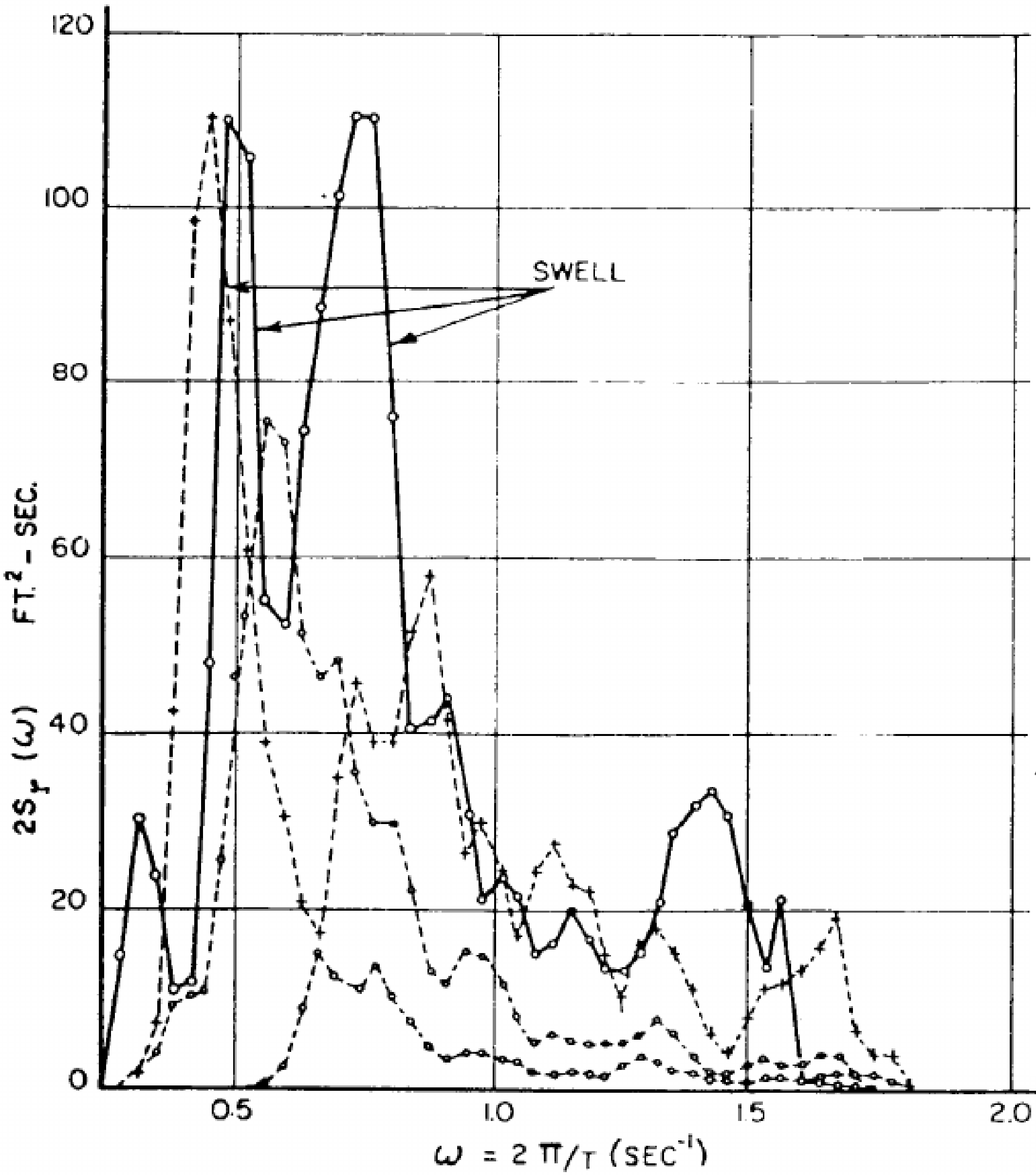
$$E \equiv \langle \eta(t)^2 \rangle = \sum_i \langle \eta_i(t)^2 \rangle = \int_0^{\infty} S(\omega)\,d\omega$$

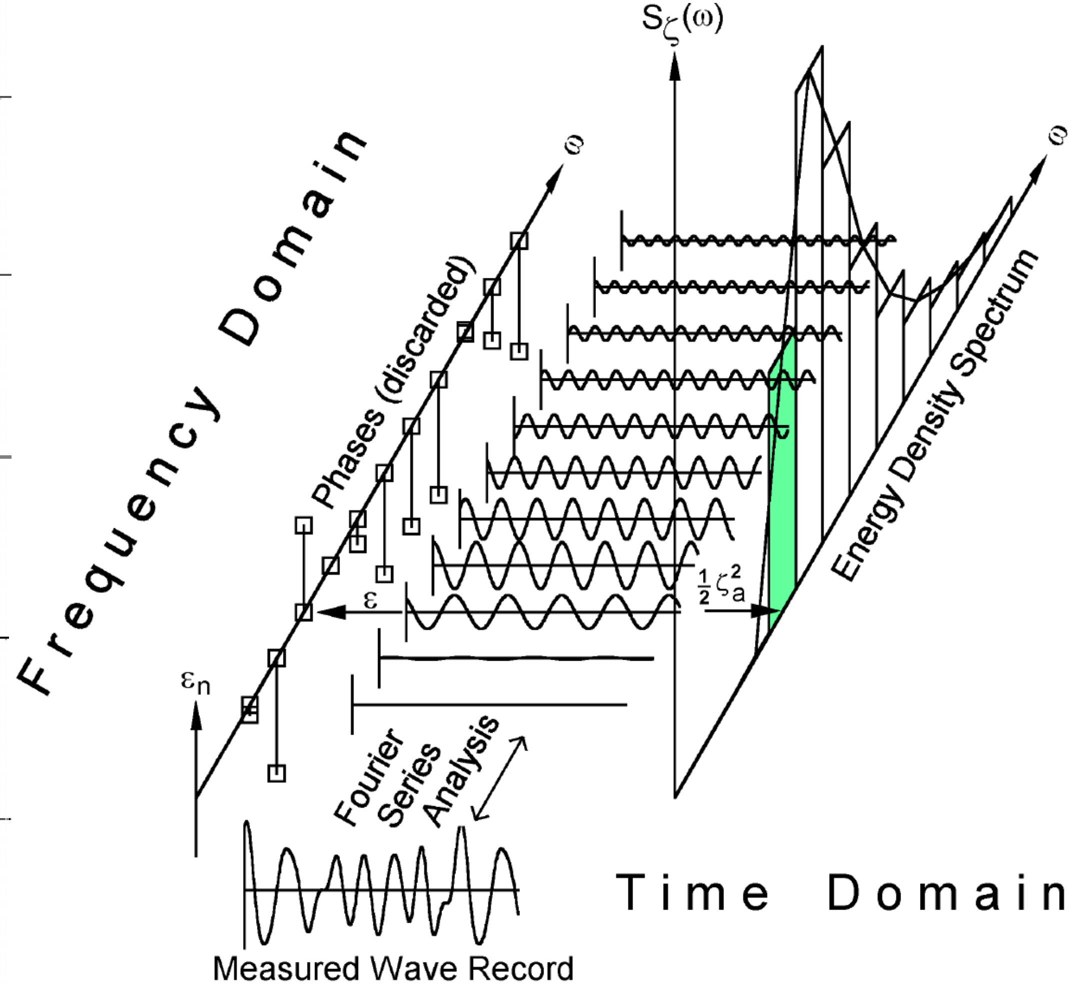


Dynamics of the environment.

Ocean waves – a short-term model

Examples of measured typical sea spectra are presented below (on the left)[*Principles of naval architecture* , v.3]., while the scheme of wave record analysis is at right [Journee J.M.J: *Offshore Hydromechanics*].





Dynamics of the environment.

Ocean waves – a short-term model

As the total variance of the wave system *E* is known, the statistical measures of the wave height depending on the frequency of appearance can be determined:

Averaged appared wave height, crest to trough $<2\overline{\eta}>$ is:

$\langle H_w \rangle = 2.5\,E^{1/2}$, or average amplitude $\langle \overline{\eta} \rangle = 1.25\,E^{1/2}$

Significant wave height, the average of the 1/3 highest waves: $\langle H_{w,1/3} \rangle = 4.0\,E^{1/2}$

And, the average of the 1/10 highest waves: $\langle H_{w,1/10} \rangle = 5.1\,E^{1/2}$

The statistical theory of extreme values used in conjuction with the narrow-band assumption results in the estimates for the *expected highest* in a sample of *N* succesive wave heights:

$$N = 100; \quad \langle H_{w,1/100} \rangle = 6.5\,E^{1/2}$$
$$N = 1000; \quad \langle H_{w,1/1000} \rangle = 7.7\,E^{1/2}$$
$$N = 10\,000; \quad \langle H_{w,1/10000} \rangle = 8.9\,E^{1/2}$$

However, if a large number of samples of the stated size are taken, 5 percent of them would be expected to have maximum heights as follow:

$$N = 100; \quad \langle H_{w,1/100} \rangle = 7.8\,E^{1/2}$$
$$N = 1000; \quad \langle H_{w,1/1000} \rangle = 8.9\,E^{1/2}$$

Dynamics of the environment.

Ocean waves – a short-term model

Approximation of wave spectra.

For design of ships and floating structures, the most often two types of idealized spectral functions are used:

- The Pierson-Mostkowitz Spectrum

- The JONSWAP Spectrum

(from DNV-rp-c205)

Dynamics of the environment.

Ocean waves – a short-term model

Exercise 5

Draw a graph of the spectrum of waves for given data:

- The Pierson-Mostkowitz Spectrum

- The JONSWAP Spectrum

Draw a graph of an exemplary waveform for a given point x and a specified range of time. Calculate the following statistical values: Hw1/3 ; Hw1/10 ; Hw1/100 ; Hw1/1000

Hs=9.01 m

Tp=11.3 s

del_t = 600 s

Response Amplitude Operator

Response Amplitude Operator (RAO) is an engineering statistic, or set of such statistics, that are used to determine the likely behavior of a ship (or floating structure) when operating at sea. Known by the acronym of RAO, response amplitude operators are usually obtained from models of proposed ship designs tested in a model basin, or from running specialized CFD computer programs, often both. [*http://en.wikipedia.org/wiki/Response_amplitude_operator*]

RAOs are usually calculated for all ship (floating structure) motions and for all wave headings.

RAOs are effectively transfer functions used to determine the effect that a sea state will have upon the motion of a ship (or floating offshore structure.)

Generation of extensive RAOs at the design phase allows to determine the modifications to a design that may be required for safety reasons (i.e., to make the design robust and resistant to capsizing or sinking in highly adverse sea conditions) or to improve performance (e.g., improve top speed, fuel consumption, stability in rough seas).

Together, the RAOs and hydrodynamic database provide certain assurances about the behavior of a proposed ship design. They also allow the designer to dimension the ship or structure so it will hold up to the most extreme sea states it will likely be subjected to (<u>based on sea state statistics</u>).

Response Amplitude Operator

The RAO transfer function is only defined when the ship (or floating structure) motions can be assumed to be linear. The above forces can then be assembled into an equation of motion:

$$\left[M+A(\omega)\right]\ddot{x}+B(\omega)\dot{x}+C\,x=F(\omega)$$

Where *x* is a degree of freedom (coordinate of rigid body motions), ω is the oscillation frequency, *M* is the structural mass and inertia *A*(ω) is the added mass (frequency dependent), *B*(ω) is the linear damping (frequency dependent), *C* is the restoring force coefficient and *F*(ω) is the harmonic excitation force proportional to *x* and the wave amplitude η_a.

The Response Amplitude Operator is also referred to as response amplitude characteristics. The response amplitude characteristics for *x* degree of freedom is defined as [Journee *OFFSHORE HYDROMECHANICS*]:

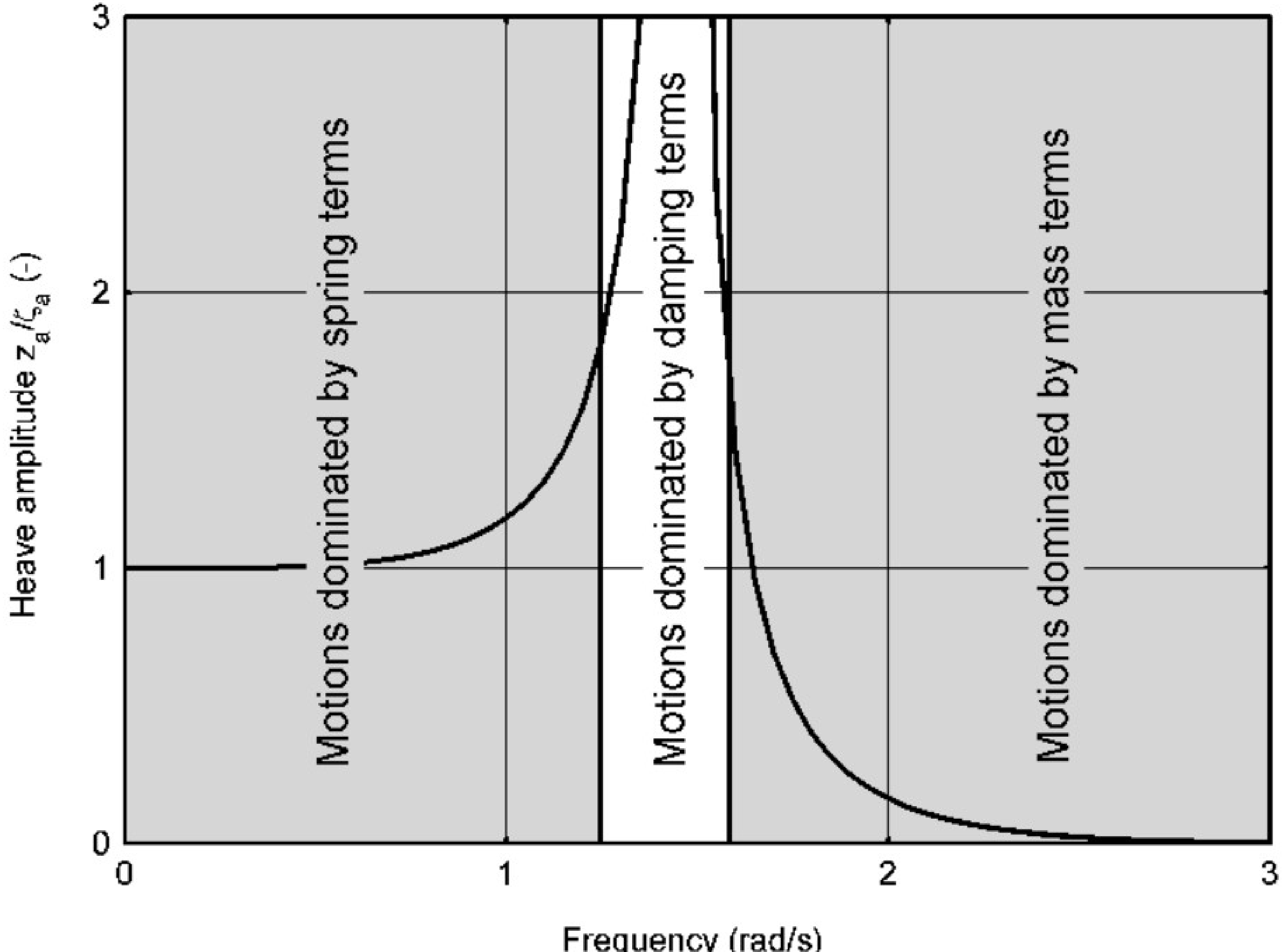
$$RAO(\omega)=Y_{x,\eta}(\omega)=\frac{x_a}{\eta_a}$$

where: x_a - amplitude of movements on the *x* coordinate

η_a - wave ampltude

Response Amplitude Operator

Typical graph of the heave ampltude characteristics z_a/η_a for vertical circular cylinder is shown below [Journee *OFFSHORE HYDROMECHANICS*]:



The graph shows the three frequency areas with respect to motional behavior

Response Amplitude Operator

The figure (on the previuos slide) shows that with respect to the motional behavior of this cylinder three frequency areas can be distinguished:

1. The low frequency area, $\omega^2 << C/(M + A)$, with vertical motions dominated by the restoring spring term.

This yields that the cylinder tends to ”follow” the waves as the frequency decreases; the *RAO* tends to 1.0 and the phase lag tends to zero. At very low frequencies, the wave length is large when compared with the horizontal length (diameter) of the cylinder and it will ”follow” the waves.

2. The natural frequency area, $\omega^2 = C/(M + A)$, with vertical motions dominated by the damping term.

This yields that a high resonance can be expected in case of a small damping. A phase shift of $-\pi$ occurs at about the natural frequency, $\omega^2 = C/(M + A)$. This phase shift is very abrupt here, because of the small damping B of this cylinder.

3. The high frequency area, $\omega^2 >> C/(M + A)$, with vertical motions dominated by the mass term.

This yields that the waves are ”losing” their influence on the behavior of the cylinder; there are several crests and troughs within the horizontal length (diameter) of the cylinder.

Response in Irregular Waves

The wave energy spectrum was defined in the previous lecture as:

$$S_\eta(\omega)\,d\omega = \frac{1}{2}\eta_a^2(\omega)$$

Analogous to this, the energy spectrum of the heave response $z(\omega, t)$ can be defined by:

$$S_z(\omega)\,d\omega = \frac{1}{2}z_a^2(\omega) = \left|\frac{z_a}{\eta_a}(\omega)\right|^2 \frac{1}{2}\eta_a^2(\omega) = \left|\frac{z_a}{\eta_a}(\omega)\right|^2 S_\eta(\omega)\,d\omega$$

Thus, the heave response spectrum of a motion can be found by using the transfer function of the motion and the wave spectrum by:

$$S_z(\omega) = \left|\frac{z_a}{\eta_a}(\omega)\right|^2 S_\eta(\omega)$$

The moments of the heave response spectrum are given by:

$$m_{n,z} = \int_0^\infty \omega^n S_z(\omega)\,d\omega$$, with: $n = 0; 1; 2; ...$

where n = 0 provides the area, n = 1 the first moment and n = 2 the moment of inertia of the spectral curve.

The signicant heave amplitude can be calculated from the spectral density function of the heave motions, just as was done for waves. This significant heave amplitude, defined as the mean value of the highest one-third part of the amplitudes, is:

$$\bar{z}_{a,1/3} = 2\sqrt{m_{0z}}$$