Definitions

Valid argument An argument is valid if its conclusion is true whenever all its premises are true.

Sound argument A sound argument is a valid argument whose premises are all true.

Tautology A tautology is a sentence which is always true.

Sentence A sentence is a combination of atomic sentence symbols (A, B, . . .) names, variables and quantifiers generated according to the rules of formation.

Consistent sentence A consistent sentence is a sentence which is true in at least one interpretation.

Rules of Inference

$\dfrac{\begin{array}{c}\bigcirc \\ \neg\bigcirc\end{array}}{\times}$	$\dfrac{\checkmark(\bigcirc \to \triangle)}{\neg\bigcirc \quad \triangle}$	$\dfrac{\checkmark(\bigcirc \wedge \triangle)}{\begin{array}{c}\bigcirc \\ \triangle\end{array}}$	$\dfrac{\checkmark(\bigcirc \vee \triangle)}{\bigcirc \quad \triangle}$	$\dfrac{\checkmark(\bigcirc \leftrightarrow \triangle)}{\begin{array}{cc}\bigcirc & \neg\bigcirc \\ \triangle & \neg\triangle\end{array}}$
$\dfrac{\checkmark\neg\neg\bigcirc}{\bigcirc}$	$\dfrac{\checkmark\neg(\bigcirc \to \triangle)}{\begin{array}{c}\bigcirc \\ \neg\triangle\end{array}}$	$\dfrac{\checkmark\neg(\bigcirc \wedge \triangle)}{\neg\bigcirc \quad \neg\triangle}$	$\dfrac{\checkmark\neg(\bigcirc \vee \triangle)}{\begin{array}{c}\neg\bigcirc \\ \neg\triangle\end{array}}$	$\dfrac{\checkmark\neg(\bigcirc \leftrightarrow \triangle)}{\begin{array}{cc}\neg\bigcirc & \bigcirc \\ \triangle & \neg\triangle\end{array}}$

Universal instantiation (UI)	$\dfrac{\forall x \ldots x \ldots}{\ldots \text{any name} \ldots}$

Existential instantiation (EI)	$\dfrac{\exists x \ldots x \ldots}{\ldots \text{new name} \ldots}$

Rules for ¬	$\dfrac{\checkmark\neg\forall x \bigcirc}{\exists x \neg\bigcirc}$	$\dfrac{\checkmark\neg\exists x \bigcirc}{\forall x \neg\bigcirc}$	$\dfrac{\checkmark\neg\neg\bigcirc}{\bigcirc}$	$\dfrac{\begin{array}{c}\bigcirc \\ \neg\bigcirc\end{array}}{\times}$

Rules of Formation

0. P_a, R_{cd}, A, or B where $a, b, c, d...$ are names and $P, R, A, B, ...$ are predicates.
1. If $\square$ is a sentence, so is $\neg\square$.
2. If $\square$ and $\triangle$ are sentences, so are $(\square \wedge \triangle)$, $(\square \vee \triangle)$, $(\square \leftrightarrow \triangle)$, $(\square \to \triangle)$
3. If $...n...$ is a sentence, so are $\exists x...x...$ and $\forall x...x...$

Truth Tables

A	B	$A \wedge B$	$A \vee B$	$A \to C$	$B \leftrightarrow C$
t	t	t	t	t	t
t	f	f	t	f	f
f	t	f	t	t	f
f	f	f	f	t	t

$A \to B$ is logically equivalent to $\neg A \vee B$
$A \leftrightarrow B$ is logically equivalent to $(A \to B) \wedge (B \to A)$

Validity and Consistency

Using Truth Tables

Argument Validity: An argument is valid if and only if the premises and the negation of the conclusion are inconsistent.

Using Refutation Trees

Argument Validity: If the refutation tree is inconsistent, the argument is valid.
Consistency: A set of sentences is consistent if the sentences can all be true at the same time. The set is inconsistent if the sentences are never all true at the same time. When all paths of a refutation tree close, the sentences are inconsistent. If there are paths open, they are consistent.
Tautology: Create a refutation tree for an argument (negate it), if the tree closes, it is a tautology.
Sentence Equivalence: For sentences A and B, you can check if they are consistent by forming a refutation tree for $\neg(A \leftrightarrow B)$ and see if all paths close.

Someone sings	$\exists x Sx$
All singers paint	$\forall x(Sx \to Px)$
Someone paints	$\exists x\, Px$

1. $\surd_2$ $\exists x Sx$ Premise
2. $\forall x(Sx \to Px)$ Premise
3. $\surd_1$ $\neg\exists x Px$ $\neg$ Conclusion
4. $\forall x \neg Px$ 3.$\neg\exists$
5. Sa 1.$\exists$
6. $Sa \to Pa$ 2.$\forall$
7. $\neg Sa$ 6.$\to$ $\times$ | Pa 6.$\to$
8. $\neg Pa$ 4.$\forall$ $\times$