Code Appendix for: The Value of Formalism: Re-examining External Costs and Decision Costs with Multiple Groups

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Abstract

Several authors have examined the optimal k-majority rule using a variety of criteria. We formalize and extend the original argument laid out by Buchanan and Tullock (1962) using a decision theoretic analysis from the perspective of an individual voter. Unlike previous formalizations, voters in our study are members of one or more groups. This allows us to examine cases wherein different voters have starkly different interests. Furthermore, voters in our study can err in their judgments of proposals allowing us to model potential irrationalities in the choice of an optimal k-majority rule. We consider both up or down votes on a single proposal as well as votes over a series of proposals. We find that the optimal k-majority rule depends on a number of parameters, most notably the number of rounds needed to create a proposal that will pass. Group membership has almost no affect. Furthermore, if two groups are at odds, then the external cost function can actually rise over some range of k; if voters err systematically in their judgment, more inclusive k-majority rules, such as unanimity rule, can fail to pass Pareto preferred proposals. Our results should help advance a classic work in Public Choice.

Code Appendix

This appendix provides pseudo-code for the R programs used to run the simulations detailed in the paper.[1] The basic building block of a simulation is an *iteration*.[2] Each *iteration* begins with the same set of initial parameters that are used for J iterations.[3] The costs reported are means averaged across the J iterations.

Each iteration proceeds in the following order:

1. The per round decision costs, c, is assigned (sometimes a constant).

2. Let g label a given group. The group level parameters are assigned, including the size g, the mean and standard deviation of the utility distribution ($N(\mu_g, \sigma_g)$), the mean and standard deviation of the error distribution ($N(m_g, s_g)$), and the change in each group's mean utility ($\alpha_{g,r}$).

3. A *series of proposals*, indexed by r, is run according to a "successive procedure." For each round r:

   (a) Voter utilities and errors are generated based on their group level parameters:

   - $u_{i,r}$ for each $i \in g$, representing utility from passage of the proposal in round r;
   - $e_{i,r}$ for each $i \in g$, representing error in judgement in round r.

   (b) Perceived utility, $u_{i,r} + e_{i,r}$, is calculated and stored for each voter.

[1] The code needed to replicate all of the findings in the paper can be found at: `https://github.com/robiRagan`.

[2] The vocabulary used in the psuedo-code includes the following. A **Round of Voting** refers to a case where the deliberative body votes up or down on a proposal. A **Series of Proposals** refers to the same deliberative body going through multiple *rounds of voting*, considering proposals one at a time, using a successive procedure. An **Iteration** is one pass through a entire *series of proposals*.

[3] We used 1,000 iterations in the paper.

(c) Voting occurs based on each voter's perceived utility:

$$p_{i,r} = \begin{cases} 0 & \text{if } u_{i,r} + e_{i,r} \leq 0 \\ 1 & u_{i,r} + e_{i,r} > 0. \end{cases}$$

(d) The number of *yea* votes are tallied: $yeas_r = \sum_i p_{i,r}$.

(e) For each k-majority rule in round r, whether or not the proposal passes is calculated and stored:

$$passes_{k,r} = \begin{cases} 1 & \text{if } yeas_r \geq k \\ 0 & \text{if otherwise.} \end{cases} \quad \text{for } k = \{1, ..., N\}$$

(f) The potential external cost, $E_{i,r}$, is calculated for each voter:[4]

$$E_{i,r} = \begin{cases} 0 & \text{if } u_{i,r} > 0 \\ |u_{i,r}| & \text{if } u_{i,r} \leq 0. \end{cases}$$

(g) If all k-majority rules have passed a proposal, or the maximum allowed number of rounds have been reached, then no more proposals are considered and the series of proposals ends. If not:

i. $\alpha_{g,r}$ is added to $\mu_{g,r}$ for all sub-groups.

ii. The program advances to the next round starting at step 3a.

4. When the series of proposals ends for all k-majority rules, the following values are calculated.

[4] These costs are "potential" external cost, because an external cost is only incurred if a proposal passes. Although we keep track of potential external costs each round, we only report external costs for a k-majority rule in the round that a proposal passes (or the program stops due to a pre-specified number of rounds).

- R_k, which is the round each k-majority passed a proposal.[5]
- The decision costs for each k-majority rule:

$$D_k \quad = \quad c \cdot R_k.$$

- The external costs for each k-majority rule:[6]

$$\begin{aligned} E_{typical,k} &= mean_i(E_{i,R_k}) \\ E_{worst,k} &= max_g(mean_i(E_{i,R_k,g})) \\ E_{best,k} &= min_g(mean_i(E_{i,R_k,g})). \end{aligned}$$

- The Total costs for each k-majority rule:

$$\begin{aligned} T_{typical,k} &= E_{typical,k} + D_k \\ T_{worst,k} &= E_{worst,k} + D_k \\ T_{best,k} &= E_{best,k} + D_k. \end{aligned}$$

5. This ends one *iteration* of the model. We report the decision costs, external costs, and total costs averaged across the J iterations.

[5] This is the lowest r, where $passes_{k,r} = 1$, for a given k-majority rule. If no proposal passed in any round, then the final round is used.

[6] Technically, we keep track of the $E_{typical,k}$, $E_{worst,k}$, and $E_{best,k}$ each round and update their values based on whether a proposal passed that round. We keep this process going for all k-majority rules across all rounds, then reference the results from round R_k for each k separately. This made writing the code easier.