CHAPTER SIX

Endogenous Parties, Interaction of Different Decision Rules

We have so far made the preposterous assumption that political parties come to us as gifts from God or Nature. We did this to get started in our investigations of party competition, but we all know this is not true. We know that political parties are endogenous *outputs* of the process of party competition, not exogenous inputs to it. Any halfway realistic model of multiparty competition must treat the set of competing political parties as endogenous. This has of course not escaped the attention of scholars modeling party competition in the classical analytical tradition, but it has proved a hard nut for them to crack. As a consequence, the inevitable trade-off between rigor and realism has been pushed even further away from realism. “Party entry” and “citizen candidates” have been analyzed in extraordinarily sparse settings with one policy dimension, minuscule electorates, and at most two extant parties. One of the core aims of this chapter is to build on intuitions from this work and, liberated by our use of agent-based modeling rather than classical analysis, develop more realistic and interesting models in which the set of competing parties is a completely endogenous output of the process of party competition.

We also got things going in chapter 5 by making the very implausible assumption that party leaders are decision-making clones of each other, with every leader using precisely the same decision rule in the same intractable complex dynamic setting. We also know that this is not true. We know that different party leaders address themselves to the same setting using different decision-making styles. This is part of what is interesting about politics and is not something we should assume away. The second core aim of this chapter, therefore, is to model party competition when different party leaders use different decision rules in the same setting. We do this by building on an approach pioneered in a different context by Robert Axelrod (Axelrod 1980a, 1997, 1980b). This involves long-running computer “tournaments” that allow us to investigate the performance and “robustness” of decision rules in an environment where any politician using any rule may encounter an opponent using either the same decision rule or some quite different rule. We are interested in how each decision rule performs *against itself* and against opponents using the same rule and also in how each rule performs against opponents using *different* decision rules. We are *most* interested, however, in how a decision rule performs against anything the competitive environment might throw against it, including agents using decision rules that are difficult to anticipate and/or comprehend.

We define and investigate a large and eclectic set of decision rules for party leaders in the next chapter. In this chapter, our concern is to develop a scalable model of dynamic multiparty competition that describes interactions among politicians who choose from a set of different decision rules. We also take a big step toward a more realistic theoretical account of party competition by endogenizing the set of competing parties rather than setting this exogenously.

Modeling Competition among Agents Using Diverse Rules

Politics as a Tournament of Decision Rules

When a set of party leaders can choose any of a set of decision rules, the number of possible leader-rule combinations can be huge. It is given by the well-known formula for "combination with repetition."[1] With three decision rules currently specified and a number of parties that varies between 2 and 12, this gives 451 possible leader-rule combinations. This is not a gigantic number. We can specify exhaustive simulations of party competition under every possible rule combination. The problem is that this brute force method is not scalable. The number of possible leader-rule combinations increases explosively with both the number of decision rules and the number of party leaders. For example, Fowler and Laver investigated a system with typically 9 parties choosing from a set of 29 different rules (Fowler and Laver 2008). There are 124,403,620 different leader-rule combinations in this setting. Even with superfast modern computers, we run smack into a computational wall if we set out to investigate every one of these combinations in an exhaustive yet rigorous way. Clearly we must adapt our approach.

We do this by building on work by Fowler and Laver, which in turn built on a famous series of computer tournaments run by Robert Axelrod (Axelrod 1980a, 1997, 1980b; Fowler and Laver 2008). The Axelrod tournaments were designed to investigate strategies for playing the it-

[1] If r is the number of different decision rules and n is the number of parties who can choose these, then the number of different decision rule combinations (not paying attention to party names) is

$$\frac{(n+r-1)!}{n!\,(r-1)!}$$

erated two-person Prisoner's Dilemma (PD) game. Scholars were asked to submit decision rules for playing this game. These rules were pitted against each other in a series of computer simulations, which scored rules according to the payoffs they received. The first tournament (Axelrod 1980a) attracted fourteen entries. The winner was Tit-for-Tat: do to your opponent in round *t* whatever your opponent did to you in round *t*–1. There was then a second tournament; new entries could obviously take account of the published results of the first tournament (Axelrod 1980b). Tit-for-Tat won again, now beating sixty-two entries. In these tournaments, decision rules were predefined and immutable. In a more recent tournament, Axelrod explored the *evolution of new rules* for playing the iterated two-person PD game. He started with a set of random rules, as opposed to a set predefined by others, and applied the standard genetic operators of crossover and mutation to these over successive iterations, rewarding successful decision rules with higher fitness scores and hence higher reproduction probabilities (Axelrod 1997). Rules resembling Tit-for-Tat often emerged from this evolutionary process. Completely new types of successful rules also evolved, which beat Tit-for-Tat and involved strategies no game theorist had submitted to earlier tournaments.

Fowler and Laver (2008) adapted Axelrod's approach to the problem of assessing the performance of different decision rules pitted against each other in dynamic multiparty competition. They took the Laver (2005) ABM, programmed a version of this as a computational test bed, and published the test bed code. They invited scholars to submit new position-selection rules and offered a $1,000 prize for the rule most successful in winning votes in very long-running simulations. The four rules investigated by Laver (Hunter, Aggregator, Sticker, and Predator) were declared as pre-entered in the tournament but ineligible to win the prize. The Fowler-Laver tournament made three important substantive innovations. It specified a distinction between *interelectoral* and *electoral* periods of political competition It specified a *de facto threshold* that determined the continued survival of political parties. And it *forced the birth of new parties*, at random policy locations, using decision rules randomly chosen from the rule set under investigation. The latter is an artifact of the tournament method designed to "churn" rule combinations on a regular basis. We move beyond this below to a more realistic model of endogenous party birth. We see the first two innovations as substantively realistic features of any model of multiparty competition.

Distinguishing "Campaign" and "Election" Ticks

We have thus far assumed party competition to be a continuous process with no particular feature, such as an election, that distinguishes one day in politics from any other, one iteration of the model from the next. Elections do happen, however, so this is substantively unrealistic. Fowler and Laver (2008) made a distinction between

campaign ticks, during which politicians make decisions about party policy in response to published information, such as opinion poll feedback, about levels of party support;

election ticks, during which, in addition to decisions made during campaign ticks, voters make decisions about which party to vote for, rewards and punishments are administered, existing parties may die, new parties may be born.[2]

The distinction between campaign ticks and election ticks is substantively realistic. We thus modify our model to specify an exogenously fixed electoral cycle, with an election timing parameter, Ψ, specifying the frequency of election ticks relative to campaign ticks.[3] There is an election every Ψ ticks of the model. At the end of each of the Ψ-1 campaign ticks that happen between elections, party leaders choose policy positions and estimated levels of party support are revealed in published opinion polls. Politicians are not, however, subject to explicit rewards and punishments at the end of a campaign tick. In line with the wording of questions in real public opinion polls, published levels of party support at the end of campaign ticks reflect how people *would have voted if there had been an election that day*; they therefore offer crucial feedback to candidates. Campaign ticks allow parties to adapt and to evaluate their actions, but only at the end of an election tick do the chickens come home to roost. Only at elections do outcomes have real consequences for the success or failure of political parties.

Fowler and Laver specified an election every twenty model ticks. Since this value of Ψ was a published feature of model rules, the tournament was fair to all rule designers. Of course it was not "fair" to all conceivable decision rules, some of which could be at a relative disadvantage in an environment in which $\Psi = 20$.[4] This is why, while we replicate the Fowler-Laver tournament environment, we also treat election timing as a substantively interesting feature of the competitive environment. We thus treat Ψ as a free parameter in model runs, varying this within the range [10, 25].[5]

[2] For stylistic reasons, and using the same terminology as the NetLogo programming environment, we often use the word "tick" to refer to an iteration of the model.

[3] We thus do not model *endogenous* election timing. On this important matter see Smith (2004).

[4] A rule that explores the policy space for twenty ticks and then makes use of this information every twenty-first tick, for example, would be at an exquisite disadvantage when $\Psi = 20$.

[5] More precisely, we set Ψ at an integer value randomly chosen from a uniform distribution on the [10, 25] interval. We also investigated effects of sampling values of Ψ on the

Modeling Fitness, Survival, and Death

Any model of party competition that involves the "birth" of new parties with some positive probability axiomatically implies "death" of some existing parties. Otherwise the number of parties grows inexorably toward an absurd situation with an infinite number of parties. However it is clearly *not* the case in the real world that political parties die, or even become more likely to die, when they get old. Many political parties die young, while established parties often outlive their human founders. This led Laver and Schilperoord to model party "death" as a failure to maintain support above some critical "survival threshold," which is a key feature of each individual party system (Laver and Schilperoord 2007). We can see clearly that such thresholds do exist in the real world, since new parties are indeed born while the number of parties does not continue to grow inexorably. However, we find rather little in the political science literature, either on party death per se or on de facto party survival thresholds more generally.

We do of course know that there are typically thresholds embedded in the electoral formulas used for mapping votes cast into seats gained—for example the explicit 5 percent threshold in Germany. There are also implicit thresholds in all political systems that arise from interactions between electoral formulas, constituency size, and constituency-level concentrations of party support (Cox 1997). All of this, furthermore, is quite different from more general de facto survival thresholds for political parties, which refer to levels of voting support below which parties simply stop competing and go away. We know, for example, that de facto thresholds arise, among other things, from the candidate nomination process (how do you get yourself on the ballot?), the political financing process (who can give you how much money?), and the system of access to mass media (can you get your message across if you do not have any money?). We know informally that factors affecting party survival can arise from the cost and role of media advertising, public relations, private opinion polling, and all of the other things that can make running a really effective political campaign such an expensive proposition. The combination of these de facto survival thresholds clearly does have an impact. We know this axiomatically because party systems in different countries do experience the formation of new parties, while at the same time "sustaining" a long-run mean number of surviving parties that does not inexorably trend upward. And we know that de facto survival thresholds must differ among countries, since there are very different long-run

[5, 50] interval in exploratory work but found that there was no measureable effect once $\Psi > 25$. We found that setting $\Psi < 5$ could produce atypical results. Since the computational budget increases in linear proportion to Ψ, we confined subsequent experiments to sampling Ψ on the [10, 25] interval.

mean numbers of parties in different countries. We find two main parties in current U.S. politics at the federal level. There were ten parties in both the Dutch Tweede Kamer and the U.K. House of Commons after their respective general elections in 2010. This is despite superficially very similar electoral formulas in the United States and the United Kingdom and superficially very different formulas in the United Kingdom and the Netherlands.

Laver and Schilperoord (2007) modeled a diverse set of substantive effects by defining a single de facto survival threshold for political parties, specifying this in their simulations as 10 percent of the vote. This survival threshold, which we label τ, sets an axiomatic upper bound of $1/\tau$ on the number of surviving parties.[6] Hence, we treat τ in what follows as a substantively important party system parameter, specifying suites of simulations with Monte Carlo parameterizations in which the value of τ for each run is randomly picked from a uniform distribution on the [0.05, 0.30] interval. At the upper end of the values of τ we investigate, we expect to find something close to the two-party competition analyzed so intensively by many classical formal models. At the lower end of this range, the axiomatic upper bound on the number of parties is twenty, though this can arise only if all parties are of identical size. As we will see when we report computational results, the typical number of parties when the threshold is 0.05 is much fewer than this.

Rather than thinking of political parties as ceasing to exist the very second they fall below some survival threshold, it is natural in a dynamic setting to think of people as having prior beliefs about key features of party competition, such as the electoral support of different parties, and updating these beliefs when new information is revealed to them. We model this updating process by building on an approach used by Laver and Schilperoord (2007). We specify a recursive algorithm for how agents update information about the "fitness" of individual political parties, denominating fitness in terms of long-run vote share. Quite simply, a party's fitness after election e is its fitness at election e-1, updated by new information about its success at winning votes at election e.[7] The scale

[6] When the survival threshold is 0.10, for example, there cannot possibly be more than ten surviving parties; if there are eleven parties, simple arithmetic tells us that at least one of them must get less than one-tenth of the vote and thus be below the survival threshold.

[7] This recursive updating algorithm generates a measure of party fitness that is mathematically equivalent to the exponentially backward discounted sum of past party vote shares. In the context of party competition, our updating model is directly analogous to an updating model proposed by Erikson, MacKuen, and Stimson for *voter perceptions of party policy positions* (Erikson et al. 2002). Their notion is that a voter's prior perception of a party's policy position is updated in this way, but not completely replaced, by new information about party policy—for example a new party manifesto. They also use a memory parameter, α, to measure how much a voter's prior belief about a party position is updated by new information.

of this update is determined by a parameter, α_f ($0 \le \alpha_f \le 1$), that reflects the extent of agent "memories" of parties' past fitness.[8] More precisely, if v_{pe} is p's observed vote share at election e, and f_{pe} is party p's fitness after election e, then,

$$f_{pe} = \alpha_f \bullet f_{p(e\text{-}1)} + (1 - \alpha_f) \bullet v_{pe} \tag{6.1}$$

Parties "die" when their updated fitness, f_{pe}, rather than their instantaneous vote share, v_{pe}, falls below the survival threshold τ. When α_f ("fitness-alpha") is zero there is a "goldfish memory" regime in which every day is the first day and agents remember nothing at all from the previous election, measuring current party fitness only in terms of current party performance. Under this regime, parties die the instant they fall below the survival threshold. This is the deeply implicit assumption of current static formal models of party competition. When $\alpha_f = 1$ there is a memory regime in which agents *never* update their estimates of party fitness from their original priors. Old parties never die under this regime. We expect values of α_f in the real world to be somewhere in between these extremes and use our computational experiments to investigate the effect of different values of α_f. Our Monte Carlo model parameterizations thus randomly picked a real value of α_f for each run from a uniform distribution on the [0.00, 0.90] interval.[9]

The simple two-parameter (α_f, τ) model of party fitness and survival that we specify here seems to us to be an elegant and realistic way to capture the evolving dynamics of party success and failure, and we build on it in subsequent chapters. From now on, all party systems we model have endogenously evolving sets of surviving parties. Since both party fitness and τ are denominated in vote shares we must keep in mind that,

[8] Note that we do not index α_f by i, assuming all agents in the system have the same memory model.

[9] We found in our methodological investigations that setting $\alpha_f > 0.9$ generated serious problems with both model burn-in and convergence on a stochastic steady state. In addition, we also confronted an operational problem that could generate peculiar interaction effects when running models with high survival thresholds and low memory parameters. These arose because it is possible for a new party to create a situation in which all parties are driven simultaneously below the survival threshold. For example, with three evenly balanced parties, and a survival threshold of 0.30, a new party entering at the "right" location can force the four resulting parties into a situation in which they each get about 0.25 of the vote and, if the memory parameter is low so updates are weighted heavily, all parties fall simultaneously below the survival threshold leaving no survivor. In a long-running simulation this is almost certain to happen. Since this seems absurd, we added a model assumption to address it. Parties whose fitness falls below the survival threshold die, provided the result is a system with two or more parties. In situations where simultaneous party deaths would result in either one or zero surviving parties, below threshold parties die in random order until only two parties survive, whereupon the two surviving parties are left in contention.

in common with almost all classical formal models of party competition, this gives a vote-seeking orientation to the competitive environment we model. We do not see this as unrealistic, while any evolutionary setting involves exogenously determined fitness criteria. Death, alas, is not a matter of opinion, while none of us can ultimately avoid death because we do not want to die. In the context of this book, however, this does mean that we should keep in mind that vote-seeking decision rules such as Hunter are programmed to increase their agents' vote shares; rules such as Sticker and Aggregator are not. This means, for example, that an Aggregator party that dies because it does not gather enough votes may not have failed *in its own terms*, which involve representing the wishes of current party supporters, however few these might be. If these supporters are not numerous enough, the party may die nonetheless.

Endogenous Party Birth

The birth of new parties is self-evidently an endogenous product of political competition. As we have seen, orthodox static spatial models of party competition have been extended to analyze the comparative statics of actual or anticipated "entry" by new parties into an existing party configuration. Key features of this work are reviewed by Shepsle; more recent writing, mostly by economists, is discussed by Dhillon and by Austen-Smith and Banks (Austen-Smith and Banks 2005; Dhillon 2005; Shepsle 1991). It is not our job to re-review this burgeoning literature, though we note that these models must be extraordinarily spare if they are to gain any sort of traction using classical formal analysis. Models of endogenous parties fall into three broad categories: those that describe new parties in terms of the *entry* of new agents from outside the system (Osborne 1993; Palfrey 1984), those that focus on *realignment* of existing coalitions of politicians into new party configurations (Ray and Vohra 1999; Levy 2004), and those that involve changes of state of political agents from "citizen" to "politician" (Osborne and Slivinski 1996; Besley and Coate 1997). Choice of modeling assumptions depends upon whether party competition is modeled in an electoral or a legislative setting. From the perspective of *electoral* competition, incentives to form new parties include the value of a party label in communicating policy positions to voters (Snyder and Ting 2002) and the desire of politicians to commit to positions other than their own ideal points when offering policy positions to voters (Levy 2004; Morelli 2004). If the focus is primarily on *legislative* politics, then parties may be seen as cartels formed to control aspects of legislative decision making (Cox and McCubbins 2005; Eguia 2007) or simply as emergent groups of like-minded legislators (Krehbiel 1993).

Our focus is this book is on *electoral* politics. Furthermore, we have already specified a "two-breed" model of political competition that distinguishes ordinary decent voters from the politicians who compete for their support. For reasons we take to be self-evident, we prefer to describe the emergence of new political parties in terms of *changes within* the political system as opposed to *entry or invasion by outside agents*.[10] We therefore exploit one of the key intuitions of the existing formal literature and model the "birth" of new political parties as a response by disgruntled voters to the existing configuration of party policy positions. Our model is thus an agent-based implementation of the core idea of "citizen candidate" models of new political parties (Osborne and Slivinski 1996; Besley and Coate 1997). We leave for future work the development of a more explicit model of *intraparty politics* that might form the basis of a dynamic model of continuously evolving coalitions of legislators according to which new parties may be formed as a result of splits or fusions of existing parties.

Voter Dissatisfaction

Our model of endogenous party birth assumes that new parties are formed by disgruntled voters. We must thus characterize the disgruntlement of voters with a configuration of party policy positions. In effect, we already did this in chapter 3, when specifying a metric for measuring "distances" between policy positions and a loss function for describing the effect of such policy distances on agent utility. We specified the distance between an agent's ideal point and some other policy position using a Euclidean metric and defined the utility of a policy position at j for a voter with an ideal point at i as the negative quadratic distance between the two policy positions.[11] We went on to define the satisfaction of a voter with the *overall configuration of policy positions* in terms of the quadratic distance between her ideal point and the policy position of the *closest party*.

As we noted when defining party fitness, it is substantively implausible in a dynamic setting to assume that voters respond only to the current instantaneous state of the system. It is much more plausible to assume they use their observations at tick t to update rather than completely determine their evaluations of the party system. Crudely speaking, if the past one hundred opinion polls reported the support of some party as being in the range of 20–30 percent, and some new poll reports that this support has fallen to 2 percent, we expect voters to discount this new information

[10] New politicians did enter the political fray from other counties in some postcommunist party systems, for example, but this is not in any way typical.

[11] $U(i, j) = -d(i, j)^2$, where $d(i, j)$ is the Euclidean distance between i and j.

to some extent and not to forget everything that has gone before. They may lower their estimate of current party support, but they will not immediately and completely update their estimate to that of the new poll. In what follows, therefore, we follow Laver and Schilperoord (2007) and model voter disgruntlement by specifying a simple updating model that is directly analogous to our updating model for party fitness. This defines d^*_{ce}, citizen c's updated dissatisfaction with the party system at election e, in the following recursive fashion:[12]

$$d^*_{ce} = \alpha_b \cdot d^*_{c(e-1)} + (1 - \alpha_b) \cdot d_{min}\,(i_c, j_e)^2 \qquad (6.2)$$

In this updating model, α_b ("birth-alpha") is a memory parameter describing how a voter updates her dissatisfaction with party policy positions and $d_{min}(i_c, j_e)$ is the distance between her ideal point and her closest party. If $\alpha_b = 0$ there is a "goldfish memory" regime. Voters treat every day as the first day and remember nothing from the past. A voter's updated dissatisfaction is simply the quadratic Euclidean distance between her ideal point and the current position of her closest party. If $\alpha_b = 1$, the voter never updates her dissatisfaction with the configuration of party positions, no matter what (crazy) positions any party adopts at election e. When $0 < \alpha_b < 1$, then the memory parameter α_b determines the extent to which each voter updates her dissatisfaction with the party system on the basis of her dissatisfaction with the current configuration of party positions. Laver and Schilperoord (2007) set $\alpha_b = 0.50$. This implies that a voter's updated dissatisfaction eight elections earlier contributes about 1 percent of the updated dissatisfaction in the current election. In exploratory work, we added α_b, and hence another dimension, to our parameter space and investigated the effects of different values of α_b. Since we found no statistically significant effect of α_b on model outputs of interest, we conserved our computational resources for more interesting problems and followed Laver and Schilperoord (2007) by setting $\alpha_b = 0.50$ for all runs.

A Model of Endogenous Party Birth

We are now in a position to specify a "citizen candidate" ABM of endogenous party birth. We assume every voter always has the implicit, albeit improbable, option of forming a new party by changing state from voter to party leader. The probability, p_{ce}, that citizen c changes state to party leader following election e increases in direct proportion to her updated dissatisfaction with the configuration of party positions. Thus $p_{ce} = \beta \cdot d^*_{ce}$,

[12] Note we assume voters get disgruntled only on the basis of party choices they have at election time. They do no get disgruntled on the basis of party positions between elections.

where β is the "birth" parameter, the sensitivity of each citizen's probability of becoming a politician to her updated dissatisfaction with the evolving configuration of party positions. The birth parameter β is scaled to the units in which policy distances, and thus d^*_{ce}, are measured. Because they are associated with an expected number of new party births per election, substantively realistic values of β are a function of how we calibrate the model to real time. Different values of β have the effect of controlling how *fast* the system evolves, rather than *how* it evolves in a substantive sense. For this reason, although we investigated the effect of different values of β in our exploratory work, we do not treat β as a free parameter in the simulations that follow. Rather, we fix it at a value that generates new party births at what we take to be a substantively plausible rate.[13]

Experiment and Run Design

Every computational experiment we report in the rest of this book involves interactions of party leaders who may use different decision rules, including rules with stochastic components. These interactions take place in evolutionary settings, also with stochastic components, in which some surviving parties may "die" and new parties may be "born." Our diagnostic work, described in chapter 4, unambiguously implies that the model runs in these experiments are stochastic processes for which a time average does *not* provide a representative estimate of μ.[14] We therefore execute several run repetitions until the process reaches steady state and calculate an ensemble average of each of the values of the output variables we collect from each of the repetitions. We find that one thousand repetitions, implying a run sample size of one thousand observations, are sufficient to pass all five of our diagnostic checks.

To determine burn-in for these processes, as we describe in chapter 4, we must use the graphical method on the most extreme run of this experiment. In fact, we have already examined this case in the empirical burn-in subsection of chapter 4. As we note there, the most extreme run is when

[13] To give an intuitive sense of our calibration of the party birth model, we use the following algorithm to determine the probability that a new party is born in any given election. One of 2,809 discrete voter locations within 3 *SD* units of the voter centroid is selected at random. This location "sprouts" a new party with a probability $\beta \cdot d^*_{ce}$. We set $\beta = 1$. The median value of d^*_{ce} for the 2,809 voter locations, for the evolved six-party system that tends to arise when $\tau = 0.10$, is of the order of 0.10–0.20. Therefore, this generates a new party roughly every five to ten elections, which we feel is not unrealistic.

[14] Specifically, the R-hat statistics calculated for our summary measures, mean eccentricity and ENP, never fell below 1.1 even after very long-run repetitions of fifty thousand elections.

the survival threshold is set at 0.05 and there are many surviving parties. As we see from Figure 4.5, the summary variables, mean eccentricity and ENP, appear to be in steady state after 200 elections. We err on the side of caution and consider that a run repetition has reached steady state after 250 elections.

To summarize our run design, we employ a mean estimation strategy for stochastic processes for which a time average does not provide a representative estimate of μ. For each run, we execute one thousand separate run repetitions for 251 iterations, each iteration being an election tick in the model.[15] For each run repetition, we collect the values of all output variables at election 251. Finally, we calculate an ensemble average over the one thousand repetitions to generate a mean estimate of μ for each of the output variables for each run. We use this run procedure for all subsequent simulations we report in this book.

Turning to experimental design, we continue to employ the Monte Carlo method with one thousand runs per experiment. Each experiment therefore involves one million run repetitions in all—one thousand runs per experiment multiplied by one thousand repetitions per run.[16] Each of the one thousand runs has a Monte Carlo parameterization in which parameter settings are randomly chosen from ranges we have argued are substantively meaningful:

- Ψ: the number of campaign ticks per election; run parameter settings are randomly chosen from a uniform distribution of integers on the [10, 25] interval;
- τ: the de facto survival threshold—run parameter settings are randomly chosen from a uniform distribution of floating point numbers on the [0.05, 0.30] interval;
- α_f, the memory parameter in the model agents used to update party fitness—run parameter settings are randomly chosen from a uniform distribution of floating point numbers on the [0.00, 0.90] interval.

Moving beyond the assumption that distributions of ideal points are perfectly symmetric about the voter centroid, our computational work in the rest of this book is based on the asymmetric ideal point distributions that are aggregations of two subpopulations, as described in chapter 3. This distribution of voter ideal points has two parameters:

[15] We therefore do not count the campaign ticks between elections as iterations.

[16] Our computational budget for this experiment, given that we reach steady state by iteration 251, is therefore 1,000 runs × 1,000 repetitions × 251 elections = 251 million elections. Since each election on average requires 17.5 campaign ticks, this budget implies approximately 4.4 billion simulated campaign ticks.

n_l / n_r, the relative size of subpopulations, is sampled from a uniform distribution of floating point numbers on the interval [2.00 1.00];

μ_r (= $-\mu_l$), the subpopulation ideal point means, are sampled from a uniform distribution of floating point numbers on the interval [0.00, 1.50].

Multiparty Competition with Diverse Decision Rules

From this point on we investigate competitive environments where the set of surviving parties is endogenous and different party leaders typically use different decision rules. Many of our more interesting results concern the relative performance of different rules. As well as aggregate party system outputs we specified and reported above, therefore, we also look at the typical number of party leaders using each decision rule, the average vote share won by the set of parties using particular rules, and the mean policy eccentricity of parties using particular rules.

Vote Shares of Parties Using Each Rule

Figure 6.1 summarizes the first set of results derived from the experiment specified above. As before, we summarize the nonlinear relationships contained in this huge amount of simulated data using fractional polynomial prediction plots. The gray bands show 95 percent confidence intervals around these plots for the sets of data points generated by the simulation, arising from one thousand repetitions of one thousand model runs, each run with a Monte Carlo parameterization. The narrowness of these confidence intervals shows us that these estimates are very precise, vindicating our run design and estimation procedures.

Measuring success in terms of aggregate vote shares of sets of parties using each decision rule, the vote-seeking Hunter rule is by far the most successful—a conclusion that does not depend in any way on the precise parameterization of the environment for party competition. Taking into account all parameterizations of the competitive environment that we investigated in this experiment, mean vote shares of sets of parties using each rule were Hunter, 0.70; Aggregator, 0.21; and Sticker, 0.09.[17] Although it is not very surprising to find that the vote-seeking rule does best at finding votes, our simulations do demonstrate quite clearly that Hunter's Pavlovian vote-seeking algorithm works well. Of the two rules not explicitly programmed to seek votes, Aggregators were much more successful than Stickers.

[17] Standard errors of these means were, respectively, 0.003, 0.003, and 0.001.

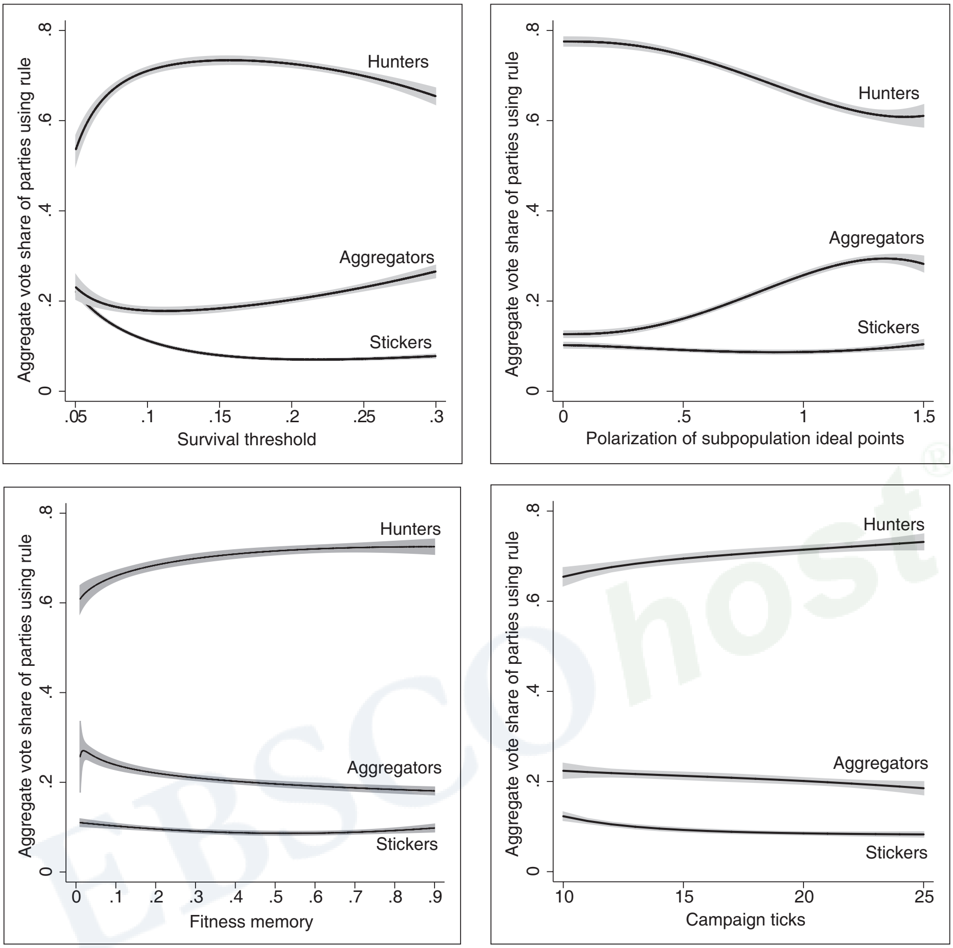


Figure 6.1. Mean vote share of surviving parties by rule, τ, Ψ, α_f and polarization of subpopulation ideal points. Each data point summarized in these plots represents the run mean estimate of the average vote share for each of the three decision rules for each of the one thousand runs of the experiment. Each run mean estimate is calculated by executing one thousand separate run repetitions for 251 elections, taking the results from the 251st election as the estimate of the steady-state decision rule vote shares for each repetition, and then taking the average of these one thousand repetition estimates.

Figure 6.1 also plots the relationships between the relative performance of decision rules and parameterized features of the competitive environment that we vary systematically in the computational experiment. The lower panels show that fitness memory, α_f, and election timing, ψ, have distinct if modest effects on the relative success of party decision rules.[18] Hunters fare better when party fitness updates more slowly (that is, for high values of α_f). This is because they make random exploratory moves when punished with lower support; these random moves carry no guarantee of success and may well, in the short run, result in *lower* levels of support. When $\alpha_f = 0$, a party's fitness is simply its current vote share; no account is taken of past success, and the party will be killed off the instant its vote share falls below the threshold. With higher values of α_f, the competitive environment is more "forgiving" of short-term dips in support, provided these are corrected. Parties are killed off only if their vote share stays below the threshold for a number of elections. The continuously adapting Hunter rule also tends to be less successful in environments with shorter election campaigns that allow parties less time to adapt, for example to the birth of a new political party close to them in the policy space.

The top two panels highlight the big news in these results, however. Party leaders using the Aggregator rule tend systematically to win more support, and those using Hunter to win less support, when survival thresholds are higher and when voter ideal points are more polarized. The relative success of the Aggregator rule is doubly enhanced when there are higher levels of polarization, if survival thresholds are also higher so that there are fewer parties. This is a classic interaction effect between polarization of the electorate and the number of parties that survive, given the survival threshold. Figure 6.2 plots this effect systematically. The gray bands summarize the performance of each decision rule when there are typically four parties or more. They show no significant relationship between rule performance and polarization in these settings. The black bands summarize the same relationship when there are three or fewer parties. Decision rule performance now responds sharply to electoral polarization. Aggregators now perform substantially better, and Hunters substantially worse, as electorates become more polarized. The black bands show that, with highly polarized subpopulations and few parties, Aggregators perform almost as well *at winning votes* as vote-seeking Hunters.

This happens because of the systematic emergence of the type of situation of shown in Figure 5.8. When survival thresholds are high, there tend to be only two or three parties, quite commonly three. In this event and with polarized electorates, one of the two subpopulations can axiomatically be "served" by just a single party. *If this party uses a vote-seeking rule such as Hunter*, then there is nothing in the dynamics of party competition to deter it from systematically moving away from "its" subpopulation and toward the other parties, in search of more votes. This is bad

[18] The relative size of the two voter subpopulations has no noticeable effect and is not shown here. This plot is shown in Figure E6.1 in the electronic appendix.

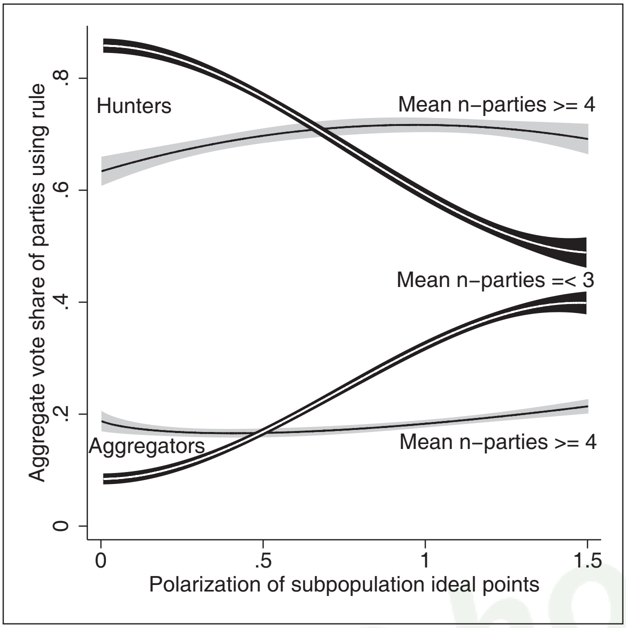


Figure 6.2. Aggregate vote share of parties using different decision rules, by ideal point polarization and number of surviving parties.

for representation of subpopulation ideal points, as we will soon see, but it is also bad for the party, which is now just one of three parties competing for vote share in the overall population.

In somewhat counterintuitive contrast, if the lone party serving one of the subpopulations uses an Aggregator rule, then it *tends to stay close to "its" subpopulation ideal point centroid and faces no incentive to move away from this*. The Aggregator party keeps well clear of the other two parties and the other subpopulation. Somewhat surprisingly, this is actually good for its vote share since it now monopolizes an entire subpopulation. This presages important and counterintuitive results that run throughout the rest of this book. We can systematically identify situations in which, if you are a party leader, insatiably seeking more votes *may be bad for your vote share*. The only way for this thought-provoking conclusion to emerge is to specify and analyze dynamic models of party competition in which different party leaders use different decision rules.

Numbers of Surviving *Parties Using Each Decision Rule*

Figure 6.3 plots the effect of the survival threshold, τ, on the number of surviving parties using each rule. Recall that $1/\tau$ sets an axiomatic up-

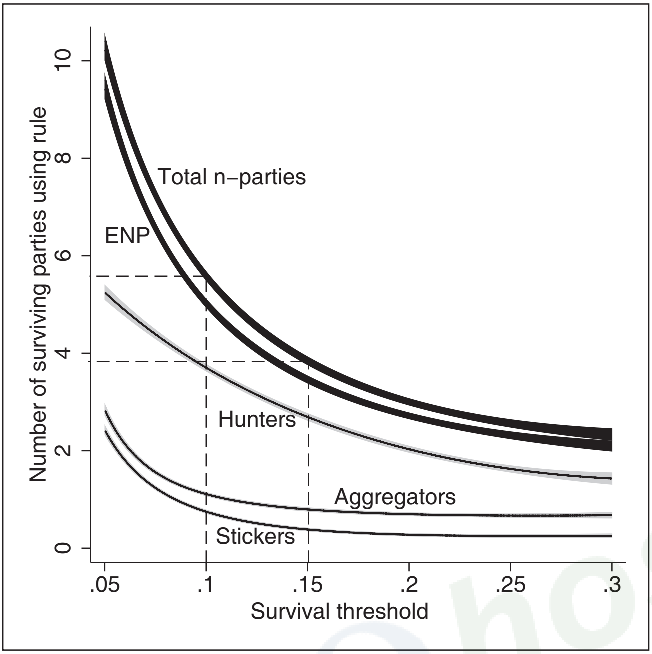


Figure 6.3. Mean number of surviving parties, by decision rule and survival threshold.

per bound on the total number of surviving parties. We thus expect a reciprocal relationship between τ and the total number of parties, and the upper black band of Figure 6.3 shows that this is what we find.[19] We see, however, that the number of surviving parties is typically much less than this axiomatic upper bound, which would be ten parties with a survival threshold of 0.10, for example. This is because the upper bound can be achieved only when all parties have identical and constant sizes, while the typical situation *in a dynamic party system* is that party sizes vary considerably over time.

Figure 6.3 gives us important substantive insights into dynamic party systems with endogenous parties. It shows, for example, that a survival threshold of 0.10 of the total vote tends to be associated over the long run with a five- or six-party system, and an effective number of parties (ENP) of about five. A threshold of 0.15 tends to be associated with a

[19] Plotting the number of parties or ENP against $1/\tau$, we get a straight line. See Figure E6.2 in the electronic appendix.

four-party system and an ENP of about 3.5; and so on. If we believe our model (and of course we do), this allows us, for any real party system that interests us, to "back out" the unobservable party survival threshold from the number of surviving parties we observe over the long run. If we tend to observe six surviving parties, this implies a de facto threshold of about 0.10; a four-party system implies a threshold of about 0.15; and so on.

The other striking pattern in Figure 6.3 is that, in an environment where parties "die" if their updated fitness falls below the survival threshold, far more surviving parties use Hunter than use any other rule. Lowering the de facto survival threshold radically increases the number of surviving parties using any rule, but until thresholds get very low and there are six or more surviving parties, it particularly increases the number of parties using relatively successful rules such as Hunter. This is the first sign of *evolutionary* dynamics in the party systems we model, dynamics that will color most of the rest of this book. These dynamics arise because relatively unfit parties die, relatively fit parties survive, so *the set of surviving parties is relatively fit*. We can access this type of conclusion, of course, only if we model continuously evolving dynamic systems with endogenous political parties.[20]

Life Expectancy of Political Parties

Closely related to party survival is party life expectancy—typical party ages at death in an evolving party system in which the surviving parties are those that stay above the survival threshold. Figure 6.4 plots the effects of the party survival threshold and the rate of fitness updating, on the median longevities of parties whose leaders use different decision rules.[21] Hunter parties tend to survive for *much* longer as the survival threshold decreases and as party fitnesses update more slowly (that is, as α_f increases). This effect is disproportionately greater for Hunter than for other rules.

Very tight confidence intervals in the left-hand plot show that the relationship between survival thresholds and party longevities is, as might be expected, extraordinarily crisp. While a more "forgiving" fitness regime, which updates more slowly, is less likely to kill off Hunters that have briefly fallen below the survival threshold in random searches for

[20] Figure E6.2 in the electronic appendix shows that, in line with results reported in Figure 6.1, the rate of updating of party fitness also has some effect on the number of parties using Hunter, while polarization of subpopulation ideal points or campaign length have no significant effect.

[21] We use the median rather than the mean as a summary of party life spans because, as with any survival process, the exponential distribution of party life spans is highly skewed.

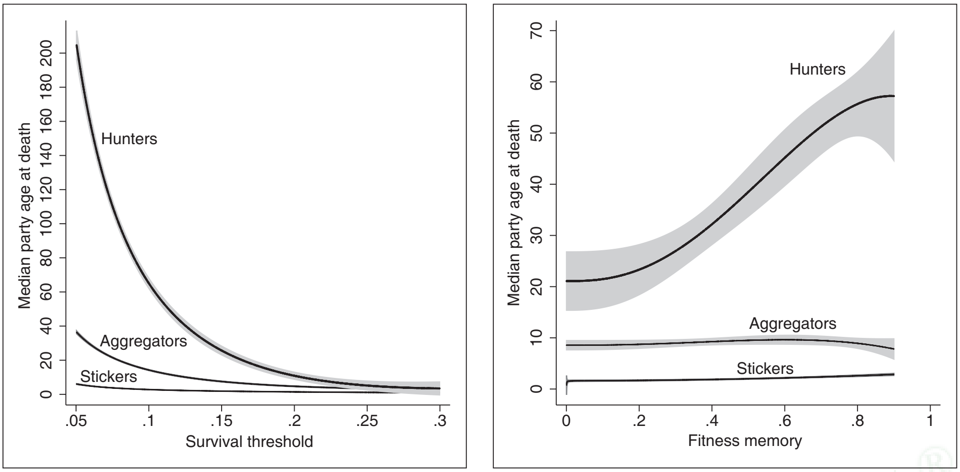


Figure 6.4. Median party age (elections survived) at death, by decision rule, survival threshold, and fitness memory.

higher voter shares, wider confidence bands around Hunter longevity in the right-hand plot reflect the very sharp effect of different party survival thresholds on Hunter longevity holding fitness memory constant. In short, there are more Hunter parties, as we saw in Figure 6.3, leading to a greater level of success for the Hunter rule, as we saw in Figure 6.1, because *Hunter parties are much more likely to survive than parties using other rules*, especially when survival thresholds are low.[22]

Since new parties choose decision rules at random in this environment,[23] and since parties using the Hunter rule are much more likely to survive than other parties, there is a preponderance of Hunters in the set of surviving parties. This is the key to the success of Hunter in the party systems we investigate here. It is not that surviving Hunter parties tend to win more votes—for the most part they are competing with other Hunter parties after all. The relative success of the Hunter rule arises because, quite simply, *there are more surviving Hunter parties* in this evolutionary setting.

[22] Figure E6.3 in the electronic appendix shows that polarization of subpopulation ideal points and the number of campaign ticks between elections have no substantial effect on party longevity.

[23] We turn in chapter 8 to a setting in which rule replication probabilities are endogenous.

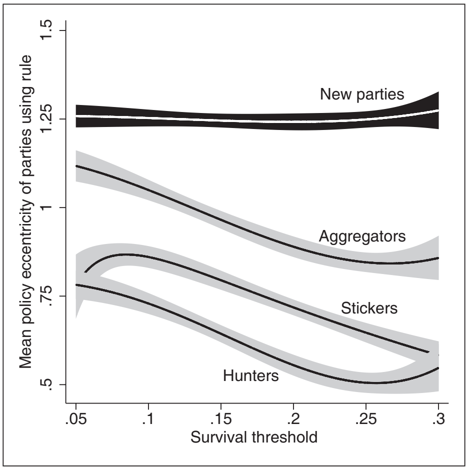
Policy Positions of New and Surviving Parties

Analyzing typical party policy eccentricities in "one-rule" party systems, we found in chapter 5 that parties using the vote-seeking Hunter rule systematically pick more central policy positions than parties using other rules. The gray bands in the left panel of Figure 6.5 plot mean policy eccentricities of parties using different decision rules, now in an environment where the set of surviving parties and the set of decision rules they use are both endogenous. Hunter parties, now competing directly with parties using different decision rules, still systematically choose policy positions that are closer to the center than other parties, regardless of the parameterization of the competitive environment. The black band shows the typical policy positions of endogenous party births. Strikingly, we see that party births tend systematically to be at policy positions that are significantly more eccentric than those of surviving parties, whatever decision rule these parties use. Averaged across all random parameterizations of the competitive environment, mean policy eccentricities of *surviving* parties using various decision rules were Hunter, 0.61; Aggregator, 0.94; Sticker, 0.74. The mean policy eccentricity of new party births was 1.25.[24]

Although in no way designed to do this, our model generates dynamic party systems in which *new parties typically emerge at relatively eccentric locations in the policy space and typically adapt over time to take more central policy positions*. For Stickers, who never change position, this involves more eccentric Stickers tending to die and less eccentric Stickers tending to survive. For parties using other rules, in addition to the increased likelihood that more eccentric parties will die, the more central location of surviving parties arises because, following party birth, party leaders tend to adapt their policy positions centripetally. This seems to us to be substantively plausible, as well as being a clear-cut empirical implication of our model. Previewing results we report in chapter 11, we do indeed find that typical policy positions of surviving parties in real party systems are systematically less eccentric than those of new party births.

The right panel of Figure 6.5 shows that party policy positions are *sharply sensitive to the centroids of subpopulation ideal points*. As voter subpopulations become more polarized, so do typical party policy positions. This is true regardless of the party decision rule, though Hunter parties still systematically choose policy positions closer to the center than other parties. The dashed diagonal line in Figure 6.5 shows what would happen if party policy eccentricities were precisely the same as

[24] Standard errors of each of these mean estimates were 0.01.



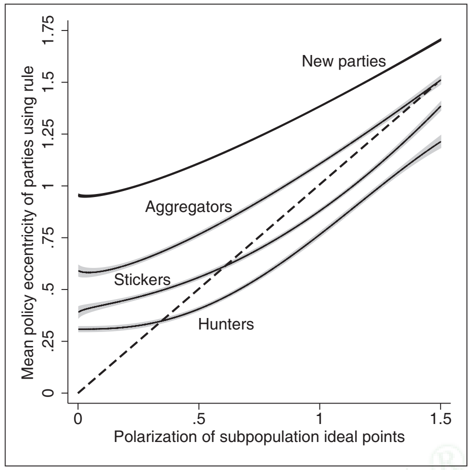


Figure 6.5. Mean policy eccentricity of surviving and new parties, by τ and the polarization of subpopulation ideal points.

those of subpopulation ideal point centroids. Reprising results reported in chapter 5, we find that when voter polarization is low and ideal point densities are therefore unimodal, typical party policy positions are more eccentric than the voter centroid. As voters become more polarized, surviving Aggregators tend to pick policy positions that approach subpopulation ideal point centroids. It is also clearly the case that Hunter parties, while always tending to be more central than Stickers and Aggregators, do also tend to pick policy positions that respond in a very precise way to the centroids of subpopulation ideal points.

The results plotted in the right panel of Figure 6.5 are substantively important. Ever since the original Hotelling-Black-Downs model of one-dimensional spatial party competition, substantive interest in modeling party competition has focused on the extent to which, both in models and in reality, political parties tend to converge on the ideological center ground. We saw in chapter 5 that, even without making any extra assumption to constrain party movement in some way, convergence on the center is *not* generic in dynamic multiparty competition in multidimensional policy spaces. We now see that, when voters are structured into relatively polarized subpopulations, adaptive parties tend to converge on *subpopulation* voter centroids when setting party policy positions in dynamic multiparty competition. Thinking of these distinct subpopulations as social or ethnic groups, for example, then Figure 6.5 suggests that these groups may tend to become *better* served by "their own" political parties as subpopulations become more polarized—in effect because party competition increasingly becomes an intragroup rather than an intergroup activity.

Representativeness of the Party System

A core substantive and normative concern of this book is the representativeness of configurations of party policy positions that typically emerge in different competitive environments. We know from chapter 5 that representativeness of the party system is sharply affected by both the number of parties and the polarization of the electorate. We have now made the number of parties an endogenously evolving feature of party competition, and we saw in Figure 6.3 that this is sharply affected by the survival threshold. Competitive environments with lower survival thresholds sustain more parties. The top panel of Figure 6.6 both plots and summarizes each of the one thousand data points generated by our experiment and shows the crystal clear pattern resulting from all of this.[25] Representativeness of the party system is sharply affected by the survival threshold; higher survival thresholds imply less representative party systems.

We also see a systematically increasing scatter of points as the survival threshold increases. This happens because, as we show below, higher thresholds imply fewer parties and, when there are fewer parties, polarization of the electorate also has a huge effect on representativeness, holding the threshold constant. When there are lower thresholds and consequently more parties, in contrast, ideal point polarization has almost no effect on the representatives of emergent configurations of party policy positions.[26]

The bottom-left panel in Figure 6.6 shows the impact on party system representativeness of the "sea change" we observed in Figure 5.10. In polarized electorates, when the number of parties drops below four, one of the two subpopulations must be "served" by at most one party. If this party is a Hunter, as Figure 6.3 shows us it is most likely to be, then it will not be punished when it searches for more votes by moving party policy away from the centroid of voter ideal points in the subpopulation it is currently serving. As we saw from the example in Figure 5.8, the short-run effect of this is an entire subpopulation of voters with no party representing their ideal points. Given our model of endogenous party birth, this situation is now likely to be remedied at some point by a party birth in the "deserted" subpopulation, but if survival thresholds

[25] Recall that each data point is estimated from one-thousand-run repetitions.

[26] This is one of the main examples of such heteroscedasticity in the data generated by our simulations.

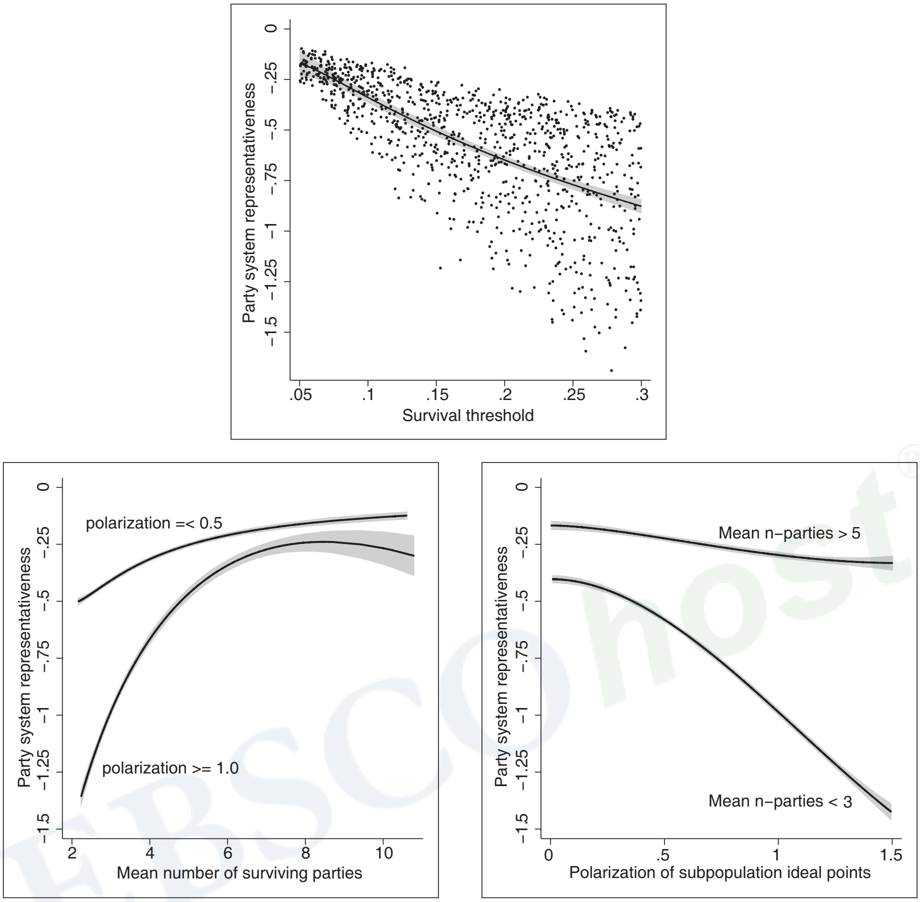


Figure 6.6. Mean party system representativeness by τ, mean number of surviving parties, and polarization of ideal point centroids.

are high the cycle of desertion will then repeat itself. With four or more parties in contention, there is typically more than one party serving each subpopulation. In this event, party system representativeness both is relatively high and shows little variation, being unaffected by other party system parameters.[27]

All of this can be seen clearly in the bottom-left panel of Figure 6.6, which plots the relationship between the typical number of surviving

[27] Figure E6.5 in the appendix plots the effects of other party system parameters on representativeness.

parties and party system representativeness, distinguishing electorates with high ($\mu_r \geq 1.0$) and low ($\mu_r \leq 0.5$) ideal point polarization. When polarization of the electorate is low, the aggregate distribution of voter ideal points is unimodal (if asymmetric), and we see that the number of surviving parties has a relatively small effect on representativeness of the evolved configuration of party policy positions. In contrast when polarization is high and the aggregate population distribution is bimodal, the steep slope of the lower band in this panel shows that the number of surviving parties has a sharp effect on party system representativeness.

This effect is further illustrated in the bottom-right panel of Figure 6.6, which plots representativeness against subpopulation polarization, distinguishing settings where the typical number of surviving parties was fewer than three from those where it was more than five. The lower band shows that, when there are typically fewer than three surviving parties, representativeness of the emergent set of party policy positions declines steeply as voter polarization increases. In contrast, when there are typically more than four surviving parties, party positions tend to represent voter ideal points well, no matter how polarized the voting population. As we have seen, this is because party competition tends to evolve into competition for votes within one or the other subpopulation, with effectively no competition among parties representing different subpopulations. A striking empirical example of this can be found in Northern Ireland, where elections are played out between a set of parties competing for the support of nationalist/Catholic voters and a quite different set competing for the support of unionist/Protestant voters.

Conclusions

We move far beyond our baseline model of party competition in this chapter, to model evolving dynamic multiparty systems in which the set of *surviving* political parties is an endogenous output of party competition, and where the leaders of different surviving parties may use different decision rules to set party policy positions. This takes us deeply into a theoretical realm in which rigorous and systematic investigations are feasible only using the new technology of computational agent-based modeling, as opposed to traditional pencil and paper formal methods.

This extension of our model has revealed interesting and important substantive conclusions. Some of these concern the endogenous set of surviving parties. Since the de facto survival threshold imposes an axiomatic upper bound on the number of surviving parties, we expect to find that higher survival thresholds imply fewer surviving parties (Figure 6.3) with shorter life expectancies (Figure 6.4). The set of surviving parties is conditioned but not determined by the survival threshold since, in a dynamic party system, (1) party support shares vary continuously and (2) the memory regime in the competitive environment affects the extent to which short-term dips in party support below the threshold are "forgiven" if these are subsequently reversed. Given all of this, the results summarized in Figure 6.3 allow us to back out the unobservable de facto survival threshold of any real party system from the observable number of surviving parties over the long run. We see that, if we typically observe four surviving parties over the long run, for example, this implies a de facto survival threshold of about 15 percent of the vote. Similarly, the two-party system we observe in the United States implies a survival threshold of about 30 percent of the vote.

Over and above the effect of survival thresholds on the set of surviving parties, our new conclusions in this chapter concern an interesting and substantively important interaction effect—between survival thresholds and the degree of ideal point polarization in the electorate. As we saw from Figure 6.1, Hunter parties remain very successful at winning votes when they compete with parties using other decision rules. But we also saw from Figure 6.2 that the *relative success of Hunter parties is strongly conditioned by an interaction between survival thresholds and electoral polarization*. Vote-seeking Hunters perform relatively worse at finding votes, and Aggregators are relatively better at doing this, when survival thresholds are high, so there are few parties, *and* the electorate is polarized. In these settings Aggregator parties, programmed only to please their *current* supporters, may win more votes over the long run because they do not, in search of more support in the short term, desert a subpopulation for which they are the sole party in contention.

This interaction between survival thresholds and polarization strongly conditions our results on representativeness of emergent configurations of party policy positions. Figure 6.5 shows that, whichever decision rule party leaders use, *party policy positions sharply track subpopulation ideal point centroids* as electorates become more polarized. This strongly affects representativeness, as we can see in Figure 6.6. The lower panels of Figure 6.6 show the interaction between survival thresholds and polarization of the electorate in two different ways. The key point is that, when survival thresholds are low and there are four or more parties in contention, the dynamics of competitive party position taking do *not* affect the representativeness of emergent configurations of party policy positions in any substantial way, even in highly polarized electorates. In such polarized settings, party competition tends to resolve into competition among parties within two subpopulations; there is little or no competition among parties in different subpopulations. For the same high level of electoral polarization, however, high survival thresholds that typically result in three or fewer parties can have disastrous effects on representativeness, as vote-seeking parties face incentives to "desert" any subpopulation within which they are currently the sole contender.

Putting on the uniform of social planners concerned above all to enhance the representativeness of emergent configurations of party policy positions, the interaction between the de facto survival threshold and voter polarization is very significant. It implies that engineering low survival thresholds—by changing the electoral system, laws on party financing, or rules dealing with access to mass media at election time, for example—*is of particular importance when the voting population is polarized*. In such polarized settings, Figure 6.6 shows us that high survival thresholds, and the resulting low number of surviving parties, have particularly severe effects on representativeness. In contrast, if voters are relatively unpolarized, then the number of surviving parties does not have a huge effect on representation. Our results thus provide strong support for arguments in favor of using proportional representation electoral systems in divided societies (Lijphart 1994, 1999; Lijphart and Waisman 1996).

Another way of thinking about this is to imagine electorates that become more, or less, polarized for reasons exogenous to our model. Think of a two-party system such as we find in the federal politics of the United States, for example, which self-evidently must have a high de facto survival threshold. If both parties are insatiable vote seekers, then the emergent configuration of party policy positions will become increasingly less representative if voters become more polarized. Conversely, if parties seek only to please their current support base, in effect using an Aggregator rule, polarization of the electorate will not have the same adverse effects on party system representativeness, as the parties' policy positions track the diverging ideal point centroids of their support base. Furthermore, if one party were to use a vote-seeking rule while the other was an Aggregator, the vote-seeking party would desert its own support base and "chase" the Aggregator well away from the center ground. Under our model of endogenous parties, however, a new party birth (or rebirth) would occur in the deserted subpopulation. But then the cycle would repeat itself again, and again