Spatial voting theory: A review of literature.

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Preface

This work is intended to provide an overview of the building blocks of the spatial voting theory. The work focuses on the properties of the majority preference relationship, the median-voter theorem, and then proceeds to discuss the extensions of the candidate policy location model. The goal is to present the material in a coherent framework, state the main results as theorems, and, whenever possible, link the results with empirical research. Another goal is to illustrate the theoretical findings with relatively simple examples.

The reader may find this work to be complementary to such textbooks as Drazen's "Political Economy in Macroeconomics" or Persson and Tabellini's "Explaining Economic Policy". Although the analysis of economic policy is not the main intent of this work, several examples are provided.

The reader is assumed to know the basics of the normative public choice theory, such as the Arrow impossibility theorem. Whenever necessary, the author refers to textbooks such as Mueller's "Public choice", MasCollel, Whinston, and Greene's "Microeconomic Theory", or other sources.

Contents

Introduction

The most basic result of the theory of electoral competition is due to Hotelling (1929) and Downs (1957). Known as the median-voter theorem, it predicts that competing candidates will tend to select identical policy platforms, equal to the preferred policy of the median voter. However, in most real-life elections, the candidates take (or claim to take) different political platforms. A considerable amount of theory has been developed to explain that contradiction. In this work I try to provide a brief overview of the argument.

Section 1 contains a review of the main results that relate to the problem of selecting a policy through a voting procedure. I formulate the policy location game, in which two or more candidates propose policy outcomes that are then voted upon by a set of voters. Throughout the work, the preferences of the voters are assumed to satisfy certain regularity conditions that are defined early on. In subsection 1.1 I give an overview of the main assumptions about voter preferences made in spatial models. In 1.2 I define the policy location game and the electoral success function that translates the share of votes cast in favor of a candidate's policy into the utility of the candidate. Several properties of that function are investigated. It turns out that there is only one function that satisfies the property of monotonicity and symmetry if the number of candidates is greater than two. Subsection 1.3 presents the well-known median voter theorem of Duncan Black. Subsection 1.4 reviews the main results of Charles Plott, Richard McKelvey, and other authors on the existence of equilibria in two-candidate games with a majority electoral success function with a multidimensional policy set. In general, there is no pure-strategy equilibrium in a two-candidate location game, as the set of voter ideal points must meet a very restrictive condition in order for the majority preference relationship to be transitive. Subsections 1.5, 1.7, and 1.8 study several solution concepts that may explain the actual stability of observed voting outcomes.

From Section 2 on, the policy set is usually taken to be one-dimensional. Subsection 2.1 extends the model to include simultaneous policy choice by several candidates. In Subsection 2.2, I review the works of Thomas Palfrey, Joseph Greenberg, Shlomo Weber, and other authors that model a two- or several-candidate policy location game when there is the threat of entry by a new candidate once the policies are chosen.

Section 3 deals with the issue of a voter's participation in large elections — of the most recalcitrant problems of the rational choice theory. I present a history of the dilemma, starting with the "calculus of voting" approach of William Riker and Peter Ordeshook, then present several rational choice models that try to explain why some voters choose to vote and others choose to abstain when the voting is a costly act and the chances of influencing the election's outcome are very small. The game-theoretic arguments are not fully conclusive; modeling turnout in an elections of a real-life scale may require making alternative behavioral assumptions. Some of that literature is also reviewed. Finally, I outline several analytical shortcuts that can be made to provide for the possibility of voter abstention when studying other aspects of spatial voting models.

The apparent stability of voting outcomes led to the development of voting models where the outcome is not deterministic with respect to the policies proposed by the candidates. Section 4 gives a review of a group of models candidates differ not only in their policy positions, but also in their general level of competence — a characteristic known as *valence*. Subsection 5.1 amends the basic spatial voting model to provide for candidates with different exogenous valences, and discusses the main implications different valence. Subsection 5 looks at a basic probabilistic voting model, where a candidate is uncertain about how the voters will evaluate her valence. Subsections 4.2 and 4.3 investigate the consequences of candidates having different levels of valence in a deterministic world. Some empirical evidence of probabilistic voting behavior is provided in Subsection 5.4.

In Section 6, I look at some of the questions that may arise if one assumes that the voters are not fully informed about the positions of the candidates. The main issue is whether the candidate's policy positions and the outcomes of the elections are identical to those when the voters are fully informed.

Section 7 presents an overview of the spatial voting models where the candidates are motivated not only by winning the elections, but also by helping to realize some policy objective (and thus have preferences over policies just like voters do). The early works of Randall Calvert and Donald Wittman suggested that a candidate has to compromise between winning the elections and achieving her preferred policy (which may be different from that of the median voter). I then present the basic citizen-candidate models that consider a candidate to be unable to propose a policy other then her best, and instead focus on a voter's decision on whether to become a candidate in order to affect the policy that will be realized after elections.

The list of stakeholders in an election is not limited to voters and candidates. There are special interest groups, who, like voters, want the winning candidate to implement some policy, but have much greater influence over the identity of the winning candidate. For example, special interest groups are a source of campaign contributions that the candidates can use to advertise their positions. In Section 8 I sample the works that model the interaction between candidates and special interest groups. Section **??** looks at the role of political parties in spatial models. Parties either arise as coalitions, or are distinct economic agents with their own goals.

1 Preliminaries and the basic spatial voting model.

Spatial voting theory studies the interaction between two types of economic agents: voters and candidates. Voters collectively choose from a set of policy proposals that are made by the candidates. A voter is interested in choosing the proposal she likes best. A candidate is interested mainly in having her proposal supported by the greatest number of voters.

1.1 Preferences of the voters.

There is a set of agents that we call *voters*. In the models that we will study, there will be either an odd number or a continuum of voters. Each voter i is characterized by a complete, transitive and reflexive preference relationship $\succ_i$ on some set $X \subset \mathbf{R}^n$. We call this set the set of acceptable policies.

We first identify an important class of preferences for single-dimensional policy sets.

Definition. Let $n = 1$. Let u_s be the utility function of voter s. The preferences of the voter are *single-peaked* if there exists $v_s \in X$ such that for every $y_1, y_2 \in X$ such that $v_s > y_1 > y_2$ or $y_2 < y_1 < v_s$, we have $u_s(y_1) > u_s(y_2)$. The alternative v_s is the *bliss point* or the *ideal policy* of voter s.

Example. The Meltzer-Richards model[1]. There is a continuum of individuals, with the utility of individual s being

$$u(c_s, l_s) = c_s + \sqrt{1 - l_s}, \tag{1.1}$$

where c_s and $1 - l_s$ is the consumption and leisure of s. l_s is the labor supply of the individual; the total amount of time available for labor and leisure is equal to unity. Each individual

[1]See Metzler and Richards (1981) and Persson and Tabellini (2000, ch. 2).

maximizes her utility subject to the budget constraint

$$c_s \leq (1-t)\frac{l_s}{\alpha_s} + T, \tag{1.2}$$

where α_s is the amount of time it takes individual s to earn an income of 1, t is the tax rate, and T is the lump-sum transfer from the government. The utility-maximizing choice of l_s given t and T is

$$l_s^* = 1 - \frac{\alpha_s^2}{4(1-t)^2} \tag{1.3}$$

For simplicity's sake we allow l_s to be negative.

Assuming $\alpha_s < 2(1-t)$, the utility of the individual given optimal choices of consumption and labor is

$$u(c_s^*, l_s^*) = T + \frac{1-t}{\alpha_s} + \frac{\alpha_s^2}{4(1-t)}. \tag{1.4}$$

Suppose that all tax proceeds are redistributed equally among individuals. We have

$$T = \int t l_s^* ds = t - t\frac{\bar{\alpha}^2 + \sigma_\alpha^2}{4(1-t)^2}, \tag{1.5}$$

where $\bar{\alpha}$ and σ_{α^2} are the mean and variance of α_s. The utility of individual as a function of the tax rate is

$$u_s^*(t) = t - t\frac{\bar{\alpha}^2 + \sigma_\alpha^2}{4(1-t)^2} + \frac{1-t}{\alpha_s} + \frac{\alpha_s^2}{4(1-t)}. \tag{1.6}$$

One can check that this value is single-peaked in t for $t \in [0,1]$. Moreover, the t at which this value is maximized increases with α.

For a multi-dimensional policy set, we will generally restrict our attention to *Eucledian preferences*, described by a utility function

$$u_i(y) = -\phi(\|v_i - y\|) \tag{1.7}$$

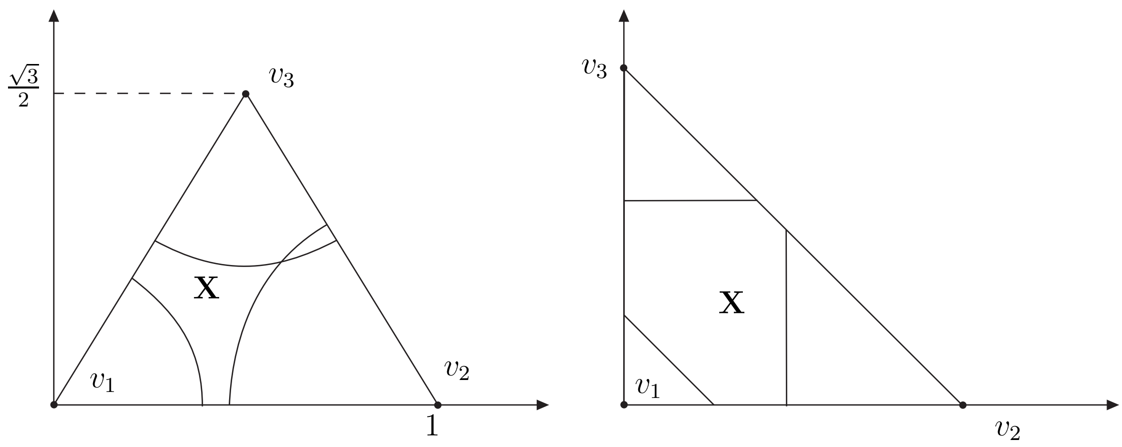
where v_i is the ideal policy of voter i, and ϕ is a twice differentiable function, $\phi(0) = 0$, $\phi'(\cdot) > 0$, $\phi''(\cdot) \geq 0$, $\|\cdot\|$ denotes Eucledian distance. We will call this the *disutility function* of voters. The ideal policies of voters are distributed on $\mathbf{R^n}$ according to some distribution function $F(\cdot)$. If this distribution is atomless, we denote its density by $f(\cdot)$.

This assumption simplifies our analysis without much loss of generality, as we are free to choose the set X.

Example. Suppose that there are three voters, who choose how to divide 1 unit of wealth among themselves. Let X be an equilateral triangle, with vertices $v_1 = (0,0)$, $v_2 = (1,0)$, $v_3 = (1/2, \sqrt{3}/2)$. An element $y \in X$ may represent such a division, with voter $i = 1,2,3$ receiving $1 - |v_i - y|$. Division v_i would be the ideal policy for voter i. See Figure 1(a). The same set of alternatives can be given by the triangle X' with the vertices $v_1' = (0,0)$, $v_2' = (0,1)$, and $v_3 = (1,0)$, if under an alternative $y' \in X'$ voter 1 receives y_1, voter 2 receives y_2, and voter 3 receives $1 - y_1 - y_2$.

There are several other types of preferences worth mentioning.

Example. Equation (1.7) implies that the voters are identical up to their ideal policies. However, the voters may have different marginal rates of substitutions for deviations from the



(a) Eucledian preferences (b) Another popular representation

Figure 1: Policy space, ideal points, and indifference curves for the 3-person division of 1 unit of wealth.

ideal policy along different policy dimensions. For example, for $n = 2$ we may want to study a case when one group of voters views deviations along the first dimension to be relatively more important. Elliptic preferences are a generalization of the preferences described by equation (1.7) that accounts for such inter-voter differences. Let A be a positive definite matrix. The the utility of voter s with ideal policy y be $u_s(y) = -\phi(\|v_s - y\|_{A_s})$, where $\|x\|_{A_s} = xA_sx$. The utility given by (1.7) is a special case for $A_s = I$.

Example. Under city-block preferences, for $x \in \mathbf{R}^n$ we have $\phi(x) = \sum_i a_i|x_i|$, where a_i are constants. The distance between the two points is defined as the sum of the distances along each dimension. This corresponds to the case of constant marginal rate of substitution between distances along each dimension (Rae and Taylor, 1971).

1.2 The policy location game.

Consider the following game. There are N agents that we will call *candidates*. Each candidate i chooses an element y_i from the set of acceptable alternatives $X \subset \mathbf{R}^n$. We call y_i the *policy platform* of candidate i. The set X is then the set of acceptable platforms, or policy set. By default we assume that the policy platforms are chosen simultaneously.

We assume that, after all candidates choose their policy positions, each voter s makes a voting decision $t_s \in \Delta^n$. We say that voter s supports candidate i with probability p_{si}.

The voteshare of each candidate is the expected mass of voters supporting that candidate. It is naturally defined as

$$V_i = \int p_{si} dF(v_s). \tag{1.8}$$

In order to close the model, we need to define two more things. First, one must provide for a way to convert voteshare into candidate payoffs. Let the payoff of candidate i be determined as follows:

$$U_i = G_i(V), \tag{1.9}$$

where $G(\cdot) : \Delta^N \to \mathbf{R}^n$ is the electoral success function (ESF) that translates voteshare into candidate utility, and $V \in \mathbf{R^n}$ is the vector of candidate voteshares.[2] Candidate utility

[2]If abstentions are allowed, then the function's domain is the $N+1$-dimensional simplex.

can have several interpretations. If the candidates are contesting a single office, their utility may be the the probability of winning the office multiplied by the value of office rents. If the "candidates" are actually political parties contesting seats in a legislature, the utility of a party might be the number of seats won.

Second, we must define the decision rule t for each voter s. In the following two sections, we consider these two points in detail.

It is useful to identify some of the desirable properties that might be possessed by an ESF. We follow the axiomatic approach proposed by Zakharov (2005)[3].

First, we want to require that the resulting utilities of the candidates do not depend on their identities. This property can be formalized as follows. If this symmetric property is satisfied, then the utilities obtained by the candidates depend only on their voteshares. If the property is violated, the identities of the candidates matter. Second, we require that the utility of each candidate does not decrease with her voteshare. Third, we want the function to be additive — the utility of each candidate should depend on her voteshare only. If this final assumption is violated, then no two candidates can increase their combined utility by exchanging votes. Thus we assume that the candidates have exhausted all coalition-building opportunities. Here is the formal definition of these properties:

Definition. The electoral success function is *symmetric* if for every $V \in \Delta^N$, and for every permutation of an N-dimensional vector $S(\cdot)$, we have $G(S(V)) = S(G(V))$. The electoral success function is *pairwise monotonic* if for every V, for every i, j, $G_i(V(t))$ is increasing in t, where $V_i' = V_i + t$, $V_j' = V_j - t$, $V_k' = V_k$ for $k \neq i, j$. The electoral success function is *fully monotonic* if for every i and for every $r \in \mathbf{R}^n$ such that $\sum_j r_j = 0$ and $r_i > 0$, $G_i(V + tr)$ is increasing in t.

The electoral success function is *additive* if for all $V, W \in \Delta^N$, for all i, j, we have

$$V_i + V_j = W_i + W_j \Leftrightarrow G(V)_i + G(V)_j = G(W)_i + G(W)_j \tag{1.10}$$

It can be shown that any fully monotonic electoral success function is also additive.

If there are only two candidates, then any monotonic ESF that is symmetric around the 45 degree ray satisfies all of the above properties. If the number of candidates is greater than two, then full monotonicity becomes a very restrictive condition. In fact, only one election success function satisfies the properties of anonymity and full monotonicity:

Theorem 1 *Let $N = 2$. Then, ESF is symmetric and monotonic if and only if $G_1 = g(V_1)$ and $G_2 = 1 - g(V_2)$, where $g(\cdot)$ is some increasing function with $g(\frac{1}{2}) = \frac{1}{2}$.*

Let $N \geq 3$. Then, ESF is symmetric and fully monotonic if and only if $G(V) = V$.

Example. Proportional representation.

$$G(V) = V. \tag{1.11}$$

Under a party list type electoral system, the number of seats won by a political party is approximately proportional to the share of popular vote received by that party. If we assume that the utility of a political party is measured in seats, (1.11) is a reasonable approximation of party utility[4].

[3]This approach is similar to Skaperdas (1996) and Clark and Riis (1998) axiomatization of contest success functions. The difference between an ESF and a contest success function is that the domain of the latter is $\mathbf{R}^n$, as we assume that in a contest there is no restriction on the sum of individual efforts.

[4]See Cox (1997, Ch. 3) or Mueller (2003) for an overview of actual electoral systems.

Example. Plurality rule.

$$G_i = \begin{cases} \frac{1}{\#\{i|V_i=\max\{V_1,\cdots,V_n\}\}}, & V_i = \max\{V_1,\cdots,V_n\} \\ 0, & V_i < \max\{V_1,\cdots,V_n\}. \end{cases} \tag{1.12}$$

Under a first-part-the-post electoral system, the election is won by the candidate with the greatest voteshare. Here we make an additional assumption that, if there is no single candidate with the largest voteshare, benefits are equally split among the candidates with the largest voteshares. The plurality rule obviously violates the additivity property if there are more than two candidates.

Example. Proportional representation with floor requirements.

$$G_i = \begin{cases} \frac{V_i}{\sum_{j|V_j \geq r} V_j}, & V_i \geq r \\ 0, & V_i < r \end{cases} \tag{1.13}$$

for some $r > 0$. In some countries, a political party can obtain seats in a legislature only if its share of popular vote exceeds a certain threshold.

The additive property for this ESF is violated. First, two parties can increase their combined utility if one of the parties has enough seats to clear the floor requirement, and the other party is very close to clearing the floor requirement. Consider 3 parties, A, B, and C, with 60%, 31%, and 9% of the vote. If the floor requirement is 10%, then the combined share of seats of parties B and C is 34,1%. At the same time, the share of seats that will be won by a coalition between parties B and C is 40%.

The second point is that the additional share of seats that a party can obtain by increasing its voteshare at the expanse of a party that fails to clear the threshold is smaller for larger parties. Consider 3 parties, A, B, and C, with 60%, 30%, and 10% of the vote, respectively. If the floor requirement is 15%, then party A obtains $\frac{2}{3}$ of seats, while party B obtains $\frac{1}{3}$. If party A increases its voteshare by 1% at the expense of party C, then the number of seats it holds will increase by approximately 0,33%. An equal increase in the voteshare of party B will increase the number of seats it holds by 0,66%. At the same time, if party A increases its utility by 1% at the expense of party B, its voteshare will increase by 0,66%. The utility of party B will increase by the same amount if it increases its voteshare by 1% at the expense of party A.

1.3 Median voter theorem.

We can now proceed to characterize the Nash equilibrium of the policy location game described above. We start by considering a simple example.

Example. Let $N = 2$, $n = 1$, and the ESF is proportional. The ideal points of the voters are uniformly distributed on the interval $[0, a]$. If the position of the candidates are $y_1 < y_2$, then all voters with $v < \frac{y_1+y_2}{2}$ support candidate 1, while the voters with positions $v > \frac{y_1+y_2}{2}$ support candidate 2. It is straightforward to show that in the only equilibrium we have, the positions of both candidates correspond to the policy position of the median voter: $y_1 = y_2 = \frac{a}{2} = y_{med}$.

There is the well-known example from Hotelling (1929), where two firms compete to to sell a homogeneous product to customers evenly spread along a linear market. If each customer had unelastic demand, no price competition is allowed, and firms have constant marginal costs, then the payoff to each firm is proportional to the number of customers who buy its product. If customers have positive transportation costs, each customer buys the product from the nearest firm, and in equilibrium both firms are located at the median customer.

This result can be formalized as follows[5].

Theorem 2 *(median voter theorem). Let $N = 2$ and $n = 1$. Then for any $G(\cdot)$, $F(\cdot)$, in any Nash equilibrium we must have $y_1, y_2 \in Y_{med}$, where*

$$Y_{med} = \left\{ y | F(y) = \frac{1}{2} \right\} \cap X. \tag{1.14}$$

Suppose that we can order the ideal policies of the voters (say, on a left-right scale). A median policy is such that it is opposed by no more than one half of the voters. If voters are discrete and their number is odd, then there is a unique equilibrium position corresponding to the one median voter. If the number of voters N_v is even, then for any policy platform in the set $(v_{\frac{N_v}{2}}, v_{\frac{N_v}{2}+1}) \cup X$, there are $\frac{N_v}{2}$ voters with ideal policies to the left and to the right of that policy platform. If there is a continuum of voters distributed according to an atomless density, there exists a unique median policy unless the voters can be divided into two disjoint groups, each of weight exactly $\frac{1}{2}$.

This is the main result in the spatial theory of voting and candidate location. It is based on simple and intuitive assumptions A large part of this work will focus on the theory explaining why the candidates might actually want to propose different policy platforms.

1.4 Nontransitivity of majority rule.

The Arrow (1963) impossibility theorem states that for any rule defining social preference relationship over a set of alternatives, there exists a profile of voter preferences such that the social preferences are not transitive. The median voter theorem tells us about the restrictions that we should impose on the preferences of voters to guarantee that the majority preference relationship is transitive[6].

One of the assumptions of the median voter theorem is that the policy space is one-dimensional. If the number of policy space dimensions n is 2 or greater, then the majority preference relationship is generally not transitive, and there is generally no equilibrium of the type described by the median voter theorem, even if the preferences of the voters are still described by functions of the type (1.7)[7].

Consider policies $y_1, y_2 \in X$. We will say that policy y_1 is majority preferred to policy y_2 if the voteshare of candidate 1 is greater than $\frac{1}{2}$, and denote $y_1 \succ y_2$.

Definition. Let $y \in X$ be such policy alternative that for no policy alternative $y' \in X$ we can have $y' \succ y$. Then y belongs to the core of the policy location game.

An element of the core is also known as the Condorcet champion or the dominant policy. In any pure-strategy equilibrium in a game with two candidates, the policy positions of both candidates must belong to the core.

[5] In fact, this statement holds for a family of utility functions that is more general than (1.7). The result, due to Black (1958) and Arrow (1963), is that it is sufficient for candidate utility to be single-peaked. A related concept are the single-crossing preferences of Gans and Smart (1996).

[6] In the framework of this work's model, we say that policy y_1 is majority-preferred to policy y_2 if $N = 2$ and $G_1 > G_2$.

[7] According to May (1952), if there are only two alternatives, then the majority preference is the only one that satisfies several plausible assumptions. For a discussion of this and other results see Mueller (2003, chapters 5 and 24) or MasCollel, Whinston, and Greene (1995, chapter 21).

Example. Let $n = 1$. If there is an odd number of voters, then the core consists of a single element — the median voter's ideal policy. If there is an even number of voters N, the core is the interval $[v_{\frac{N}{2}}, v_{\frac{N}{2}+1}]$.

If the policy set X is bounded, then a sufficient condition for the existence of the core is the transitivity of the preference relationship $\succ$. In a number of works, starting with Plott (1967), it was shown that if the number of policy space dimensions is two or greater, the conditions for the transitivity are very restrictive[8]:

Theorem 3 *Suppose that there is an odd number of voters N. Call voter y the total median if on every line passing through i, there is an odd number of voters, and for every such line, y is the median voter with respect to other voters on that line. Then, the core exists if and only if there is a total median, which is the only element of the core.*

If this condition is satisfied (such as in Figure 2(a)), then there is always the same number of voters to the left and to the right of any line that passes through y. Now consider an arbitrary policy x. Voters that prefer y to x lie to the right of the line L, as shown on Figure 2(a). Consider the line M that is parallel to L and passes through y. By our assumptions, more than one half of all voters (including y) lie to the right of M. Thus y is preferred to x, and is the Condorcet champion. If there is no median (such as in Figure 2(b)), then for every policy y it is possible to construct a policy x such as $x \succ y$.

The following result is due to Davis, de Groot, and Hinich (1972):

Theorem 4 *Suppose that the conditions of Theorem 3 are satisfied. Then, the majority preference relationship is transitive.*

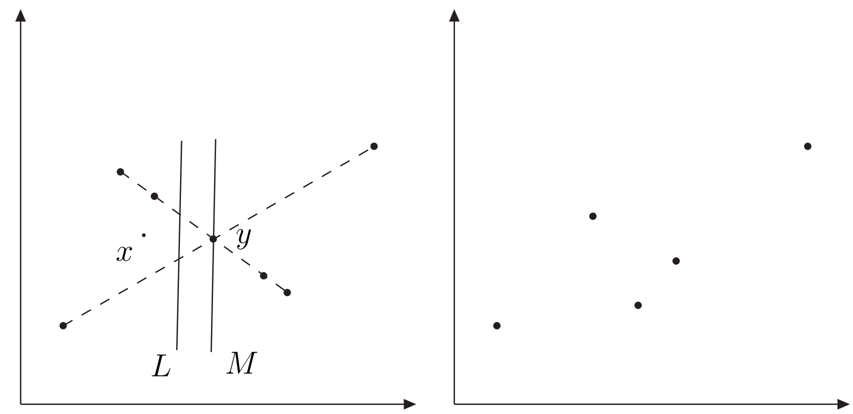
Example. The earliest and best-known example of majority rule nontransitivity is the Condorcet paradox. Suppose that there are three voters and three alternatives, x, y, and z. Voter 1 prefers alternative x to y and y to z, Voter 2 prefers y to z and z to x, and Voter 3 prefers z to x and x to y. Then, the majority rule would choose x over y, y over z, and z over x. preferences majority rule would It is straightforward to check that these preferences the voters cannot be represented by a one-dimensional Eucledian (or even single-peaked) utility function.

Example. For city-block preferences, Rae and Taylor (1971) demonstrated that the Condorcet winner always exists for $n = 2$. For higher dimensions, the existence of the Condorcet winner is not assured.

Example. Suppose that $n = 2$, and there are three voters with positions v_1, v_2, and v_3, as on Figure 3. Let y be some policy. Then voters 1 and 2 will prefer to y any policy in region a (bounded by indifference curves of the two voters), voters 1 and 3 will prefer any policy to region b, and voters 2 and 3 will prefer any policy in region c. Since y is an arbitrary policy, the core is empty.

Example. Consider Hotelling's example for a two-dimensional city. If the city is radially symmetric, then every line passing through its center divides it into two parts of an equal area.

[8] Davis, DeGroot, and Hinich (1972) derived similar conditions for a continuum of voters. See also Slutsky (1977a). Greenberg (1979) investigated the existence of the Concorcet champion if the issue must be decided by a majority $0 < \alpha < 1$. It was shown that if α is sufficiently large, the Concorcet champion exists. See also Slutsky (1979). Kramer (1973) and Schofield (1977) derived local conditions for transitivity of majority preferences for voters with preferences represented by differentiable utility functions, and found them to be very restrictive.



(a) Voter y is the total median. (b) There is no total median.

Figure 2: The existence of total median for $n = 2$.

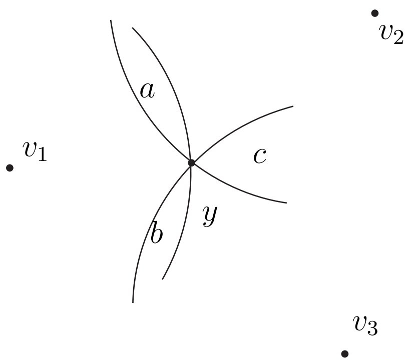


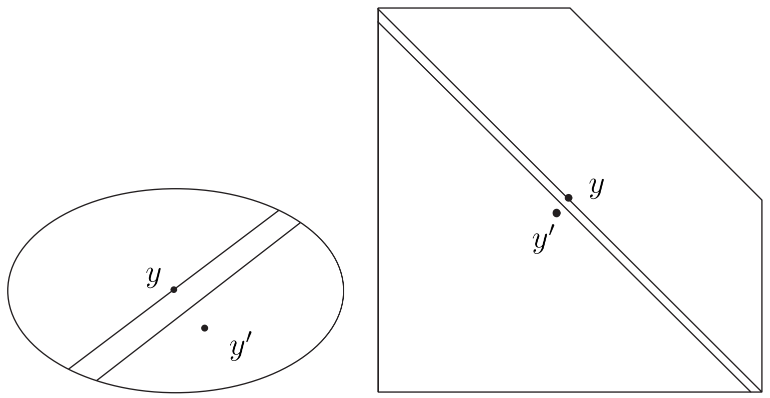
Figure 3: Intransitivity of majority preference for 3 voters, $n = 2$.

In an equilibrium, both firms are located at the center, and the payoff to each firm is equal to one half. Any deviation from the center, however, results in a loss. This can be seen from Figure 4(a). Suppose that $y_1 = y_2 = y$. Suppose that one firm deviates to y'. The line between points y and y' represents the customers who are indifferent between the firms. The area on the y' side of that line will always be smaller than that on the y side.

If the city is not radially symmetric, then the equilibrium does not exist. By definition, for any location there exists a line that passes through that location and divides the city into two parts of unequal area. Thus, for any location of the opponent firm, it is possible to choose a location that will attract more than one half of the customers (see Fig. 4(b)).

Example. Let $n = 2$. Suppose that there is a continuum of voters, consisting of three equilateral triangles of equal size, joined at one vertex, as shown on Figure 5. The points A, C, and E are located at an equal distance from the center; the points B, D, and F are also located at some other distance from the center. The arrangement is such that the line separating the voters supporting A and F is parallel to the base of the two triangles that form the set of voters. If A and F are sufficiently close to the center, then A is preferred to F and B is preferred to A. But then C is preferred to B, and preferences form a cycle.

[8]The analysis of mixed strategy equilibrium involves the notion of the so-called uncovered set. Policy A is said to cover policy B if A is majority–preferred to B and for every C that is majority–preferred to A, C is also majority–preferred to B. Thus if both candidates choose policies simultaneously, then B a dominated position, since by choosing A a candidate can do at least as well. Hence the uncovered set is the set of all policies that



(a) A radially symmetric city: the unique center exists.
(b) Not a radially symmetric city: the center does not exist.

Figure 4: The existence of the median voter if the policy set is two-dimensional.

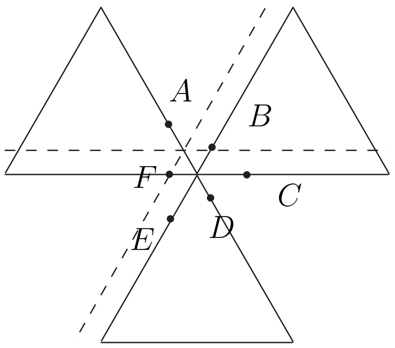


Figure 5: Nontransitivity of majority preference if the policy set is not radially symmetric

If the Condorcet champion does not exist, the transitivity of the majority preference breaks down, since there must exist some alternatives $x_1, \cdots, x_n$ such that $x_1 \succ \cdots \succ x_n \succ x_1$. Otherwise, every finite set of alternatives has the element that is preferred to every other alternative in this set.

One might try to analyze such cycle sets generated by the majority preference relationship.

Definition. The preference relationship $\succ$ divides the policy set X into disjoint sets $X_1, \cdots, X_m, X'$[9] if

1. For every i, for every $x, y \in X_i$, there exist $x_1, \cdots, x_l \in X_i$ such that $x \succ x_1 \succ \cdots \succ x_l \succ y$.
2. For every $i = 1, \cdots, m-1$, for every $x \in X_i$, $y \in X_{i+1}$, we have $x \succ y$.

Thus, a majority path can be constructed between any two elements belonging to the same cycle set, but between elements belonging to two different sets only a one-way majority path is possible. The set X' is the set of alternatives over which there is no cycle.

The top cycle set X_1 obviously exists. Moreover, every element of this set must be majority-preferred to every element of the set X'.

Early literature, such as Buchanan (1968), Tullock (1967) or Sen (1976), suggested that the the size of the top cycle set X_1 may be shrinking the number of voters increases — thus making

are not covered by any other policies. Works in this direction include, among others, Cox (1987b), McKelvey and Ordeshook (1976), McKelvey (1986), Miller (1980), and Shepsle and Weingast (1984).

[9]Here we assume, for simplicity's sake, that there is a finite number of such sets.

it a natural candidate for a social choice function that represents a preference relationship under which the society is indifferent between policies belonging to the same cycle set.

However, the results yielded by a more rigorous analysis questioned the appropriateness of the top cycle set as a solution concept. Consider the following theorem, proved by McKelvey (1976).

Theorem 5 *Let $N \geq 3$ and $n \geq 2$. Then, for the majority preference relationship, there is only one cycle set $X_1 = X$.*

This theorem states that the cycle set generated by the majority voting procedure is the whole set of acceptable policies X. Thus, starting from an arbitrary position, it is possible to arrive at any outcome — even a Pareto-dominated one.

Example. The "shining path" (citation needed). Consider a three-member committee. There are 4 policy proposals. Under proposal A, the payoffs to committee members are $A = (10, 10, 10)$. The other three proposals are $B = (11, 11, 0)$, $C = (12, 0, 1)$, and $D = (1, 1, 2)$. Clearly, B is majority-preferred to A, C to B, and D to C. At the same-time, A is Pareto-superior to D.[10]

Subsequent research produced more results on the failure of the majority preference. Cohen (1978) extended the previous result to elliptical preferences. Schofield (1978) and McKelvey (1979) demonstrated between any two policies there exists a continuous path that is arbitrary close to *any* continuous path connecting the two policies, if the dimensionality of the policy set is large enough. The former work required that $n \geq \frac{N}{2}$, while the latter work relaxed that restriction. Additional dimensionality conditions for cycling with odd or even number of voters were derived by Schofield (1983).

1.5 Some rationale for stability of majority voting

The apparent stability of the decisions made by majority voting in a variety of political institutions poses a theoretical challenge, formulated initially by Tullock (1981).

One solution concept that allows for the existence of a pure-strategy equilibrium in the policy location game is the bicameral core of Hammond and Miller (1987). Suppose that there are two groups of voters with Eucledian preferences, and that in order for alternative x to succeed against alternative y, x must be majority-preferred to y by each of the two voter groups. This argument can apply to policies that need approval of both chambers of a bicameral parliament — hence the concept's name. Thus, either alternative x succeeds against y, y succeeds against x, or the support of the chambers is split between the alternatives. The bicameral core consists of all alternatives y such that there is no $y' \in X$ can defeat majority-defeat y within each group of voters.

The bicameral core can be shown to exist for any two odd-numbered groups of voters. In the example on Figure 6, there are 2 groups of voters: group V with positions v_1, v_2 and v_3, and group W with positions w_1, w_2, and w_3. The bicameral core consists of all alternatives on the interval between policies P and R. For any alterantive not on the line connecting v_2 and w_2, there exists an alternative on that line that is majority-preferable by both groups of voters (such as alternative y is preferable to y'). For any point on the line but outside of the

[10]It is straightforward to show that if the number of committee members is N and K votes are required to decide on the issue, then a Pareto-dominated outcome can be reached in no more than $\lfloor \frac{N}{N-K} \rfloor + 1$ steps, where $\lfloor x \rfloor$ denotes the integer part of x.

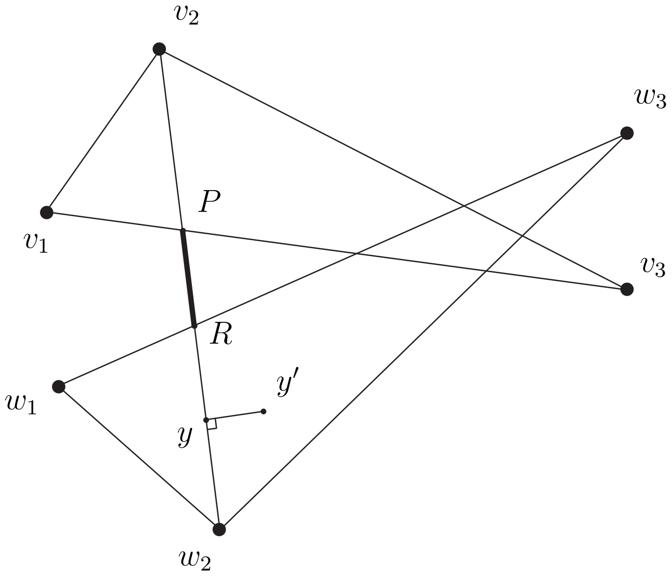


Figure 6: The bicameral core

RP interval is majority-preferable either by P or by R, such as R is preferred to y by voters 1 and 2 from group V, and by voters 1 and 2 from group W. Thus the bicameral structure adds stability to the policy location game.[11]

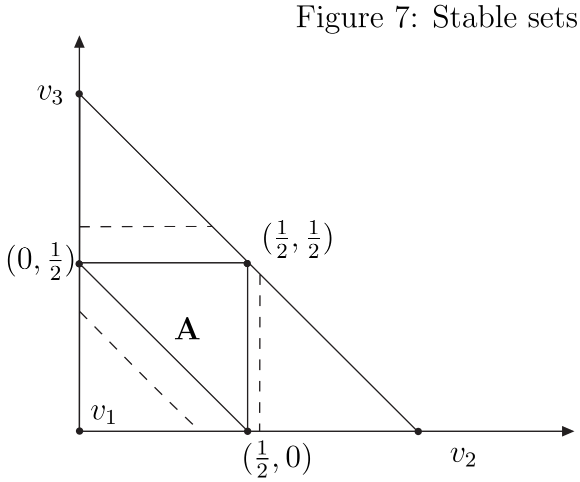
Another solution concept is the von Neumann-Morgenstern stable sets.

Definition. S is a von Neumann-Morgenstern stable set if for every $y \notin b$, there exists $x \in S$ such that $x \succ y$.

This approach is somewhat problematic because the stable set is not unique. However, one may try to look for the smallest possible stable set.

Example. Let $N = 3$, with voter preferences as on Figure 1(b). Then any set $\{(y_1, y_2)|y_2 \leq \frac{1}{2} + a, y_1 \leq \frac{1}{2} + a, y_1 + y_2 \geq \frac{1}{2} + a\} \cup X$, $a \geq 0$, is a stable set.

Figure 7: Stable sets in the divide-the-dollar game.



The triangle A on Figure 7 depicts the smallest possible stable set. Indeed, a stable set in

[11]The Hammond and Miller (1987) work also investigates the core when one of the voters has vetoing power, and under other institutional structures. The more recent work on the bicameral core includes Lynch and Schnieder (2005), for a comparison of the bicameral core and uncovered set solution concepts, and Miller, Kile, and Hammond (1996), for experimental evidence supporting the bicameral core theory.

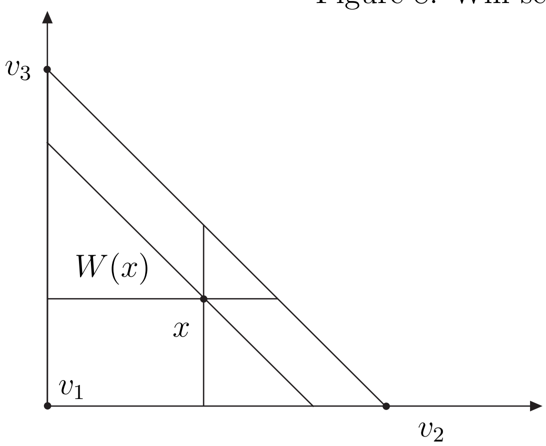
this example must be convex, and must include the alternatives where one of the voters receives zero.

Yet another concept is the Copeland winner.

Definition. The win set $W(x)$ is the set of all alternatives y such that $y \succ x$. The Copeland Winner (or strong point) is the alternative x with the smallest win set $W(x)$.

It is the alternative that is beaten by the fewest number of alternatives. Unlike the von Neumann-Morgenstern sets, the strong point is generally unique. The strong point coincides with the Condorcet winner, if the latter exists.

Figure 8: Win set in the divide-the-dollar game.



Example. Let the voter preferences be as in the previous example. The win set $W(x)$ is $\{y|y_1 \geq x_1, y_1+y_1 \leq x_1+x_2\} \cup \{y|y_2 \geq x_2, y_1+y_1 \leq x_1+x_2\} \cup \{y|y_1 \geq x_1, y_2 \geq x_2\}$ (see Figure 8). The area of the $W(x)$ is $\frac{1}{2}(x_1(1-x_2)+x_2(1-x_1)+(1-x_1)(1-x_2)$. It is straightforward to check that $W(x)$ is minimized at the equal division $x_1 = x_2 = \frac{1}{3}$[12].

1.6 The minimax set and alternative majority rules.

If the elections are decided by a simple majority, then any alternative can almost always be defeated by some other alternative. However, if some other decision rule is employed, a Condorcet champion may exist.

Example (Mueller, 2003, p. 100). Suppose that $n = 2$ and the voters are uniformly distributed on an equilateral triangle (see Fig. **??**). Then point g cannot be defeated by any alternative if a $\frac{5}{9}$ majority is required for one alternative to defeat another. If a $\frac{8}{9}$ majority is required, any alternative in the region $ABCDEF$ cannot be defeated.

We now give a definition of a solution concept proposed by Kramer (1977).

Definition. For alternatives $x, y \in X$, denote by $n(x, y)$ the fraction of voters who prefer y to x. Let $v(x) = \max_y n(x, y)$ be the largest share of voters who may prefer some other alternative to x. Then, the minimax number is

$$v_m = \min_x v(x), \tag{1.15}$$

[12]The Copeland winner always exists. Shapley and Owen (1985) and Grofman et al (1987) show that the Copeland winner can be easily calculated in N is odd, $n = 2$, and voter ideal points located at the vertices of a convex polygon.

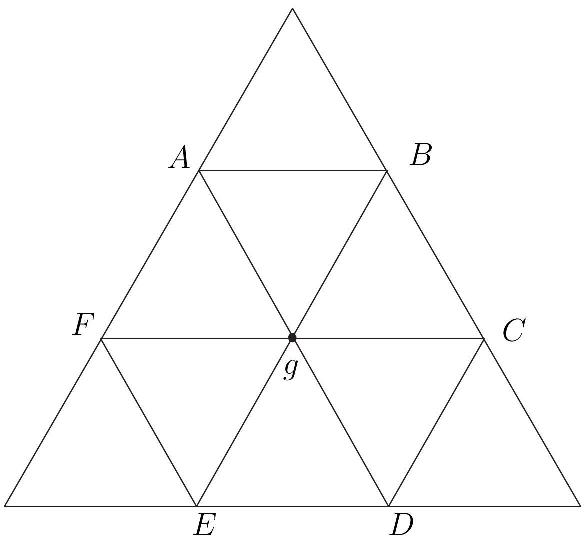


Figure 9: Unbeatable alternatives under various majority rules.

while the minimax set is

$$M = \{y|v(y) = v_m\}. \tag{1.16}$$

In the previous example, the minimax number is $v_m = \frac{5}{9}$ — under a less stringent majority rule any alternative (even g) can be defeated by some other alternative.

The minimax set in the previous example consists of the single alternative g. However, if the number of voters is finite, the minimax set usually consists of a large number of elements.

Example. On Figure 10, any alternative in area A can be defeated by 5 votes, any alternative in area B — by at most 3 votes, in area M — by at most 1 vote.

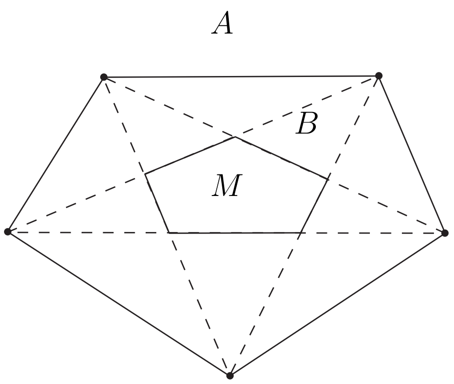


Figure 10: Minimax set for a finite number of voters.

Kramer (1977) observed that the function $v(y)$ defined a social preference relationship, with the minimax set being the top element. Indeed, if the Plott symmetry conditions are satisfied, then the minimax set consists of only one alternative — the Condorcet champion.[13]

[13]Kramer (1977) proposed a dynamic model of election where each subsequent policy alternative defeats its predecessor by the largest possible margin. The main theorem proven in that work is that for any such sequence (where $n(x, x_{t+1}) = v(x_t)$) each successive element is closer to the minimax set than its predecessor. The sequence eventually enters the minimax set. Aizerman (1981) reviews literature on dynamic voting models; additional works in this field include Chebotarev (1984) and Novikov (1985).

Snyder and Ting (2006)

The minimax number v_m defines the majority rule that guarantees the existence of a Condorcet champion. The first estimate of an upper bound of v_m was produced by Greenberg (1978):

Theorem 6

$$v_m \leq \frac{n}{n+1}. \tag{1.17}$$

It follows that if the number of policy dimensions is large, we need to employ a near-unanimous majority rule in order to guarantee the existence of a Condorcet winner. Caplin and Nalebuff (1998) produced a stronger result[14]:

Theorem 7 *Let X be convex and the density of ideal points $f(y)$ be a concave function. Then,*

$$v_m \leq 1 - \frac{1}{e} \approx 0.63. \tag{1.18}$$

The concavity condition on $f(y)$, however, is restrictive. It fails if the society is divided into several groups or clusters, with the voters belonging to each cluster having relatively similar policy preferences, but with great differences among the policy preferences of the voters belonging to different clusters. This is especially likely to happen if one of the policy dimensions involves redistribution of wealth between some two groups of voters.

1.7 The uncovered set.

The existence of a pure-strategy Nash equilibrium in a two-candidate location game is a knife-edge result that requires the existence of the total median. An important solution concept for the case when the total median does not exist is the uncovered set.

Definition. Let $x, y \in X$. Alternative x is said to cover alternative y if $x \succ y$ and for every $z \in X$ such that $z \succ x$, we have $z \succ y$.

If x does not cover y, then either $y \succ x$, or there exists z such that $y \succ z \succ x$[15]. If x covers y, then y can defeat x in no less than three steps. The *uncovered set* is thus the set $\{x \in X | \text{there is no } y \in X \text{ such that } y \text{ covers } x\}$.

In a mixed-strategy two-candidate equilibrium, the support of each candidate belongs to the uncovered set. If alternative y is covered by some x, then no candidate will play y, since every alternative that is defeated by y is also defeated by x.

Cox (1987b) and McKelvey (1986) demonstrated that the uncovered set exists, and is smaller if the set of alternatives X is closer to being symmetric. More exactly, the uncovered set is contained within a circle of radius $4r$, where r is the radius of the yolk[16]. As the set of alternatives approaches a symmetric set, the uncovered set collapses to the core.[17]

Example. Suppose that the total median exists. Then the uncovered set obviously consists of only one element — the total median, which covers every other alternative.

[14]See also Caplin and Nalebuff (1991).

[15]Related notions are the admissible set proposed by McKelvey and Ordeshook (1976), and the competitive solution of McKelvey, Ordeshook, and Winer (1978).

[16]The yolk is the smallest disc intersecting all median lines.

[17]The the uncovered set as a solution concept was proposed by Miller (1980). Penn (2002) and Miller (2006) contain a review of literature on this topic.

Example. Consider a three-person division game. The set of alternatives is $X \in R^3$, $x_1 + x_2 + x_3 = 1$, where $x_i \geq 0$ is the payoff to voter i. Then the uncovered set is the set of all divisions where at least two voters get positive payoffs.

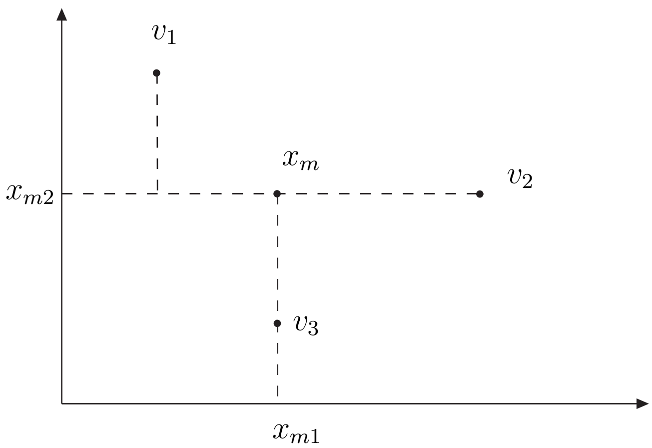
Analytic methods were used to derive the upper bounds of the uncovered set. Its exact shape, however, is difficult to derive analytically. Numeric methods, such as in Bianco et. al. (2006), have shown the uncovered set to be large in most finite-voter examples, eroding the set's value as a solution concept. The exceptions are distributions of voter preferences that are close to being symmetric in the Plott (1967) sense, with one voter located centrally relative to the other voters.

1.8 Issue-by-issue voting

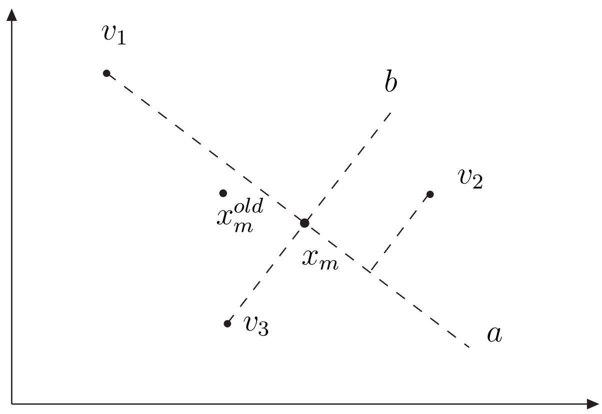
If the policy space is multi-dimensional, a well-defined voting outcome outcome can be achieved if the resulting policy is chosen one dimension at a time.

Let there be an odd number of voters with positions $v_1, \cdots, v_N \in \mathbf{R}^N$ be the positions of voters. If the preferences of the voters are Eucledian or elliptic, then they are single-peaked along each dimension. The structure-induced equilibrium of Shepsle (1979) is the policy $v_m(v_1, \cdots, v_N)$ such that its ith component is the median along the ith dimension. For Eucledian preferences, the voting outcome does not depend on the order in which the issues are voted on[18].

There are two problems with this solution concept. First, one can manipulate the solution by allowing the committee to vote on a linear combination of policy.



(a) Voting along issues 1 and 2



(b) Voting along a linear combination of issues 1 and 2

Indeed, suppose that $G(v)$ is a distance-preserving linear transformation of the policy space. Then, in general, we will have $G(v_m(v_1, \cdots, v_N)) \neq v_m(G(v_1), \cdots, G(v_m))$. On Figure 11(a) we choose positions along axes 1 and 2. On Figure 11(b), we choose positions along perpendicular rays a and b, and get a different equilibrium.

The second problem is that the equilibrium may be Pareto-inefficient (Slutzky, 1977b).

Finally, there may be more than one voting equilibria if the policy preferences of the voters are not Eucledian. In the Eucledian case, each voter's preferred policy along each dimension is independent of the policies that will be realized along other dimensions. Thus the order in which the issues are voter upon is irrelevant. This does not hold for a more general case (such

[18] See Feld and Grofman (1987) for a geometric proof of the uniqueness of the median.

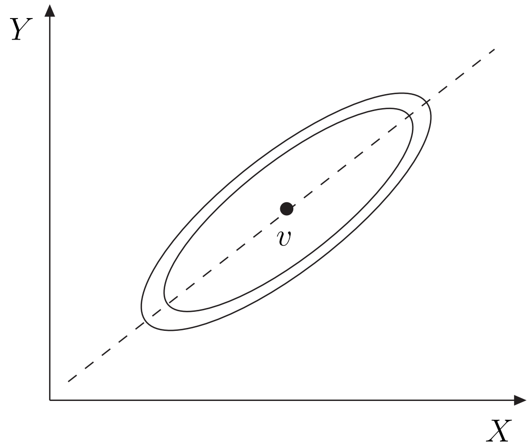


Figure 11: Elliptic preferences

as elliptic preferences). On Figure 11, the voter's preferred policy along each dimension X depends on the policy that will be realized along dimension Y, and vice versa. Thus her vote on X is likely to depend on the expectations of Y. A number of works investigate the role of expectations in structure-induced equilibria in these more general settings. See Enelow and Hinich (1983), Denzau and Mackay (1981), Enelow (1984), and Epple and Kadane (1991).

2 Extensions.

2.1 Number of candidates.

The properties of the majority preference relationship produced a dilemma. On one hand, the majority preference is intransitive for $n > 1$, and leads to a median-voter equilibrium for $n = 1$ if the relatively plausible single-peakedness assumption is satisfied. On the other hand, empirical evidence suggests that the policies selected by the actual candidates are both polarized and relatively stable. In this section we extend the model to include competition among more than two candidates, and consider the case when the candidates choose their policies sequentially.

Following Eaton and Lipsey (1975), Cox (1987a) and Osborne (1993) we start by looking for equilibria in a one-dimensional model with a number of candidates greater than two. We assume that the voters are sincere. Thus, for analytic convenience we take the distribution $F(\cdot)$ to be atomless, with support of $[0, 1]$.

First, we derive a necessary condition for the existence of an equilibrium. Denote by $w(y)$ the number of candidates with position y. Let $L(y)$ be equal to $F(y) - F(\frac{y+y_l}{2})$, where y_l is the position of the candidate to the left of candidate with position y, or 0 if y is the leftmost candidate position. Similarly, define $R(y)$. We have the following:

Theorem 8 *Suppose that $n = 1$, G_i is monotonic, and $y_1, \cdots, y_N$ are candidate positions in a pure-strategy Nash equilibrium. Then, the following must hold:*

1. *For all i, $w(y_i) \leq 2$.*

2. *For $y = \min_i y_i$ and $y = \max_i y_i$, we have $w(y) = 2$.*

3. *If $w(y) = 2$, then $L(y) = R(y)$.*

The proof of this statement is simple. If any number of candidates shares a position y, then their combined voteshare must be $R(y)+L(y)$. If $R(y)>L(y)$, then any candidate can choose a position $y+\epsilon$, obtaining $R(y)>\frac{R(y)+L(y)}{w(y)}$. If $w(y)>2$, then $R(y)>\frac{R(y)+L(y)}{w(y)}$ even if $R(y)=L(y)$. If $y=\max y_i$ and $w(y_i)=1$, then $V_i=1-F(\frac{y+y_l}{2})$, and candidate i can benefit by moving to the left. Finally, if $y_i \neq y_j$ and $L(y_i)>L(y_j)$, then candidate j can benefit by choosing $y_i+\epsilon$.

Example. Let $N=2$. Then, for any $F(\cdot)$ it follows that the conditions of Theorem 8 are satisfied only for $y=y_1=y_2$ such that $L(y)=R(y)$, which is obviously the statement of the familiar median voter theorem. For $N=3$ it follows that there can be no pure-strategy equilibrium. For any y_1, the optimal responses of candidates 1 and 3 would be to select $y_1=y_2-\epsilon$, $y_3=y_2+\epsilon$, where ϵ is an arbitrarily small number. The voteshares of the candidates would be $G_1=F(y_2)$, $G_3=1-F(y_2)$, and $G_2=0$. Candidate 2 would respond by leapfroging one of the other candidates[19] (see Figure 12(a)).

For a large number of candidates, the type of the electoral system may be important. Literature distinguishes between two types of candidate behavior:
Definition. The candidates are said to be *share-maximizers* if $G(V)=V$. The candidates are *plurality-maximizers* if[20]

$$G_i = V_i - \max_{i \neq j} V_j. \tag{2.19}$$

Under the first assumption, a candidate is better off whenever her voteshare increases, regardless of how the voteshares of the other candidates are distributed. Due to Theorem 1 and its corollary, this is only possible if the electoral system is proportional. Under the second assumption, the candidate's benefit is equal to her *plurality* — the difference between her voteshare and the highest voteshare among other candidates.[21]

If the candidates are share maximizers, then, in addition to the conditions of Theorem 8, we must have $L(y_i)=L(y_j)$ and $R(y_i)=R(y_j)$ whenever $w(y_i)=w(y_j)$ for some $i \neq j$. If this condition fails, say due to $R(y_i)>R(y_j)$, then candidate j may increase her payoff by choosing the position $y_i+\epsilon$.

If the candidates are plurality maximizers, then this condition may be violated in equilibrium.

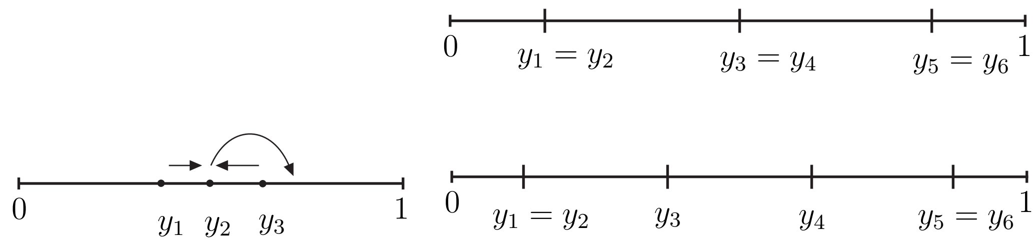
Example. Let $N=4$ and $F(\cdot)$ be uniform. In equilibrium, we have $y_1=y_2=\frac{1}{4}$ and $y_3=y_4=\frac{3}{4}$. For $N=5$, we have $y_1=y_2=\frac{1}{6}$, $y_3=\frac{1}{2}$, and $y_4=y_5=\frac{5}{6}$ for $N=5$. Note that in the five-candidate case, the voteshare of candidate 3 is greater than the voteshares of other four candidates. For $N \geq 6$, the conditions of Theorem 8 admit a continuum of equilibria, with $y_1=y_2=\frac{1}{6}-\frac{x}{3}$, $y_3=\frac{1}{2}-x$, $y_4=\frac{1}{2}+x$, and $y_5=y_6=\frac{5}{6}+\frac{x}{3}$, for any $x \in [0,\frac{1}{8}]$. See Figure 12(b).

The continuum of equilibria in the last example is a result of the assumption that the ideal policies of the voters are distributed uniformly. Consider three candidates with positions $y_1<y_2<y_3$. The voteshare of candidate 2 is equal to $F(\frac{y_3+y_2}{2})-F(\frac{y_1+y_2}{2})$. If the distribution

[19] The absence of equilibrium for in a three-candidate spatial competition was noted as early as in Chamberlain (1933). Rigorous proof of this and other multiagent results appeared in Eaton and Lipsey (1985) and Denzau, Katz and Slutsky (1985). The latter work dealt with candidates who maximize rank, which is based on the weighted average of the number of candidates with greater and equal voteshare.

[20] See Aranson, Hinich, and Ordeshook (1974) for a discussion of various candidate objective functions.

[21] Some literature also distinguishes strict plurality maximizers and simple plurality maximizers, who are indifferent between any two outcomes in which their plurality is strictly positive.



(a) $N = 3$ (b) Limiting cases for $N = 6$

Figure 12: Equilibria in the Downs model with different number of players.

is uniform, this value is equal to $\frac{y_3+y_2}{2}$, and does not depend on y_2. For any other distribution, it does.

Theorem 8 also applies if the policy space is a circle, with the exception that condition 2 is no longer relevant. Thus for $N = 2$ and uniformly distributed preferences, any two positions are equilibrium. For $N = 3$ there is a continuum of equilibria. As the number of candidates increases, the equilibrium becomes similar to that of the linear model.

For distributions of preferences different from the uniform distribution, equilibria generally do not exist for $N \geq 4$.

Example. Let $N = 4$ and $F(\cdot) \neq const$. The following conditions must be satisfied in equilibrium: $y_1 = y_2$, $y_3 = y_4$, $2F(y_1) = F(\frac{y_1+y_3}{2})$, $1 - F(\frac{y_1+y_3}{2}) = 2(1 - F(y_3))$, and $F(\frac{y_1+y_3}{2}) = \frac{1}{2}$. The first four are due to conditions 2 and 3 of the above theorem. The last one is to guarantee that candidate 1 will not choose position $y_3 - \epsilon$ or candidate 2 will not choose $y_1 + \epsilon$. After simplification, these translate into $F(y_1) = \frac{1}{4}$, $F(y_3) = \frac{3}{4}$, and $F(\frac{y_1+y_3}{2}) = \frac{1}{2}$. We should not expect them to hold unless some restrictive assumptions (such as symmetry around median) are imposed on $F(\cdot)$.

A general nonexistence result for plurality-maximizing candidates was proven by Osborne (1993, 1995)[22]. It was assumed that each candidate has an option of not running in an election, and ranks the election outcomes as follows: the candidate is the unique winner (with the highest voteshare), is one of several winners, the candidate withdrew from the election, and is not among the winners. Formally, we consider the policy set of each candidate to be $X \cup \{\text{out}\}$, and the election success function to be

$$G_i = \begin{cases} V_i - \max_{j \neq i} V_j, & y_i \neq \text{out} \\ -\epsilon, & y_i = \text{out} \end{cases} \tag{2.20}$$

The result obtained by Osborne (1993, 1995) is as follows:

Theorem 9 *Suppose that the electoral success function is given by (2.20). If $n \geq 3$, then no pure-strategy Nash equilibrium exists for almost any atomless distribution $F(\cdot)$.*

If the dimensionality of the policy set is greater than 1, then the existence of pure-strategy equilibria for more than two candidates is also unlikely. Shaked (1975) proved that the equilibrium does not exist for $N = 3$ and $n = 2$ for any distribution $F(\cdot)$. Eaton and Lipsey (1975) and Lomborg (2006) used numeric methods to look for equilibria for $n = 2$ and $N \geq 4$, and found none.

[22]See also Cox (1987, 1990a).

Example. Let $N = 3$, $n = 1$, and voters be uniformly distributed on $[0, 1]$. Let $V_i(y_1, y_2, y_3)$ be the voteshare of candidate i given the policy positions of the candidates. Suppose that $y_1^* \leq y_2^* \leq y_3^*$ satisfy the following conditions:

$$V_1(y_1^*, y_2^*, y_3^*) \geq V_1(y_1^*, y_2^{**}, y_3^*) \tag{2.21}$$

and

$$V_1(y_1^*, y_2^*, y_3^*) \geq V_1(y_1^*, y_2^*, y_3^{**}), \tag{2.22}$$

where y_2^{**} and y_3^{**} are the positions of candidates 2 and 3 maximizing V_2 and V_3 given y_1^*, y_3^* and y_1^*, y_2^*, respectively. Similar conditions are satisfied for Candidates 2 and 3.

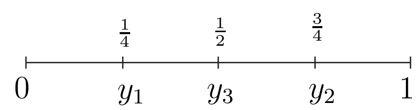
Suppose that $y_1^* = a$, $y_2^* = \frac{1}{2}$, $y_3^* = 1 - a$. Then we have $V_1 = V_3 = \frac{1}{4} + \frac{a}{2}$, $V_2 = \frac{1}{2} - a$.

2.2 Order of entry.

A number of works considered 2-stage games where several candidates simultaneously select policies at stage 1, then an additional candidate selects her policy at stage 2.

The rationale offered by Palfrey (1984) and Shepsle and Cohen (1990) is that certain political parties are *established* in the sense that their positions remain relatively unchanged over a period of several elections. At the same time, the positions of the established political parties have to be stable with respect to the threat of entry by a newcomer. Candidates will not choose identical positions if they do not choose positions simultaneously.

Example. Following Chamberlain (1933) and Smithies (1941). Consider an election with three candidates, where two candidates select positions simultaneously, Candidates 1 and 2 select their platforms simultaneously, while Candidate 3 selects her platform at a later stage, after observing the policy platforms of the first two candidates. Take $G(V) = V$ and voters uniformly distributed on $[0, 1]$. The optimal response for candidate 3 is to select $y_3 \in (y_1, y_2)$ if $y_2 - y_1 \geq 2\max\{y_1, 1 - y_2\}$. The payoff to the candidate would be $\frac{y_2 - y_1}{2}$. Otherwise, candidate 3 must choose $y_1 - \epsilon$ if $y_1 > 1 - y_2$, and $y_2 + \epsilon$ if $y_1 < 1 - y_2$. The payoffs in these two cases would be y_1 and $1 - y_2$, respectively. Given this strategy of candidate 3, the equilibrium strategies for candidates 1 and 2 are to select $y_1 = \frac{1}{4}$ and $y_2 = \frac{3}{4}$ (see Figure 13(a)).



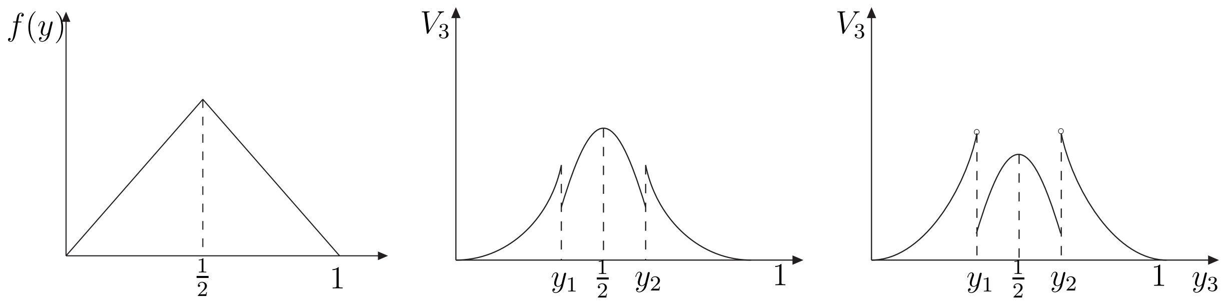
(a) Equilibrium

Figure 13: Policy divergence with a threat of entry of third candidate

A number of works studied the sequential firm location problem with customers being uniformly distributed along a line. If the number of subsequently entering candidates is greater than two, then some additional assumptions must be made. Most important, one has to provide an alternative definition of a candidate's voteshare if that candidate is located between some other two candidates. In the setting we used so far, the voteshare of Candidate 3 was equal to $\frac{y_2 - y_1}{2}$ and did not depend on her exact location, given that $y_1 < y_3 < y_2$.

One way to deal with this problem is to assume that the voter is more likely to abstain if her preferred candidate's policy platform is far away from her own favorite policy. If so, the last candidate to choose her policy position will always locate in the middle of the neighboring candidates. Such a setting (with the first two candidates locating at the endpoints of the policy segment) was analyzed by Rotschild (1979). In this setting he proved the intuitive result that early entrants obtain larger voteshares. Similar results for the firm location problem were obtained by Prescott and Visscher (1979) and Hay (1976).

There is a number of technical issues involved with extending the spatial voting model to include nonuniform voter densities. Suppose, for instance, that there are two incumbents and one entering candidate. For y_1 and y_2, the voteshare of candidate 3 may be discontinuous in y_3. Consider see Figure 14. For Figure 14(c), V_3 has no well-defined maximum. Thus y_3, the best response of candidate 3, is not a well-defined function.



(a) Distribution of voter density (b) The voteshare of the entrant depending on her position (c) The voteshare of the entrant depending on her position – no optimal response exists

Figure 14: Entrant's position and voteshare

In order to account for such irregularities, Palfrey (1984) used a slightly different definition of equilibrium (which he called *limit equilibrium*). For two incumbents and one entrant, he demonstrated the following:

Theorem 10 *Let $G(V) = V$, voter preferences distributed on $[0,1]$, and density $f(\cdot)$ be differentiable, unimodal, and symmetric around $\frac{1}{2}$. Then, limit equilibrium exists, with*

$$F(y_1) = 1 - 2F\left(\frac{1}{4} + \frac{y_1}{2}\right) \text{ and } y_2 = 1 - y_1. \tag{2.23}$$

The voteshare of the entrant was shown to always be smaller than that of an incumbent. In the Figure 14(c) example, in a limit equilibrium candidate 3 would randomize over policies y_1 and y_3. Candidates 1 and 2 would be aware of that when choosing their policy platforms.

Weber (1992) analyzed an extension of Palfrey's model, relaxing the assumption that the distribution of voter preferences is symmetric. Weber introduced a different equilibrium notion he called the hierarchical equilibrium, and showed that it exists as long as the voter density is single-peaked.

The equilibrium notions of Palfrey and Weber are similar to Nash equilibrium. Each incumbent candidate's policy position must be the best response to the policy position of the other incumbent candidate, as well as to the entry by the third candidate. An equilibrium of a different type was proposed by Greenberg and Shepsle (1987)[23].

Definition. The set of candidate policies $A = \{y_1, \cdots, y_K\}$, $y_i \neq y_j$ is a K-equilibrium if for any $y_{K+1} \notin A$, for any $i = 1, \cdots, K$, we have $V_{K+1} \leq V_i$.

This defines an arrangement of policy platforms by K incumbents that is sufficient to keep an entrant from getting more votes than any one of the incumbents. There are two conditions. First, the entrant is not allowed to propose a policy that has already been proposed by one of

[23]See also Greenberg and Weber (1985).

the incumbents. Second, if the entrant and one of the incumbents have equal voteshare, the incumbent wins the seat.

Greenberg and Shepsle proved the following impossibility result:

Theorem 11 *For any $K \geq 2$, there are distributions $F(\cdot)$ such that no K-equilibria exist.*

The following example is from Shepsle and Cohen (1990).

Example. Suppose that $K = 2$ and there are 15 voters with positions 0.05, 0.10, 0.15, 0.20, 0.47, 0.48, 0.49, 0.50, 0.51, 0.52, 0.53, 0.80, 0.85, 0.90, and 0.95. For any pair of locations y_1, y_2, there is a y_3 such as the number of votes received by candidate 3 is greater than the number of votes received by at least one of the other candidates.

There are exceptions to this nonexistence result. Weber (1990) analyzed the existence conditions for discrete voters and demonstrated that the the equilibrium exists if and only if the number of voters does not exceed $2K + 1$. Cohen (1987) obtained existence conditions for symmetrically distributed continuous voters and $K = 2$.

Theorem 12 *Suppose that $F(\cdot)$ is an atomless distribution with continuous unimodal density $f(\cdot)$ symmetric around 0. Let $F(y) = \frac{3}{4}$. If $K = 2$ and $f(0) \leq 2f(y)$, then $(-w, w)$ is a unique K-equilibrium.*

These conditions imply that Palfrey's limit equilibrium exists whenever there is a K-equilibrium for $K = 2$. Both equilibria are symmetric, so one may be interested in comparing the degree to which the policy of candidates 1 and 2 would diverge. Shepsle and Cohen (1990) argue that the divergence would be smaller in a limit equilibrium than in the K-equilibrium, since in the latter equilibrium candidates 1 and 2 have to make sure that the entrant not only fails to win, but also fails to get the second place.

Example. Let $f(x) = 2x$ for $x \leq \frac{1}{2}$ and $f(x) = 2 - 2x$ for $\frac{1}{2} < x \leq 1$. In the limit equilibrium defined by (2.23) we have $y_1 = \frac{\sqrt{10}-1}{6} = .3604$ and $y_2 = 1 - y_1 = .6396$, while in the K-equilibrium for $K = 2$ we have $y_1 = \frac{\sqrt{2}}{2} = .3535$, $y_2 = 1 - y_1 = .6465$.

The difference between these two equilibria depends on the degree to which the distribution of voter preferences is concentrated around its mode. Cohen (1987) and Shepsle and Cohen (1990) suggested that the concentration increases, the difference becomes less and less significant.

Other works studying spatial voting models with candidate entry include Osborne (1993, 2000), who looked at the entry problem with plurality-maximizing candidates. Chisik and Lemke (2004) analyzed Nash equilibria for several incumbent candidates, one entrant, and uniformly distributed voter preferences. An emerging body of literature models party positions using numeric methods. See Kollman, Miller, and Page (1992a, 1992b, 1998), Laver (2005), and Laver and Schilperoord (2006).

2.3 Sincere and strategic voting.

So far we implicitly assumed that voters support candidates they like best, without regard for their chances of getting elected and influencing the policy outcome. In the language of our model, this conjecture can be formulated as follows.

Definition. Under *sincere* voting, $p_{si} = \frac{1}{A}$ if there are exactly $A-1$ other candidates $j_1, \cdots, j_{A-1}$ such that

$$u_s(y_i) = u_s(y_{j_k}),\ k = 1, \cdots, A-1, \tag{2.24}$$

and for no k we have

$$u_s(y_i) < u_s(y_k). \tag{2.25}$$

Otherwise, $p_{si} = 0$.

Following this rule, a voter supports the candidate who delivers the highest utility. If there are several such candidates, the voter's support is split equally among these candidates.

Potentially, we may want to analyze a more sophisticated voting behavior. In particular, we may expect that a voting decision maximizes the individual's expected payoff, given the voting decisions of other voters. We give a definition.

Definition. Under *strategic* voting, the voting decisions t form a Nash equilibrium in a one-stage game where voters choose t given candidate platforms $y_1, \cdots, y_N$.

The first result is for two-candidate games.

Theorem 13 *Let $N = 2$. Then, the voting decision of a strategic voter is identical to the voting decision of a sincere voter.*

If the number of voters is greater than two, the equilibria may be different given the two types of voting behavior.

Example. There are 5 voters with ideal policies $v_1 = v_2 = 0$, $v_3 = \frac{5}{6}$, $v_4 = v_5 = 1$, and 3 policy platforms: $y_1 = 0$, $y_2 = \frac{5}{6}$, $y_3 = 1$. Voting is decided according to plurality rule. Voter disutility is $\phi(x) = x$. If the votes are sincere, then policies y_1 and y_2 are realized with probability $\frac{1}{2}$ each. The expected utility of voter 3 is then $-\frac{1}{4}$.

Now suppose that voter 3 is strategic. If she supports policy y_3, then this policy is realized with probability 1, and the utility of the voter is $-\frac{1}{6}$. Acting strategically, the voter supports the next-best policy y_3 in order to influence the outcome of the voting, instead of supporting her favorite (but hopelessly outnumbered) policy y_2.

There are two implications of voters behaving strategically. The first one has already been mentioned: if a rational voter faces a positive cost of participation and the number of voters is large, then in an equilibrium only a small fraction of voters will participate in the election.

The second implication is that a voter will not necessarily support the candidate with the nearest political platform. The actions of a voter now depend on two factors. First, it her utility from the implementation of the political platform of each candidate. Second, it is her belief regarding the magnitude by which each candidate's chances of being elected will increase if she decides to vote for that candidate[24]. A voter can support only a fixed number of candidates (depending on the ballot structure), and will thus use her votes optimally. Thus, the outcome of the voting may be belief-dependent.

Myerson and Weber (1993) argued that these beliefs themselves may depend upon the policy positions taken by the candidates. For example, a centrist candidate who for some reason moves too far to the right may no longer be considered by the public to be a front-runner and a viable contender. She will lose votes not only because of a change in her positions, but also because the voters will perceive her to be a candidate whose chances of winning cannot be

[24]A simple Nash equilibrium model of strategic voting without beliefs was analyzed in Feddersen, Sened, and Wright (1990).

improved by their support.

Example (from Myerson and Weber, 1993). Suppose that there is a continuum of voters are uniformly distributed on $[0,1]$. There are 3 candidates who simultaneously propose policies y_1, y_2, and y_3 on the policy space $[0,1]$. Let $y_1 \leq \frac{1}{2}$ and $y_1 \leq y_2 \leq y_3$. The elections are decided according to a winner-take-all rule, with the runner-up receiving a small consolation award.

The candidates are office-motivated. The candidate who obtains most votes receives a utility of 1. If two candidates are tied for the first place, then each receives a utility of $\frac{1}{2}$. Let $q_{ij} = q_{ji} \in \{0,1\}$ be the beliefs that candidates i and j will rank no worse than first and second in terms of voteshare. We assume that a voter with the preferred policy v will support candidate i if and only if there exists j such that

$$q_{ij} = 1 \text{ and } |y_i - v| \leq |y_j - v|. \tag{2.26}$$

Thus we assume that the voter will only vote for a candidate if she expects that candidate to obtain at least a second place. If two candidates are expected to be tied for a second place, we assume that the voter supports the candidate with the closest political platform. If two candidates are expected to be tied for the second place and have identical policy platforms, we assume that the voters support either candidate with probability $\frac{1}{2}$.

In an equilibrium, we must have policy platforms y_1, y_2, y_3, and beliefs that are consistent with the voting decision defined by (2.26). The policy platforms must be such that no candidate can increase her voteshare given the policy positions of other candidates and beliefs.

Consider the following policy platforms

$$y_1 < \frac{1}{2},\ y_2 = \frac{1}{2},\ \text{and } y_3 = 1 - \frac{y_1}{2}. \tag{2.27}$$

Suppose that the beliefs are formed as follows:

$$q_{23} = 0,\ q_{13} = \begin{cases} 0, & y_3 < 1 - \frac{y_1}{2}, \\ 1, & y_3 \geq 1 - \frac{y_1}{2}, \end{cases} \text{ and } q_{12} = \begin{cases} 1, & y_3 < 1 - \frac{y_1}{2}, \\ 0, & y_3 \geq 1 - \frac{y_1}{2}, \end{cases} \tag{2.28}$$

It is straightforward to check that the policy positions (2.27) are an equilibrium if the voter's decision rule is (2.26) and the beliefs are (2.28). If the policy platforms are as given, then voters with positions $v \leq \frac{1}{2} + \frac{y_1}{4}$ will support candidate 1, while voters with positions $v > \frac{1}{2} + \frac{y_1}{4}$ will support candidate 3. No voter will support candidate 2, thus fulfilling the belief that candidate 2 will never finish above third place. Candidate 1 will win because she is closer to the median voter. Candidate 3 cannot choose a position closer to the median voter's because if she does, all voters will behave as if only the race between candidates 1 and 2 is taken seriously.

This result depends on the specification of beliefs. We assume that if Candidate 3 moves too close to the center, she will no longer be considered a viable contender. Myerson and Weber conjecture that in a real political arena, this may happen if the candidate is labeled an "opportunist" by some political pundit, thus causing a shift in public's perceptions.

In this example, we have an equilibrium with candidates choosing different policy platforms, and one candidate winning. This result depends on the assumption that each candidate has only one vote.

It can be demonstrated[25], for example, that under approval voting, in any equilibrium we will have all three candidates clustered at the median.

Example. Suppose that in the previous example, each voter has a second vote that she will use if some candidate's policy platform is more attractive to the voter than the policy platform of the candidate who received the voter's first vote. Suppose that each voter believes that there is always a positive probability that any pair of candidates can finish first and second. Even if that probability is very small, the voter will cast both votes that will go to the two candidates most preferred by the voter. Candidate 1 will be the top choice for voters with positions $v \leq \frac{y_1+y_2}{2}$ and the second choice for voters with positions $v \in [\frac{y_1+y_2}{2}, \frac{y_1+y_3}{2}]$. Counting both first and second votes, the voteshare of candidate 1 will be $V_1 = \frac{y_1+y_3}{2}$. Similarly, the voteshare of candidate 3 will be $V_3 = 1 - \frac{y_1+y_3}{2}$. The voteshare of candidate 2 will be 1 unless candidates 1 and 3 occupy the median position, which they will do.

2.4 Directional policy leadership

One strand of literature attributes policy divergence to the fact that the electorate may be divided into several groups based on their attitudes toward some set of policy or nonpolicy issues. According to Rabinowitz and Macdonald (1989), voters may evaluate policies according to a directional measure and support parties that take relatively intense or extreme positions on their side of the issue.

As an example consider the issue of whether the provision of a certain good (say, health care) should be public or private. On one hand, a voter's utility is determined by the degree to which health care is provided by the state. For each voter there exists an optimal combination of public and private health care provision (determined largely by the voter's income). On the other hand, the voter may also have an idealogical position on whether health care should be public or private. From that standpoint, a voter who believes that health care should be public will prefer a candidate who proposes a 100% public health care system. The voter's total utility is thus a combination of these two factors (often called proximity and directional factors).

Consider a model adapted from Iversen (1994), Merrill and Grofman (1997), and Adams and Merrill (1999)[26]. There are N groups of voters. Within each group there are many voters with different policy preferences. The utility of a voter from group i with policy preference v from the election of candidate j with policy platform y_j is assumed to be as follows:

$$u_v(y_j) = -(1-\beta)(y_j - v_i)^2 + \beta v_i y_j + V_{ij} \tag{2.29}$$

Thus a voter is motivated by both proximity and directional factors ($y = 0$ is assumed to be both the "neutral" position on the issue and the population median). The value $0 \leq \beta \leq 1$ represents the degree to which a voter is motivated by the general direction of a candidate's policy. If $\beta = 0$, then the voter is motivated by candidate's position only (as in the standard Downs case). If $\beta = 1$, then the voter is motivated only by the direction of the candidate's policy (this case is considered in Rabinowitz and Macdonald, 1989[27]). V_{ij} denotes partisan proclivity of voter group i toward candidate j (for example, a voter may evaluate a candidate based on whether she belongs to the same class, ethnic, or religious group).

For two candidates and $V_{ij} = 0$, the candidate's positions converge on the median voter. The positions may diverge if candidates enjoy partisan support. Adams and Merrill (1999) have

[25]The model investigated in Myerson and Weber (1993) is more complex. First, a *voting equilibrium* is defined for every set of candidate platforms. The definition of the equilibrium centers around the notion of *pivot probability* p_{ij} — the voter's belief that Candidates i and j will tie for the first place. If voter k supports Candidate i, then the voter's expected utility gain relative to nonvoting would be $G_{ki} = \sum_j p_{ij}(U_{ki} - U_{kj})$, where U_{ki} is the utility of voter k in the case when the policy of Candidate i is implemented. In an equilibrium, a rational voter will choose a candidate with the largest G_{ki}. The beliefs p_{ij} should be consistent with the decisions of the voters. A similar setting was also considered in Fey (1997).

[26]Directional voting models first appeared in Rabinowitz and Macdonald (1989). For a survey of alternative formalizations of voter choice under proximity and directional factors see Merrill and Grofman (1998).

[27]A slightly different formalization is considered in Matthews (1979).

analyzed a three-candidate scenario with two groups of population – one supporting Candidates 1 and 2, and the other supporting Candidate 3 (more exactly, $V_{11} = V_{12} = V_{23} = 3$, $V_{13} = V_{21} = V_{22} = -3$). The preferences of Group 1 were distributed on $[-5, 0)$, the preferences of Group 2 — on $[0, 5]$. The authors have shown that Candidates 1 and 2 locate to the left of the population median, and to the left of the median of Group 1. The position of the candidates is thus more "extreme" than that of its constituent electorate.[28]

2.5 Measuring policy positions and voter preferences

What we can observe are the position of voters, candidates, or parties on a large number of issues. There are three sources of such data.

First, it is the roll call data of the legislators. The voting record of each member of a legislature is known. Factor analysis can be used to determine whether (and to what degree) the differences in voting records can be explained by different positions on a one- or two-dimensional scale. In a series of influential works, Poole and Rosenthal (1985, 1987) established that most differences in the roll-call data for the US congress in 1979–1982 can be expressed in terms of a position on a one-dimensional (liberal-conservative) scale.

A product of factor analysis of roll-call data are the estimated positions of individual candidates on the one- or two-dimensional issue space. If the roll call data is available over an extended time period, it is possible to test whether individual positions remain stable. Poole and Rosenthal (1991) estimated that the positions of individual legislators in the U.S. Congress were almost always stable over the post World War 2 period; thus the changes in voting patterns were almost always due to the retirement of legislators through retirement or defeat (Poole and Rosenthal, 1991).

2.6 Experimental evidence of cycling

The key works of Plott, McKelvey, and other authors establish the intransitivity of the majority voting preference, and provide the conditions that have to be satisfied in order for the core to exist.

Starting with Fiorina and Plott (1978), these results were tested in a laboratory setting. According to the review by McKelvey and Ordeshook (1990), the experimental research focuses around two broad questions: whether a committee with preferences over a two-dimensional policy set will select a policy that is the Condorcet winner (and which policy will be selected if the core is empty), and whether the positions of candidates in a two-candidate elections converge to the core (if it exists).

Fiorina and Plott (1977) conducted a series of experiments testing the proposition that the committee will eventually select the Condorcet winner in case the core is not empty. In an experiment, the five-subject committee voted on which point on the blackboard to select. Each subject was informed about her payoff function (that represented Eucledian, elliptic, or city-block type of preferences). The ideal points of the committee members satisfied the Plott symmetry condition.

At the beginning of the experiment, the subjects were informed of the status quo (which was outside of their Pareto set). Each member of the committee could offer an alternative to the status quo. If the alternative was recognized by the experimenter, it was put to vote. If it was supported by a majority of members, it became a new status quo. The procedure continued

[28]Rabinowitz and MacDonald (1989) tested a directional voting model with voter utilities of the type (2.29) for 1984 Presidential elections. It was found that the model with unrestricted β is better at predicting the choice of voters.

until a proposal to end the proceedings was approved by a majority of committee members; no threats or promises of monetary transfers between subjects were allowed.

In general, the results of this and other experiments confirmed the hypothesis that the core solution, when it exists, should be the committee's final choice (either identically or approximately).

Committee experiments were also used to test the viability of various solution concepts in the absence of the Condorcet winner (such as the competitive solution, tested by McKelvey, Ordeshook, and Winer, 1978). The outcomes of this and other experiments are almost always located inside the Pareto set, and are usually clustered (although not as densely as in the case when the core exists).

The second set of works tested whether the policies chosen by two candidates would converge to the core. In a typical experiment, there were two types of subjects (voters and candidates). The voters were assumed to be sincere. Several works, including Plott (1977) and McKelvey and Ordeshook (1985b), conducted such tests. The key feature of such works was that the voters were not always perfectly informed about the positions of the candidates.

3 The possibility of voter abstention

Allowing voter turnout to be determined endogenously and investigating the factors that affect it is a major theoretical challenge — possibly one of the greatest in the theory of rational choice. In fact, Morris Fiorina (1989) identified the general failure of rational voting models to explain turnout as the paradox that "ate the rational choice theory".

3.1 The calculus of voting

The main reason why rational choice theory has struggled to explain turnout is that the benefit of participating in large elections is very small due to the small chance of swinging the outcome in favor of one's preferred candidate. Consider a voter who is deciding whether to vote in an election. If a voter is rational, then her decision whether or not to participate is based on a tradeoff. On one hand, by casting her vote she will increase the chances that her preferred candidate will be elected (and thus influence the policy that will be realized). On the other hand, there are certain fixed costs (travel, time, gathering information, etc.) that a voter may have to incur in order to cast her ballot. Assuming that these factors enter linearly into the voter's utility function, one can represent the voter's expected utility as

$$R = pB - C, \tag{3.30}$$

where p is the probability of casting the decisive vote (or pivotal probability), B is the utility difference between the alternatives offered by the two candidates, C are the fixed costs of voting.

The well-known "paradox of rational voter" is a consequence of the fact that the probability p is very small if the number of voters participating in an election is large[29]. Hence, if the fixed cost of voting C is positive, the net benefit of voting R will be negative even for relatively large values of B. Hence, a rational individual will not vote.

[29] The relationship between p and the number of voters depends on the model specification. If the number of voters is $2N+1$, and a voter believes that every other voter supports Candidate 1 some known probability P, then the probability that her vote is the probability that of $2N$ other voters, N support Candidate 1 and all other voters support Candidate 2. This is calculated according to

$$Q(N,P) = \binom{2N}{N} P^N (1-P)^N.$$

This paradox was first remarked upon by Anthony Downs (1957) in his well-known work. He did not address the issue directly, attributing widespread voting to extra-theoretic (and irrational) factors.

In terms of equation (3.30), it may be argued that the cost of voting C is negative. In that case, the net benefit from participation in an election will be positive for all voters. Thus, negative net voting costs cannot explain why some people vote while others do not.

Riker and Ordeshook (1968)[30] suggested that the equation (3.30) may actually be of the form

$$R = pB - C + D, \tag{3.31}$$

where D denotes fixed benefits of voting (such as the satisfaction from the fulfillment of one's citizen duty). If $D > C$, then the net cost of voting is negative, and we should have positive turnout. However, as Ferejohn and Fiorina (1974) noted, this argument fails to answer the main question: why some citizens vote while others don't. In order for equation (3.31) to have any explanatory value in a large elections, $C - D$ must be very close to zero. Otherwise, a less than perfect turnout is possible only if the value of D is different for different voters, so the differences in turnout are assumed away rather than explained.

3.2 Game-theoretic models with rational voters.

The basic game-theoretic argument for voting by a rational individual can be formulated as follows. If no one is voting, then the outcome of the election will depend on the choice of any single voter who decides to vote. Other voters will become active as long as the benefit of voting exceeds the cost. Thus it is possible to have an equilibrium where all voters are rational and some level of voting activity is present. In such an equilibrium, every active voter's expected benefit of voting will be no less than the cost. Nevertheless, this argument is insufficient, since the participation rate in large electorates will be very small

Ledyard (1981, 1984) examined the extension of this logic to the Downsian candidate location model. For elections with a large number of voters the positions of the candidates converge to the median voter's ideal policy. But if the positions of the candidates are identical, the voters will have no reason to vote, resulting in zero turnout.

Palfrey and Rosenthal (1983) considered two groups of voters (one supporting the issue that is voted upon, and another one opposing it), with costs of voting being equal across all voters. It was demonstrated that for each group there is a substantial probability of winning the vote, as turnout will be lower in large groups and higher in small groups. High probability of turnout in both groups occurs if the two groups are of equal size (and thus there is a high probability of each voter being decisive).

High-turnout equilibria of this kind exist even if the cost of voting varies across the voters. However, a subsequent work Palfrey and Rosenthal (1985) demonstrated substantial turnout is impossible if one introduces uncertainty about the costs of voting for other voters. In that case, only the voters with negligible voting costs will participate, and turnout will be small. The latter two works took candidate platforms to be exogenous.

According to the Stirling formula, this value is on the order of $\frac{1}{\sqrt{N}}$ when $p = \frac{1}{2}$ but is very small for $p \neq \frac{1}{2}$ (on the order of e^{-cN} for some $c > 0$). A generalization of this result for a random number of voters was obtained by Myerson (2000).

However, if the voter believes that the probability P is a random variable, then the probability of being pivotal is on the order $\frac{1}{N}$, since as N becomes large, it is only the likelihood of $p = \frac{1}{2}$ that counts. This result was demonstrated by Chamberlain and Rotschild (1982).

[30] See also Davis, Hinich, and Ordeshook (1970) and Aldrich (1983) for developments of this model.

An alternative way to endogenize candidate positions was looked at by Feddersen (1992). The voters were taken to be strategic. Each voter's decision was to support some policy position on the issue space. If there is a positive cost of voting and the prevailing policy is chosen according to the plurality rule, then in a Nash equilibrium there are exactly two policies that receive votes.

A more recent strand of literature views abstention as a result of rational behavior of voters who are not perfectly informed about their benefit from the election of a particular candidate. It was argued that if voters are not perfectly informed about their preferences, some voters might abstain even if the cost of voting is zero. It was argued that the less informed voters may abstain in order to allow the more informed voters to decide the outcome.

Feddersen and Pesendorfer (1996) examined a model with voters of two preference types deciding which of the two candidates to elect. The following example, adapted from Feddersen (2004), can be used to illustrate some of the logic involved. There are two candidates, 4 voters, and 2 possible states of nature. In state 1, voters 1,2, and 4 prefer candidate 1, and voter 3 prefers candidate 2. In state 2, voters 1,2, and 4 prefer candidate 2, and voter 3 prefers candidate 1. It is assumed that voters 1,2, and 3 know the realization of the state of nature, and voter 4 does not. If the election is decided by majority voting (with a coin flip in the case of a tie), then the optimal decision for voter 4 is to abstain and let voters 1 and 2 (who have similar preferences) make the right choice for him. The key issue in the analysis of such models is whether the outcome is identical to the outcome under perfect information.

3.3 Alternative behavioral assumptions.

Most theoretical research on turnout focuses on the formulation of alternative assumptions regarding the motivation behind voter's actions.

Under the expressive voter hypothesis, offered by Fiorina (1976), a voting decision does not depend on the probabilities of candidates winning, but only on their value to the voter, and on the net cost of voting. The act of voting is thus an expression of approval or disapproval, not an attempt to influence the outcome of the elections[31].

A decision-theoretic model of Ferejohn and Fiorina (1974) follows this course. The authors assumed that an individual voter has no way of estimating the value of p. The voter chooses the action which minimizes potential regret over choosing the wrong action. Suppose, for example, that $k(i)$ is the number of votes from other voters received by alternative i, there are two alternatives A and B, and there is an odd number of voters. Then each voter considers the following states of nature: $k(A) \geq k(B)+1$, $k(A) = k(B)$, $k(A) = k(B)-1$, and $k(A) < k(B)$. The voter chooses the action (support A or B or abstain) that minimizes the maximum possible difference between the payoffs from the chosen action and from all other actions. The decision of a regret minimizer does not depend on the probability p, as long as it is different from zero. This may offer an explanation to positive turnout, but also provides ground for criticism.

Several works try to reconcile rational choice theory with the facts by assuming that the otherwise fully rational voters somehow overestimate the pivotal probability p. The evidential decision model of Grafstein (1991) treats every voter as thinking that her action will influence the actions of all other voters. In this setting both the perceived probability of being pivotal and turnout are higher. In a model by Kanazawa (1998), backward-looking voters similarly associate their past voting behavior with the outcome of the previous election. The results are substantial turnout and an evolutionary-model type behavior of voters.

[31]This claim is certainly contradicted by strategic behavior on the part of the voters. The studies surveyed by Cox (1997) suggest that the share of the voters who exhibit strategic behavior is significant but is far from majority.

Eldin, Gelman, and Kaplan (2005) considered voters with "social" preferences. If a voter cares not only for her own utility, but also for the utility of every other individual in the society, then the magnitude of B in equation (3.31) increases with the number of voters, offsetting the decrease in p. Thus for any number of voters, some voters may abstain, and some participate in the election. A similar approach, followed by Schachar and Nalebuff (1999), is to divide the population into leaders and followers. A leader exerts effort in order to convince the followers to support a particular candidate; a follower bases her decision (whether and for whom to vote) on the observable effort of the leaders. The authors then

Yet another assumption regarding voter motivation is that each voting decision is made in order to maximize the aggregate utility of the group of voter to which she belongs. The voter is taken to believe that other voters belonging to the same group are identically motivated and follow the same voting rule. Works in this vein include Harsanyi (1980), Feddersen and Sandroni (2002), and Coate and Conlin (2005). The first of these works looked at one group of voters that would benefit from the passing of some issue. If there is a threshold level of turnout and a positive cost of voting, then group utility is maximized with turnout just high enough for the issue to pass, and people with the highest voting costs abstaining. The latter two works further develop the same idea, looking at several groups of voters with heterogeneous voting costs. An important distinction is whether a voter seeks to maximize the utility of the society as a whole, as in Feddersen and Sandorini (2002) or only that of her own group, as in Coate and Conlin (2005).

A related vein of literature considers voters belonging to different social groups, with a third type of economic agent — the group leader, who can offer material benefits to group members for votes in support that group's candidate[32]. Works following this approach include Morton (1991) and Uhlaner (1989). It can be shown that the benefits offered by the group leaders may cause individually rational voters may act as if they are group utility maximizers. Turnout may be substantial even in large elections, where the probability p of being a pivotal voter is negligible.

3.4 Indifference and alienation.

Spatial voting models with rational voters and positive voting costs provide limited insight into the candidate positioning problem because of the complexity of such models and the lack of adequate behavioral assumptions. One possible shortcut may be to assume that a voter's decision whether or not to vote depends in some known way on the position of the voter's ideal policy relative to the policy platforms of the candidates. There are two verisimilar assumptions that were suggested as early as by Smithies (1941) and Downs (1957).

Voter *indifference* refers to the conjecture that the probability of voting declines with the utility difference between the two candidates. Thus, a voter will attend the elections only if her vote *may* make a significant difference for the voter. Under voter *alienation*, the probability of voting declines with the distance between the voter's ideal policy and the policy of her preferred candidate[33].

In a two-candidate spatial model, indifference and alienation hypotheses can be formulated as follows.

Assumption 1 *(indifference). Voter i abstains if $|u_{i1} - u_{i2}| < c$.*

[32]In Leighley's (1996) empirical study of group mobilization efficacy, mobilization efforts exercised within a group were shown to be positive factor in voter's political activity.

[33]For a broader discussion on the reasons for voter abstention, see Ragsdale and Rusk (1993). Empirical evidence for voter alienation is given in Brody and Page (1973) and Plane and Gershtenson (2004).

Assumption 2 *(alienation). Voter i abstains if* $\max\{u_{i1}, u_{i2}\} < d$.

The indifference hypothesis is consistent with the assumption that voters maximize utility (3.30) given exogenous beliefs regarding P. The alienation hypothesis is not consistent with voter utility maximization. On the contrary, a voter who maximizes utility of the type (3.30), and whose disutility from policy distance is convex, should be more likely to vote if her ideal policy is far apart from the positions of the two candidates. The alienation hypothesis is consistent with a utility-maximizing voter who believes that P is exogenous, and that some status quo policy is realized if neither candidate is elected.

The primary factor that determines the effect of voter indifference and alienation on the equilibrium of the candidate positioning game is the the shape of the distribution of voter preferences, and on the candidate objective function.[34]

Voter indifference by itself is not sufficient to dislodge the median-voter equilibrium. A change in the policy position of one of the candidates has a symmetric effect on the number of voters who abstain because of indifference.

Example. Let $n = 1$ and $y_1 < y_2$. Suppose that voter ideal points are distributed on $[0, 1]$ according to some distribution function $F(\cdot)$. Under the indifference assumption, the voteshares of the two candidates are

$$V_1 = F(\bar{y}_1),\ V_2 = 1 - F(\bar{y}_2), \tag{3.32}$$

where $\bar{y}_1$ and $\bar{y}_2$ are given by

$$\phi(\bar{y}_1 - y_1) - \phi(y_2 - \bar{y}_1) = c \text{ and } \phi(y_2 - \bar{y}_2) - \phi(\bar{y}_2 - y_1) = c. \tag{3.33}$$

We have

$$\frac{\partial \bar{y}_1}{\partial y_1} = \frac{\partial \phi'(\bar{y}_1 - y_1)}{\phi'(\bar{y}_1 - y_1) + \phi'(y_2 - \bar{y}_1)} > 0 \text{ and } \frac{\partial \bar{y}_2}{\partial y_1} = \frac{\phi'(\bar{y}_2 - y_1)}{\phi'(y_2 - \bar{y}_2) + \phi'(\bar{y}_2 - y_1)} > 0. \tag{3.34}$$

As $\bar{y}_1 + \bar{y}_2 = y_1 + y_2$, a change in y_2 will have similar effects on the location of $\bar{y}_1$ and $\bar{y}_2$. Thus both the voteshare V_1 and the plurality $V_1 - V_2$ of candidate 1 increases with y_1, and the positions of both candidates converge to the median. [35]

If the distribution of voter ideal points is symmetric and unimodal, the median voter equilibrium will also be preserved under voter alienation (see Davis, Hinich, and Ordeshook, 1970, and McKelvey, 1975). A candidate who decides to move her platform closer to the median voter' bliss point faces a tradeoff: on one hand, she gains some votes closer to the center of the political spectrum (the "moderate" voters), but on the other hand she may be bound to lose some votes on the far left (those of the "extreme" voters). If the distribution is unimodal, and the voter density at the peak of the distribution is sufficiently high, then both candidates converging to the mode is a local equilibrium. If the distribution is symmetric and the median and the mean coincide with the mode, then the equilibrium is a global one.

[34] See Davis, Hinich, and Ordeshook (1970), Comanor (1976). The probabilistic voting model of Hinich, Ledyard, and Ordeshook (1972a,b) allowed for a less that perfect turnout, but the convergence result depended on the probability of voting being concave in voter utility.

[35] This may not be the case if the candidates have different valence. Then, in general, we will not have $\frac{\partial \bar{y}_1}{\partial y_1} = \frac{\partial \bar{y}_2}{\partial y_1}$. See Zakharov (2006).

Example. Suppose that voter preferences are distributed on $[0, 1]$ with a positive, symmetric, differentiable, and unimodal density $f(y)$. We further assume that $f(0) = f(1) = 0$. Let

$$P_1(y_1, y_2, v) = \begin{cases} 1 & \text{if } y_1 - a \leq v \leq y_2 + a \\ 0 & \text{otherwise} \end{cases} \tag{3.35}$$

be the probability that a voter with the ideal policy v supports Candidate 1. Thus all voters who are more that $a < \frac{1}{2}$ units to the right of the rightmost candidate candidate abstain. The two candidates maximize votehsare. For $y_1 < y_2$, the voteshares are given by

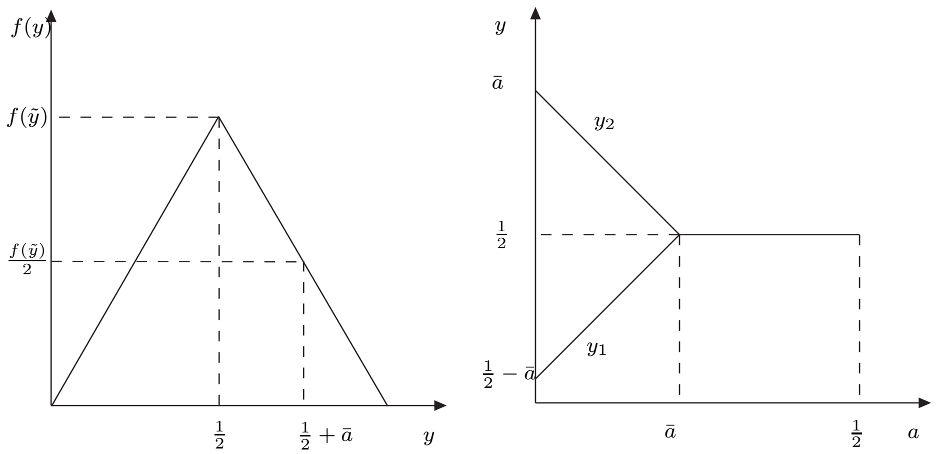
$$V_1 = \frac{y_1 + y_2}{2} - F(\max\{0, y_1 - a\}),\ V_2 = F(\min\{1, y_2 + a\}) - \frac{y_1 + y_2}{2}. \tag{3.36}$$

The first-order conditions for voteshare maximization are

$$\frac{\partial V_1}{\partial y_1} = \frac{1}{2} f\left(\frac{y_1 + y_2}{2}\right) - f(y_1 - a) = 0 \text{ and } \frac{\partial V_2}{\partial y_2} = f(y_2 + a) - \frac{1}{2} f\left(\frac{y_1 + y_2}{2}\right) = 0. \tag{3.37}$$

It follows that the equilibrium is symmetric, with $y_1 = \frac{1}{2} - b$, $y_1 = \frac{1}{2} + b$, where b is given by $\frac{1}{2} f(\frac{1}{2}) = f(\frac{1}{2} + a + b)$. If no such b exists, we have the median-voter equilibrium $y_1 = y_2 = \frac{1}{2}$. Note that the second-order conditions are also satisfied if the equilibrium exists.

An increase in a will lead to a decrease of policy distance between the candidates. If a exceeds a certain critical value (determined by $\frac{1}{2} f(\frac{1}{2}) = f(\frac{1}{2} + \bar{a})$), then the positions will converge at the median (see Figure 15).



(a) Voter density (b) Candidate's positions

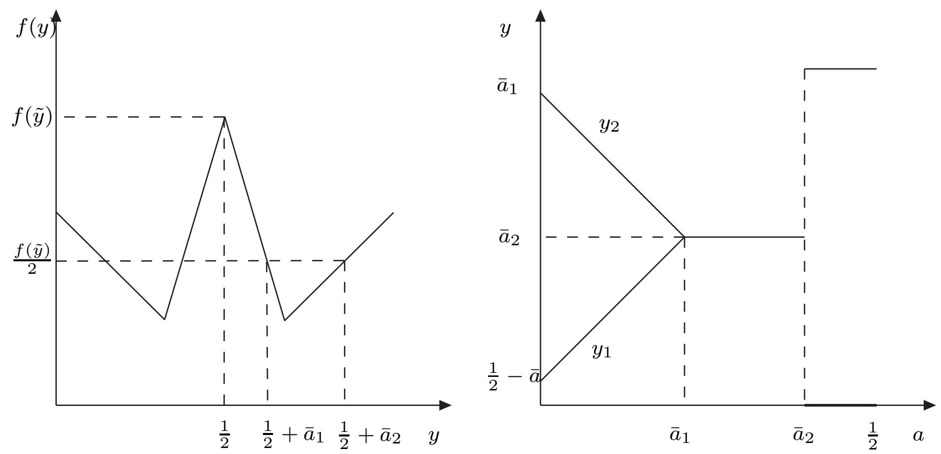
Figure 15: Downsian equilibrium under voter abstention, density has a single mode at $\tilde{y} = \frac{1}{2}$.

Thus the density around the median voter must be sufficient in order for the median-voter equilibrium to exist. For example, a bimodal distribution with the median voter located at the trough may not produce the median–voter result.[36]

If the distribution has several modes, then the location of the candidate platforms in a symmetric equilibrium may depend on a discontinuously (see Figure 16).

This may happen for the following reason. Consider a voter that is on the verge of abstaining. Suppose that the voter density in the neighborhood of that voter is decreasing as one moves

[36]The argumentation in Downs (1957) was not rigorous. The incorrect conclusion was that unimodality centered around the median voter is necessary and sufficient for the convergence of policy platform. However, the overall idea survives this criticism.



(a) Voter density (b) Candidate's positions

Figure 16: Downsian equilibrium under voter abstention, density has several modes.

away from the median voter. In that case the marginal benefit of moving one's policy away from the median is decreasing. An increase in a will reduce the benefit of moving one's policy position away from the median, provided that the opponent moves symmetrically or that the voter density in the neighborhood of the indifferent voter is constant.

If the density around the voter on the verge of abstaining is increasing with the distance from the median, then the marginal benefit of moving one's policy position away from the median is increasing with policy distance. It follows that the candidates will either select the median voter's position, or choose diverging positions at the ends of the policy spectrum. The second scenario is more likely if a is larger.[37]

The voting probability function (15) may seem overly simplistic, not allowing for any kind of uncertainty (perhaps over voting costs of individuals). However, that makes it possible to work with a rather general class of distribution functions $f(\cdot)$. Introducing a more general voting probability function would require imposing corresponding limitations on the distribution of voter preferences in order to make the analysis tractable.

Example. The voting probability function suggested by Kirchgässer (2003) was as follows:

$$P_1(y_1, y_2, v) = \begin{cases} \frac{y_2-y_1}{y_1+y_2-2v}, & v < y_1, \\ \frac{y_1+y_2-2v}{y_2-y_1}, & y_1 \le v \le \frac{y_1+y_2}{2}, \\ \frac{2v-y_1-y_2}{y_2-y_1}, & \frac{y_1+y_2}{2} \le v \le y_2, \\ \frac{y_2-y_1}{2v-y_1+y_2}, & y_2 \le v. \end{cases} \tag{3.38}$$

This function accommodates for both the indifference and the alienation of voters (see Figure 17). If the voters are uniformly distributed on $[-1, 1]$, voteshare-maximizing candidates will select diverging positions, with $y_1 = -a$, $y_2 = a$, where $\ln(-a) - a = \frac{1}{2}$.

3.5 Empirical evidence of voter abstention.

There is an overwhelming body of empirical literature on voter participation, some of it testing several hypotheses relevant to the spatial theory.

The best-studied prediction is that turnout depends on the closeness of the election, as in a closer election the probability of casting the decisive vote is higher. Mueller (2003, ch. 14)

[37] If the distribution $f(\cdot)$ is asymmetric, a local equilibrium in which candidates select different policy platforms may not be a global equilibrium (Comanor, 1976).

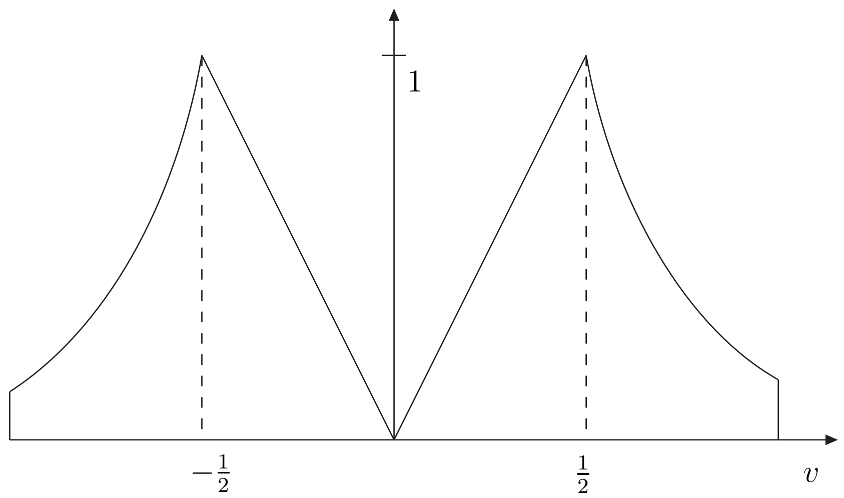


Figure 17: The Kirchgässer (2003) voting probability function for $y_1 = -\frac{1}{2}$ and $y_2 = \frac{1}{2}$.

and Geys (2006) contains a review of relevant literature). Most tests support the hypothesis, although the evidence is sometimes contradictory, such as in Kirchgässer and Zu Himmern (1997) study of German General Elections for 1983–1994.

The so-called "mobilization hypothesis" provides an alternative explanation to the (possible) positive relationship between turnout and election closeness. It can be argued that if the elections are more closely contested, then the competing candidates are mobilized to procure additional turnout (see, for example, a study by Cox and Munger, 1989, linking campaign spending and election closeness).

The two conjectures linking the likelihood of turnout and the policy positions of the candidates are known as *indifference* and *alienation* hypotheses. Under the indifference hypothesis, a voter casts her ballot if and only if there is sufficient difference between payoffs that the candidates offer to the voter. Hence, a voter who is indifferent between the candidates will abstain. Under the *alienation hypothesis*, a voter will abstain if she is sufficiently dissatisfied with the policies promised by either of the candidates.

Until recently, there were relatively few works investigating indifference and alienation hypothesis. The in the earliest work on the subject (Brody and Page, 1973) the authors analyzed survey data collected after 1968 U.S. Presidential election. It was found that the likelihood of the respondent having voted was greater if her evaluation of her most preferred candidate was more favorable. A similar relationship supporting the indifference hypothesis was also found. Later works by Zipp (1985) and Plane and Gershtenson (2004) also found support for both indifference and alienation hypotheses using survey-level data.

In a recent paper, Adams, Dow, and Merrill (2006) used a conditional logit model to estimate the alienation and indifference components of abstention. The authors found that for the 1980-1992 Presidential elections both factors contributed to depressed turnout, with no substantial partisan differences in their effects.

4 Candidate-specific characteristics.

In the models that we studied so far, candidates differed (from the voter's standpoint) only in the policy that they would implement upon election. The voting decision of each voter was based only on the proximity of each candidate's policy proposals to that voter's ideal point (and to the ideal points of other voters, in case the voters are strategic).

However, we may want to account for other factors that affect a voter's evaluation of a candidate — such as a candidate's credibility, honesty, intelligence, experience, track record, connections, and personal charisma. Starting with Stokes (1963) this is generally referred to as candidate *valence*. As Mueller (2003, p. 240) nicely put is, valence issues are "issues for which all voters agree that more is better than less".

A candidate may have a similar advantage if there is less uncertainty about the policy that she will implement once elected (see Section 6 for a more detailed discussion). Thus being an incumbent is doubly beneficial: voters are more certain about a candidate's policy position, and perceive him to have some experience on the job.

In the first subsection, we investigate some of the issues related to the introduction of valence into spatial voting models. The second subsecton explores the literature that treats valence characteristics as exogenous and random from the viewpoint of competing candidates. In the third subsection, we review deterministic spatial models with valence, and look at ways to endogenize the valence characteristics of candidates.

4.1 Valence issues — some basic results.

We modify the utility function (1.7) to account for valence. The utility of voter s if the candidate i is elected is

$$u_s(y_i, e_i) = e_i - \phi(\|v_s - y_i\|), \tag{4.39}$$

where y_i denotes the policy position of candidate i, and e_i denotes the value of his valence characteristics.

The first set of implications concerns the existence of an equilibrium for a two-candidate majority vote. The following result is due to Ansolabehere and Snyder (2000):

Theorem 14 *Let $e_1 < e_2$, $\phi(x) = x^2$, and the elections are decided by a winner-take-all rule. Then an equilibrium pair of strategies y_1, y_2 exist if and only if $r < \sqrt{e_2 - e_1}$, where r is the size of the yolk.*

In the resulting equilibrium, candidate 1 obtains a position under which his voteshare is greater than one half regardless of the position of candidate 2. Immediately it follows that such an equilibrium always exists for $n = 1$ (since the yolk consists of the unique median point). Indeed, for candidate 1 it is sufficient to occupy the median voter's point to guarantee victory. For $n \geq 2$, such an equilibrium is possible if the distribution of voter preferences is sufficiently close to being symmetric.

The second set of implications is for proportional electoral systems. If the candidates have different valence, the changes in the positions of the candidates have different effects on the voteshares of the candidates. For example, let $n = 1$ and $N = 2$. Consider the change in the location of the voter who is indifferent between candidates 1 and 2. The position of such a voter is given by

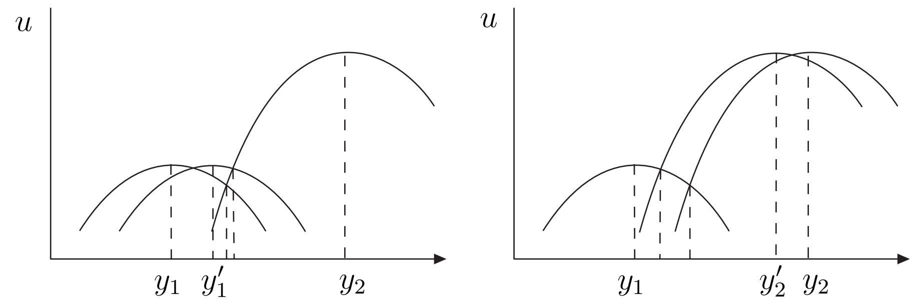
$$e_1 - \phi(|\tilde{y}_{12} - y_1|) = e_2 - \phi(|\tilde{y}_{12} - y_2|). \tag{4.40}$$

Applying the implicit function theorem, we get

$$\frac{\partial y_{12}}{\partial y_i} = \frac{\phi'(|\tilde{y}_{12} - y_i|)}{\phi'(|\tilde{y}_{12} - y_1|) + \phi'(|\tilde{y}_{12} - y_2|)}. \tag{4.41}$$

for $y_1 < \tilde{y}_{12}$ and $y_2 > \tilde{y}_{12}$. If $e_1 < e_2$, then $\tilde{y}_{12} - y_1 < y_2 - \tilde{y}_{12}$, so $\frac{\partial y_{12}}{\partial y_1} < \frac{\partial y_{12}}{\partial y_2}$. See Figure 18.

This result requires the second derivative of the disutility function $\phi(\cdot)$ to be positive (for example, due to the assumption that the voters are risk-averse).



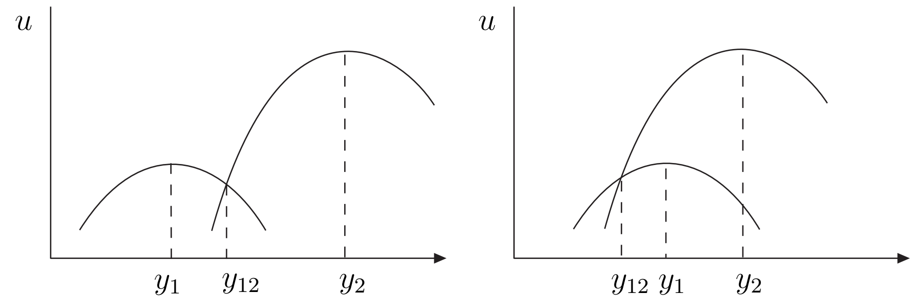
(a) Low-valence candidate — small change in $\tilde{y}_{12}$ (b) High-valence candidate — large change in $\tilde{y}_{12}$

Figure 18: The effects of a change in the position of a candidate on the location of the indifferent voter.

If candidates 1 and 2 are sufficiently close, then equation (4.40) suggests that we may have $\tilde{y}_{12} < y_1 < y_2$, like on Figure 19. Admitting such possibility, we can arrive at some surprising results. The equation (20) for candidate 1 becomes

$$\frac{\partial y_{12}}{\partial y_i} = \frac{-\phi'(|\tilde{y}_{12} - y_i|)}{\phi'(|\tilde{y}_{12} - y_1|) - \phi'(|\tilde{y}_{12} - y_2|)} < 0. \tag{4.42}$$

This implies that the best response of candidate 2 is to select $y_2 = y_1 + \phi^{-1}(e_1 - e_2)$[38]. Thus we can obtain an equilbrium in which the policies of the candidates are different.



(a) $y_1 < \tilde{y}_{12} < y_2$ (b) $y_{12} < y_1 < y_2$

Figure 19: Indifferent voter in a 2-candidate spatial model with valence.

Example. Let $n = 1$, $N = 3$, $G(V) = V$, $\phi(x) = x^2$, $e_1 = e_3 = 0$, $e_2 = e$, and the density $f(y)$ be unimodal, with a mode at $y = 0$. It is relatively easy to verify that the unique equilibrium is $y_1 = -\sqrt{e}$, $y_2 = 0$, and $y_3 = \sqrt{e}$.

4.2 Exogenous valence under deterministic voting.

Candidate valence was traditionally treated as an exogenous characteristic over which the candidate has no control. It is relatively straightforward to see that the 2-candidate Downsian election game has no pure-strategy equilibrium if one of the candidates has a valence advantage. Indeed, the candidate with the greater valence would be guaranteed the entire vote if she selects the position identical to her rival's. The rival, in turn, would always be able to secure at least some vote by moving some distance away from the advantaged candidate's position[39].

[38] Note that this result also holds for $\phi''(\cdot) < 0$.

[39] This was noted in Groseclose (2001), Aragones and Palfrey (2002), Berger, Munger, and Potthoff (2000), and a number of other works. If one candidate's advantage is large enough, then a pure–strategy equilibrium

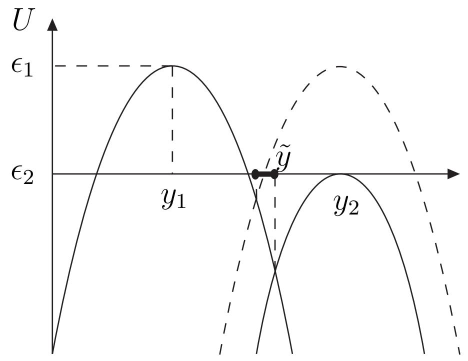


Figure 20: The position of the indifferent voter in the Downsian model where Candidate 1 has valence ϵ_1, and Candidate 2 has valence $\epsilon_2 < \epsilon_1$. The thick line indicates the voters whose preferred policies are closer to the policy position of Candidate 2, but who vote for Candidate 1 because of her higher valence.

Potentially there are two ways to analyze the two-candidate spatial voting model given exogenous valence of the candidates. The first one is to assume that the candidates are motivated by policy as well as by voteshare. Following Groseclose (2001), suppose that there are two candidates with preferred policies -1 and 1, voters are distributed uniformly on the interval $[-1, 1]$, and that Candidate 1 has a valence advantage of ϵ[40].

The utilities of the candidates would be

$$U_1 = -\lambda(-y-1)^2 + (1-\lambda)\frac{\tilde{y}+1}{2}, \tag{4.43}$$

and

$$-\lambda(1-y)^2 + (1-\lambda)\frac{1-\tilde{y}}{2}, \tag{4.44}$$

where y is the position of the prevailing candidate and

$$\tilde{y} = \frac{y_1+y_2}{2} + \frac{\epsilon}{2(y_2-y_1)} \tag{4.45}$$

is the indifferent voter's position[41].

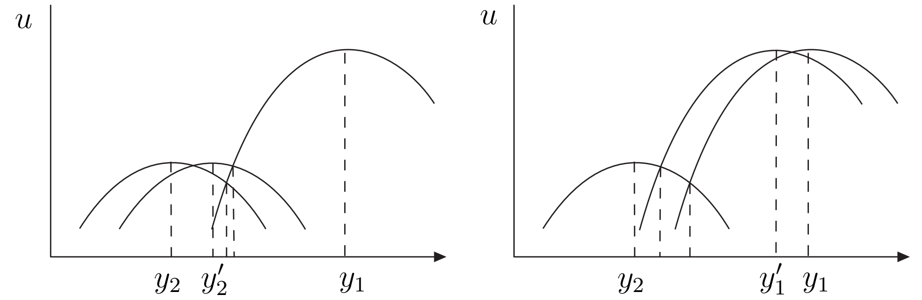
is possible when the candidate with the valence advantage is located at the center, and the disadvantaged candidate (who always loses) can locate anywhere (Ansolabehere and Snyder (2000)). Local conditions for a pure-strategy equilibrium for the case when candidate valence has both fixed and random components were obtained by Schofield (2004a).

A number of studies attributed candidate valence to partisan attachment by voters (Adams, 1998). If a candidate has an entrenched position on some policy issue, then the voters perceive her to be more credible. So it might be argued that a centrist candidate who moves to imitate her far right rival's position risks eroding some or all of the credibility that she may currently have.

Also, candidate valence is not necessarily a one-dimensional characteristic. One candidate may be perceived to be more competent on foreign policy issues, while his rival may be viewed as more honest. Wittman (2001), for example, considered candidates to have several exogenous valence characteristics over which the voters may have different preferences. It was shown that if the voter preferences satisfy certain conditions (less restrictive than the single-peakedness assumed in the spatial models), then a pure-strategy equilibrium may exist.

[40] A different way of introducing valence into model was proposed in Sahaguet and Persico (2006). In a two-candidate election game that they considered, a candidate's policy proposal amounted to a redistribution of a fixed amount of resource (budget) among a continuum of voters. The candidate's valence was the size of the budget that the voters believe would be available for redistribution once once that candidate is elected. Thus, candidate valence affected candidate's budget constraint, not directly the utility of voters.

[41] It is the solution to the equation $\epsilon - (\tilde{y}-y_1)^2 = -(y_2-\tilde{y})^2$. In this setting, policy-motivated candidates can commit to their proposed policy platforms, which is a strong assumption.



(a) Disadvantaged candidate changes position – little effect on the position of the indifferent voter

(b) Candidate with valence advantage changes position– large effect on the position of the indifferent voter

Figure 21: The effect of a change in a candidate's position on the position of the indifferent voter

A move toward the opponent's position entails both the benefit of greater voteshare and the potential cost of the prevailing policy being further away from one's own preferred policy. Assuming an interior solution, in a pure-strategy equilibrium the positions of both candidates are such that for each candidate, the marginal benefit and cost described above are equal.

The marginal voteshare benefit from moving toward the median voter is greater for the candidate with the valence advantage (see Figure 21). This is because the indifferent voter is located closer to the disadvantaged candidates. If a candidate's distance to the voter is close and that candidate shifts her position toward the voter' the voter's utility increases by a relatively small amount. It takes only a small move in the voter's position in order to equate the utilities from both candidates. Thus a change is the policy platform of the candidate with the valence advantage has a greater effect on the indifferent voter's position.

Thus the candidate with the valence advantage must also have a higher marginal cost of moving one's policy toward the opponent's position. Thus we should expect that the candidate with the valence advantage should locate closer to the center than her opponent[42].

Mixed equilibrium for office-motivated candidates was analyzed by Aragones and Palfrey (2002). The authors assumed that the candidates have linear disutility, and can choose policy positions from a discrete set of evenly spaced admissible policies X. The number of elements in that set was assumed to be inversely proportional to the magnitude of the valence advantage ϵ. The authors have demonstrated that as ϵ tends to zero (and thus the number of elements in the set of admissible policies approaches infinity), then the set of admissible policies over which both candidates randomize their policy platforms converges to the median voter's preferred policy.

[42]There is potential difficulty with this argument, since by moving toward the center the candidate increases his chances of being elected (and of implementing his proposed policy). Thus the effect of such a move on that candidate's expected disutility depends on the distribution of voter preferences.

Also, it is not obvious whether a policy-motivated candidate's platform will be more moderate or more extreme if her valence advantage increases. On one hand, a greater valence advantage allows a candidate to capture more votes by moving one's policy closer to the median; on the other hand, the candidate can now shift the policy closer to her bliss point without losing any votes.

For the general disutility and voter density functions, Groseclose (2001) has shown that as valence advantage increases from zero, the advantaged candidate moves toward the median voter, while the other candidate moves away.

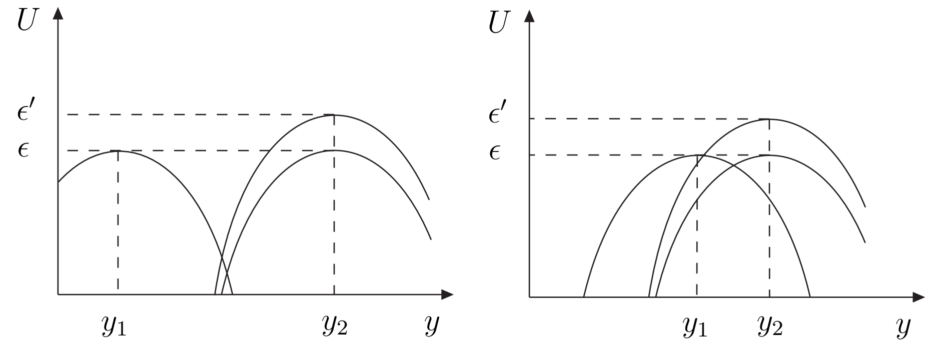
There is empirical evidence that the policies proposed by the disadvantaged candidates are indeed more extreme (Fiorina, 1973, Ansolabenehere, Snyder, and Stewart, 2001). Additional theoretical evidence is given in Aragones and Palfrey (2002). The effect may persist if the candidates enter sequentially (Cutrone, 2005).

4.3 Endogenous valence.

A number of works assume that candidates have some control over their level of valence. Zakharov (2005) assumed that a candidate can increase her level of valence by spending resources during the election campaign — for example, by increasing the candidate's name recognition through billboard advertising, advertising that candidate's good track record, or undermining opponent's reputation.[43]

It was shown that if the office-motivated candidates have the opportunity to increase their valence after they commit to their policy positions, then the policy positions that they will select will not be identical.

Indeed, suppose that the policy proposals of the two candidates are very close (for example, near the median voter's position). Then, the voters evaluate the candidates mostly according to their valence. If one of the candidate gains even a small valence advantage over her rival, then her voteshare will increase by a large amount (see Figure 22). Thus the marginal benefit of increasing one's valence is large, and the marginal (and thus the total) cost of increasing one's valence is also large. If the policy proposals of the candidates are identical, then the marginal benefit of increasing one's valence is infinite. In an equilibrium, the candidates will spend all their available resources on valence.



(a) The distance between the policy positions of the candidates is large. An increase in the valence of Candidate 2 increases her voteshare by a small amount.

(b) The distance between the policy positions of the candidates is small. The voteshare of Candidate 2 increases by a large amount.

Figure 22: The effect of an increase in a candidate's valence on the voteshare of that candidate.

If the electoral system has some degree of proportionality (or there is uncertainty with respect to the position of the median voter), then the rational candidates would be willing to move away from the median, sacrificing some voteshare in order to decrease the costs of obtaining valence.

Consider a 2-stage game. At stage 1 each candidate i chooses her political platform $y_i \in X \in \mathbf{R}^n$. At stage 2 each candidate observes the other's policy platform, and chooses a level of valence e_i, incurring a cost $c(e_i)$. The gains to each voter are as given by equations (4.39). The final payoff to each candidate is

$$U_i = G_i(V) - c(e_i). \tag{4.46}$$

[43]This setting is different from that of Harrington and Hess (1996), where the authors assumed that the a candidate can exert resources in order to make her opponent's position appear more distant form the median. However, in their setting the amount of votes earned by an extra dollar of campaigning depends only on the position of the indifferent voter, not on the distance between the policies of the candidates.

The first result was that of nonexistence. If the policy preferences of the voters are sufficiently homogeneous, then no Nash equilibrium exists. Formally,

Theorem 15 *Let $N = 2$, $n = 1$, and $G(V)$ be continuous. Then, for every disutility function $\phi(\cdot)$ and cost function $c(\cdot)$ there exists a such that for every density $f(\cdot)$, such that $f(y) = 0$ for $y \neq [b_1, b_2]$, $b_2 - b_1 < a$, there exists no pure-strategy Nash equilibrium for the 2-stage candidate location game with valence.*

The equilibrium certainly does not exist if there is only one voter with policy preference y. If both candidates select $y_1 = y_2 = y$, then at stage 2 the marginal benefit of obtaining valence is infinite if $e_1 = e_2$. Since in any pure-strategy Nash equilibrium the marginal benefit must equal marginal cost, the cost must also be infinite. But then a candidate will be better off with zero valence and zero voteshare.

Second, the author derived the conditions for the existence of a local Nash equilibrium.

Theorem 16 *Let $N = 2$, $n = 1$, and $G(V) = V$. Suppose that the voters are distributed continuously. The sufficient conditions for a local Nash equilibrium in the game described above are*

$$
\begin{aligned}
f'(\tilde{y}) &= 0 && (4.47)\\
f''(\tilde{y}) &> 0 && (4.48)\\
\frac{\phi'(d)^3}{\phi''(d)} - c'^{-1'}\left(\frac{f(\tilde{y})}{2\phi'(d)}\right) f(\tilde{y}) &= 0 && (4.49)\\
e_1 = e_2 &= c'^{-1}\left(\frac{f(\tilde{y})}{2\phi'(d)}\right), && (4.50)
\end{aligned}
$$

where $\tilde{y} = \frac{y_1+y_2}{2}$ and $d = \frac{y_2-y_1}{2}$.

Example. Suppose that the voters are uniformly distributed on $[0, a]$, $\phi(x) = \frac{x^2}{2}$, and $c(x) = \frac{x^2}{2b}$, for some $b > 0$. If the policies of the candidates are such that $0 < \frac{y_1+y_2}{2} < a$, then the locally optimal level of valence, chosen at the second stage, is given by the equation (4.50):

$$
e_1 = e_2 = \frac{ba}{2(y_2 - y_1)}.
$$

The utilities of the candidates at stage 1 are given by

$$
U_1 = \frac{y_1 + y_2}{2a} + \frac{b}{8a^2(y_2 - y_1)^2}, \quad U_2 = 1 - \frac{y_1 + y_2}{2a} + \frac{b}{8a^2(y_2 - y_1)^2}.
$$

The first-order utility maximization conditions are

$$
\frac{\partial U_1}{\partial y_1} = \frac{1}{2a} - \frac{b}{4a^2(y_2 - y_1)^3} = 0, \quad \frac{\partial U_2}{\partial y_2} = -\frac{1}{2a} + \frac{b}{4a^2(y_2 - y_1)^3} = 0,
$$

or

$$
y_2 - y_1 = \left(\frac{b}{2a}\right)^{\frac{1}{3}}.
$$

In this particular model, the indifferent voter can be located anywhere on $[0, a]$. The marginal benefit and marginal cost of moving toward the opponent's position depend only on the distance to the opponent because the distribution of voter preferences is assumed to be uniform, and the electoral system is assumed to be proportional. As the voter density $\frac{1}{a}$ increases, the marginal benefits of increasing valence rise, and the candidates shift their policy positions move away from the indifferent voter in order to compensate for increasing cost of obtaining valence.

Assume that the equilibrium is symmetric: $\frac{y_1+y_2}{2} = \frac{a}{2}$. The utility of the each candidate is then given by

$$U_1 = U_2 = \frac{1}{2} - 2^{\frac{7}{3}} a^{-\frac{4}{3}} b^{\frac{1}{3}}.$$

We have $U_1 \geq 0$ if and only if

$$a \geq 2^{\frac{5}{2}} b^{\frac{1}{4}}.$$

If a is too small (and the voters are sufficiently homogeneous), then this condition is not satisfied, so in any local Nash equilibium, at least one of the candidates will have negative utility, and no proper Nash equilibrium exists.

For the general cost and disutility functions, the author analyzed the existence of local Nash equilibria (LNE). It was shown that the location of the indifferent voter depends only on the distribution of voter preferences and corresponds to a local minimum of the voter density function. If the density function is monotone and a local minimum does not exist, then in an equilibrium, the policy of one of the candidates in located at the boundary of the set of acceptable policies. It can be shown that such "extreme" candidate receives the minority of the vote.

Similar reasoning was used to explain product differentiation in the product location problem. See, for instance, Hay (1976), d' Aspermont, Gabszewicz, and Thisse (1979), Prescott and Visscher (1979), Shaked and Sutton (1982), and Economides (1989). The utility from the consumption of a product is assumed to depend on the taste (or location) of the consumer, and on the product's price. Each of the two or more firms decides which product (with what location or taste) to produce. After making that choice, the firms are free to participate in Cournot price competition. If two firms produce products that are perfect substitutes, then the loss of profit due to price competition is large. Thus a firm might be compelled to produce a differentiated product at the cost of losing some of the consumers.

In the spatial voting theory setting, a model similar to Zakharov (2005) was developed independently by Carrillo and Castanheira (2002). The model was different in two respects: the candidates could choose only from three policy positions (left, center, and right), and the valence could assume only two values: 0 and 1. A candidate can exert effort in order to influence the voter's belief that the value of her valence is 1. Under certain conditions, the candidates can deviate from the central position.

5 Probabilistic voting theory.

The concept of probabilistic voting was developed, among others, in the works of Hinich, Ledyard, and Ordeshook (1972a,b), Hinich (1977), and Coughlin and Nitzan (1981), as a remedy to the inherent instability of multidimensional deterministic voting models.

The principal assumption of a spatial probabilistic voting model is that a candidate is not fully aware of the effect that her policy will have on the utility and decision of a voter. Thus, from a candidate's perspective, the action of each voter (and the candidate's share of vote) is a random variable conditional on the ideal policy of the voter, the platforms of all candidates, and other observable factors.

The academic literature names several reasons for such uncertainty.[44] In real-life elections, the information received by candidates (or parties) is often gathered from opinion polls and is

[44]One survey of such literature is Coughlin (1991).

subject to measurement error. The preferences of individual voters with regard to the candidates are based not only on candidate's policy or ideology, but on a large number of subjective factors (such as the candidate's appearance, manners, perceived competence). A candidate cannot predict the response on an individual voter to a change in her policy platform. A voter may continue to support the same candidate, decide to vote for a different candidate, or abstain, depending on voter's characteristics that the candidate cannot observe. Rather, candidates perceive the electorate as several groups of voters, with the proportion of voters within each group supporting each candidate depending on candidate position.

The models of probabilistic voting imply the existence of an equilibrium where all candidates select identical policy positions. The analysis of such models usually serves to answer one of the four questions. The first one is how the convergent equilibrium depends on model specifications. The second and third are of the local and global stability of the convergent equilibrium. The fourth question relates to the existence of other equilibria. Finally, the predictions of probabilistic models can be tested against empirical data.

5.1 The convergent equilibrium.

Hinich, Ledyard, and Ordeshook (1972a,b) were among the first to look for equilibria in a positioning game with probabilistic voting. Let there be K voters, and P_{ij} be the probability that voter i supports candidate j. The expected voteshare of candidate j is then

$$U_j = \sum_{i=1}^{K} P_{ij}. \tag{5.51}$$

Suppose that the policy platforms of the candidates are $y_1, y_2 \in \mathbf{R}^n$. The following result has been obtained.

Theorem 17 *Let the following assumptions are satisfied:*

1. *The utility voter i obtains from the election of candidate j, u_{ij}, is continuous and concave in y_j.*
2. *The probability P_{ij} is nondecreasing and concave in u_{ij} and is nonincreasing and convex in $u_{i,-j}$, where the index $-j$ denotes the other candidate.*
3. *$P_{ij}(u_{ij}, u_{i,-j}) = P_{i,-j}(u_{i,-j}, u_{ij})$.*

Then in a game with payoffs given by (5.51), there exists a unique pure-strategy Nash equilibrium with $y_1^* = y_2^*$.

This theorem is a consequence of a Nash equilibrium existence result for continuous games.

Example. Suppose that $y_1, y_2 \in \mathbf{R}^1$,

$$u_{ij} = -(v_i - y_j)^2, \tag{5.52}$$

and

$$P_{i1}(u_{i1}, u_{i2}) = P_{i2}(u_{i2}, u_{i1}) = P(u_{i1} - u_{i2}), \tag{5.53}$$

where P is an increasing differentiable, concave function. The first-order conditions for vote-share maximization are

$$\begin{aligned} \frac{\partial V_1}{\partial y_1} &= -2\sum_{i=1}^{K}(y_1 - v_i)P'(u_{i1} - u_{i2}), \\ \frac{\partial V_2}{\partial y_2} &= -2\sum_{i=1}^{K}(y_2 - v_i)P'(u_{i2} - u_{i1}). \end{aligned} \tag{5.54}$$

This system has a solution at

$$y_1^* = y_2^* = \bar{v} = \frac{1}{n}\sum_{i=1}^{K} v_i. \tag{5.55}$$

Both candidates select policy platforms at the position equal to the mean of voter ideal policies. The intuition behind this result is straightforward. The marginal effect of y_1 on u_{i1} increases with the policy distance $|v_i - y_1|$. Hence, if $y_1 = y_2$, then the probability of voting P_{i1} is more sensitive to changes in y_1 if the policy distance $|v_i - y_1|$ is greater. Thus the candidates put higher weigh on the voters with more distant positions. If the disutility is quadratic in policy distance, then the weights are linear, and the candidates select the mean voter's ideal policy.

Lin, Enelow, and Dorussen (1999) have extended this logic to the case when there are more than two voters, and considered the effect of several alternative distance measures.

Example. Let $y_1, y_2 \in \mathbf{R}^n$. Suppose that the voters have city-block preferences over policy: $u_{ij} = \sum_{k=1}^{n} |v_{ik} - y_{jk}|$. Then, in the equilibrium all candidates select the dimension-by-dimension median.

Enelow, Endersby, and Munger (1990) analyzed a model where each voter i believes that candidate j will actually carry out policy $l_{ij}y_j$, where $0 \le l_{ij} \le 1$ is interpreted as the competence level of candidate j as perceived by voter i. Thus the policy $y = 0$ is interpreted as the status quo — that is, the policy that will be carried out by a totally incompetent candidate. The utility of each voter i from the election of candidate j is

$$u_{ij} = e_{ij} + \phi(v_i - l_{ij}y_j). \tag{5.56}$$

The first-order conditions for voteshare maximization by candidates 1 and 2 are

$$\begin{aligned}
\frac{\partial V_1}{\partial y_1} &= \sum_i l_{i1}\frac{\partial \phi(v_i - l_{i1}y_1)}{\partial l_{ij}y_1} g_i(e_{i1} - e_{i2} + \phi(v_i - l_{i1}y_1) - \phi(v_2 - l_{i2}y_2)) = 0 \\
\frac{\partial V_2}{\partial y_2} &= -\sum_i l_{i2}\frac{\partial \phi(v_i - l_{i2}y_2)}{\partial l_{i2}y_2} g_i(e_{i1} - e_{i2} + \phi(v_i - l_{i1}y_1) - \phi(v_2 - l_{i2}y_2)) = 0.
\end{aligned} \tag{5.57}$$

Solving these equation requires $G(\cdot)$ to be distributed uniformly on some relevant interval. If there are two voters with $v_0 = 0$, $v_1 = 1$, the conditions (5.57) translate into

$$y_1 = \frac{l_{21}}{l_{11}^2 + l_{21}^2} \text{ and } y_2 = \frac{l_{22}}{l_{12}^2 + l_{22}^2}. \tag{5.58}$$

The candidates anticipate the discount that will be applied to their positions, and adjust them accordingly. There are several ready observations. First, if each voter views both candidates as equally competent, the candidates select identical policy positions. If the voters value the competence of each candidate differently, the positions of the candidates may diverge. In the authors' example, a candidate who is perceived to be soft on crime issues may "overshoot her mark, advocating tougher penalties for a long list of offensives" in order to accommodate for the discounting of her positions by voters.

The equation (5.53) defines the probability of voting as a function of the difference of candidate utilities. An alternative specification is the binary Luce choice model:

$$\frac{P_{i1}}{P_{i2}} = \frac{u_{i1}}{u_{i2}} \tag{5.59}$$

and

$$P_{i1} + P_{i2} = 1. \tag{5.60}$$

Solving equations (5.59) and (5.60) we obtain

$$P_{ij} = \frac{u_{ij}}{u_{i1} + u_{i2}} \tag{5.61}$$

for $j = 1, 2$. This is a two-candidate case of a choice model satisfying the independence of irrelevant alternatives condition that the ratio of probabilities that some voter supports some two candidates does not depend on the utilities that the voter obtains if some other candidates are elected.

Coughlin and Nitzan (1981a, 1981b) analyzed a probabilistic voting model with the voting probability functions given by (5.61). The convergence result they obtained is stated as follows.

Theorem 18 *Suppose that the following conditions are satisfied:*

1. *There are K voters with probabilities of voting given by (5.60).*
2. *For each i, j, u_{ij} is a continuous differentiable function of y_j.*
3. *The candidates maximize expected plurality*

$$U_j = \sum_{i=1}^{K} (P_{ij} - P_{i,-j}). \tag{5.62}$$

Then, y_1^*, y_2^* is an equilibrium if and only if they both maximize

$$W_j = \sum_{i=1}^{K} \log u_{ij}. \tag{5.63}$$

Thus the equilibrium policies of the candidates maximize the Benthamite social welfare function. It follows that if u_{ij} are concave and $u_{ij} = u_{i,-j}$ for $y_1 = y_2$, then we have a unique equilibrium $y_1^* = y_2^*$.[45] .

Lindbeck and Weibull (1987) obtain a convergence result for a different specification of the model. They define a policy alternative as a redistribution of a fixed resource among the voters. The preferences of a voter are defined explicitly as a utility function that is concave in the amount redistributed to the voter, and linear in the random element. It was found that, under some additional assumptions, equilibrium redistribution maximized the expected sum of voter utilities.

5.2 Conditions for local convergent equilibrium.

In the two examples above, the existence of the equilibrium depended on the assumption the function P is concave in $u_{i1} - u_{i2}$ (and, hence, in y_1). However, this function cannot be concave everywhere if the domain of y_1 is unrestricted. Thus the conditions necessary for at least a local Nash equilibrium is that $P(\cdot)$ is concave at 0.[46]

One can define the function $P(\cdot)$ more explicitly, and check whether (and under what conditions) this function is concave.

[45] See also Samuelson (1984)

[46] See Slutsky (1975) and, most recently, Kirchgässner (1999).

Example.

Suppose that $N = 2$ and there are K voters, with ideal policies $v_1, \cdots, v_k$. The utility of voter i is

$$u_{i1} = e_1 - \phi(\|v_i - y_1\|) \tag{5.64}$$

if candidate 1 is elected, and

$$u_{i2} = -\phi(\|v_i - y_2\|) \tag{5.65}$$

if candidate 2 is elected. The value e_1 is the valence advantage of candidate 1 over candidate 2. We assume that (as far as the candidates are concerned) e are independent random variables, identically distributed with a distribution function $F(\cdot)$ and a differentiable density $f(\cdot)$. We assume that the candidates maximize their expected voteshares that are given by

$$U_1 = \sum_{i=1}^{K}(1 - F(\phi(\|v_i - y_1\|) - \phi(\|v_i - y_2\|))) \text{ and } U_2 = \sum_{i=1}^{K} F(\phi(\|v_i - y_1\|) - \phi(\|v_i - y_2\|)). \tag{5.66}$$

The first-order conditions for voteshare maximization are

$$\begin{aligned} - \quad & \sum_{i=1}^{K} (\nabla\phi(\|v_i - y_1\|)) f(\phi(\|v_i - y_1\|) - \phi(\|v_i - y_2\|)) = 0 \\ & \sum_{i=1}^{K} (\nabla\phi(\|v_i - y_2\|)) f(\phi(\|v_i - y_1\|) - \phi(\|v_i - y_2\|)) = 0, \end{aligned} \tag{5.67}$$

where ∇ denotes the gradient vector. If $n = 1$ and $\phi(x) = x^2$, then the first-order conditions are satisfied at $y_1 = y_2 = \bar{v}$. The second-order conditions sufficient for a local maximum of y_1 at $\bar{v}$ given $y_2 = \bar{v}$ and vice versa are

$$\frac{\partial^2 U_1}{\partial y_1^2} = -2\sum_{i=1}^{N}(f(0) - 2(y_1 - v_i)^2 f'(0)) < 0 \tag{5.68}$$

and

$$\frac{\partial^2 U_2}{\partial y_2^2} = -2\sum_{i=1}^{N}(f(0) + 2(y_1 - v_i)^2 f'(0)) < 0. \tag{5.69}$$

This translates into a condition on the dispersion of v_i:

$$|2f'(0)|D_v < f(0). \tag{5.70}$$

The condition (5.70) roughly means that the equilibrium exists if the model is sufficiently close to being deterministic. If the voters are sufficiently clustered together, then the mean voter is a local equilibrium. If the voters are far apart or if the density of ϵ is small, the mean voter is not an equilibrium.

Schofield (2006) obtained a generalization of condition (5.70) for $N \geq 2$ and $n \geq 2$. He assumed that the utility of voter i if candidate j wins is

$$U_{ij} = \lambda_j + \epsilon_{ij} - \beta\|y_j - v_i\|, \tag{5.71}$$

where λ_j is the exogenous valence of candidate j, ϵ_{ij} is the random variable distributed according to the distribution function $\Psi(h) = e^{-e^{-h}}$.

Zakharov (2006) generalized the condition (5.70) to account for voter indifference.[47] Suppose that voter i supports Candidate 1 if

$$-(v_i - y_1)^2 \geq -(v_i - y_2)^2 + c, \tag{5.72}$$

[47]Enelow and Hinich (1984b) use a different specification of indifference and alienation.

supports Candidate 2 if

$$-(v_i - y_1)^2 + c \leq -(v_i - y_2)^2, \tag{5.73}$$

and abstains otherwise. The voteshares of the two candidates are

$$V_1 = \sum_{i=1}^{N}(1 - F((v_i - y_1)^2 - (v_i - y_2)^2 + c)) \tag{5.74}$$

$$V_2 = \sum_{i=1}^{N} F((v_i - y_1)^2 - (v_i - y_2)^2 - c). \tag{5.75}$$

If candidate i maximizes

$$U_i = (1 - \lambda)V_i + \lambda(V_i - V_{-i}) = V_i - \lambda V_{-i}, \tag{5.76}$$

and the density $f(\cdot)$ is symmetric around zero mean, then a local Nash equilibrium at $y_1 = y_2 = \bar{v}$ exists if

1. $g'(c) \geq 0$ or
2. $g'(c) < 0$ and

$$D_v < \frac{g(c)(1 + \lambda)}{-2g'(c)(1 - \lambda)}, \tag{5.77}$$

The equilibrium is more likely to exists if the density of ϵ is multimodal, with the voters with the realizations of ϵ at the modes of the distribution not abstaining. Thus a local equilibrium becomes less likely if c is large. Finally, the local convergent equilibrium exists if the variance of voter ideal policies is small or the candidates are plurality maximizers.

5.3 Conditions for global convergent equilibrium.

Research suggests that the local convergent equilibrium fails to be a global equilibrium if the mean and the median of the distribution of voter ideal points are sufficiently distant[48].

Example. Let $n = 1$, $N = 2$, and let there be 3 voters with the ideal policies $v_1 = v_2 = 0$ and $v_3 = b$. Suppose that $\phi(x) = x^2$. For each voter i, e_i is logistically distributed:

$$P(e_i \leq x) = \frac{1}{1 + e^{-x}}. \tag{5.78}$$

Given the locations of candidates 1 and 2, the probability that voter i will support candidate 1 is

$$P_i = \frac{1}{1 + e^{(y_1 - v_i)^2 - (y_2 - v_i)^2}}. \tag{5.79}$$

The expected voteshares of the candidates are

$$U_1 = P_1 + P_2 + P_3 = \frac{2}{1 + e^{y_1^2 - y_2^2}} + \frac{1}{1 + e^{(y_1 - b)^2 - (y_2 - b)^2}},\ U_2 = 3 - P_1 + P_2 + P_3. \tag{5.80}$$

The convergent equilibrium is given by

$$y_1 = y_2 = y^* = \frac{1}{3}(v_1 + v_2 + v_3) = \frac{b}{3}. \tag{5.81}$$

[48]The conditions for the existence of the mean voter equilibrium were derived, among others, by Hinich (1978), Kramer (1978), and McKelvey and Patty (2006).

Let $y_2 = y^*$. The utility of candidate 1 is then

$$U_1^* = \frac{2}{1 + e^{y_1^2 - \frac{b^2}{9}}} + \frac{1}{1 + e^{(y_1 - b)^2 - \frac{4b^2}{9}}}.$$

The equilibrium given by (5.81) is stable if and only if $y_1 = y^*$ is the global maximum for U_1^*. This is true for $|b| < 3.123$. Otherwise, there is a different global maximum, and the equilibrium does not exist.

Example. Let e_i be distributed as

$$P(e_i \leq x) = \begin{cases} 0, & x < -\delta \\ \frac{x+\delta}{2\delta}, & -\delta \leq x \leq \delta \\ 1, & x > \delta. \end{cases} \tag{5.82}$$

The first-order condition for the maximization of voteshare is

$$\frac{\partial U_1}{\partial y_1} = \frac{1}{2\delta} \sum_{i:|V_i| \leq \delta} (2y_1 - 2v_i) = 0 \tag{5.83}$$

or

$$y_1 = \bar{y} = \sum_{i:|V_i| \leq \delta} v_i. \tag{5.84}$$

Similarly setting $\frac{\partial U_2}{\partial y_2} = 0$ we obtain

$$y_2 = \sum_{i:|V_i| \leq \delta} v_i. \tag{5.85}$$

If the range of the voter ideal points is smaller than δ, then $y_1 = y_2 = \bar{y}$ is a strong Nash equilibrium (meaning that neither candidate will deviate from the policy position $\bar{y}$, regardless of the position of her opponent). A strong equilibrium exists because the probability of a voter i supporting candidate 1 over candidate 2 is linear in the difference between the utilities $\phi(y_1 - v_i) - \phi(y_2 - v_i)$. However, if the distance between the mean and the median voter is greater than δ, both firms locate at the median in the equilibrium.

Enelow and Hinich (1982a) extended the probabilistic voting model to include different distribution functions $G(\cdot)$ for different voters. If there are only two voters of weight $w_1 < w_2$ and the e_i are distributed normally, with zero mean and variance σ_i^2, then the conditions for the existence of a global Nash equilibrium can be derived explicitly.

Theorem 19 *Let $\phi(x) = x^2$, $v_1 = 0$, and $v_2 = 1$. Put*

$$\bar{y} = \frac{w_2 \sigma_1}{w_1 \sigma_2 + w_2 \sigma_1}. \tag{5.86}$$

Then, if $\sigma_1^2 > 2(1 - \bar{y})^2$ and $\sigma_2^2 > 2(1 - (1 - \bar{y})^2)$, the equilibrium exists, and $y_1 = y_1 = \bar{y}$.

It the conditions for the existence of the equilibrium are not satisfied, then both candidates choose $y_1 = y_2 = 0$.

The policy $\bar{y}$ is a weighted mean of the ideal policies $v_1 = 0$ and $v_1 = 1$. This, Enelow and Hinich argued, underscores a difference between representative democracy, where policies are proposed by candidates and voted for by voters, and direct democracy, where citizens select policies directly. Representative democracy, it follows, should lead to a more balanced outcome, while direct democracy may result in the "tyranny of the majority" (in this example, the majority of the voters would prefer $y = 0$). Moreover, the weight of a group decreases with the group's variance σ. Thus, a minority group should be better represented if it placed a relatively greater weight on policy issues rather than on valence issues.

5.4 Empirical evidence

The predictions of the probabilistic voting models can be tested empirically. Enelow, Endersby, and Munger (1993) used data from the National Election Survey to estimate the parameters of the individual utility functions for a model given in Section 5.1[49]. The estimation allowed to compare different model specifications and see which one better fits the observed data.

For each respondent of the survey, l_{ij} was calculated as a voter's weighted response to several questions concerning the candidate — such as whether she is "moral", "intelligent", or "commands respect". The authors carried out a principal-component analysis of the difference between each voter's evaluation of Republican and Democratic candidate on each such valence issue. Since it was found that the most of the variation was explained by a single issue, the authors took $e_{i1} = l_{i1}$ and $e_{i2} = l_{i2}$.

For each spatial issue (such as abortion or government spending) the respondent has to estimate her own position, the position of each candidate, and the status quo on a 7-point scale. Each respondent's utility function was assumed to be elliptic:

$$u_{ij} = e_{ij} - \sum_k a_{ik}(A_{ijk} - v_{ik})^2, \tag{5.87}$$

where v_{ik} is the position of voter i on issue k, and

$$A_{ijk} = SQ_{ik} + l_{ij}(y_{ijk} - SQ_{ik}) \tag{5.88}$$

is the estimate of voter i's perception of what the policy on issue k will be if candidate j is elected. The value a_{ik} is the salience of issue k to voter i, SQ_{ik} is voter i's perception of the status quo on issue k, y_{ijk} is voter i's perception of position of candidate j on issue k.

The authors then analyzed the following logit regression:

$$login(P_i) = u_{i1} - u_{i2},$$

with the dependent variable being the vote of the respondent, and obtained maximum likelihood estimates of a_k for each spatial issue k (taken to be uniform across voters) and $e_{i1} - e_{i2} = e_i$ for each voter i.

The main question that the authors sought to answer was whether the extended model with nonuniform l_{ij} predicts individual votes and candidate positions better than the standard Enelow and Hinich's probabilistic voting model, for which $l_{ij} = 1$ for each i and j. The authors estimated three models: for which l_{ij} are allowed to vary, for which $l_{ij} = 1$ for both candidates j, and for which $l_{ij} = 1$ for the incumbent candidate only.

All three models predicted individual votes fairly well. For example, for the 1980 Presidential elections, the model with fixed l_{ij} correctly predicted in $93,7\%$ of the cases, while the success rate for the model with $l_{ij} = 1$ was $93,7\%$.

The second issue was whether the generalized model better predicted the positions of the candidates.

It is possible to test whether the voting decisions are uncertain at an individual level (possibly due to uncertainty about the positions of the candidates), or only at the group level (due to a large electorate with diverse valence evaluations of the candidates).

[49] Erikson and Romero (1990) estimated the less general probabilistic voting model of Enelow and Hinich (1984). Schofield, Nixon, and Sened (1998) tested a multiparty probabilistic voting model using the survey data for the 1992 and 1996 elections to the Israeli Knesset. As one of the results, the authors found no significant differences between models that included party-specific valence and those that did not. Similar analysis of later Knesset elections was carried out in Schofield (2004b).

Burden (1997) analyzed the results of several surveys, where each respondent was asked either to name the candidate who she was going to vote for, or to specify the probability of voting for a certain candidate. Responses to both probabilistic and deterministic questions were found to be equally good at predicting the individual vote, signifying the existence of individual-level uncertainty.

5.5 Divergent equilibria and empirical estimation.

Virtually all analytic results in probabilistic voting theory are concerned with convergent equilibria. At the same time, the theory says very little about the existence of equilibria where the candidates choose different policy positions. The work of Lin, Enelow, and Dorussen (1999) suggests that any such equilibrium may be Pareto-inferior to the convergent equilibrium. They used numeric methods to find that divergent equilibria tend to be local, and disappear if the variance of the random elements ϵ_j in the model is large enough.

Schofield, Sened, and Nixon (1998) and Schofield (2006) used numeric simulation of divergent Nash equilibria were to test whether the observed positions of political parties correspond to the assumption that parties select policy platforms in order to maximize their expected voteshare.

In his analysis of the 1996 Israel Knesset elections, Schofield (2006) used survey data to estimate the positions of the voters and obtain the policy space, and then used expert opinion to estimate the positions of the political parties in that policy space. After that, he assumed that the voter utility is of the form (5.71), and obtained maximum likelihood estimates of λ_j and β, given the known voter choice. He then used numeric methods to obtain divergent Nash equilibria for the parties with the estimated valences.

Both in a Nash equilibrium and in the observed data, the high valence parties are located closer to the mean voter point. However, the predicted positions are closer to being convergent. This led the author to speculate that valence is endogenous, and that political parties choose more extreme positions in order to obtain support from party activists, who, in turn, mobilize the electorate to vote for the party and thus contribute to the party's valence.

Other works

6 Information and the spatial model

Every election is inherently uncertain, as no voter can be certain of the effect that the election of a candidate might have on her welfare. Downs (1957) and Grofman and Withers (1993) list some of the consequences of costly and incomplete information.

First, voters who are uncertain of a policy's effect on their welfare are susceptible to persuasive advertisement. Second, a voter may not have the time and ability to evaluate the positions of a candidate on all relevant policy issues. Instead, the voters gauge the candidate's ideological position from her position on a sample of issues[50]. The ideological position is then used as a proxy to estimate the candidate's position on all other issues[51] Third, the voters are risk-averse. For that reason, there will be a bias toward status quo. Fourth, the voters are

[50]See Popkin (1993) for a discussion of such "informational shortcuts".

[51]Enelow and Hinich, (1982b, 1990) and Coughlin and Hinich (1984) conjecture that the candidates compete by selecting one-dimensional ideological labels. The voter associate each label with a position (or a distribution density) in a multi-dimensional issue space. The key question is which ideology-issue mappings generate single-peaked voter preferences over candidate ideologies.

more certain of the position of an incumbent candidate, whose record in office is known. This stimulates parties and candidates to be consistent in their policies.

6.1 Risk-averse voters and candidate location

The degree of uncertainty surrounding the platforms of the candidates affects the choice of a risk-averse voter. Consider the following example.

Example. Suppose that there are two candidates, the policy space is one-dimensional, and the disutility of the voters is quadratic. Let the position of candidate 2 be a random variable with variance σ_y^2 and mean $\bar{y} > 0$. The position of candidate 1 is $y_1 = 0$. The payoff to voter v from the election of candidate 1 is

$$u_1(v) = -v^2. \tag{6.89}$$

from the election of candidate 2 —

$$u_2(v) = -E(y-v)^2 = -v^2 + 2v\bar{y} - \bar{y}^2 - \sigma_y^2. \tag{6.90}$$

The location of the indifferent voter is given by

$$v = \frac{1}{2}(\bar{y} + \frac{\sigma_y^2}{\bar{y}}). \tag{6.91}$$

It follows that the voteshare of candidate 2 decreases with σ_y^2.

A reduction in uncertainty σ_y^2 amounts to an increase in the valence of a candidate. Thus the results for a two-candidate policy location game parallel those for a two-candidate model with valence (see McKelvey, 1980, or Dalen, Moen, and Riis, 2003). If the uncertainty is the same for the two candidates, then the positions of the candidates converge to the median voter's. If the variances are not identical, there is no equilibrium (see Shelsle, 1972).

The risk-averse voters are less likely to select a candidate whose position is uncertain. The degree of uncertainty may depend on how close the candidate's announced position is to the position that the voters expect the candidate to announce. Thus if the voters are expecting a candidate to select a certain policy platform (this may well be expected from an incumbent), then that candidate may not choose to deviate from that platform since otherwise the voters will be less certain about the policy that she will pursue once she is elected.

Consider an argument loosely following that of Bernhardt and Ingerman (1985). There is a continuum of voters uniformly distributed on $[0,1]$. There are two candidates. The candidates simultaneously selects policy platforms $y_1 < y_2$. When elected, Candidate 1 implements policy $y_1 + d_1$ with probability $\frac{1}{2}$ and policy $y_1 - d_1$ with probability $\frac{1}{2}$. Likewise, Candidate 2 implements policy $y_2 + d_2$ with probability $\frac{1}{2}$ and policy $y_2 - d_2$ with probability $\frac{1}{2}$. Voter disutility is quadratic. Thus the utility of voter v from the election of Candidate i is

$$u_i(v) = -\frac{1}{2}(y_i + d_i - v)^2 - \frac{1}{2}(y_i - d_i - v)^2 = -(v - y_i)^2 - d_i^2. \tag{6.92}$$

The location of the indifferent voter is determined from

$$v = \frac{y_1 + y_2}{2} + \frac{D_2 - D_1}{2(y_2 - y_1)}, \tag{6.93}$$

[51] The value of information may be different for different groups of voters. An example given by Noll (1993) suggests that if the voter's costs of gathering information are nontransferable, and the information on the candidate's positions that the voters gain is private, then the outcome will be biased in favor of those voters whose utility is more responsive to the policy outcome.

where $D_1 = d_1^2$, $D_2 = d_2^2$. The first-order conditions for utility maximization are

$$\begin{aligned}
\frac{\partial U_1}{\partial y_1} &= \frac{1}{2}\left(1 + \frac{1}{(y_2 - y_1)^2}\left(-\frac{\partial D_1}{\partial y_1}(y_2 - y_1) - D_1 + D_2\right)\right) = 0 \\
\frac{\partial U_2}{\partial y_2} &= -\frac{1}{2}\left(1 + \frac{1}{(y_2 - y_1)^2}\left(\frac{\partial D_2}{\partial y_2}(y_2 - y_1) + D_1 - D_2\right)\right) = 0 \qquad (6.94)
\end{aligned}$$

Suppose that the voters expect Candidate 1 to select $y_1 = 0$ and Candidate 2 to select $y_2 = 1$, and that D_1 and D_2 are linear in deviation from the expected policy: $D_1 = ay_1$, $D_2 = a(1-y_2)$. Solving the equations (6.94), we obtain

$$y_1 = \frac{1-a}{2}, \qquad (6.95)$$

$$y_2 = \frac{1+a}{2}. \qquad (6.96)$$

We can see that the more the voters are sensitive to the deviations from expected policies, the smaller is the distance by which a candidate's policy platform will deviate from the policy that she is expected to announce. For $a \geq 1$ we obtain the corner solutions $y_1 = 0$, $y_2 = 1$.

For a more rigorous approach to this argument, one would have to consider the degree to which the candidate will deviate from the announced policy to be her private information. The voters will then try to evaluate the candidate's type from her choice of policy platform. Works in this direction include Banks (1990) and Alesina and Cukierman (1990), discussed in Section 7.

6.2 Information pooling

Voters may be nonuniform with respect to the amount of information that they have. An important theoretical question is whether this lack of information is important to the outcome of the elections.

A series of works by McKelvey and Ordeshook (1985a, 1985b, 1986, 1990) looked at how an uninformed voter can use poll data to ascertain the position of the candidate. The following example is adapted from Grofman and Withers (1993).

Example. There are 2 candidates with positions $y_1, y_2 \in [0, 1]$ and a continuum of voters, with ideal points uniformly distributed on $[0, 1]$. Each voter knows her own ideal policy v and the location of the indifferent voter $\tilde{y} = \frac{y_1+y_2}{2}$. The voters also know which of the two candidates is further to the left.

Under these assumptions, a voter will vote for Candidate 1 if her preferred policy is $v < \tilde{y}$, and for Candidate 2 otherwise. It is straightforward to check that $|y_1 - v| < |y_2 - v|$ if and only if $v < \frac{y_1+y_2}{2}$.

In this one-dimensional example, uninformed voters use poll data to determine the positions of the candidates. One of the reasons why the poll data may be available is that a fraction of the voters is informed and knows the positions of the candidates.

This result should not hold if the number of dimensions is two or greater. However, McKelvey and Ordeshook have shown that, under a number of assumptions, a series of polls produces a result that converges to that for the fully informed voters.

In the equilibrium concept that the authors used, all parties made optimal decisions given the beliefs, with the beliefs being consistent with the observed data. These theoretical results suggest that voters need little information in order to vote correctly.[52]

7 Policy motivation and citizen candidacy.

So far, we considered the candidates to be office-motivated. The candidates were assumed to be indifferent with respect to the policy that is implemented by the winning candidate. The sole goal of a candidate was winning office. Once elected, the candidate would obtain a fixed material gain, or simply enjoy her new status (or gather the so-called "ego rents").

An alternative view is that the candidates are policy-motivated — that is, their desire to win the elections is motivated by the need to have a certain policy objective achieved. Winning the office is thus only a means to achieve that objective.

Indeed, a candidate (or a political party) usually represents one or several constituencies that have policy interests; sometimes a candidate *is* such a constituency. A political platform of such a candidate is always some sort of a compromise between the maximization of (expected) office rents and the constituency interest.

7.1 Policy-motivated candidates

Policy motivation of candidates was considered one of the reasons why the candidates' policies may fail to converge to the median voter.

Consider an adaptation of the model by Calvert (1985). Let the voters be distributed uniformly on the interval $[0, 1]$. Assume a winner-take-all ESF. The utility of Candidate $i \in \{1, 2\}$ is given by:

$$U_i = \lambda P_i(y_1, y_2) - (1 - \lambda)\left(P_1(y_1, y_2)u(v_i - y_1) + P_2(y_1, y_2)u(v_i - y_2)\right).$$

Here, λ is the weight of the office rent in the utility of the candidate, $u(\cdot)$ is the disutility function, $P_i(y_1, y_2)$ is candidate i's expected share of office rents given candidate platforms y_1 and y_2, and v_i is the ideal policy of candidate i.

We consider two cases. In the first case, the electoral system is the plurality-rule and the distribution of voter preferences is known to both candidates. Since the location of the indifferent voter is

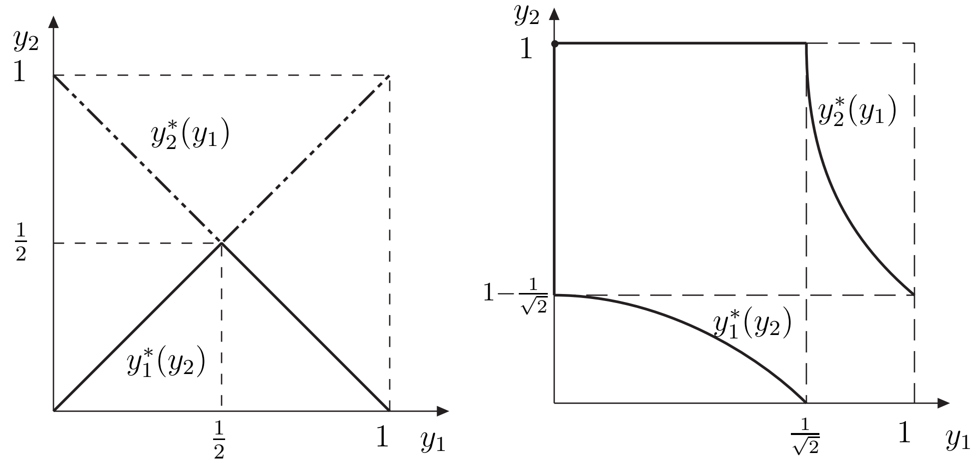
$$\tilde{y} = \frac{y_1 + y_2}{2},$$

we have

$$P_1(y_1, y_2) = \begin{cases} 1, & \frac{y_1+y_2}{2} > \frac{1}{2} \\ \frac{1}{2}, & \frac{y_1+y_2}{2} = \frac{1}{2} \text{ or } y_1 = y_2 \\ 0, & \frac{y_1+y_2}{2} < \frac{1}{2} \end{cases} \tag{7.1}$$

[52] An alternative interpretation is that the positions of the candidates are known, but the voters themselves do not know where their own ideal polities lie. In a model of Konrad (2004), the candidates can benefit by informing certain voters about their preferences. Consider the following example. There are 2 candidates, 1 and 2, and uninformed 100 voters. According to poll data, 51% support candidate 1, while 49% support candidate 2. The probability that Candidate 1 is the better candidate for an uninformed voter is thus 51%, and all voters support Candidate 1. Now suppose that Candidate 2 identifies three voters who benefit more from the election of Candidate 1, and reveals their identities to the electorate. Now, the probability that one of the 97 remaining uninformed voters is better off if Candidate 1 is elected is only 49.5%. Thus the uninformed voters will support Candidate 2. The author argued that given the possibility of such "inverse campaigning", the policy proposals discriminate against small groups of voters.

It is straightforward to check that the only equilibrium (for any λ) would be the one with $y_1 = y_2 = \frac{1}{2}$. If Candidate 2 is located to the right of the median, then the optimal response of Candidate 1 would be to select a policy just right enough to win the elections. That policy will be better (from Candidate 1's point of view) than the policy of Candidate 2. If Candidate 2 is to the left of the median, then Candidate 1 will have no incentive to change the election outcome (but Candidate 2 will be better off by moving her policy back to the right). The only stable point is when both candidates select the median position[53]. The optimal response functions of both candidates are illustrated on Figure 23(a).



(a) Winner-take-all system, any λ, any $a_1 < \frac{1}{2} < a_2$.

(b) Proportional representation, certainty equivalence, $\lambda = 0$, $a_1 = \frac{1}{4}$, $a_2 = \frac{3}{4}$.

Figure 23: Best response to opponent's policies for policy-motivated candidates with uniformly distributed voters

The policy positions of the candidates converge to the median voter's ideal policy even if they are policy-motivated because a candidate's policy has any chance of being implemented only if that candidate's voteshare exceeds one half. Thus the candidate's tastes are irrelevant to her choice of policy platform.

Now suppose that for any voteshare, there is a positive probability that a candidate will win the election. This may be due to two factors. First, the exact location of the median voter may not be known to the candidates. Second, the electoral system may exhibit some degree of proportionality. A candidate faces a tradeoff. On one hand, a move toward the median voter increases the expected voteshare and the probability that the candidate's policy platform will be implemented. On the other hand, the policy is now further away from the candidate's ideal policy, and will deliver her less utility in the case that candidate wins the election.

Suppose that the candidates are elected according to the proportional representation rule. The probability that a candidate is elected is equal to her voteshare, and the functions P are given by

$$
\begin{aligned}
P_1 &= \frac{y_1 + y_2}{2} \\
P_2 &= 1 - \frac{y_1 + y_2}{2}.
\end{aligned}
\tag{7.2}
$$

Suppose that $a_1 = 0$ and $a_2 = 1$. Then the first-order conditions for the maximization of

[53]This argument can be generalized to the multidimensional policy space (as long as the policy set is radially symmetric).

U_1 and U_2 are

$$\begin{aligned} y_1 &= \frac{\lambda}{2(1-\lambda)} \\ y_2 &= 1 - \frac{\lambda}{2(1-\lambda)}. \end{aligned} \tag{7.3}$$

If $\lambda < \frac{1}{2}$, then the policy positions of the candidates diverge in equilibrium[54].

If the disutility is not linear, then the policy platforms of a candidate may be between her position and the position of the median voter. Suppose that $\lambda = 0$ and $u(x) = x^2$. Then, the utilities of the candidates are given by

$$U_1 = -\frac{y_1 + y_2}{2}(v_1 - y_1)^2 - \left(1 - \frac{y_1 + y_2}{2}\right)(v_1 - y_2)^2$$

and

$$U_2 = -\frac{y_1 + y_2}{2}(v_2 - y_1)^2 - \left(1 - \frac{y_1 + y_2}{2}\right)(v_2 - y_2)^2.$$

We assume that the candidates are symmetric: $v_1 = \frac{1}{2} - a$, $v_2 = \frac{1}{2} + a$. In that case, the equilibrium policies selected by the candidates would be

$$y_1 = \frac{1}{2} - \frac{a}{2a+1}, \; y_2 = \frac{1}{2} + \frac{a}{2a+1}.$$

If the candidates are not motivated purely by office, then the equilibrium outcome depends on the way that the policy that is implemented is determined from the policy platforms of the two candidates. The assumption that was made so far is that the platform of the winning candidate is implemented with probability one. The alternative assumption is that some weighted average of the two platforms is implemented. A natural choice of weights would be the voteshares of the candidates (see, for instance, Ortuño-Ortín, 1997). The utility function of candidate i under such certainty equivalence[55] assumption would be

$$U_i = \lambda P_i(y_1, y_2) - (1-\lambda)u(v_i - P_1(y_1, y_2)y_1 - P_2(y_1, y_2)y_2). \tag{7.4}$$

If such a compromise policy is implemented, then the candidates may propose policies that are extreme than their own preferred policies. Suppose that $v_1 = \frac{1}{4}$, $v_2 = \frac{3}{4}$, and $u(\cdot) = |\cdot|$. The response functions of the candidates are depicted on Figure 23(b). We can see that in the equilibrium, the policies of the candidates are $y_1 = 0$ and $y_2 = 1$. The extreme policies are selected in order to influence the expected outcome of the election, which is still closer to the median that the candidate's ideal policy.

7.2 The commitment problem and information

The idea that candidates in an election are policy-motivated was propagated by the often-cited works of Calvert (1985) and Wittman (1977, 1983). A major criticism of this approach was put forward in Alesina (1988). The author noted that the previous works implicitly assumed that the candidates are able to make binding commitments to the policies that they propose.

[54]In the original work, R. Calvert considered general forms of P and candidate disutility from policy distance. It was shown that as λ tends to zero, the optimal response $y_1(y_2)$ of Candidate 1 to Candidate 2's approaches y_2.

[55]Alternatively, one can assume that the policy of the winning candidate is implemented, and that the probability of winning is equal to that candidate's voteshare. Both assumptions will produce identical outcomes as long as the candidates are risk-averse.

It was argued that the winning candidate, once elected, may be tempted to renege on her announced policy platform and implement her own ideal policy. If the voters are rational and informed about policy preferences of the candidates, then they should evaluate candidates not on their announcements (which are not credible), but on their ideal policies (which the candidates cannot control). A candidate can commit to her announcement only if she expects to be punished in the case she deviates. One example of such a punishing a mechanism was given in Alesina (1988).

Example. Let $N = 2$, $n = 1$. There is a winner-take-all system, with the median voter's ideal policy is uniformly distributed on $[0, 1]$. The preferred policies of the candidates are $a_1 = 0$ and $a_2 = 1$. The voters are sincere, and the utility of candidate i is given by

$$u_i = -(a_i - y)^2, \tag{7.5}$$

where y is the policy that will be realized. In a subgame-perfect Nash equilibrium, the candidate's policies are $y_1 = a_1$, $y_2 = a_2$, with each candidate winning with probability $\frac{1}{2}$. This outcome is Pareto-inferior to the one where both candidates can commit to policies $y_1 = y_2 = \frac{1}{2}$. The expected utility of each candidate is $-\frac{1}{2}$ in the first case, and $-\frac{1}{4}$ in the second. However, if candidate 1 chooses $y_1 = a_1 = 0$, when both the voters and the other candidate expect her to choose $y_1 = \frac{1}{2}$, her payoff would be $-\frac{1}{8}$.

Now consider an infinitely-repeated game, with the utilities being discounted at the rate $\delta < 1$. The expected discounted payoff of each candidate is

$$U_i = -\sum_{t=1}^{\infty} E(a_i - y_t)^2. \tag{7.6}$$

Suppose that there is an agreement between the candidates that they should choose $y_1 = y_2 = \frac{1}{2}$. Their discounted payoffs would be $U_1 = U_2 = -\frac{1}{4}\frac{1}{1-\delta}$. If one of the candidates violates from the agreement, no further agreement is possible (so in all subsequent periods both candidates will act opportunistically, choosing $y_i = a_i$. The payoff of the voter who deviates from the agreement is $-\frac{1}{8}$ in period 1, and $-\frac{1}{2}$ in each of the subsequent periods. Thus the condition that both candidates will commit to a Pareto-optimal $y_1 = y_2 = \frac{1}{2}$ is

$$-\frac{1}{4}\frac{1}{1-\delta} > -\frac{1}{8} - \frac{1}{2}\frac{\delta}{1-\delta} \text{ or } \delta > \frac{1}{3}. \tag{7.7}$$

Under this gentlemen's agreement, the candidates will be better off because they are risk-averse, and the agreement insured them against risk. However, it requires that the voters believe that the winning candidate wouldn't revert to her ideal policy. If the candidates discount their utility slowly, each candidate would be worse off by reverting, since we have assumed that once she does, no further agreement can be reached.[56]

Alesina (1988) argued that there may be reputational costs of deviating from an announced (or agreed-upon) policy. But that also means that there are costs of announcing a policy that is different from one's ideal policy (as one would be tempted to deviate once the elections has been won).

Banks (1990) analyzed a two-candidate model where the true policy position of each policy-motivated candidate was private information. The two candidates had to make announcements

[56] Ferejohn (1986) considered a similar mechanism. There, the threat of the loss of credibility motivated incumbents to exert additional effort in office.

of their positions to reveal some of that information to the voter. There were explicit costs of making policy announcements, with the cost increasing in the distance between the true and the announced policy. Each candidate faced a tradeoff: in order to convince the voters that she is a moderate (and increase her chances of winning), she had to make a costly announcement.[57]

Convincing the voters that one is a moderate candidate may be costly because it would require making some costly action prior to the elections — such as an extremist incumbent pursuing a moderate policy in his first term. Alesina and Cukierman (1990) looked at a model with are 2 candidates, incumbent and challenger. There are 2 periods. Candidates derive utility from both office and policy. The policy preferences of the candidates are single-dimensional and quadratic. The incumbent selects the policy in period 1. Then, elections take place. The voters vote, taking into account only their expectations of the ideal policies of the candidates (any announcements are not considered to be credible).

The ideal policy of the challenger for the second period is known; the ideal policies of the incumbent for periods 1 and 2 are positively correlated random variables. Thus the incumbent faces a tradeoff. On one hand, she may want to choose a position as close to her ideal policy as possible. On the other hand, her choice may reveal to the voter the some information regarding her preferred policy. If her first-period preferred policy happens to be extreme, and if this is private information, then the candidate may want to choose a more moderate first-period policy in order to convince the voter that her second-period preferred policy will be moderate, and obtain a better chance of being reelected.[58]

This and other works suggest that some candidates may be better off if the voters are more ambiguous about the policy that they will implement[59].

7.3 Citizen candidacy

Until now, we assumed that the candidates either were indifferent to policy, or had exogenous policy preferences. We then considered how the policies that the candidates select would depend on their policy preferences. The principal theoretical weakness of this approach is that it requires solving (or assuming away) the commitment problem.

If that problem cannot be solved, then one has to investigate the determinants of the candidates' policy preferences. The "citizen candidate" approach assumes that each candidate is a voter who chose to run (at a cost) for office in order to influence the outcome of the elections in her favor (and, possibly, to obtain some office rents). Such a setting would naturally produce

[57]One of the reasons why deviations from announced policies may be costly is the loss of reputation by the deviating candidate. Aragones, Palfrey, and Postlewaite (2005) looked at a scenario there candidates have to make policy announcements before each elections. If an announcement is not incentive-compatible, or if the candidate has reneged on her announcement in the past, the voter will believe that the candidate will implement her ideal policy. See also Aragones and Postlewaite (1999) and Casamatta and de Donder (2003).

[58]In the one-period model of Cukierman and Tommasi (1998), the incumbent can commit to any policy platform of his choice, while the challenger is restricted to choosing only her favorite platform. The ideal policies of both candidates are not known to voters, but are correlated. Thus the incumbent faces a tradeoff, as her choice of policy reveals some of the information on the platform that the opponent will implement if elected.

[59]Glazer, Gradstein, and Konrad (1998) suggested a different reason why candidate's policy motivation can give rise to strategic behavior. The authors conjectured that changing the policy is costly both for the winning candidate and for the public, and argued that a candidate who makes a policy proposal faces a tradeoff. On one hand, a policy platform closer to the median voter's ideal policy is acceptable to a larger share of voters. On the other hand, adopting an extreme policy gives the candidate an advantage in the next elections (when she will be an incumbent). If her proposed and implemented policy is sufficiently distant from the ideal policy of the challenger, then the latter will be expected to change the policy in the case she is elected. This will be to the detriment of the voters, who will be more likely to support the incumbent.

a divergence of policy platforms. Indeed, a voter is not likely to bear the cost of running if there is already a candidate with policy preferences similar to that of her own.

One can then investigate how the number of candidates and their policy positions depend on the cost of entry, office rent, and the distribution of voter preferences. The credit for this approach is shared by two well-known works, published nearly simultaneously. Osborne and Slivinskii (1996) assumed sincere voters and a linear policy space, while Besley and Coate (1997) looked at a more general model, with strategic voters and general-form individual preferences over a finite number of alternatives. The results yielded by the two works are complementary.[60]

There are several other works of the same vein. Hamlin and Hjortlund (2002) analyzed citizen candidacy for sincere voters under proportional representation (when the policy is determined as a weighted average of candidate positions).

Policy divergence in a citizen candidate model is an outcome of the candidate's inability to credibly commit to a policy other than her own ideal policy. The policy, however, may be influenced by some third party — for example, by a lobbyist who compensates the winning candidate. The implication of such behavior for strategic voters was analyzed Besley and Coate (2001).

7.4 Sincere voters, linear policy space

Osborne and Skivinskii (1996) considered a continuum of sincere voters with ideal policies uniformly distributed on $[0,1]$. Before the election takes place, each voter decides whether to become a candidate and to run for office. The decisions of whether to run are taken simultaneously. The winner is determined by the plurality rule.

If a voter decides to run, she incurs a fixed cost of running c. If she wins the elections, she implements her preferred policy and obtains an office rent b. Thus the utility of a candidate with the ideal policy v is

$$U_v = b - c \tag{7.8}$$

if she is the sole winner of the election, and

$$U_v = -|y - v| - c \tag{7.9}$$

if she loses, where y is the policy position of the winning candidate. The voters (and candidates) are risk neutral. If $k > 1$ candidates obtain the same maximum amount of vote, then office rents are divided equally, and each candidate's payoff is

$$U_v = \frac{1}{k} \sum_{V_i = \max V} |y_i - v| + \frac{b}{k} - c, \tag{7.10}$$

If a voter does not run, her utility is

$$U_v = -|y - v| \tag{7.11}$$

if there is only one winning candidate, and

$$U_v = -\frac{1}{k} \sum_{V_i = \max V} |y_i - v| \tag{7.12}$$

[60] In a subsequent work, Besley and Coate (1998) looked at a two-period voting model, with preferences and policies defined in familiar terms of tax rates, personal consumption, and public good provision. The main result was that citizen candidacy does not always lead to a Pareto-optimal decision with regard to the production of a public good.

if there are k winning candidates.

If a voter decides to become a candidate, then her ideal policy automatically becomes her policy position. A candidate cannot credibly commit to any other policy. We assume that if the number of candidates is zero, then all voters receive the utility of $-\infty$.

The first observation is that if the cost of running is high relative to the value of the office rent, then in an equilibrium one may have a single uncontested candidate. Indeed, suppose that that candidate is the median voter. Then, she can be successfully challenged only by another median voter. In that case, the two candidates would split the office rent. If, however, the cost of running is higher than one half of the office rent ($b \leq 2c$), then the expected utility of the entrant will be less than zero, and only one candidate will run.

If the cost of running is very high, then the single candidate may have a position that is different from that of the median voter. Such a candidate can be defeated by an entrant whose position is closer to the median than her own. However, the entrant's gain (the sum of the office rent and the utility gain from policy change) must be greater than the cost of entry.

Example. Let $b < c$, and let there be a single candidate with the position $y_1 \leq \frac{1}{2}$. Then no potential candidate with a position $y_2 < y_1$ or $y_2 > 1 - y_1$ can enter, since she will lose the elections. A candidate with a position y_2 such that $y_1 < y_2 < 1 - y_1$ will win by entering. The necessary and sufficient condition for the existence of a single-candidate equilibrium is that the entrant's gain, $y_2 - y_1 + b$ should be less than the cost of entry c for all y_2 such that $y_1 < y_2 < 1 - y_1$, or $y_1 > 1 - c + b$.

The higher the cost of entry, the further from the median the candidate's position can be. In order for a voter to benefit from entry, her ideal policy has to be sufficiently distant from the incumbent's (so that there is a sufficient policy gain to cover the net cost of entry $c - b$). However, with a position too distant she will not capture the median voter, and will lose the vote.

The second observation is that in a two-candidate elections, the ideal policies of the candidates (and hence their political platforms) always diverge. The policy distance between the candidates is determined by two factors. First, the distance must be small enough to avoid successful entry by the median voter. Second, the candidates must be sufficiently far apart. If they are too close, then a candidate's loss from dropping out (her foregone share of office rents plus the loss due to policy distance) will be smaller than the cost of entry.

Example. Let there be two candidates in an equilibrium. Then we must have $y_1 + y_2 = 1$, because otherwise the voteshares of the candidates will not be equal, so one of them will win with certainty, and the other one will benefit from withdrawing. The median voter will not be able to enter if $\frac{y_2 - y_1}{2} < y_1 + \frac{y_1 + \frac{1}{2}}{2}$, or $y_1 > \frac{1}{6}$. Neither candidate will benefit by dropping out if $\frac{1}{2} - y_1 + \frac{b}{2} > c$. Both of the above conditions are satisfied for some $y_1 + y_2 = 1$ if the cost of running is not too high: $c < \frac{1}{3} + \frac{b}{2}$.

The third observation is that multicandidate equilibria are possible (including equilibria with three candidates, which cannot exist in the standard Downs model). If the cost of entry is sufficiently low, then one can have an equilibrium where all candidates receive an equal share of the vote.

Example. Consider three candidates with positions $y_1 = \frac{1}{3} - a$, $y_2 = \frac{1}{3} + a$, and $y_3 = 1 - a$ for some $0 < a < \frac{1}{3}$. Let $b = 0$. It is straightforward to check that the voteshare of each candidate will be equal to $\frac{1}{3}$. There are two two conditions for the existence of a two-candidate equilibrium. The first one is that no voter can improve her welfare by entering. Suppose that a candidate enters with a position y between y_1 and y_2. She obtains a voteshare equal to a. The voteshare of candidate 3 will still be equal to $\frac{1}{3}$, so candidate 3 will win outright, resulting in a payoff of $a + y - c - 1$ to the entrant. The entering candidate would have received $-2a$ had she not entered. Thus the no-entry condition is $c > 4a - \frac{2}{3}$.

The second condition is that no running candidate can improve her welfare by dropping out. The utilities of candidates 1, 2, and 3 are

$$U_1 = U_2 = U_3 = -\frac{1}{3}(2a + \frac{2}{3}) - c.$$

It is easy to check that each candidate will obtain a utility of $-2a$ by withdrawing. Thus the second condition is that the cost of entry is sufficiently low: $c < \frac{2}{3} - \frac{4}{3}a$.

One can also imagine a three-candidate equilibrium in which the voteshare of one of the candidates is strictly less than that of her rivals. Despite losing with certainty, the candidate does not withdraw since she still influences the identity of the winner (and the outcome of the elections).

Example. Suppose that there are three candidates such that $y_1 < y_3$, $y_1 > 1 - y_3$, and $y_2 = 1 - \frac{y_1+y+3}{2}$. Then the voteshares of Candidates 1 and 3 will be $\frac{1}{2} - \frac{y_3-y_1}{2}$, and the voteshare of Candidate 2 will be $\frac{y_3-y_1}{2}$. If $y_3 - y_1 < \frac{1}{3}$, then Candidates 1 and 3 will win the elections and Candidate 2 will lose. The expected utility of Candidate 2 will be $-c - \frac{y_3-y_1}{2}$. However, if Candidate 2 drops from the elections, then Candidate 1 will win with certainty, and Candidate 3's utility will change to $-(1 - \frac{y_1+y_3}{2} - y_1) = -1 + \frac{3y_1}{2} + \frac{y_3}{2}$. That will be a decrease as long as $1 - c > \frac{y_1+y_3}{2}$[61].

7.5 Discrete strategic voters, plurality rule

The citizen candidate model of Besley and Coate (1997) looked at the implications of strategic voting behavior. A strategic voter's decision which candidate to support (or whether to abstain) is her best choice given the choices of other voters. Thus a strategic voter may not support a close but hopeless candidate, knowing that she can help a ensure a victory of a more distant (but acceptable) candidate.

Consider N voters with weights w_i, and ideal policies y_i, $i = 1, \cdots, N$. The elections are modeled as a two-stage game. At the first stage, each voter decides whether to become a candidate. Denote the decision of voter i by $r_i \in \{0, 1\}$.

The decisions whether or not to run are made simultaneously and are observed immediately after. At the second stage, each voter decides which candidate to support. Let $v_i \in \{0, 1, \cdots, N\}$ be the voting decision of the voter i, with $v_i = 0$ indicating that the voter abstains. Thus the strategy of voter i for this two-stage game is $(r_i, v_i(r_1, \cdots, r_N))$.

As before, the payoff to a voter with the ideal policy v is given by (7.8)-(7.12)[62] .

The conditions for the existence of a single-candidate equilibrium under strategic voting are identical to those for sincere voting. If there are only two candidates in equilibrium, then every voter's dominant strategy is to support the nearest candidate (that is, to "vote sincerely"). Indeed, a voter who supports a more distant candidate will not decrease her utility by voting

[61] Formally, an equilibrium would require an additional condition that a candidate to the left of Candidate 1 should not be able to enter.

[62] Besley and Coate (1997) also considered a more general setting. The preferences of the voters are given by a $N \times N$ matrix defining the utility of each candidate's policy to each voter.

for the candidate with a closer position. A voter will abstain only if she receives the same utility from the election of both candidates (if such a voter is sincere, then she will vote for either candidate with equal probability). Thus it is sufficient that the voteshare of any entering candidate be less than the cost of entry.

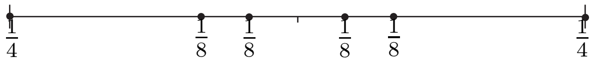


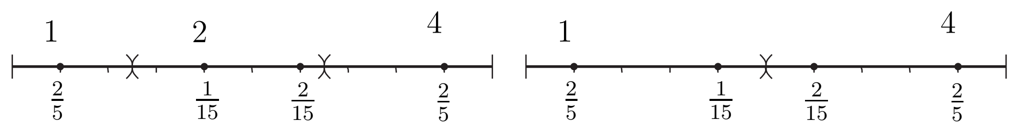
Figure 24: No two-candidate equilibrium exists under strategic voting.

The conditions for the existence of a two-candidate equilibrium are less restrictive if the voting is strategic. The candidates still be at some minimum distance from the median voter in order to benefit from competing against one another. However, the candidates no longer have to fear the entry by a third candidate representing the median voter. If such candidate enters, the voters with the ideal positions close to the median will not support the entering candidate. If any single voter supports the median candidate, then she will cause the "wrong" one of the two original candidates to be elected with certainty (we remember that in any two-candidate equilibrium the candidates have equal chances of being elected).

Example. There are 6 voters of weight $w_1 = w_6 = \frac{1}{4}$ and $w_2 = w_3 = w_4 = w_5 = \frac{1}{8}$, with the ideal policies $y_1 = 0$, $y_2 = 4$, $y_3 = 5$, $y_4 = 7$, $y_5 = 8$, and $y_6 = 12$ (see Figure 24). Suppose that Voters 1 and 6 are running for office, with Voters 1, 2, and 3 supporting Voter 1's candidacy and Voters 4, 5, and 6 supporting Voter 6's candidacy. This cannot be an equilibrium under sincere voting. Voter 3 can become a candidate and win the election, because Voters 2, 3, 4, and 5 will vote for her. However, this will be an equilibrium if the voters are strategic. Suppose that Voter 3 becomes a candidate. If Voter 2 supports that candidate (as she would were she sincere), then Voter 6's candidacy will win the elections with certainty, and Voter 2's expected payoff will decrease from -6 to -8. Similarly, the candidacies of Voters 3, 4, and 5 will not receive any support.

Intuitively, the existence of "spoiler" should less likely under strategic voting. However, this can still happen.

Example. There are four voters with positions $y_1 = 1$, $y_2 = 4$, $y_3 = 6$, and $y_4 = 9$ and weights $w_1 = w_4 = \frac{2}{5}$, $w_2 = \frac{1}{15}$, and $w_3 = \frac{2}{15}$. If the cost of running c is sufficiently small, then under sincere voting there is a 3-candidate equilibrium with participating candidates 1, 2, and 4. Candidates 1 and 4 have equal chances of winning the election, while 2 is certain to lose. But this will also be an equilibrium under strategic voting: Voter 2 will run and vote for herself in order to prevent Candidate 4 from winning. If she does, then she will have the support of Voter 3.



(a) 2 participates in the election, 1 and 3 share the office. (b) 2 does not participate in the election, 4 wins the office.

Figure 25: Sincere voting by strategic candidates in a three-candidate equilibrium.

In general, given strategic voting we should expect fewer equilibria with a large number of candidates. However, we should expect rational voters may act more sincerely if they are able to coordinate their actions within a coalition. Consider the two-candidate example. If Voter 3 enters, and Voters 2,3, 4, and 5 decide to coordinate their actions and support Voter 3's candidacy, then Voter 3's candidate will win with certainty, and all 4 voters will be better off.

7.6 Sincere voters, proportional representation

We next consider citizen candidacy under a proportional representation electoral system.

A departure from the winner-take-all setting requires providing a mechanism that translates the positions and voteshares of all candidates into the policy that is realized. We take the resulting policy to be the weighted average of the candidate positions, with the weights being candidate voteshares. We assume that the voters have linear disutility and zero office rents.

The main difference between plurality rule and proportional representation is that under sincere voting and proportional representation one cannot have more than two candidates.[63] Suppose that there are two candidates (Left and Right), and that the third candidate with some intermediate enters. Then, the resulting policy is subject to two effects. First, it should move to the right, as the voteshare of the Left candidate will decrease in favor of the entrant. For the same reason, it should also move to the left, as the voteshare of the Right candidate will also decrease. If policy preferences are uniformly distributed and the policy is determined proportionally, then the two effects will be equal in magnitude, and entry will not affect the resulting policy. Thus, the entry decision will not be justified.

Under plurality rule, in any two-candidate equilibrium the voteshares of the candidates were equal. This does not have to be the case given proportional representation, as a candidate may influence the policy outcome even if her voteshare is strictly smaller than that of her opponent. Suppose that the policies of the candidates are y_1 and y_2. Then the utilities of the candidates are

$$U_1 = -\left(y_1 \frac{y_1 + y_2}{2} + y_2(1 - \frac{y_1 + y_2}{2}) - y_1\right) - c,$$

and

$$U_2 = -\left(y_2 - y_1 \frac{y_1 + y_2}{2} - y_2(1 - \frac{y_1 + y_2}{2})\right) - c.$$

By dropping out either candidate will obtain $-(y_2 - y_1)$. Thus the condition that Candidate 1 will not drop out is

$$c < (y_2 - y_1)\frac{y_1 + y_2}{2}.$$

For Candidate 2, such condition is

$$c < (y_2 - y_1)\left(1 - \frac{y_1 + y_2}{2}\right).$$

Barring entry from a third candidate, any two candidates can run in an equilibrium if the cost of running is small enough[64].

[63] This is contrary to the well-known Duverger (1964) prediction that proportional representation should lead to a greater number of political parties.

[64] Another consequence of the resulting policy being the weighted average of the policies of the two candidates is that the policy to which a candidates may now be able to commit in not necessarily her own ideal policy. A candidate may choose to pursue a more extreme policy in order to shift the resulting policy closer to her ideal policy.

Consider two candidates who commit to policy positions a_1 and a_2. The resulting policy outcome is $\tilde{a} = a_1\frac{a_1+a_2}{2} + a_2(1 - \frac{a_1+a_2}{2}) = a_2 + \frac{a_1^2-a_2^2}{2}$, and the utilities of candidates are $U_1 = |\tilde{a} - \frac{1}{4}|$ and $U_2 = |\tilde{a} - \frac{3}{4}|$, where $\frac{1}{4}$ and $\frac{3}{4}$ are the ideal policies of the candidates 1 and 2, respectively. If we solve the maximization problem for each candidate, we obtain the only (and stable) equilibrium at $a_1 = 0$ and $a_2 = 1$.

The proportional representation setting admits multiple equilibria if the distribution of voter preferences is not linear. In that case, the entry of an intermediate candidate will have an effect on that candidate's ideal policy, possibly justifying the entry decision.

8 Special interest politics

Winning an elections requires resources that must be spent in order to increase the name recognition of the candidate, inform the voters of her policy platform, and so forth. This presents an opportunity for a special type of stakeholders, the special interest groups, to influence policy (directly or indirectly) by supplying funds to the candidates in exchange for the right choice of policy platform. The theoretical interest lies in explaining how the existence of such groups affects the final policy outcome and the identity of the winning candidate.

8.1 Noncontingent contributions

One way to look at a campaign contributions is to consider is to be an effort by a special interest group to help the election of a specific candidate, whose policy platform the group considers to be the most acceptable. The contributions made to the candidates depend on the policy platforms of the candidates. The candidates, on their part, select policies expecting the groups to make the contributions.

Literature had suggested two ways by which a campaign contribution can increase a candidate's voteshare. First, such as in Austen-Smith (1987), the candidate can use contributions in order to decrease the uncertainty of the risk-averse voters about her position.

Consider the first example from Section 6. Candidate 2 can increase her voteshare by reducing σ_y^2. Note that in this model, reducing the variance is identical to increasing the valence of the candidate (as in Zakharov, 2006).

The second way contributions can help to increase a candidate's voteshare is through persuasive advertising[65]. Some voters, it can be argued, do not base their voting decision on the policy positions of the candidates, but rather react passively to political advertisement. The candidates can use campaign funds in order to sway these "uninformed" voters in what is essentially a rent-seeking contest. The next example follows the model of Baron (1994).

Example. A model with noncontingent contributions. Suppose that there are two office-motivated candidates with policy positions $y_1 \leq y_2 \in [0, 1]$ and two groups of voters. The first group of size $\lambda < 1$ are the informed voters. The informed voters have single-peaked policy preferences with ideal policies uniformly distributed on $[0, 1]$. An informed voter supports the candidate with the nearest policy platform.

The second group of size $1-\lambda$ are the uninformed voters. The probability that an uninformed voter supports candidate i is $\frac{c_i}{c_1+c_2}$, where c_i is the campaign budget of candidate i. Thus the uninformed voters are divided between the candidates in a rent-seeking fashion. The voteshares

[65] A nonspatial model of persuasive advertisement was developed by Skaperdas and Grofman (1995). It was assumed that the voters belong to one of the three groups: supporters of candidates 1 and 2, and the undecided voters. The one-shot election game began with some initial distribution of voters. The candidates could exert resources either to change a fraction of the undecided voters into supporters (positive campaigning), or to change the supporters of the opponent into undecided voters. The authors found that the closeness of the race and the number of candidates affected the way the candidates divide a fixed campaign budget between positive and negative campaigning.

of the two parties are

$$V_1 = (1-\lambda)\frac{c_1}{c_1+c_2} + \lambda\frac{y_1+y_2}{2}$$

and

$$V_2 = (1-\lambda)\frac{c_2}{c_1+c_2} + \lambda\left(1-\frac{y_1+y_2}{2}\right).$$

Campaign budget of Candidate 1 is fixed at $c_1 = \bar{c}$. Campaign budget of Candidate 2 comes from the contribution made by a special interest group.

The elections are played as a two-stage game. First, Candidate 2 selects policy platforms $y_2 > \frac{1}{2}$. The policy platform of Candidate 1 is assumed to be fixed at the median voter's ideal point. Then, the special interest groups makes a contribution c_2. After that, voting takes place and benefits to all parties are realized.

We begin analyzing this model by solving the special interest group's problem. Suppose that the ideal policy of the group is $a = 1$. Assume that the probability that a candidate wins the office is equal to her expected voteshare. An interest group is policy-motivated, with linear disutility. The payoff to the interest group is then

$$\tilde{U} = -V_1(1-y_1) - V_2(1-y_2) - c_2, \tag{8.13}$$

or

$$\tilde{U}_2 = -((1-\lambda)\frac{c_1}{c_1+c_2} + \lambda\frac{y_1+y_2}{2})(1-y_1) - ((1-\lambda)\frac{c_2}{c_1+c_2} + \lambda(1-\frac{y_1+y_2}{2} - y_2)) - c_2. \tag{8.14}$$

The first-order maximization condition is

$$c_2^* = \sqrt{(1-\lambda)(y_2-y_1)c_1} - c_1. \tag{8.15}$$

We assume that c_1 is small enough for c_2^* to be positive. The intuition behind this relationship is quite simple. If the positions of the candidates are identical, the special interest group cannot influence the final policy outcome by making campaign contributions. If the positions are different, then the group is better off supporting a candidate with the closest policy position.

This behavior of the special interest group is anticipated by candidate 2 when she propose her policy platforms. The candidate faces a tradeoff. On the one hand, locating near the median voter helps capture the informed electorate. On the other hand, the candidate must differentiate her position from that of her opponent in order receive campaign contributions, which are needed to sway the uninformed voters.

The voteshare of Candidate 2 is

$$V_2^* = (1-\lambda)\left(1-\frac{\sqrt{c_1}}{\sqrt{(1-\lambda)(y_2-y_1)}}\right) + \lambda\left(1-\frac{y_1+y_2}{2}\right). \tag{8.16}$$

The first-order maximization condition is

$$\frac{\partial V_2^*}{\partial y_2} = \frac{1-\lambda}{2}(y_2-y_1)^{-\frac{3}{2}} - \frac{\lambda}{2} = 0. \tag{8.17}$$

It follows that

$$y_2 = y_1 + \left(\frac{1}{\lambda}-1\right)^{\frac{2}{3}}. \tag{8.18}$$

For expository purposes, we considered a single interest group and a single voteshare-maximizing candidate. Most works analyze models with more than one lobby and two or more candidates. There are several theoretical predictions. Notably, a group will only contribute to the candidate with the closest policy position.

8.2 Contingent contributions

In the previous example, the special interest groups could not make any credible promises to the candidates prior to the selection of policy platforms by the candidates. A number of works considered the opposite. A special interest group is able to present the candidates a menu of contracts specifying how much a candidate will receive for proposing a specific policy. After that, the candidates select policies, and the elections take place. The contribution received by the candidate can then be used to increase her voteshare through advertising.

In a well-known work, Grossman and Helpman (1996) predict that the candidates will select divergent positions. The candidate that has an exogenous advantage[66] is expected to attract more contributions from the special interest groups, who at least compensate the candidate's loss of voteshare from the selection of unpopular policies.

8.3 Lobbying and policy-motivated candidates

If the candidates are policy-motivated, the contributions can be used to compensate them for the losses associated with deviating from one's preferred policy.

Besley and Coate (2001) looked at lobbying in a citizen-candidate setting[67]. After the election results are realized, lobby groups may bargain for policy with the winning candidate. We are interested in knowing how the number and policy preferences of the lobbyists affects the identities of the running candidates and the policy that is realized in equilibrium.

One of the results of is that we should not expect the lobbies to have a significant affect on the policies that will be implemented in equilibrium. This is because the voters anticipate the degree to which each candidate's policy will be affected by lobbying, and will compensate for that influence. We should expect this to happen if the range of voter types is sufficiently broad, and the equilibrium utility of the winning candidate does not depend on her policy preference too greatly (either due to compensation from the lobby groups or from there being more than one winner).

Example. There is a continuum of citizens with policy preferences uniformly distributed on $[0, 1]$. The policy disutility of citizens and candidates is quadratic in policy distance. Thus the utility of the winning candidate is

$$U = -(y - v)^2 - c,$$

and the utility of a voter is

$$U = -(y - v)^2,$$

where y is the realized policy and v is the preferred policy.

If there is no lobby, in equilibrium we have candidates with policy preferences

$$y_1 + y_2 = 1$$

subject to the participation constraint[68]

$$\sqrt{2c} \leq y_2 - y_1.$$

[66] The authors use a concept different from valence.

[67] See also Coate (2001). An alternative approach at candidate motivation was taken by Glazer and Grafstein (2001). The authors assumed that the candidates select policy platforms in order to maximize contributions from the citizens. In equilibrium, the candidate policy platforms diverge. They diverge further if the caps on contributions are introduced. See also Che and Gail (1998) for the analysis of effort caps in the framework of an all-pay auction.

[68] For exposition purpose, we assume that only two candidates can run in equilibrium, and that they neglect the possibility of entry by a third candidate.

Thus a pair of policies proposed in a two-candidate equilibrium is given by

$$\left(\frac{1}{2} - y, \frac{1}{2} + y\right), \text{ where } y \geq \sqrt{\frac{c}{2}}. \tag{8.19}$$

Now suppose that there is a lobby group with quadratic disutility and preferred policy a, where $0 \leq a \leq 1$. It offers the winning candidate a take-it-or-leave-it contract (y, b), specifying a payment b in exchange for the implementation of policy y. The candidate with preferred policy y_s will accept the contract as long as

$$(y - y_s)^2 \leq b. \tag{8.20}$$

Thus the lobbyist's optimization problem is

$$\max_{(y,b)} U_l = -(a - y)^2 - b$$

subject to the participation constraint (8.20). This problem has the solution

$$\bar{y}_s = \frac{a + y_s}{2}. \tag{8.21}$$

As the lobbyist is assumed to have all the bargaining power, the winning candidate is indifferent between the implementation of her preferred policy y_s and the implementation of y_s with the payment b.

We want to know what pair of policies $(\bar{y}_1, \bar{y}_2)$ can be realized in equilibrium. There are three conditions that we have to impose on $(\bar{y}_1, \bar{y}_2)$. First, candidate voteshares in a two-candidate equilibrium must be equal. Since the voters are assumed to be strategic, believing that candidate i will end up implementing policy $\bar{y}_i$, we must have

$$\frac{\bar{y}_1 + \bar{y}_2}{2} = \frac{1}{2}. \tag{8.22}$$

Second, the policies that the candidates propose must be in the range $[0, 1]$. Without loss of generality we take $y_1 < y_2$ and $a \leq \frac{1}{2}$. Thus

$$y_2 = 2(1 - a) - y_1, y_1 \in [0, 1 - a]. \tag{8.23}$$

From conditions (8.23) we get

$$\bar{y}_2 = 1 - \bar{y}_1, \ \frac{a}{2} \leq \bar{y}_1 \leq \frac{1}{2}. \tag{8.24}$$

Third, each candidate's ideal policy must be sufficiently far apart from the policy that will be implemented if she withdraws. Since we assumed that $a < \frac{1}{2}$, Candidate 2 has more to lose if she withdraws. Candidate 1 gains $-(\bar{y}_2 - y_1)^2$ if she withdraws, and $-\frac{1}{2}(\bar{y}_2 - y_1)^2 - c$ if she does not. Thus the condition that Candidate 1 will not withdraw is

$$\sqrt{2c} \leq |\bar{y}_2 - y_1| = |1 - \frac{a}{2} - \frac{3}{2}y_1|. \tag{8.25}$$

This result rests on two assumptions regarding the nature of the special-interest groups. First, the work (implicitly) assumes that the special-interest group is not able to commit to any menu of contracts in advance. Second, it is assumed that the lobbying special-interest group has all the bargaining power. Thus the contract that will be offered to the winning candidate will just compensate that candidate for the departure from her ideal policy. All potential candidates (or at least those whose ideal policies fall within a certain range) thus become purely office-motivated.

The voters know which policy will be implemented by each of the winning candidates. The voters are strategic, so the majority of votes will be received by the candidate whose effective position is closer to the median voter's. If no candidate's effective position is equivalent to the median voter's optimal policy, then such a candidate will enter and win the elections. Thus in an equilibrium the median voter's optimal policy is selected, and all running candidates will end up implementing identical policy positions. The lobby's power is thus effectively offset by citizens who support candidates with diametrically opposing preferences. Possible exceptions arise only when the population's preferences are sufficiently narrow, and if the only lobby's preferences are too extreme[69].

The chief positive result was that the existence of lobby groups does not have a significant impact on the equilibrium policy. It is also likely to produce the median-voter result characteristic of Downsian frameworks in which the candidates are office-motivated.

9 Political parties in spatial models

The economic agents we considered so far were either individual candidates contesting an executive job, or monolithic political parties contesting seats in a legislature.

However, coalitions of political parties, and political parties themselves, are collections of agents with diverse interests. Any action taken by a party (or a coalition) must be acceptable to all its members, to the extent that none of the members would be better off leaving the party (possibly forming a new one in the process).

There are two ways to model a political parties.

First, a political party can be considered a coalition of individual candidates. The issue is whether such coalition (formed before or after the elections) is stable, and whether the formation of coalitions before or after the elections affects the identity (and policy platforms) of the candidates (or parties) contesting seats in a legislature. For example, a political party can be modeled as a mechanism that allows candidates with identical policy platforms to reduce the cost of running. Theoretical predictions can be measured against several observed patterns. One such prediction is the Riker (1962) minimum winning coalition hypothesis. According to the hypothesis, a coalition of parties in a legislature forming a cabinet should contain just enough parties to form a majority. Because of the zero-sum nature of the process, smaller parties would be ejected from the coalition. Another prediction is that a coalition should be connected — it must not leave out a party if there are parties in the coalition with positions both to the left and to the right of the position of that party (Axelrod, 1970).

Second, a political party can be considered as a separate entity that has its own policy and/or office goals, that are different from office or policy goals of the candidates who are affiliated with that party.

9.1 Coalitions made before elections

There might be several reasons why coalitions might be formed prior to an election. Levy (2004) suggested that coalitions can act as commitment mechanisms for individual policy-motivated candidates. A single candidate's promise to implement a policy different from her preferred policy is not credible, which limits the number of votes she can obtain. For example, a far-right politician's promise to carry out a centrist policy will be viewed with suspicion since the

[69]The authors also demonstrate that the existence of more than lobby results in inefficient rent-seeking.

voters will believe that, once elected, she will pursue a far-right policy instead. However, that politician may join a political party with a more moderate platform that has the support of a larger number of voters, and attempt to shift the party's platform in the direction of her preferred policy.

In the author's model, there are two conditions imposed on the equilibrium. First, the voters must believe that a political platform offered by a coalition is credible. For that, there should be no other platform that can make all members of the coalition better off. Thus, the coalition can propose any platform that belongs to its Pareto set.

Second, the coalition must be stable. A coalition is not stable if it can be broken up into two sub-coalitions with different political platforms (subject to the first condition), and if all members of at least one these sub-coalitions are better off as a result. In particular, no single member of any coalition should benefit from running independently (thus coalitional stability is a stronger concept than Nash equilibrium).

Parties forming before the elections can lead to different policy outcomes, as illustrated in the following example.

Example. Let $N = 3$ and $n = 2$. There are three policy-motivated candidates with ideal policies $a_1 = (0, 0)$, $a_2 = (1, 0)$, $a_3 = (1, 1)$. There are three groups of voters with ideal policies $v_1 = (0, 0)$, $v_2 = (1, 0)$, $v_3 = (1, 1)$, of weight $w_1 = 0,3$, $w_2 = 0,4$, and $w_3 = 0,3$. The voters are sincere. The winner is determined by the plurality-rule system.

If no coalitions are allowed, then the voters believe that each candidate will implement her own ideal policy if elected. Candidate 2 will obtain the greatest voteshare and win the elections (moreover, it will be the Condorcet winner among the three policies).

Now suppose that Candidates 1 and 3 can form a coalition that offers a policy that belongs to the segment with the endpoints $(1-1/\sqrt{2}, 1-1/\sqrt{2})$ and $(1/\sqrt{2}, 1/\sqrt{2})$. Any such policy will be preferred to $a_2 = (1, 0)$ by both Candidates 1 and 3. Also, any such policy will be preferred to a_2 by voters of groups 1 and 3, and will defeat a_2 in an elections. Thus the coalition will be stable, and will be supported by the majority of voters[70].

In an interpretation of this example, provided by the author, there were 3 groups of voters: a small group of rich voters, and two groups of poor voters, divided according to some principle — regional or religious, perhaps. There are two policy dimensions: how much should the rich group be taxed and how the tax proceeds should be distributed between the two poor groups. In the absence of political parties, the larger poor group is the Condorcet winner, with the prevailing policy being high taxes and a relatively even distribution of wealth between the two groups. If political parties are allowed, however, the rich candidate may form a coalition with the smaller poor candidate and propose a low-tax policy where the entire tax proceeds are diverted to the smaller poor group.

One of the results obtained in Levy (2004) is that the formation of coalitions if feasible only if the number of potential candidates is not too large relative to the number of the dimensions of the policy space.

Example. No coalition formation is possible if there are three voter groups (and three candidates), but only one policy dimension. Indeed, suppose that $a_1 < a_2 < a <_3$ and $w_1 > w_2$, $w_1 > w_3$, and $w_3 < \frac{1}{2}$. If the coalitions are not possible, candidate from group 2 (who occupies the median position) wins the election, because he is supported by voters from groups 2 and 3. A coalition between candidates from groups 1 and 3 is not possible. Suppose that the coalition forms, and proposed a policy platform $y \leq a_2$. Then, voters from group 3 will support group 2's candidate instead. Also, no coalitions are possible if for some i we have $w_i \geq \frac{1}{2}$.

9.2 Coalitions made after elections and legislative bargaining

There are several reasons why modeling the behavior of parties in a parliamentary democracy is a challenging task. First, in order to have some influence on policy, the party must be a member of the cabinet-forming coalition. Its ability to enter into a coalition with other parties depend on the campaign policy promises made by the party to its voters and contributors, on the ideological position of the party and its voters, and on the number of seats that the party won in the legislature.

Second, the number of seats and the party's voteshare, in turn depends on the policy position of the party. The possibility of strategic behavior on the part of the voters further complicates the issue (even if the voters consider the party's campaign promises to be credible).

We first look at the problem of the formation of the winning coalition given policy positions of the legislators.

Example. Suppose that in a legislature there is an odd number N of parties with ideal policies $a_1, \cdots, a_N \in \mathbf{R}$. Suppose that party i makes a coalition offer that consists of a policy y and a redistribution $g_1, \cdots, g_N$ of a fixed amount of wealth G. If the offer is accepted, j's payoff is

$$U_j(g_j, y) = g_j - (a_j - y)^2. \tag{9.26}$$

The proposal is implemented if it is accepted by the majority of parties. Otherwise, the status quo is implemented, and everyone receives zero payoff.

The problem of party i is as follows:

$$\max U_i = g_i - (a_i - y)^2 \tag{9.27}$$

with the constraints

$$\#\{j : U_j(g_j, y) \geq 0\} \geq 0. \tag{9.28}$$

Assuming G is large enough, in the solution of this problem we have

$$y = \sum_{j \in S_i \cup \{i\}} a_i \text{ and } g_j = \begin{cases} (a_j - y)^2, & j \in S_i \\ G - \sum_{k \in S_i} g_k, & j = i \\ 0, & j \notin S_i \cup \{i\}, \end{cases} \tag{9.29}$$

where S_i is the set of $\frac{N-1}{2}$ legislators with positions closest to a_i.

The party i acts as the agenda setter or *formateur.* The coalition that forms as the result is both minimal and connected[71], while each party in the coalition (with the exception of the formateur) receives zero payoff.

This is a very simple one-stage legislature bargaining model that can be developed in several ways. Most obviously, the bargaining process may consist of a finite or an infinite number of steps, with a different party acting as the formateur at each stage. The process terminates if someone's coalition proposal is accepted.

The classical model of coalition bargaining of Baron and Ferejohn (1989), legislative bargaining is either a finite or infinite-stage process, with the agenda setter selected randomly at each stage. The coalition proposal involves a redistribution of a unit of wealth among the parties. The authors considered both closed and open systems. It the latter case, a (randomly

[71] The coalition is connected since all parties have identical weight, and the amount of wealth that is needed to compensate a party depends only on the policy distance between the party's policy position and the proposed policy.

drawn) party can propose an amendment to the existing proposal. It was shown that if amendments are possible, the coalition proposals would provide for more equal distribution of wealth among parties.

If the order in which policy proposals are made is predetermined, there is much room for strategic behavior. The influential work of Austen-Smith and Banks (1988) looks at a three-party legislature. The authors assume that there were three parties that contest seats in a legislature. After the elections, the parties must agree on some one-dimensional policy, and on some way to divide a fixed office rent.

The legislative bargaining is taken to be a three-stage process, with the largest party making its proposal at Stage 1, rent; if the proposal is not accepted by the majority of the parties, then at Stage 2 a similar proposal is made by the second-largest party. If the issue is not resolved by Stage 3, some default proposal is accepted. A proposal consists of the a one-dimensional choice of policy and a division of the office rent.

Example. There are three parties in a legislature, with ideal policies $a_1 < a_2 < a_3$. No party has a majority of seats.

The first policy proposal is made by Party 2. If the proposal is rejected by a majority, then Party 1 makes a similar proposal. If Party 1's proposal is rejected, Party 3 makes a proposal. If the agreement is not reached after that, a caretaker government is formed, and some default policy is implemented, with the utility of each party being zero. Denote y_i the policy proposal of party i, g_{ij} — the transfer that party i proposes to party j, such that

$$g_{i1} + g_{i2} + g_{i3} = G, \;\; g_{ij} \geq 0, \tag{9.30}$$

where G is the fixed amount of office rent to be distributed. We assume that G is large enough to allow positive payoffs to any party in any two-party coalition. The utility of Party i is

$$U_i = g_i^* - (a_i - y^*), \tag{9.31}$$

where y^* is the resulting policy, and g_i^* is the resulting transfer to Party i.

We solve the bargaining problem through backward induction. Party 3 will propose a coalition $\{2, 3\}$. The optimization problem of Party 3 would be

$$\max_{y_3, g} g_{33} - (y_3 - a_3)^2 \tag{9.32}$$

subject to constraints (9.30) and Party 2's participation constraint

$$g_{32} - (y_3 - a_2)^2 \geq 0. \tag{9.33}$$

The solution to this problem is

$$y_3^* = \frac{a_2 + a_3}{2}, \;\; g_{31}^* = 0, \;\; g_{32}^* = \frac{(a_3 - a_2)^2}{4}, \;\; g_{33}^* = G - \frac{(a_3 - a_2)^2}{4}. \tag{9.34}$$

Now consider the problem of Party 1. It will propose a coalition $\{1, 2\}$. The coalition proposal must offer Party 2 at least 0, which Party 2 will get under Party 3's coalition offer. The optimization problem will be similar to Party 3's, with the solution

$$y_1^* = \frac{a_2 + a_1}{2}, \;\; g_{11}^* = G - \frac{(a_2 - a_1)^2}{4}, \;\; g_{32}^* = \frac{(a_2 - a_1)^2}{4}, \;\; g_{13}^* = 0. \tag{9.35}$$

Party 2 must choose between coalitions $\{1,2\}$ and $\{2,3\}$. In the first coalition, Party 1 must get at least $G - \frac{(a_2-a_1)^2}{4}$. In the second coalition, it is sufficient to offer Party 3 a utility slightly above zero. Thus, Party 2 will propose the coalition $\{2,3\}$, with the coalition offer being

$$y_2^* = \frac{a_2+a_3}{2},\ g_{21}^* = 0,\ g_{22}^* = G - \frac{(a_3-a_2)^2}{4},\ g_{23}^* = \frac{(a_3-a_2)^2}{4}. \tag{9.36}$$

That offer is accepted by Party 3.

The authors argued that the order in which the coalition offers are made is determined by the share of seats that the parties hold. Under the bargaining mechanism described here, the prevailing coalition consists of the parties of highest and lowest weight. If follows that the winning coalition is minimal, but not necessarily connected.

At the next stage of their analysis, the authors solved the equilibrium voting strategies, and, finally, looked at the policy location problem of the parties. In an equilibrium, the smallest party located in the middle, while the two parties on the outside received an equal share of votes (unlike in the previous example). The authors have shown that in an equilibrium, there will be always some strategic voting.

Example. Suppose that the order in which the parties make proposals is determined by a fair coin toss if the voteshares of the parties are equal. Let the positions of parties be symmetric, with $a_2 = 0$, $a_3 = 2a$, and $a_1 = -2a$ for some $a > 0$. Suppose that the voteshares of Parties 1 and 3 are equal, and greater than those of Party 2. Then, the policies $-a$ and a are realized with probability $\frac{1}{2}$ each.

The expected payoff of voter with the ideal policy $0 < v < a$ is $-\frac{1}{2}(v+a)^2 - \frac{1}{2}(a-v)^2 = -v^2 - a^2$ if Parties 1 and 3 have equal voteshares, and $-(a-v)^2 = -v^2 - a^2 + 2av$ if Party 3 obtains the largest voteshare. Thus, the voter has an incentive to vote strategically.

There is a large literature on the issues relating to the formation of cabinets. A model of similar structure was analyzed by Baron (1993). The key distinction was that the voting was sincere, and that the parties maximized the aggregate welfare of its supporters.[72]

Most works in this vein, such as Baron (1991), Austen-Smith and Banks (1990), and Laver and Shepsle (1990), focus on the cabinet-formation part of the story, foregoing the policy location and voting stages. A major issue in coalition formation research is explaining the existence of minority governments. Diermeier and Merlo (2000) argued that uncertainty can lead to minority and surplus government (which are impossible in a deterministic setting of Austen-Smith and Banks).[73]

The stability of parliamentary governments was studied in a large body of empirical literature, including Laver and Schofield (1990), Warwick (1994), and others. The main factors affecting the government's duration were the size and ideology of the parties in the coalition. Those consisting of the single majority party were the most stable; a significant fraction (up to one third) of coalitions were formed by one minority party, although such governments

[72] Alesina and Rosenthal (1996) argue that the voters anticipate the bargaining process between different branches of the government. This may result in ticket-splitting — the same voter may support a Democratic presidential candidate but a Republican legislature.

[73] All models assume that there is some mechanism that determines the order in which parties make coalition offers. Strom, Bulge, and Laver (1994) review some of the limitations on the coalition formation process. Such constraints may include constitutional rules stipulating the minimum size of the coalition (such as in Germany), the parties that must be present in a coalition (such as in Belgium), or the composition of the coalition in case cabinet formation process reaches a stalemate.

were among the least stable. Most (but not all) coalitions were minimal winning coalitions. Governments formed by such coalitions were also relatively stable.

A number of recent empirical studies investigate the extent to which the voters take into account the parties' chances becoming a member of the winning coalition. Bawn (1999) found evidence of such strategic behavior in German elections. In particular, the voters were more likely to vote for a party that is expected to be in the government. Blais et. al. (2006) found evidence of strategic voting in Israel. Using telephone survey data, the authors found that the choice of a voter depended not only on her preferences over parties, but also on her preferences over potential coalitions. For the 2003 Israeli elections, the authors attributed as much as 10% of the voteshare of the Likud party to strategic voters who preferred far-right nationalist and religious parties.

9.3 Cost sharing

Political parties were also considered as a way to reduce the cost of running. Osborne and Tourky (2004) analyzed this problem in a very general setting. They considered a one-shot game among N candidates. Each player decides on whether to participate and, if so, which single-dimensional political platform to pursue. The payoffs to each player and the policy outcome somehow depend on the choices of the candidates. If some political platform is championed by more than one candidate, these candidates are said to form a political party. The authors made several assumptions.

First, they assumed that participation is costly: if a participating candidate decides not participate and the outcome does not change, then that candidate is better off.

Second, the authors assumed that a candidate moving to a larger party increases the candidate's utility if the outcome is unaffected.

Third, the authors assumed that the policy space to be one-dimensional, and that the policy outcome belongs to the median set of the proposed policies.

If these three assumptions are satisfied, then there are either two political parties of equal size, or political parties of equal size with an independent candidate between then. The result is intuitive: if three political parties exist, then there are two political parties at or to the right of the median voter; a candidate belonging to the smaller of the two parties will benefit from moving to the larger party, since it will not affect the outcome. An independent candidate would also prefer to move to a large party or drop out.

9.4 Parties as informative brands

The announcement of a policy platform by an individual politician contesting an executive office or a single-seat constituency may not be credible, since the candidate may be motivated by the implementation of some specific policy. Moreover, the candidate's policy preferences may not be readily observable by the voters. If the voters are risk-averse, such a candidate is at a disadvantage relative to a candidate whose policy position is known.[74]

Snyder and Ting (2002) suggested that political parties act as screening devices. Each political party has an announced policy position that, as they argued, is known to all voters.

A politician can join a party. The cost of party membership is greater for those candidates whose policy preferences are more distant from that of the party. There are costs of party membership the reflect the dislike of associating with someone who has different political views,

[74] Another view of information asymmetry is that the voters may not be perfectly informed about their own preferences (that depend on the state of the world), while the candidates may receive some correlated signals about it, and convey them in their policy platforms. See Heidhues and Lagerlöf (2003).

the discipline imposed by a party's leadership, or the imperfect ability of parties to screen out candidates with different idealogical views. Such costs may be prohibiting to potential candidates candidates whose positions are sufficiently different from that of the party.

Since joining a party is costly, a politician who does so conveys some information to the voter, and may gain an advantage relative to an unaffiliated candidate.

A vote maximizing party faces a tradeoff. On one hand, choosing an extreme position may only attract members with extreme policy preferences. On the other hand, taking a position near a population's median may attract a large number of candidates with varying positions, making the position of a randomly chosen party member more ambiguous. Which effect is stronger depends on the cost of joining a party.

If the parties are able to impose high membership costs on candidates with different policy positions, then all candidates that join a party will have similar policy preferences. Thus, the voters are well-informed of the policy that a party's candidate will end up implementing. In this case, the parties are expected to choose policy positions at or near the median voter[75].

If the cost of party membership is low, the parties will attract all types of candidates. One way to reduce the uncertainty about the positions of candidates affiliated with a party is to select a position near the end of a political spectrum. If a party selects a position such that there are no potential candidates to the right of that position, then the only candidates joining the party will be those with the positions to the left. The expected position of a candidate affiliated with that party will be to the left of the party's position, and the uncertainty will be smaller than in the case where the candidates affiliated with the party have positions both to the right and to the left of the party's position.

The only way to inform the voters about the positions of the party's candidates is to select an extreme position.

Example. Suppose that the voters are distributed uniformly on $[-1,1]$. Suppose that party 1 fields a single candidate with a known position of $y_1 = 0$. Party 2 has an announced position y_2. Candidate representing Party 2 is randomly chosen from the interval $y[y_2-d, \min\{y_2+d, 1\}]$, where d is a threshold that represents the signaling costs for candidates. The costs of joining the party are zero for parties with positions within the interval, and are prohibitive for parties with positions outside it.

The expected value and variance of the position of the candidate representing Party 2 are given by

$$\bar{y}_2 = \begin{cases} y_2, & y_2 + d \leq 1 \\ \frac{1+y_2-d}{2}, & y_2 + d > 1, \end{cases} \tag{9.37}$$

and

$$\sigma_{y_2}^2 = \begin{cases} \frac{d^2}{3}, & y_2 + d \leq 1 \\ \frac{(1-y_2+d)^2}{12}, & y_2 + d > 1, \end{cases} \tag{9.38}$$

The position of the indifferent voter is given by

$$v = \begin{cases} \frac{1}{2}\left(y_2 + \frac{d^2}{3y_2}\right), & y_2 + d \leq 1 \\ \frac{1}{2}\left(\frac{1+y_2-d}{2} + \frac{(1-y_2+d)^2}{6(1+y_2-d)}\right), & y_2 + d > 1. \end{cases} \tag{9.39}$$

The voteshare of Party 2 is

$$V_2 = 1 - \frac{1}{2}\min\{1, v\}. \tag{9.40}$$

[75] Consider a limiting case: A party admits only those candidates with positions equal to the party's platform. Then, the position of any candidate who is affiliated with some political party is identical to the policy position of that party. Thus, if there are two parties, we should expect both parties to locate at the median voter.

It can be shown that y_2 that maximizes V_2 increases with d. For $d \geq 1$ we have $y_2^* = 1$; for $d < 1$ we have $0 < y_2 < 1$. We have $y_2 \to 0$ as $d \to 0$.

9.5 Parties and elections in different constituencies

Another consideration is that political parties unite politicians that contest offices in different constituencies. The electorate in each constituency may have different policy preferences, thus the policy positions of the politicians that represent the party's interests may also be different. For example, on the issue of gun control a Democratic contender in Texas or Arizona is probably more conservative than her Republican colleague in New York or Massachusetts. A large number of influential politicians with different interests within a party (say, Democrats from Texas and Massachusetts) may lead to costs that the party may wish to optimize.

Eyster and Kittsteiner (2004) consider the setting where a candidate in each constituency is motivated only by winning the office, while political parties desire to capture as many constituencies across the country as possible, at the lowest possible cost. Thus a candidate that campaigns under some party's banner may be motivated to deviate from the official position of his party. However, there are certain costs attributed to this — either there are costs of party membership that increase with the distance between the candidate's ideal policy and the party platform, or there are costs of implementing a policy that deviates from that party' platform.

There are two office-motivated parties that participate in a two-stage game. At Stage 1 parties selects policy platforms y_1, y_2. At Stage 2 candidates from both parties in all constituencies propose policy platforms. The utility of candidate from party i in constituency s is

$$U_i^s = w - |y_i - v_s^i|$$

if she wins the election and

$$U_i^s = -|y_i - v_s^i|$$

if she loses. Here, v_s^i is the candidate's policy position. The losses represent the costs of deviating from the party line. The authors assume that the elections are decided according to the plurality rule. The election in each constituency can have three equilibrium outcomes. First, both candidates may select the median voter's policy $\tilde{y}$ with probability one. This happens, for instance, when party platforms are identical. Second, there may be a mixed-strategy equilibrium. Finally, if the the policy distance is large enough ($|y_1 - y_2| < 2w$), and either party is distant from the then the median voter, then the candidates select policies y_1 and y_2 with probability one (in author's terminology, the elections are uncontested). The winner is the candidate whose party's position is closer to the median voter (that candidate is said to "claim his home turf").

The expected equilibrium utility of a candidate may increase with the distance between her party's platform and the median. Thus the threat competition at the second stage may act as a deterrent and cause the political platforms of the parties to diverge similarly to d'Aspermint, Gabszewicz, and Thisse (1979), Economides (1989) and Zakharov (2005).

A somewhat similar situation was analyzed by Fauli-Oller, Ok, and Ortuno-Ortín (2003). The parties were assumed to have control over the identity of the candidates competing in an election, but no control over their policy platforms. Thus political parties delegates the decision-making authority to the candidates of their choice.

The election process is modeled as a two-stage game. At the second stage, the two candidates compete against one another in an election. Both parties and candidates are motivated only by policy. The setting of the electoral competition between the two policy-motivated candidates is similar to that of Calvert (1985), so the commitment problem is assumed away. Both candidates compromise between the chances of being elected and the policy that will be implemented.

At the first stage, each of two political parties chooses the candidate that will compete at the second stage. The party must decide on the preferred policy of the candidate. The chief result of the work was that the parties may nominate candidates whose positions are more extreme than their own. This is because policy platforms proposed by candidates are more moderate than their policy preferences. Hence to achieve a policy platform that is sufficiently far away from the median and has a good chance of winning, the party must nominate a more radical candidate.

10 Conclusion

The theoretical literature on spatial candidate location and voting is vast, and the author is fully aware that it cannot be covered comprehensively.

This is by far not a complete overview of the theoretical causes of polarization. Several issues have yet to be addressed. First, it is the role that the formation of post-election coalitions plays in the pre-election selection of policy platforms by parties and candidates (or the identities of the candidates themselves). Second, the interaction between candidates, voters and interest groups may also affect which policies the candidates choose, and whether these policies are polarized. Third, some authors (see Shultz, 1996, for instance) have investigated the role of information asymmetry in the formation of policy platforms. For example, the voters may not be perfectly informed about the policy benefits, so the position of a candidate can be used to reveal information to the voter. Finally, the issue of the measurement of polarization (either among the electorate or within a legislature) is nontrivial and deserves some discussion.

The author would be grateful for any further input from the reader.

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