7 Other Voting Rules and Considerations

7.1 Introduction

This final chapter is used to consider some additional important issues that are related to the conclusion to this point that BR clearly has an advantage over the other voting rules when all factors are considered. It was mentioned in Chapter 5 that a major criticism of BR is that it is generally perceived to be very susceptible to manipulation, such that a group of voters can misrepresent their true preferences with strategic voting in order to obtain a more preferred outcome from an election. We begin this chapter by providing a thorough analysis of this phenomenon, to see if this common belief is valid. The next section will then consider the Three-valued Scale Evaluative Voting Procedure that is a new voting rule that has been receiving attention as an extension of AV or as a particular case of Range Voting (RV), where RV sets a fixed range of allowable scores and each voter then selects a score to assign to each candidate from that range. The third section will then consider the overall validity of the conclusion that BR generally displays superior performance when attention is changed to consider the case of more than three candidates. The final section will consider the impact that voter abstention rates might have on our conclusions.

7.2 BR and Strategic Voting

The concept of strategic voting, or manipulation, can be demonstrated by returning back to the story of the two friends in Chapter 1, when a voting situation was given in Fig. 1.8 to lead to the No Show Paradox. Both of the friends had the identical preference rankings on three candidates $\{A,B,C\}$ with $A \succ B \succ C$, and this has not changed. When both participated in an election with NPER, their middle-ranked Candidate B was elected, and it was then discovered that their most-favored Candidate A would have been elected if they had not participated at all in the election.

The story continues, and Candidate B is now nearing the end of that first term, and an effort is being made for re-election. The same three candidates have entered into the race again. But, after the disappointing outcome of the first election, the two friends led a successful campaign to change the method of voting to BR. Two members have been added to the electorate, so there are now 35 voters participating in the election. The preferences of the original voters have also shifted from those in Fig. 1.8, due to the performance of Candidate B during the first term, and the voting situation that represents the current preference rankings of the electorate is displayed in Fig. 7.1.

$$\begin{array}{cccccc} A & A & B & C & B & C \\ B & C & A & A & C & B \\ C & B & C & B & A & A \\ n_1 = 6 & n_2 = 6 & n_3 = 7 & n_4 = 5 & n_5 = 6 & n_6 = 5 \end{array}$$

Fig. 7.1 An example preliminary voting situation for strategic voting.

Without actually knowing the specific details of this voting situation, one of the two friends secretly confesses to having taken the extremely wise step of studying the fascinating topic of election procedures after the awful fiasco of the first election. A general observation regarding the current election scenario is then presented by this wise friend; that preliminary polls and discussions make it look like a close race between *A* and *B*, with *C* having much less of a chance to win. Then, since both know that their true preferences are $A \succ B \succ C$, the suggestion is proposed that both should instead misrepresent their preference ranking as $A \succ C \succ B$, without discussing this questionable strategy with any other voters. The idea behind this strategy is that this will increase the relative level of support for their preferred Candidate *A* over *B*, by decreasing the BR score that *B* will obtain. There is nothing that these two voters can do to increase the actual BR score for *A*. While this maneuver will simultaneously show more support for their least-preferred Candidate *C*, that candidate is not considered to be a serious threat to *A* to begin with. So, what really happens as a result of this devious action by these two conniving friends to misrepresent their preferences with strategic voting?

Based on the preliminary voting situation in Fig. 7.1, we find that Candidate *B* is the CW, with *BMA* (18-17) and *BMC* (19-16) from the true preferences. When BR is used to determine the winner, we find

$$Score(A) = 1(12) + \frac{1}{2}(12) + 0(11) = 18,$$
$$Score(B) = 1(13) + \frac{1}{2}(11) + 0(11) = 18.5,$$
$$Score(C) = 1(10) + \frac{1}{2}(12) + 0(13) = 16.$$

The CW is therefore elected by BR for this voting situation in the absence of any misrepresented preferences.

When the two friends change their preferences from $A \succ B \succ C$ to $A \succ C \succ B$, the voting situation is modified to become

$$\begin{array}{cccccc} A & A & B & C & B & C \\ B & C & A & A & C & B \\ C & B & C & B & A & A \\ n_1 = 4 & n_2 = 8 & n_3 = 7 & n_4 = 5 & n_5 = 6 & n_6 = 5 \end{array}$$

Fig. 7.2 An example modified voting situation with strategic voting.

As a result of this misrepresentation of preferences, the BR results become:

$Score(A) = 1(12) + \frac{1}{2}(12) + 0(11) = 18,$
$Score(B) = 1(13) + \frac{1}{2}(9) + 0(13) = 17.5,$
$Score(C) = 1(10) + \frac{1}{2}(14) + 0(11) = 17.$

Candidate *A* has therefore become the winner with BR, so the two conniving friends with the preference ranking $A \succ B \succ C$ have successfully manipulated BR to obtain a better outcome for themselves by misrepresenting their preferences to vote strategically. This outcome might provide some solace to them after being victimized by the No-Show Paradox in the first election.

However, it is extremely important to recognize that successful manipulation strategies require a lot of thought and careful coordination for things to work as planned. For example, suppose that the two conniving friends had instead decided to carry this deception further, to determine that there are four other voters with the same preference ranking $A \succ B \succ C$, and then convinced all of them to join into a coalition to misrepresent their preferences as $A \succ C \succ B$. The resulting over-modified voting situation that would result from this scenario is shown in Fig. 7.3.

A	*A*	*B*	*C*	*B*	*C*
B	*C*	*A*	*A*	*C*	*B*
C	*B*	*C*	*B*	*A*	*A*
$n_1 = 0$	$n_2 = 12$	$n_3 = 7$	$n_4 = 5$	$n_5 = 6$	$n_6 = 5$

Fig. 7.3 An example over-modified voting situation with strategic voting.

The resulting candidate scores with BR for the over-modified scenario from Fig. 7.3 are given by:

$Score(A) = 1(12) + \frac{1}{2}(12) + 0(11) = 18,$
$Score(B) = 1(13) + \frac{1}{2}(5) + 0(17) = 15.5,$
$Score(C) = 1(10) + \frac{1}{2}(18) + 0(7) = 19.$

By carrying the use of strategic voting too far, some level of karma would be prevailing in this case; since things have backfired very badly on the coalition. The ultimate winner of the election with BR would now be the least-preferred Candidate *C* for every member of this conniving coalition of six ultimately less-than-strategic voters. It is very clear that successful manipulation would require a significant level of coordination among voters.

The fact that BR can be manipulated is far from being a recent observation. It was mentioned previously that BR was implemented by the French Academy of Sciences sometime after Borda presented his paper on the topic in 1770 until 1801. Many observations of strategic voting were observed, and BR was strongly criticized for this flaw. Borda responded wryly to this criticism by stating that [Black (1958), pg. 238] "My scheme is intended only for honest men". It is very interesting to note that the early criticism of this flaw with BR apparently completely overlooked the fact that this same flaw existed with all other voting rules as well. It was not until Gibbard (1973) and Satterthwaite (1975) that it became known that effectively every voting rules is susceptible to strategic voting with three or more candidates. So, to what extent can the alleged high vulnerability of BR to strategic voting be considered to be true when it is compared to other voting rules?

Before we proceed to consider the likelihood that various voting rules are vulnerable to manipulation, a few definitions are needed. *Individual Manipulation* refers to scenarios in which a single voter can change the outcome of an election with strategic voting. When a group of voters is required to accomplish this outcome, it is referred to as *Coalitional Manipulation*. The scenario that was described above when two friends were successfully using strategic voting is referred to as a scenario that displays *Naïve Behavior* on the part of voters who are not involved in the manipulation, since none of them make any effort to react to the possibility that such strategic voting might be used to their detriment.

7.2.1. Manipulation with naïve voters: The evidence against BR

A seminal paper in the study of the propensity of PR, BR, PER and NPER to be susceptible to Coalitional Manipulation with naïve voters was performed in Chamberlin (1985) for three-candidate elections. This was a Monte Carlo simulation based analysis that generated random voters' preferences from two different models: with the basic IC scenario and with a geometric spatial model of voter preferences. These results generally indicate that PER tends to be the voting rule that is least susceptible to manipulation, and that BR is the most susceptible. To illustrate the results from this study, Table 7.1 lists the percentage of voting situations for the case of 21 voters with the assumption of IC.

Table 7.1 Percentage of voting situations subject to Coalitional Manipulation with IC.

Voting Rule	Likelihood of Coalitional Manipulation
PR	81.0%
BR	96.0%
PER	23.3%
NPER	95.7%

The propensity for BR to be manipulated is very high in Table 7.1 at 96.0%, but it is comparable to NPER. The general speculation that BR is particularly sensitive to manipulation is largely supported by these results. Other secondary factors were also examined by Chamberlin, such as the average minimum number of voters that are required for a coalition to successfully use strategic voting with each voting rule. As this minimum required number of voters in a coalition increases, it would become more difficult to coordinate a successful attempt at manipulation. It is found that BR generally holds a middle-ranked position among voting rules, being neither the worst nor the best performer, based on these secondary factors.

The susceptibility of BR, PR and RV to Individual Manipulation was considered in Nitzan (1985) via Monte Carlo simulations based on the assumption of IC. The results that were obtained consistently show that the vulnerability of all three voting rules to individual manipulation increases with the number of candidates and decreases with the number of voters. For any given combination of number of voters in the range $5 \leq n \leq 90$ and number of candidates in the range $3 \leq m \leq 5$, RV is found to be more sensitive to manipulation than BR, which in turn is more susceptible to manipulation than PR. This result is at least valid for this range of a relatively small numbers of voters. Evidence has been found later to indicate that BR is not necessarily more vulnerable to Individual Manipulation than PR when the number of voters is sufficiently large.

A related study was conducted by Smith (1999) that introduces additional measures of Individual Manipulation. These measures account for the number of different voters who could each manipulate the election outcome of voting situations and the amount of improvement that these individual voters could achieve through successful manipulation. The analysis is performed with Monte Carlo simulations based on the assumption of IC, and the results confirm the generally poor performance of BR for most of these measures. The only scenario under which BR performed well in this study was for the very implausible strategy that has voters misrepresent their preferences by randomly selecting some other preference ranking. Just as in the study above by Nitzan (1985), these results apply to elections with a small numbers of candidates and voters.

A series of studies by Aleskerov et al (2011, 2012, 2015) considers the degree of manipulability of a set of voting rules that includes BR. This analysis behind these studies is slightly different that other studies of strategic voting, since it allows voting rules to be *multiple-choice* in the sense that they are not required to select a single winner by resorting to the use of some tie-breaking mechanism. All their results come from Monte Carlo simulations that are based on both IC and IAC. It turns out that taking multiple-choice into account does not significantly modify the hierarchy of the voting rules: BR remains among the group of voting rules with a rather high degree of susceptibility to Coalitional Manipulation. However, its performance is somewhat better when Individual Manipulation is considered, since BR is found to perform better than PR in that case.

A perverse voting effect that is very similar in nature to the example in the introduction in which manipulation of BR backfired for coalition of voters is observed in Laslier (2010) for Individual Manipulation. The study uses Monte Carlo simulation analysis to compare strategic aspects of PR, BR, AV and Copeland Rule (*CR*), which evaluates candidates with a score that measures the number of other candidates that it beats by PMR minus the number of other candidates that beat it. The voters do not know the particular preferences of other voters, but they can respond to a public signal that takes the form of an announced ranking of the candidates by the voting rule that is being used, where ties have been broken randomly. The allowable manipulation strategy that is available for any voter is specifically fixed. To illustrate this allowable strategy for BR, suppose that the announced ranking on four candidates $\{A,B,C,D\}$ is *ABBBCBD*, but the specific Borda Scores for the candidates are not revealed. So, it is not known if a tie exists between Candidates *A* and *B* with the Borda Scores.

Suppose that a given voter has preferences that agree with this ranking with $A \succ B \succ C \succ D$ and this voter wants to give some additional support to Candidate *A*, just in case it is tied with *B* based on the Borda Scores. Following the logic of the example in the introduction, the voter does this by switching the second-ranked *B* with the last-ranked *D* to manipulate the reported ranking to $A \succ D \succ C \succ B$. By following the same principle, a voter might have the actual preference ranking $B \succ A \succ C \succ D$ and think that Candidates *B* and *A* could be tied based on Borda Score. Their manipulated ranking would switch the second-ranked and least-preferred candidates to obtain $B \succ D \succ C \succ A$.

It is then shown that the use of this strategy by either voter could possibly lead to a substantial perverse effect from the attempted Individual Manipulation. This is the same effect that was observed in our example in the introduction when Coalitional Manipulation backfired with BR, where we now find that the least-preferred Candidate *D* would be elected by BR as a result of this strategic voting. The study then goes on to show that PR, AV and CR are all immune to the possibility of displaying this effect.

Analytical representations for the probability that Individual and Coalitional Manipulation can occur for BR and CR with IAC are obtained in Favardin et al (2002). BR is consistently found to be more vulnerable to manipulation than CR in both cases for any number of voters. To illustrate these results for the case of 51 voters, the vulnerability to Individual Manipulation is 2.1% for CR and 3.8% for BR; and the vulnerability to Coalitional Manipulation is 22.7% for CR and 47.9% for BR. A significant component of our earlier analysis has focused on the general propensity of the introduction of a degree of dependence among voters' preferences with IAC to reduce the likelihood that paradoxical outcomes are observed. We do the same thing here and note that the probability that BR is susceptible to Coalitional Manipulation with IAC for 21 voters is given in Favardin et al (2002) as 44.8%. This is dramatically smaller than the associated IC-based probability of Coalitional Manipulation with BR of 96.0% in Table 7.1, but the probability that BR can be manipulated still remains quite high with IAC. It was later shown by Wilson and Pritchard (2007) that the vulnerability of BR to Coalitional Manipulation for the limiting case as $n \to \infty$ under IAC tends to 50.25%.

All of the evidence to this point is making the case against BR look quite strong, since it has consistently been found to have a greater propensity to be manipulated than the other voting rules that it has been compared to. The long-standing conventional belief that BR is particularly susceptible to manipulation has definitely been supported. However, it is very important to recall the events of Chapter 5, when the case against BR looked very strong when it was being compared to PER based on the criterion of Condorcet Efficiency. We have found so far that BR has many good properties, so we continue on with the hope of once again being able to conclude that "The report of the demise of BR is exaggerated".

7.2.2 BR: It is not really all that bad … and sometimes it is optimal

The first glimmer of hope that BR might not be particularly vulnerable to strategic voting comes from Peleg (1979), which demonstrates with a generalized form of IC that the probability that Individual Manipulation is possible tends to zero for every $WSR(\lambda)$ in the limiting case of voters as $n \rightarrow \infty$. The study specifically concludes that no $WSR(\lambda)$, which would include BR, should experience any significant problems from Individual Manipulation with large electorates.

Baharad and Neeman (2002) then go on to extend and generalize this result to consider a number of different probability models that describe how voting situations are obtained, including models that allow for some degree of "localized correlation" among the voters' preferences. The probability that a small coalition of voters can successfully manipulate any $WSR(\lambda)$, is shown to decrease to zero as the total number of voters increases, to establish an overall "asymptotic strategyproofness" for all $WSR(\lambda)$ as $n \rightarrow \infty$. The same result is found to be valid for voting rules that must elect the CW, when there is one. But, this result does not hold for some of these Condorcet consistent rules when a CW does not exist. The analysis is concluded by stating that if voters must incur some small cost to determine how a voting rule could possibly be manipulated, then all $WSR(\lambda)$ and some Condorcet consistent rules will be highly resistant to strategic manipulation with large electorates. They go on to specifically state that these particular voting rules can be expected to perform very well "whether people are honest or not". This final comment apparently refers back to Borda's statement that was mentioned earlier regarding his voting scheme being intended for use only by "honest men". In another study, Slinko (2002) obtains a similar result to extend this conclusion to two-stage runoff procedures.

A more elementary version of manipulation is considered in Saari (1990), where the objective of manipulation is not necessarily to change the winner of an election, but just to modify the overall ranking of candidates that is obtained with a voting rule. In particular, manipulation is assumed to occur if strategic voting can cause two candidates to switch their positions in the outcome ranking. A micro-manipulation is said to occur in this context if it can be accomplished through a coordinated effort by a coalition that comprises only a small percentage of the electorate. The bounds on the size of the small percentage of voters in such a coalition is not specifically analyzed in the paper, but it is indicated that the proven results will hold when the group of strategic voters represents less than 5% of the electorate. He considers all WSR(λ) and a class of "multiple voting systems", to show that the voting rule that is least susceptible to micro-manipulation for three candidates is BR, for a sufficiently large number of voters under a slightly modified UC assumption. The optimal voting rule changes for a larger number of candidates, but the analysis shows that BR will always perform well anyway. That is, even when BR is not the best choice, it is not very far from being so.

These same results are then extended to consider uncoordinated macro-manipulations in which many small groups of voters attempt to use strategic voting, but where they do so without any coordination of their efforts. It is concluded that such an uncoordinated effort by groups to modify preference rankings on candidates will result in a cancellation of the strategic efforts between opposing groups. After all of these offsetting cancellations are accounted for, the problem reduces to an equivalent micro-manipulation problem. Saari also stresses that it is possible to justify any given voting rule in this fashion by appropriately picking some measure of susceptibility to manipulation and then selectively imposing assumptions to restrict how voters' preferences are generated.

Pritchard and Slinko (2006) revisit the notion of evaluating voting rules based on the average minimum size that is required for a coalition to be capable of successfully manipulating each of these rules. This measure was introduced earlier in our discussion of Chamberlin (1985), and the greater the value of this characteristic is for a voting rule, the more capable that rule is to avoiding manipulation. It is then shown that this measure of susceptibility to Coalitional Manipulation is not particularly meaningful for NPR, and they consequently suggest the use instead of a more technical measure that they define as the average minimum size of a "threshold coalition". Both of these measures coincide for all $WSR(\lambda)$ for the limiting case of voters as $n \to \infty$, and it is then proved for that case with IC that BR is the optimal $WSR(\lambda)$ to maximize the average minimum size of a threshold coalition for three and four-candidate elections.

The tide is definitely starting to turn in favor of BR, since these immediately preceding studies are refuting the commonly held belief that BR is particularly sensitive to Strategic Voting. And, the most conclusive piece of evidence to support BR is presented in Favardin and Lepelley (2006), where the logic behind the assumption of naïve voters is finally brought into question. Why would all other voters idly sit by, and not respond to the possible threat that they could be receiving a less-desirable outcome from an election because some coalition of voters could be utilizing strategic voting against them?

When voters are *non-naïve*, the strategic behavior of any voter can possibly be neutralized by the actions of others, as suggested in the discussion of the cancellation of strategic efforts by competing coalitions above from Saari (1990). They introduce the notion of a *strategic reaction*, which occurs when a voter, or a coalition of voters, prefers the outcome that would result from sincere voting; and where they could possibly be harmed from a threat of strategic voting by another voter, or another coalition of voters. Their response to this threat would be to react strategically with their own voting, in order to increase the probability that the sincere outcome will prevail. We illustrate this process with BR in a three-candidate election by starting with the voting situation in Fig. 7.4 that represents the sincere preferences for five voters.

A	A	B	C	B	C
B	C	A	A	C	B
C	B	C	B	A	A
$n_1 = 3$	$n_2 = 0$	$n_3 = 2$	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$

Fig. 7.4 Sincere preferences for an example of strategic reaction.

These sincere preferences result in a situation where Candidate A is the CW, with AMB (3-2) and AMC (5-0). The Borda Scores are calculated as:

$$Score(A) = 1(3) + \frac{1}{2}(2) + 0(0) = 4.0,$$
$$Score(B) = 1(2) + \frac{1}{2}(3) + 0(0) = 3.5,$$
$$Score(C) = 1(0) + \frac{1}{2}(0) + 0(5) = 0.$$

So, Candidate A is selected as the winner with these sincere preferences. The two voters with preferences $B \succ A \succ C$ really want B as the winner, and they choose to behave in the same fashion as the two conniving friends in the example from the introduction, to instead report their preferences as $B \succ C \succ A$, to produce the manipulated voting situation in Fig. 7.5.

A	A	B	C	B	C
B	C	A	A	C	B
C	B	C	B	A	A
$n_1 = 3$	$n_2 = 0$	$n_3 = 0$	$n_4 = 0$	$n_5 = 2$	$n_6 = 0$

Fig. 7.5 Manipulated preferences for an example of strategic reaction.

The resulting Borda Scores from this strategic voting would be:

$$Score(A) = 1(3) + \frac{1}{2}(0) + 0(2) = 3.0,$$
$$Score(B) = 1(2) + \frac{1}{2}(3) + 0(0) = 3.5,$$
$$Score(C) = 1(0) + \frac{1}{2}(2) + 0(3) = 1.0.$$

Candidate B would therefore be the winner as a result of the manipulation by the two voters. But, the three voters with preferences $A \succ B \succ C$ are not naïve in this case, and they are aware of the possibility that the two voters could manipulate their preferences to obtain the voting situation in Fig. 7.5. They really want their most-preferred Candidate A to be the winner, as it would have been with sincere voting. So, they strengthen the Borda Count for A over B with the strategic reaction by misrepresenting their own preferences as $A \succ C \succ B$, to arrive at the voting situation in Fig. 7.6.

A	A	B	C	B	C
B	C	A	A	C	B
C	B	C	B	A	A
$n_1 = 0$	$n_2 = 3$	$n_3 = 0$	$n_4 = 0$	$n_5 = 2$	$n_6 = 0$

Fig. 7.6 Manipulated preferences with a strategic reaction.

The Borda Scores are now calculated to be:

$Score(A) = 1(3) + \frac{1}{2}(0) + 0(2) = 3.0,$

$Score(B) = 1(2) + \frac{1}{2}(0) + 0(3) = 2.0,$

$Score(C) = 1(0) + \frac{1}{2}(5) + 0(0) = 2.5.$

Candidate *A* has now re-emerged as the BR winner, just as it was with sincere voting. Note that the Borda Score for Candidate *C* has increased from 1.0 with sincere voting to 2.5 due to the combined effects of the initial manipulation and the strategic reaction to it. The process worked in this example, to reinstate the winning candidate with BR for sincere preferences; but it could have backfired if the Borda Score for *C* had increased any more than it did to elect every voter's least preferred candidate with sincere preferences. The final outcome of this action and reaction resulted in no change from the winning candidate with sincere preferences, so the attempt at manipulation was meaningless because the voters were non-naïve. It turns out that effectively no manipulation occurred as a result, to make this a "quasi-stable" situation.

Favardin and Lepelley (2006) make a further distinction regarding how manipulation might occur. The first case represents *Homogeneous Voters*, which is similar to all of the examples that we have considered so far. That is, all of the voters included in a manipulating coalition have identical preferences rankings on the candidates. This case would be most relevant to scenarios like a political assembly, where the set of all members can be neatly partitioned into several groups according to party membership. Homogeneous preferences would be expected to prevail within each of these groups, with preferences being aligned according to the associated party ideologies. Such a scenario would typically be very amenable to communication within each of the individual groups of homogeneous voters before an election, to lead to the development of insular voting strategies for each group.

Extending the same level of communication across different groups to lead to a coalition of groups that agree to cooperatively adopt strategies that are mutually beneficial for each of these groups might be more challenging to achieve in an assembly. Such a coalition of groups would be composed of *Heterogeneous Voters*, and the possibility of across-group cooperation obviously allows for more options for Coalitional Manipulation to occur. Representations are obtained to yield limiting IAC probabilities as $n \to \infty$ that a number of voting rules are susceptible to Individual and Coalitional Manipulation with both naïve and non-naïve voters. We focus on the differences in the results that were obtained for the more interesting case of Heterogeneous Voters, since that scenario results in a greater potential level of susceptibility to manipulation.

When Individual Manipulation was considered for naïve voters, BR was found to be one of the set of voting rules with the lowest susceptibility to manipulation for all cases with more than 15 voters, which is in complete agreement with the expectations from Saari (1990) that were discussed above. The computed probabilities of susceptibility to Coalitional Manipulation with naïve voters are shown in Table 7.2 for PR, NPR, BR, PER and NPER. These results for the voting rules that we have given the most attention to were accumulated from Lepelley and Mbih (1987, 1994) and Wilson and Pritchard (2007), where it is seen BR is second only to NPR for having the highest likelihood of manipulation. This limiting probability estimate of .5025 for BR was mentioned earlier as strongly reflecting an overall poor performance.

Table 7.2 Limiting Probability of Coalitional Manipulation for Heterogeneous Voters with IAC.

Voting Rule	Naïve Voters	Non-Naïve Voters
PR	.2916	.1736
NPR	.5185	.4444
BR	.5025	.1375
PER	.1111	.0920
NPER	.4306	.0903

An extraordinary outcome happens in Favardin and Lepelley (2006) when they instead consider the results for non-naïve voters. As is the case with naïve voters, BR remains among the set of voting rules that have the lowest potential for Individual Manipulation. The probabilities of susceptibility to Coalitional Manipulation with non-naïve voters are listed in Table 7.2 for PR, NPR, BR, PER and NPER, where BR shows an absolutely remarkable decrease in its manipulation probability to .1375, and BR is now clearly superior to both PR and NPR. Both PER and NPER do perform better than BR, but these results strongly indicate that, contrary to common belief, BR is not at all particularly vulnerable to manipulation when non-naïve voters are considered. It is therefore very natural to wonder if voters really do tend to react to potential threats of manipulation in a non-naïve fashion.

Some evidence to provide an answer to this question comes from an empirical laboratory study by Béhue et al (2009), where evidence is found to support the premise that voters do indeed react to potential threats of manipulation by acting like non-naïve voters. Two types of voting situations with three candidates are compared in this study. In the first scenario, Type 1 voting situations are contrived to have one voter who can successfully manipulate the election, and where no possible reaction to this threat exists for any of the other voters. In Type 2 voting situations there is still one voter who is capable of manipulating the election, but now there is also another voter who can use a strategic reaction to that threat to restore the original winner from sincere voting. The results show that the sincere winner is elected with BR in only 33 percent of the elections that were conducted with Type 1 situations, so the voters who can manipulate the outcome without any interference clearly are taking advantage of that opportunity. And, the sincere winner is elected in 75 percent of the elections with Type 2 situations, so that the voters who have the possibility of using a strategic reaction are frequently taking advantage of that option when it is available. This provides some credible support for the premise that voters generally do tend to act in accordance with the model of non-naïve voters when a threat of possible manipulation is present.

7.2.3 Other studies of BR Manipulation

A laboratory experiment was used in Kube and Puppe (2009) to study the overall propensity of voters to engage in strategic voting with BR for three voters and four candidates. Different behavior patterns were found to exist for voters, based on whether, or not, an individual voter knew that other voters were using strategic manipulation. In each election, one single player was provided with additional information about the other players' preference rankings on the candidates. It is found that rates of attempts to manipulate outcomes are surprisingly low, even for the individual voters who possessed superior information about the other voters' preferences. However, when this individual voter was provided with both the other voters' preferences and information about how they actually voted, their manipulation rates increased significantly. This rate rose to about 50% in one scenario, to suggest that uncertainty about how other voters might be behaving plays a key role in understanding the use of strategic voting in elections. It is asserted that this particular manipulation rate for such a highly contrived scenario can only be viewed as an upper bound for actual elections. That is, typical elections that are conducted under natural conditions that are both more complex and also involve voters who are inexperienced with the complexities of manipulation should be expected to have much lower rates of strategic voting. Similar conclusions can be found in a number of other studies that attempt to measure the percentage of strategic voters in real elections, where it is found that typically only around 5% of voters do so [See for example Blais (2002)].

The final study that we include regarding the susceptibility of BR to strategic voting, takes a very different and interesting approach by starting with the observation that the manipulation of a voting rule is not necessarily a 'bad' thing. As an elementary example, Coalitional Manipulation can prevent the election of a CL with PR! This line of reasoning is pursued in Lehtinen (2007) to evaluate strategic voting under BR in the context of the overall expected value of utility that is obtained for the electorate. The analysis is performed with Monte Carlo simulation analysis for three-candidate elections.

The model that is used is based on the utilities that each voter has on the candidates, and voters' preference rankings are obtained accordingly. Some incomplete information about the sincere preferences in the voting situation of the electorate is available from "noisy signals" before the election is held, and voters can use this information to derive their beliefs about whether, or not, one candidate has a higher Borda Score than another. Expected Utility Behavior is then defined to permit voters to vote either strategically or sincerely, depending on their preferences and beliefs; and Utilitarian Efficiency is defined as the percentage of simulated elections in which the candidate with the highest sum of utilities over all voters is elected. The main finding of the study is that the use of strategic voting with Expected Utility Behavior yields higher Utilitarian Efficiencies than sincere voting does. As a result of this, the arguments that have previously been thought of as being a major drawback from using BR actually turn out to be an argument to support using it, when attention is shifted to focus on maximizing overall utility. Lehtinen remarks that strategic voting actually increases Utilitarian Efficiency for most commonly used voting rules. But, BR has two advantages over some of the other rules. Strategic voting increases overall utility, even if the voters have unreliable information regarding other voters' preferences. And, BR tends to yield high Utilitarian Efficiencies, even if only a few voters engage in strategic behavior.

Based on all of this information, a variety of arguments exist to defend BR against the commonly held perception that it is particularly susceptible to strategic voting. First, the frequency of any actual success with strategic voting is found to be low in real elections for various voting rules that are used in practice, including BR. Second, when manipulation can be accomplished, it turns out that BR is less susceptible to manipulation than other voting rules in a number of different scenarios, particularly with non-naïve voters. The conclusion that BR is less susceptible than other $WSR(\lambda)$ is definitely quite robust. Third, the impact from successfully using strategic voting in an election actually is not necessarily a negative outcome with BR, since its manipulation can be expected to result in an increase in the likelihood of electing the overall utility-maximizing candidate. We find that it is indeed true that "The report of the demise of BR is exaggerated".

7.3 The Three-Scale Evaluative Voting Rule

Three-Scale Evaluative Voting (*EV*), is similar in nature to AV, where AV is implemented by having voters assign a weight of one to each candidate that they approve of, and a weight of zero to candidates that they do not approve of. It is clear that AV was designed with the basic premise that voters can adequately represent their preferences on candidates by partitioning them into two sets that are represented by dichotomous preferences, and this was thoroughly examined in Chapter 6. The basic notion behind EV allows for a refinement of the AV assumptions, such that voters' preferences can take on three levels of acceptability to be trichotomous. Voters will then assign a weight of two to candidates that are considered to be in a preferred group, one point to candidates that voters view with indifference and zero points to candidates that are disapproved of. The placement of candidates into the indifference or neutral category could result from voters having insufficient information to form a distinct opinion about them. Voting with EV has received a lot of recent attention, and it is considered to have some nice properties [see for example Smaoui and Lepelley (2013)]. Alcantud and Laruelle (2014) develop the necessary and sufficient properties that characterize EV.

It has been mentioned before that it is difficult to explore any aspects of voting theory without finding footprints that were left behind by Condorcet, and EV is not an exception. A voting rule was suggested by Condorcet (1793) for use in electing groups of representatives from districts to the National Assembly, as part of a proposed new constitution for France. There were to be three times as many nominated candidates as the number of seats to be filled in each district. Voters were then required to partition these candidates into three groups of equal size according to their preferences, and present a ballot that listed the most preferred group of Election Votes and the middle-ranked group of Supplemental Votes. This requirement that the number of candidates in each group must be equal is a departure from EV. Candidates who were listed on a majority of ballots among Election Votes were then chosen in order according to the number of votes received. If not enough candidates received a majority of votes on the ballots in this first step, the Supplemental Votes were then included with the Election Votes in a second step to look for candidates who appeared on a majority of ballots in the combined set. As mentioned in the first chapter, this proposed constitution was defeated and Condorcet died in prison soon afterward. However, this particular voting rule was actually adopted for use in Geneva. So far, this does not describe EV except for the partition of candidates into three categories. The EV concept came when this rule was modified later.

After Condorcet's procedure had been used in Geneva, Lhuilier (1794) wrote about some difficulties that had been observed with its use, and suggested a slightly modified procedure. An interesting sidelight is that he mentions the fact that Condorcet could not be contacted regarding these newfound results. During this period of very limited communication possibilities in France, Lhuilier apparently was completely unaware that Condorcet had been imprisoned and died there. The modification that was suggested applied to the second step of the process, such that the number of Supplemental Votes for candidates would only receive one-half of the weight that was given to the Election Votes. Lhuilier goes on to show that using these BR weights for the three categories of votes would lead to a significant improvement for the results that were obtained in three-candidate elections, compared to using Condorcet's proposed method with equal weights for both groups, and this modified procedure is the same as EV. It was further suggested that different weights might be more appropriate for use in situations with more than three candidates.

It was mentioned above that EV has received a lot of recent attention that has been largely focused on showing that it has some very nice properties, but some potential difficulties with using this voting rule have also been pointed out in Smaoui and Lepelley (2013) and Felsenthal (2012). In particular, it has been noted that EV does not necessarily elect the CW. and it can exhibit a Strong Borda Paradox by electing the CL. This concern was investigated in El Ouafdi et al (2017a) that extended the earlier analysis that compared AV to other voting rules under the condition voters have dichotomous preferences. They instead compared EV to other voting rules for the case of three candidates when voters have trichotomous preferences.

The definition of EV was specifically based on the assumption of trichotomous preferences, but the other voting rules must be modified to account for this scenario. This is accomplished in a straightforward manner, based on how the candidates are distributed among the preferred, indifference and disapproved categories. Voting situations for which all three candidates are contained in the same preference category are ignored, since this effectively indicates that a voter is indifferent between all three candidates. If the candidates are partitioned into just two categories, the empty category is ignored and the remaining two categories are treated like dichotomous preferences. If there is a candidate in each of the three categories, a linear preference ranking exists. Then, the Extended Weighted Scoring Rule definitions for EPR, ENPR and EBR can be used for the cases in which candidates are partitioned into two or three preference categories. If the candidates are partitioned into two categories, then AV can be directly applied. Some accommodation must be made for AV if a linear preference ranking exists, and this is done in El Ouafdi et al (2017a) by assuming in this case that it is equally likely that the middle-ranked candidate in a voter's preference ranking will be approved or disapproved with AV.

Limiting representations were then obtained for large electorates as $n \to \infty$ with IAC for both the Condorcet Efficiency and the Strong Borda Paradox Probability that the CL is elected for each of EV, AV, EPR, ENPR and EBR. The results are listed in Table 7.3.
Table 7.3 IAC Efficiencies and Strong Borda Paradox Probabilities with trichotomous preferences.

Table 7.3 IAC Efficiencies and Borda Paradox Probabilities with trichotomous preferences.

Voting Rule	Condorcet Efficiency	Borda Paradox Probability
EV	.8652	.0098
EPR	.8487	.0223
ENPR	.7821	.0217
EBR	.9339	.0000
AV	.9153	.0011

These results indicate that EV outperforms both EPR and ENPR on the basis of both Condorcet Efficiency and the probability that a Strong Borda Paradox is observed. It is also seen that AV outperforms EV on both factors, but different results were obtained in Smaoui and Lepelley (2013) with a different set of assumptions that will be discussed later were used. The most important finding is that the consistent pattern emerges again, where BR is found to be superior to the other single-stage voting rules both on the basis of Condorcet Efficiency and the probability that a Strong Borda Paradox will be observed.

7.4 The Case of More than Three Candidates

We have focused almost completely on the outcomes from three-candidate elections, to conclude that BR looks like a very good option for use in an election, but it is obvious to wonder how robust this conclusion might be when more candidates are considered in an election. The process of obtaining exact limiting probability values with IC and IAC as $n \to \infty$ becomes quite restricted when the case of more than three candidates is being evaluated. However, significant progress has been made in this area with IAC for the case of three and four candidates.

For the case of four candidates in the limiting case for voters as $n \to \infty$, Schürmann (2013) obtains the Condorcet Efficiency of PR. Other results from El Ouafdi et al (2017b) do the same for of BR and NPR, and representations for the Condorcet Efficiency of two-stage elimination procedures are also obtained. For the case of more than three candidates, these runoff procedures retain only the two top-scoring candidates for the second stage of voting by majority rule. The runoff procedures are based on using the first-stage voting rules of Plurality Rule (PRR), Negative Plurality (NPRR) and Borda Rule (BRR). The results in Table 7.4 summarizes computed values from these representations for four candidates, and the results for the three-candidate cases from Tables 4.1 and 5.1 are included for comparison. The runoff voting rules PRR, NPRR and BRR are replaced by PER, NPER and BER for the case of three candidates.

Table 7.4 Limiting Condorcet Efficiency with IAC.

Voting Rule	Three Candidates	Four Candidates
PR	.8815	.7426
NPR	.6296	.5516
BR	.9111	.8706
PRR	.9685	.9116
NPRR	.9704	.8450
BRR	1.0000	.9961

It is interesting to note that the reduction in the rate of the Condorcet Efficiency that occurs when moving from three to four candidates with IAC is lower for BR than it is for PR, NPR, PRR and NPRR. The efficiency for BR is now closer to the two-stage PRR value, and it is higher than the NPRR value! An independent study by Bruns et al (2017) confirms the computed value for Condorcet Efficiency of PRR with four candidates.

The limiting probability of observing a Strong Borda Paradox with with PR, NPR and BR for four candidates with IAC is also considered in El Ouafdi et al (2017b). The results are listed in Table 7.5, along with their corresponding values for three candidates from Tabl**e 2.9.**

Table 7.5 Limiting Strong Borda Paradox Probability with IAC.

Voting Rule	Three Candidates	Four Candidates
PR	.0296	.0227
NPR	.0315	.0238
BR	.0000	.0000

The study by Bruns et al (2017) confirms the probability that the CL is elected by PR with four candidates, and the probabilities for observing a Strong Borda Paradox decreases for both PR and NPR in the move from three to four candidates.

When more than four candidates are considered, Monte Carlo simulation estimates are typically used in studies of the Condorcet Efficiency of voting rules. Lepelley et al (2000) performed such an analysis to obtain estimates of the limiting efficiencies of PR, NPR and BR with IC and IAC for the number of candidates in the range $3 \leq m \leq 8$, and the results are summarized in Table 7.6.

Table 7.6 Limiting Condorcet Efficiency estimates with IC and IAC.

	Voting Rule					
	PR		NPR		BR	
m	IC	IAC	IC	IAC	IC	IAC
3	.7574	.8816	.7571	.6298	.9010	.9108
4	.6416	.7429	.6415	.5517	.8702	.8706
5	.5570	.6139	.5602	.5090	.8552	.8541
6	.4858	.5198	.4946	.4730	.8450	.8471
7	.4663	.4524	.4450	.4386	.8438	.8457
8	.4123	.4088	.4378	.4101	.8362	.8428

Based on earlier discussion, it is not at all surprising to see that the IC and IAC estimates for these Condorcet Efficiencies converge to very similar values as *m* increases for each voting rule. What is surprising is the degree to which BR significantly dominates PR and NPR as *m* increases. As the number of candidates increases from three to eight, PR efficiency drops from .8816 to .4088 with IAC, which is a 53.6% decrease from the starting value with three candidates. On the other hand, BR drops from .9108 to .8428, which corresponds to only a 7.5% decrease from its starting value with IAC! The Condorcet Efficiency dominance of BR in Table 7.6 is blatantly obvious as the number of candidates increases for these single-stage voting rules.

Another Monte Carlo simulation-based analysis of the Condorcet Efficiency of voting rules with more than three candidates was conducted by Smaoui and Lepelley (2013), and it is designed to allow for a more natural comparison of AV and EV to other voting rules. Voting situations are developed on a utility-based model for obtaining preferences for each voter. For each individual voter, a random utility value is assigned to each of the m candidates from a uniform distribution over the interval $[0,1]$. Voting with PR, PRR, NPR and BR is then conducted by using the preference rankings for voters that follow from the ordinal rakings of candidates according to decreasing values of their utilities, which is consistent with IC. The CW was also determined from these ordinal rankings in all cases. When voting with AV, each voter approves of all candidates for which their utilities are in the in the range $[1/2,1]$, and disapproves of candidates with utilities in the range [0,1/2). When voting with EV, each voter categorizes all candidates with a utility in the rage [1,2/3) as being preferred, all candidates with utilities in the range [2/3,1/3) as being in the indifference group and all candidates with utilities in the range [1/3,0] as being disapproved.

The study was based on $n = 300$ voters for samples of 10,000 voting situations for which a CW was found to exist for each case, with the number of candidates in the range $3 \leq m \leq 8$. The results are summarized in Table 7.7.

Table 7.7 Condorcet Efficiencies for six voting rules with a utility based model.

m	EV	PR	PRR	NPR	BR	AV
3	.7066	.7600	.8987	.7278	.9005	.6006
4	.6699	.6376	.8002	.5960	.8719	.5390
5	.6481	.5491	.7121	.5020	.8532	.4981
6	.6322	.4831	.6430	.4317	.8391	.4678
7	.6209	.4308	.5846	.3775	.8278	.4451
8	.6118	.3893	.5360	.3348	.8190	.4257

The results of this utility-based analysis once again show that BR consistently dominates all other voting rules on the basis of Condorcet Efficiency, including the two-stage PRR. Moreover, the Condorcet Efficiency of BR remains rather high when the number of candidates increases, in contrast with PRR. We find that EV now dominates AV with this model, as opposed to what was observed with three-candidates in the results from the scenario that required voters to have trichotomous preferences in Table 7.3.

7.5 Final Conclusion: The Impact of Abstentions

The consistent message that has been received in this analysis is that BR really does perform very well, relative to other voting rules, on the basis of Condorcet Efficiency; and that the commonly held belief that BR is particularly sensitive to strategic voting is not really supported. So, what remains to be considered? The answer to this question is that the impact of abstention rates can be a critical factor in evaluating the effectiveness of voting rules. Recall what was observed during the discussion of the Forced Ranking Option, when it became very clear that there were very large differences between what happened when the Actual and the Forced Winners were considered. These differences were so glaring that it became obvious that the Forced Ranking Option should never be used.

We still define the Actual Winner based on the true preferences of the entire electorate of possible voters. When abstention is an option, we instead define the Observed Winner solely on the basis of the preferences of the subset of voters from the entire electorate who choose to participate. Gehrlein and Fishburn (1978) use an IC-based assumption $IC(\alpha_{PP})$ for three candidates to develop a representation for the limiting conditional probability that the Actual and Observed Winners coincide, given that an Actual Winner exists. Each of the six linear preference rankings is assumed to be equally likely to represent the preferences of any voter with this model, and the probability that each potential voter participates in the election is α_{PP}. Computed results from this representation are shown in Table 7.8.

Table 7.8 Limiting IC Probabilities and Efficiencies

α_{PP}	Actual = Observed	PR & NPR Efficiency	BR Efficiency
≈ 0	.3041	1/3 = .3333	1/3 = .3333
.1	.4236	.4399	.4576
.2	.4806	.4882	.5151
.3	.5290	.5277	.5630
.4	.5742	.5630	.6068
.5	.6186	.5961	.6490
.6	.6640	.6280	.6911
.7	.7126	.6595	.7345
.8	.7675	.6911	.7810
.9	.8365	.7234	.8337
1.0	1.0000	.7572	.9012

Some results from Table 7.8 are quite predictable. The Actual and Observed Winners must obviously coincide if $\alpha_{PP} = 1$. Nothing is known if no voters participate in the election, but the .3041 probability that the Actual and Observed Winners coincide as $\alpha_{PP} \to 0$ is 1/3 of the limiting probability that a CW exists with IC that we considered earlier.

Gehrlein and Fishburn (1979) use the same model to determine limiting representations for the Actual Condorcet Efficiency for general $WSR(\lambda)$ and find that the PR and NPR efficiencies are identical. Computed values are obtained from this representation for PR, NPR and BR in Table 7.8. As $\alpha_{PP} \to 0$, the voting rules are found to approach random selection procedures, with Condorcet Efficiencies equal to 1/3. As $\alpha_{PP} \to 1$ the efficiencies approach the known IC values when no abstention is allowed, from Table 4.1. All of the results in Table 7.8 show what has by now come to be expected. That is, BR outperforms the other voting for all levels of voter participation, but a very unfortunate part of this conclusion is that the BR Efficiencies also become very weak for low participation probabilities

The only remaining hope is that we can rely on the introduction of some degree of dependence among voters' preferences with IAC to increase these efficiencies to more acceptable levels. That particular aspect of this problem was studied in Gehrlein and Lepelley (2017), where the $IAC(\alpha_{PP})$ assumption was used for the limiting case of voters, where all voting situations that have the same participation proportion α_{PP} of voters from the entire electorate are equally likely to be observed. The results from that analysis are shown in Table 7.9.

Table 7.9 Limiting IAC Probabilities and Efficiencies.

α_{PP}	Actual = Observed	PR Efficiency	NPR Efficiency	BR Efficiency
≈ 0	5/16 = .3125	1/3 = .3333	1/3 = .3333	1/3 = .3333
.1	.3482	.3679	.3621	.3704
.2	.3949	.4131	.3975	.4184
.3	.4564	.4723	.4411	.4807
.4	.5357	.5481	.4928	.5600
.5	.6310	.6378	.5475	.6533
.6	.7291	.7267	.5912	.7457
.7	.8160	.7988	.6153	.8212
.8	.8887	.8476	.6255	.8737
.9	.9492	.8738	.6289	.9026
1.0	1.0000	119/135 = 0.8815	41/45 = .6296	17/27 = .9111

As $\alpha_{PP} \to 0$, the voting rules approach random selection procedures with PR, NPR and BR having Condorcet Efficiency values of 1/3, and the probability that the Actual and Observed Winners coincide is 5/16, which is 1/3 of the limiting probability that a CW exists with IAC. As $\alpha_{PP} \to 1$ the efficiencies approach the known IAC values from Table 4.1 where no abstention is allowed.

The results of this study show that the introduction of a degree of dependence among voters' preferences provides some good news, and a lot of bad news, when we compare the results of Table 7.8 and Table 7.9. The results with the $IAC(\alpha_{PP})$ model show a uniform decrease in Condorcet Efficiency for NPR, compared to the $IC(\alpha_{PP})$ results. However, NPR has already been shown to be a consistently poor performer. All of the values of the probability that the Actual and Observed Winners coincide, and the Actual Condorcet Efficiencies for PR and BR, increase with $IAC(\alpha_{PP})$ for participation rates with $\alpha_{PP} \geq 1/2$, and decrease for $\alpha_{PP} < 1/2$. As in the case with $IC(\alpha_{PP})$, BR consistently outperforms PR and NPR on the basis of Condorcet Efficiency. However, its "relatively best" performance with $IAC(\alpha_{PP})$ is even worse that we found with $IC(\alpha_{PP})$ for low levels of voter participation. So, the Condorcet Efficiency values for BR, PR and NPR are actually found to be lower with $IAC(\alpha_{PP})$ than with $IC(\alpha_{PP})$ for all $0 < \alpha_{PP} < 1/2$! This leaves little doubt that BR tends to be the best voting rule for use to maximize Condorcet Efficiency, but this matters very little when participation rates are low. While BR might be the best in that case, none of the voting rules are very good at selecting the Actual Winner.

Given this insight, what do actual participation rates look like? DeSilver (2016) lists the percentage of voting-age citizens who participated in recent elections in 35 countries that are members of the Organization for Economic Cooperation and Development. Most member countries of this group are highly-developed democratic states. The participation percentages range from 38.6% in Switzerland for 2015 to 84.3% in Turkey for 2015. The participation rates for Australia in 2016 is listed at 91.0%, and at 87.2% in 2014 for Belgium; but all voting-age citizens in both of those countries are legally obligated to report to a polling place on election days, so they were not included in the range that is reported here.

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