Challenges to the Standard Euclidean Spatial Model

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1 Introduction

Spatial models are useful to represent political competition over policy issues. If the feasible policies over a given policy issue are endowed with a natural left/right or low/high order, we can represent the set of feasible policies by a subset of the real line. Many policy issues are indeed easily ordered: tax rates can vary from 0 % to 100 %; any budgeted policy item can receive a lower or higher budget; criminal law can specify lighter or harsher sentences; etc. It is standard to assume that agents have a unique ideal policy and that given two policies below the agent's ideal policy, or given two policies above the agent's ideal policy, the agent prefers the policy closer to the agent's ideal. Preferences satisfying this assumption are *single-peaked*. If agents' preferences are single peaked over the real line, simple majority rule is transitive (Black 1948); furthermore, the median ideal policy among all the agents' ideal policies defeats any other policy if the number of agents is odd and it cannot be defeated by any other policy when preferences are aggregated by majority rule (Black 1958). Since the median policy cannot be defeated by any other, electoral competition between two candidates leads to policy convergence: both candidates choose the median policy (Downs 1957, building on Hotelling's (1929)), even if the candidates have diverging policy preferences (Wittman 1983; Calvert 1985).

Political competition usually involves multiple policy issues. Candidates propose policy bundles with one policy per issue. Multidimensional spatial models represent preferences over policy bundles: each dimension corresponds to a given issue. Starting with Davis et al. (1972), the standard approach is to assume that agents have a

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most preferred alternative in the policy space, and utilities that are decreasing in the Euclidean distance to this point, typically with a linear (Kramer 1977; Wittman 1977; Patty et al. 2009; Degan and Merlo 2009; or Eguia 2012), quadratic (Feddersen 1992; Clinton et al. 2004; Schofield and Sened 2006; or Schofield 2007b,a) or exponential (Poole and Rosenthal 1985) loss function.[1] Other theories allow for more general utility functions, but they preserve the circular Euclidean shape of indifference curves (McKelvey 1976), or they relax the assumption of circular indifference curves but maintain the restrictions that utility functions be differentiable (Plott 1967; Schofield 1978; Duggan 2007; or Duggan and Kalandrakis 2012), quasiconcave (Banks and Duggan 2008), or differentiable and quasiconcave (Kramer 1973).

I present a series of theoretical and empirical results that challenge the assumption that preferences over multiple issues can be adequately represented by utility functions that are linear, quadratic or exponential Euclidean in a multidimensional space. More generally, I present results that call into question whether preferences can be represented by differentiable or quasiconcave utility functions, let alone with Euclidean or weighted Euclidean utility functions.

I divide these theoretical and empirical challenges to standard assumptions in three classes:

I. Concerns about the concavity of the loss function, accepting the Euclidean shape of the indifference curves.
II. Concerns about the shape of indifference curves: convexity, and different weights for different dimensions.
III. Concerns about the shape of indifference curves: separability across issues.

2 Concerns About the Loss Function

Circular indifference curves are a common assumption on preferences in multidimensional spatial models. Circular indifference curves are such that two policy points which are at identical distances from an agent's ideal point are valued identically, i.e. the 'direction' of the perturbation from the agent's ideal point is inconsequential for his or her utility. This is a standard assumption on indifference curves. However, no similar consensus exists on a standard or default assumption on the loss function associated with these indifference curves. Linear or quadratic loss functions are the most commonly used (McCarty and Meirowitz 2007, Sect. 2.5). As noted in the Introduction, exponential functions are also used (Poole and Rosenthal 1985).[2] The choice of the functional form of the utility function in the various theories in the literature appears motivated by convenience or simplicity.

The choice of loss functions is consequential: important results rely crucially on the concavity of the loss function. For instance, in a probabilistic voting model of

[1] D'Agostino and Dardanoni (2009) provide an axiomatization of the Euclidean distance; Azrieli (2011) provides an axiomatization of Euclidean utilities with a quasilinear additive valence term.

[2] In support of their assumption of exponential utility functions, Poole and Rosenthal (1997) argue that (standard) concave utility functions do not fit the data well.

electoral competition with two candidates, Kamada and Kojima (2010) show that in equilibrium candidates converge to the median if voters' utility functions are concave, but candidates diverge if voters' utility functions are sufficiently convex.

Osborne (1995) warns that "*the assumption of concavity is often adopted, first because it is associated with 'risk aversion' and second because it makes easier to show that an equilibrium exists. However, [...] it is not clear that evidence that people are risk averse in economic decision-making has any relevance here. I conclude that in the absence of any convincing empirical evidence, it is not clear which of the assumptions is more appropriate*."

Seeking to test voters' risk attitude, Berinsky and Lewis (2007) assume that utility functions take the form $u_i(x, x_k) = -d(x, x_i^*)^\alpha$, where $d(x, x_i^*)$ is a weighted Euclidean distance and α is a parameter to be estimated. They find that the estimate that provides a best fit for voter choices in US presidential elections is $\hat{\alpha} \approx 1$, suggesting that it is appropriate to assume that voters' utilities are linear weighted Euclidean. They interpret this finding as evidence that voters are risk neutral, but Eguia (2009) casts axiomatic doubt on this interpretation: linear Euclidean utilities do not satisfy additive separability, so the preferences over lotteries on a given issue and hence the risk attitude of a voter with a linear Euclidean utility function depend on outcomes on other issues. In other words, voters with multi-dimensional linear Euclidean utilities are not risk neutral. With utilities that decrease in weighted Euclidean distances, additive separability (i.e. independence of preferences over lotteries on one issue with outcomes on other issues) requires that the loss function be quadratic (Eguia 2011b). The only way to reconcile additive separability (which under Euclidean indifference curves requires a quadratic loss function) with Berinski and Lewis's (2007) finding (with Euclidean indifference curves a linear loss function provides the best fit) is to discard the assumption of Euclidean indifference curves, and to check if under different shapes of the indifference curves, we obtain a best fit with a parameter for the loss function that is consistent with additive separability. This leads us to the second class of concerns: concerns about the shape of the indifference curves.

3 Concerns About Convexity of Preferences

A first concern about the assumption of utility functions that depend on the Euclidean distance is that some issues may be more important than others, and hence utilities ought to be weighted, generating elliptical (rather than circular) indifference curves in the case with two dimensions. If all voters assign the same weights to these dimensions, the problem is trivially solved, and Euclidean circles reinstated, by rescaling the units of measure of each dimension according to its weight. If different groups of voters assign different relative weights to the various dimensions, then it is not possible to rescale the dimensions so as to use unweighted Euclidean utilities, and we must instead use weighted Euclidean utilities with different weights for different voters (Miller and Schofield 2003).

A deeper concern is that preferences may not be representable by weighted Euclidean utility functions: indifference curves may have shapes that are not elliptical. Weighted Euclidean utilities represent a particular class of convex preferences. Preferences are (strictly) convex if the upper contour set defined by each indifference curve is (strictly) convex; that is, if the set of policies preferable to policy x is convex, for any x. Representable (strictly) convex preferences are representable by (strictly) quasiconcave utility functions. If preferences are not strictly convex, they cannot be represented by Euclidean utility functions, neither unweighted nor weighted ones. The curvature imposed by Euclidean utilities is simply not adequate to represent the preferences.

An alternative assumption to Euclidean preferences is city-block preferences, which define square indifference curves (with squares tilted at a 45 degree angle relative to the axes of coordinates), and are representable by utility functions that are decreasing in the l_1 distance $\|x - x^*\|_1 = \sum_{k=1}^{K} |x_k - x_k^*|$, where x_k is the policy on issue $k \in \{1, \ldots, K\}$. That is, agents with city block preferences calculate the distance between two points by adding up the distance dimension by dimension, as if traveling on a grid (that is why the l_1 or city block distance is sometimes called "Manhattan distance"), and they prefer points closer to their ideal according to this notion of distance. If preferences are city block, their utility representation is not strictly quasiconcave, and it is not differentiable. Classic results on the instability of simple majority rule (Plott 1967; McKelvey 1976) do not apply if agents have city block preferences. In fact, the core of simple majority rule is not empty under more general conditions if agents have city-block preferences (Rae and Taylor 1971; Wendell and Thorson 1974; McKelvey and Wendell 1976; Humphreys and Laver 2009).

Humphreys and Laver (2009) invoke results from psychology and cognitive sciences (Shepard 1987; Arabie 1991) to argue that agents measure distance to objects with separable attributes by adding up the distance in each attribute, which implies that if the object under consideration is a policy bundle on separable issues, agents measure distance according to the city block function.

Grynaviski and Corrigan (2006) find that a model that assumes voters have city block preferences provides a better fit of vote choice in US presidential elections than an alternative model that assumes voters have linear Euclidean preferences. Westholm (1997) finds that a model with city block preferences outperforms a model with quadratic Euclidean preferences, when aiming to predict vote choice in Norwegian elections. However, a binary comparison between city block utilities based on the l_1 metric $\|x - x^*\|_1 = \sum_{k=1}^{K} |x_k - x_k^*|$ and the linear Euclidean utilities based on the l_2 metric $\|x - x^*\|_2 = (\sum_{k=1}^{K} (x_k - x_k^*)^2)^{\frac{1}{2}}$ is unnecessarily restrictive: l_1 and l_2 are special cases of the Minkowski (1886) family of metric functions, which parameterized by δ, gives the distance between x and x^* as:

$$\|x - x^*\|_\delta = \left(\sum_{k=1}^{K} \left(x_k - x_k^*\right)^\delta \right)^{\frac{1}{\delta}}. \tag{1}$$

Rather than comparing $\delta = 1$ (linear city block) and $\delta = 2$ (linear Euclidean), it appears more fruitful to estimate parameter δ. Rivero (2011) estimates δ for several Spanish regional elections and finds that $\hat{\delta} \in (0.92, 1.17)$; none of the estimates is significantly different from $\delta = 1$, and they are all significantly different from $\delta = 2$. These tests support the use of linear city block over linear Euclidean utility functions.

Utility functions that are linearly decreasing in expression (1) are not additively separable unless $\delta = 1$. To satisfy additive separability, the utility function must be linearly decreasing in the δ power of $\|x - x^*\|_\delta$, so that

$$u\big(x, x^*\big) = -\sum_{k=1}^{K} \big(x_k - x_k^*\big)^\delta, \tag{2}$$

with linear city block utilities corresponding to $\delta = 1$, and quadratic Euclidean to $\delta = 2$. Notice that any parameter $\delta > 1$ results in strictly convex preferences and strictly quasiconcave and differentiable utility functions, while $\delta < 1$ results on preferences that are not convex, and utility functions that are neither strictly quasiconcave, nor differentiable. Ye et al. (2011) estimate parameter δ using the utility function (2) and voting data from the American National Election Studies corresponding to the 2000, 2004 and 2008 Presidential elections. However, their results are inconclusive, obtaining estimates that vary greatly across elections and, most puzzlingly, across candidates.

Further empirical work appears necessary to establish which utility functions provide a better fit, and whether the standard assumption of convex preferences is justified.

Most of the literature, and all of the discussion above, considers the set of alternatives as exogenously given: there is a subset $X \subseteq \mathbb{R}^K$ that is given, and agents have preferences over X. In this view, the question on the adequate assumption on the shape of the utility functions (Euclidean, city block, Minkowski with parameter δ) is a question on what primitive preferences over alternatives do we believe that agents have on $X \subseteq \mathbb{R}^K$.

However, the spatial representation of the set of feasible policies is itself a representation used for convenience, just as the utility functions are representations of underlying preferences. If, for instance, there are three policies x, y and z and agent i prefers x to y to z, and agent i is indifferent between y and a fair lottery between x and z, then we can map the three policies to the real line using a mapping $f : \{x, y, z\} \to \mathbb{R}$ such that $f(x) = 0$, $f(y) = 0.5$ and $f(z) = 1$ and then we can say that the agent has a linear utility function over [0, 1] with ideal point at 0. But we can represent the same underlying preferences using a mapping $g : \{x, y, z\} \to \mathbb{R}$ such that $f(x) = 0$, $f(y) = \sqrt{\frac{1}{2}}$ and $f(z) = 1$ and say that the agent has a quadratic utility function over [0, 1] with ideal point at 0. Under this perspective, we see that the shape of the utility function is an object of choice for the theorist who wishes to study an individual: using a different mapping of the set of alternatives into a vector space leads to indifference curves of different shapes. The spatial representation of the set of alternatives and the utility function we use in this space jointly determine the assumptions we make on the underlying preferences of the agent.

Once we recognize that the spatial representation of the set of alternatives is an endogenous choice made by the theorist who wishes to model preferences, we can ask new questions: can all preferences over policies be represented by Euclidean utility functions *in some space?* if not, what preferences can be represented by Euclidean utility functions? If we accept a spatial representation with great dimensionality, we obtain a positive result: any preference profile with N agents can be represented by utility functions that are Euclidean for all N agents if we let the mapping of the set of alternatives X into $\mathbb{R}^K$ contain $K \geq N$ dimensions (Bogomolnaia and Laslier 2007). If we care for the number of dimensions in our spatial representation, we do not obtain such a positive result. Suppose the policy issues are exogenously given, and we want to use no more than one dimension per issue in our spatial representation. In this case, while we can represent any single-peaked, separable preference relation of a single individual using quadratic Euclidean utility functions over an appropriately chosen spatial representation of the set of alternatives, we cannot represent the preferences of all N individuals with quadratic Euclidean utility functions in any spatial representation unless the underlying preference profile satisfies very restrictive conditions (Eguia 2011a).[3]

For any single-peaked preference profile with separable preferences, we can map the set of alternatives into $\mathbb{R}^K$ so as to represent the preferences of a given agent by quasiconcave utility functions over the chosen map. However, depending on the preference profile, any mapping that achieves this may be such that the utility representations of the preferences of other agents violate quasiconcavity and/or differentiability. Whether preference profiles in any given application are such that the preferences of all agents can be represented in some map with quasiconcave utility functions is an open empirical question.

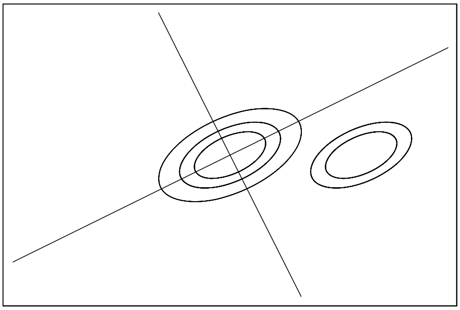
4 Concerns About Separability of Preferences

Expressions (1) or (2) above, or variations with weights for each dimension, allow us to relax the assumption that indifference curves have circular or elliptical curvature. We are free to assume any degree of curvature, including preferences that are not convex by choosing $\delta < 1$. These generalizations of the standard model from $\delta = 2$ to any $\delta > 0$ preserve the assumption that preferences are separable across issues: ordinal preferences over alternatives on a given issue do not depend on the realized outcome on other issues.

Milyo (2000b) and (2000a) notes that preferences over multiple dimensions of public spending cannot possibly be separable. Suppose a fixed unit of national income is to be allocated between public spending on policy one, public spending on

[3] Calvo et al. (2012) analyze an additional complication: agents may not agree on which alternative is to the right or left of another on a given issue. If so, we cannot use a unique spatial representation; rather, we must have subjective maps of the set of the set of alternatives, one for each agent.

Fig. 1 Obtaining separability by using a new basis of vectors



policy two, and private consumption. Decreasing marginal utility over consumption of public goods means that as public spending on policy one increases, the opportunity cost of spending on issue two also increases, so the ideal amount of expenditures on issue two must decrease with the amount spent on issue one. Preferences over public spending on issues one and two cannot be separable. This problem is easily solved by redefining the policy dimensions over which we assume that agents have separable preferences: let the first dimension be total public spending, and let the second dimension be the fraction of public spending devoted to issue one. Preferences may well be separable under this representation of the set of issues, and in any case they escape Milyo's (2000b) and (2000a) critique.

A more insidious difficulty arises if preferences are truly non-separable, not due to budgetary concerns, but because agents' ideal values on a given issue actually depend on the outcomes on other issues. For instance, it is possible that agents have non-separable preferences about immigration policy and the social safety net, preferring a more generous safety net if immigration policy is restrictive so redistributive policies benefit only natives, than if immigration policy is lax so redistributive policies would in part favor immigrants. Lacy (2001a,b, 2012) uncovers evidence of such non-separability across various pairs of issues.

If agents have non-separable preferences, but the correlation between issues is the same for all agents, then the problem is addressed by considering new, endogenous policy dimensions over which agents have separable preferences. Suppose that there are two complementary issues, such that for any agent i,

$$u(x_1, x_2) = -\left(x_1 - x_1^i\right)^2 - \left(x_2 - x_2^i\right)^2 + \left(x_1 - x_1^i\right)\left(x_2 - x_2^i\right).$$

These utility functions, depicted for two arbitrary agents in Fig. 1, are not separable over the two issues. However, if we use a different basis of vectors, as depicted in Fig. 1, and consider the new two dimensional vector space given by the two tilted axes of coordinates in Fig. 1, then agents have separable preferences over the new, endogenous dimensions.

This solution fails if agents have non-separable preferences and the correlation between preferences on different issues is heterogeneous across agents. In this case, we cannot create dimensions to make all agents separable over our newly defined dimensions. For instance, returning to non-separability between immigration and social safety net, if some agents prefer a larger safety net to help needy immigrants when immigration policy is lax, while other agents prefer a smaller safety net to not spend money on immigrants when immigration policy is lax, then we can redraw the axes to make the preferences of one group of agents separable, but in doing so, the preferences of the other group of agents remain non-separable. In very non-technical terms, agents have non-separable preferences if their indifference curves are tilted; if all agents have curves equally tilted, we can tilt the whole map to return to a standard model over newly defined dimensions.

If, on the contrary, different agents have preferences tilted in different directions, we cannot correct this problem by tilting the whole map. We need instead to introduce parameters to accommodate the correlation across issues. This is a considerable setback, similar to the problem of agents who assign different relative weights to the various dimensions -but more damaging, because we need more parameters to fix it. In order to accurately represent the preferences of agents who disagree on the weights they assign to the different dimensions we need to add one parameter per dimension per agent or group of agents who disagree on these weights, for a maximum of $(K-1)(N-1)$ new parameters if there are N agents and K dimensions. In order to represent the preferences of agents who disagree on the correlation in preferences between issues, we must add one correlation parameter per possible pair of issues and per agent or group of agents who disagree, for a maximum of $\frac{K(K-1)}{2}N$ new parameters.

While violations of separability do not affect classic results on the instability of simple majority rule as long as preferences are smooth (Plott 1967; McKelvey 1979), they affect how we can interpret and use common spatial models. Consider the structured-induced equilibrium theory (Shepsle and Weingast 1981), which proposes that the instability is solved by choosing policy dimension by dimension. In the standard structured-induced equilibrium theory, the order in which the legislature considers the various policy dimensions is irrelevant, because preferences are separable. With non-separable preferences, the order in which each policy dimension is considered affects the chosen policy outcome. For a second example, consider the ideal point estimation literature (Poole and Rosenthal 1985; Clinton et al. 2004): if preferences are not separable, estimating the ideal point of each legislator is not enough to predict vote choice.

5 Discussion

Theoretical and empirical work questions not only the standard assumption of Euclidean utility functions in multidimensional spatial models, but the more general assumptions of separable, convex and/or smooth preferences.

Standard spatial models suffer from limitations that I have not considered here. For instance, an increasing body of literature argues that we must add a candidate valence term to capture the actual preferences of voters about candidates. Valence is any quality that all voters agree is good, and makes the candidate who possesses more of it more attractive to all voters. Current research on valence seeks to endogenize it and to analyze its relation to the candidate's spatial location (Ashworth and Bueno de Mesquita 2009; Zakharov 2009; Serra 2010 and 2012; Krasa and Polborn 2010, 2012; or Schofield et al. 2011). In this chapter I analyze concerns about a basic pillar of the spatial model: the assumption that agents have preferences over a vector space that represents the set of feasible policies, preferences that can be represented by analytically convenient utility functions. Valence, dynamics, uncertainty, bounded rationality, other-regarding preferences or other improvements can be added to the basic spatial model to generate richer theories, but any theory with a spatial component must address the challenges posed in this chapter about the appropriate formalization of spatial preferences in the theory.

Further empirical work is necessary to establish whether agents have convex preferences over policy bundles with multiple policy issues. Assuming the functional form (1) or, if we want to satisfy additive separability, functional form (2) for the utility functions, empirical work must estimate parameter δ. If the estimated parameter $\hat{\delta}$ is less than 1, the consequences for theoretical work are dramatic: Preferences are not convex, and hence utility functions are neither quasiconcave, nor differentiable. Standard results in the literature that rely on these assumptions, most notably the instability of majority rule (Plott 1967; McKelvey 1976; Schofield 1978), would not apply. Whereas, results that rely on city block preferences (Humphreys and Laver 2009) or on non-differentiable utility functions (Kamada and Kojima 2010) would become more relevant, and further theoretical work would be needed to establish what results in the literature obtained under assumptions of quasiconcavity or differentiability of preferences are robust and apply in environments with agents whose preferences are not representable by quasiconcave or differentiable utility functions.

If the estimated parameter $\hat{\delta}$ is consistently greater than 1, even if it is not near 2, much of the theoretical literature will be validated. The main impact of obtaining a better estimate of δ in utility functions of the form (2) that is $\hat{\delta} \neq 2$ but $\hat{\delta} > 1$ will be to improve the fit of further empirical work on ideal point estimation models (Clinton et al. 2004; Poole and Rosenthal 1985), or vote choice models, by assuming that agents have utility functions with the curvature corresponding to the best estimate of δ within the parameterized family of utility functions (2), instead of assuming that agents have utility functions with parameter $\delta = 2$ even though parameter $\delta = 2$ provides a poorer fit for the model.

With regard to separability, violations of the assumption typically do not affect equilibrium existence or convergence results on models of electoral competition or policy choice. However, application of spatial models to specific real world polities or electorates should take into account existence evidence on non-separability across various pairs of issues (Lacy 2001a,b, 2012), so that if the models explicitly include such issues, utility functions are not assumed to be separable over them. Many spatial models do not include many issues; rather, they collapse the list of all issues onto two dimensions, one that groups economic issues (from left/pro-state to right/pro-market) and another that includes all cultural issues (from left/progressive to right/conservative). It is more difficult to determine whether preferences are separable or not over such dimensions, which are not precisely defined. Nevertheless, if future empirical work reveals evidence of a systematic correlation between preferences across economic and cultural issues, models should either seek to define new dimensions (new ways of bundling or weighing the issues) in such a way that preferences are separable over the new dimensions, or else, if this cannot be achieved, then it may be necessary to allow for non-separable preferences, estimating not only an ideal point, but also a degree of correlation between dimensions for each agent or group of agents.

Euclidean preferences have been an extremely useful tool in the development of multidimensional spatial models that can explain electoral competition, government formation and legislative policy-making. Generalizations that show that several theoretical results are robust if preferences are not Euclidean but are convex and smooth allowed us to conjecture that Euclidean preferences are only a simplifying shortcut with limited effect on our ability to understand the political processes we model. Nevertheless, we lack convincing empirical evidence that preferences are convex and smooth. If preferences are not convex and smooth, nor separable, and our theoretical models assume that they are, we are impaired in our ability to understand and predict the political processes we study.

Future empirical work shall establish whether preferences are convex and smooth, and whether we can find systematic evidence of differentiated non-separability over pairs of issues, or systematic differences in the weights assigned to different dimensions, across different groups of voters or legislators. Future (better) theories must make assumptions that are consistent with these future empirical findings.

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