The spatial modes dynamics in curved multimode integral optical waveguides

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Introduction

⊳ Integral photonics is widely used in quantum technologies. Traditionally photonic schemes are based on one-mode waveguides. However information in photonic circuits can be encoded into spatial modes of multimode integral optical waveguide, which may have benefits. Analysis of the eigenmodes is the complicated problem for curved multimode waveguides in case of the complexity of eigenmodes dynamics, because of mode mixing. As a consequence, memory costly numerical methods with high value of time of calculations need to be utilized (**3D FDTD**).

⊳ In this work, we introduce a new method of analyzing the dynamics of spatial modes by solving the differential equation for mode amplitude coefficients. As an example, object of analysis is buried planar waveguide which are defined by guiding curvature function (**fig. 1**). The core and clad are isotropic dielectrics.

$$n(\boldsymbol{\rho}) = \begin{cases} n_{core}, & |\boldsymbol{\rho}| \le |\boldsymbol{\rho}_0| \\ n_{clad}, & |\boldsymbol{\rho}_0| \le |\boldsymbol{\rho}| \le |\boldsymbol{\rho}_{max}| \end{cases}$$

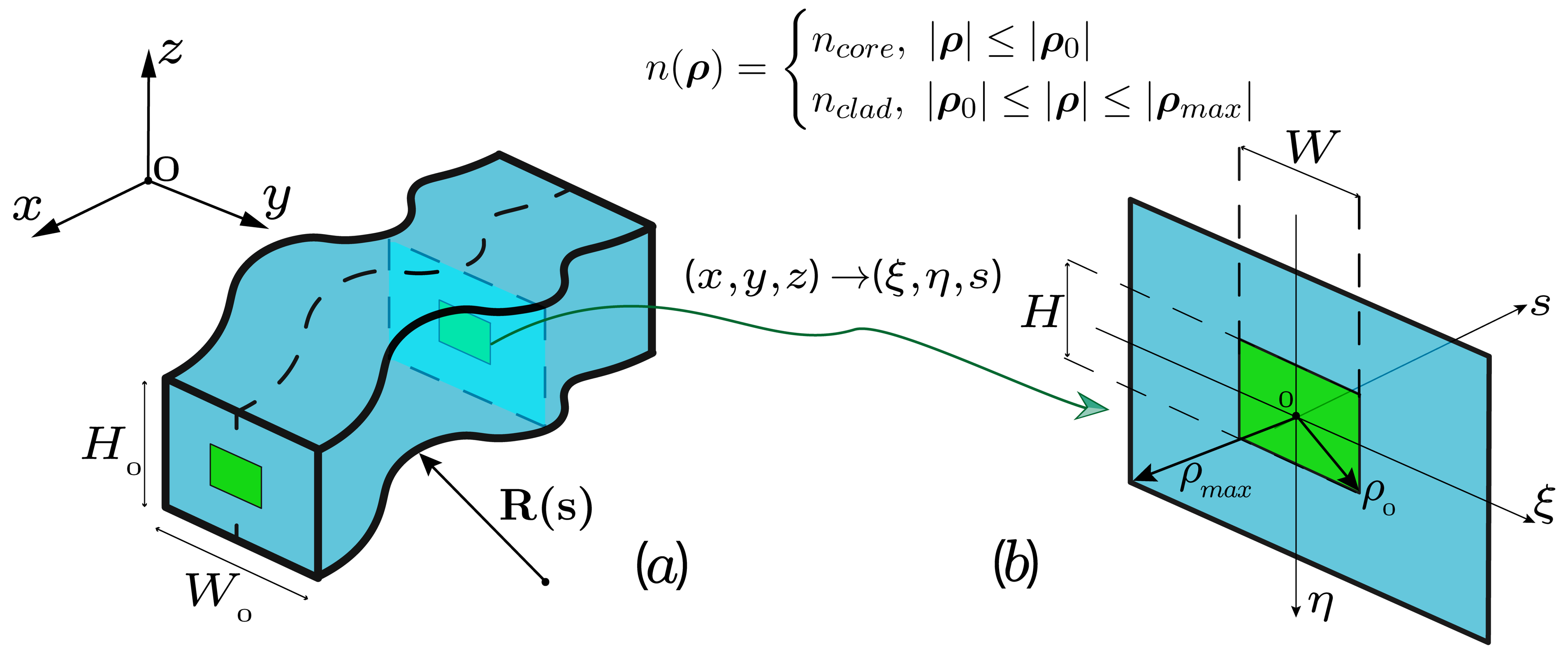

Fig. (1) (a) Buried planar waveguide with defined curvature function $\kappa(s)=1/R(s)$ along its length L.
(b) Cross section of waveguide illustrated in new orthogonal curvilinear coordinates (ξ,η,s)

Method

⊳ Following method based on solving the Helmholtz equation for electromagnetic wave, which propogates in the curved waveguide. The field in the waveguide is decomposed into series of waveguide eigenmode profiles

$$\vec{\varepsilon}\,(\vec{\rho},s) = \sum_m c_m(s)\ \vec{U}_m(\vec{\rho},s)\exp\left(i\int_0^s \beta_m(s')\ ds'\right)$$

⊳ This decomposition lead to calculations of eigenmodes and propogation constants at each value of s ($0 \le s \le L$). After calculations (**fig. 2**) the coefficients of functional matrixes $M^{(1)}$, $M^{(2)}$ in second order nonlinear differential equation need to be calculated also. Solution of this equation reveals the amplitude coefficients values at the end of the waveguide and the mixing matrix T, which describes how modes as a result of evolution in curved waveguide were coupled. The full plan of research is shown on the block scheme.

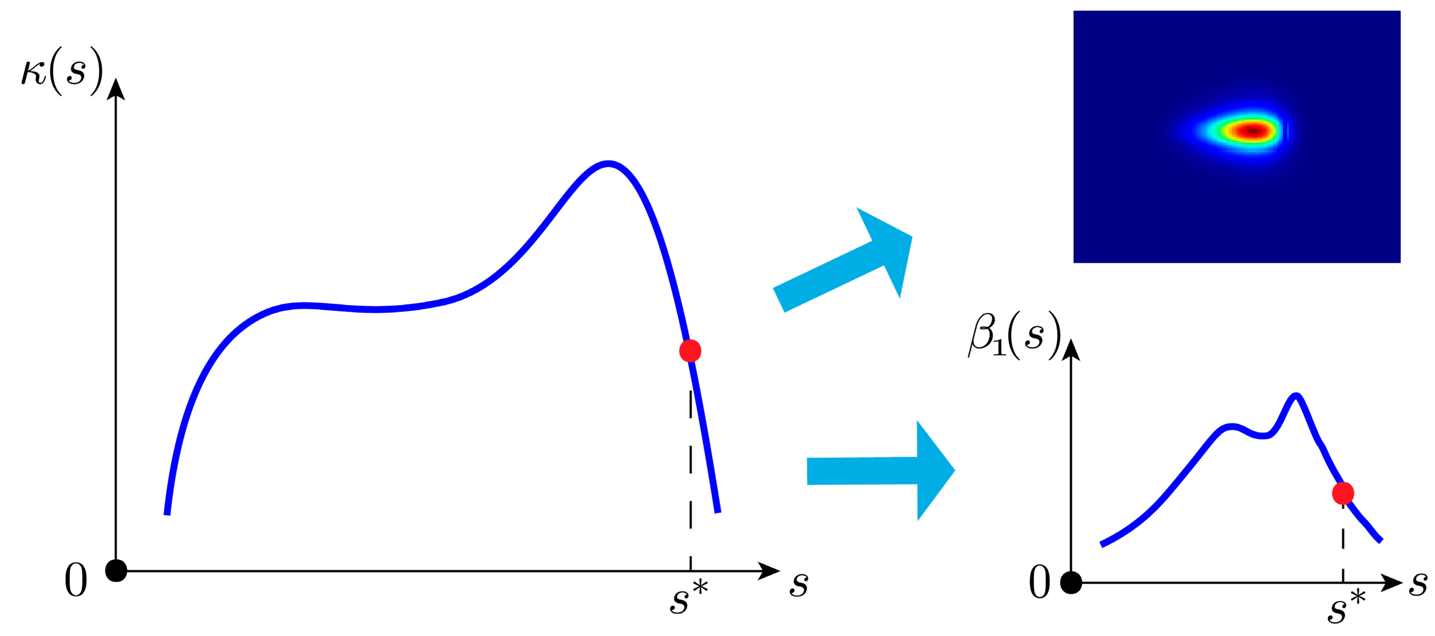

Fig. (2) The example of typical **TE 1** mode propogation constant $\beta_1(s)$ dependency from a length of a curved waveguide with corresponding eigenmode profile at $s = s^*$

Define the guiding curvature function $\kappa(s)$ for waveguide

⬇

Helmholtz equation for eigenmodes $\boldsymbol{U}_m(\boldsymbol{\rho},\kappa)$ and eigenvalues $\beta_m(\kappa)$ of curved waveguide

$$\Delta^{(\kappa)}_{\eta\xi}\mathbf{U}_m(\boldsymbol{\rho},\kappa) + \left[k_0^2 n^2(\boldsymbol{\rho}) - \frac{\beta_m^2(\kappa)}{(1+\kappa\xi)^2}\right]\mathbf{U}_m(\boldsymbol{\rho},\kappa) = 0$$

⬇

Eigenmodes and eigenvalues for different $\kappa(s)$ values

$$\boldsymbol{U}_m(\boldsymbol{\rho},\kappa), \quad \beta_m(\kappa)$$

⬇

Differential equation for amplitude coefficients $\boldsymbol{C}(s)$ dynamics

$$\boldsymbol{C}''(s) + M^{(1)}(\boldsymbol{\beta}(s),\kappa(s),s)\boldsymbol{C}'(s) + M^{(2)}(\boldsymbol{\beta}(s),\kappa(s),s)\boldsymbol{C}(s) = 0$$

⬇

The values of amplitude coefficients at the end of the waveguide and mixing matrix T

$$\boldsymbol{C}(s=L) = T\boldsymbol{C}(s=0)$$

Example

⊳ Parameters of a researched waveguide cross section: the core is Si with $W = 2\ \mu m$, $H = 0.22\ \mu m$; the clad is SiO_2 with $W_0 = 14\ \mu m$, $H_0 = 6.22\ \mu m$. The wavelength for simulation is $1.55\ \mu m$. The calculations of eigenmodes and propogation constants were carried out by **FDFD** eigensolver developed by ourselfs, which fully corresponds with solutions from **ANSYS Lumerical**. As an example curvature function we use paraboloic bend (**fig. 3, (a, b)**).

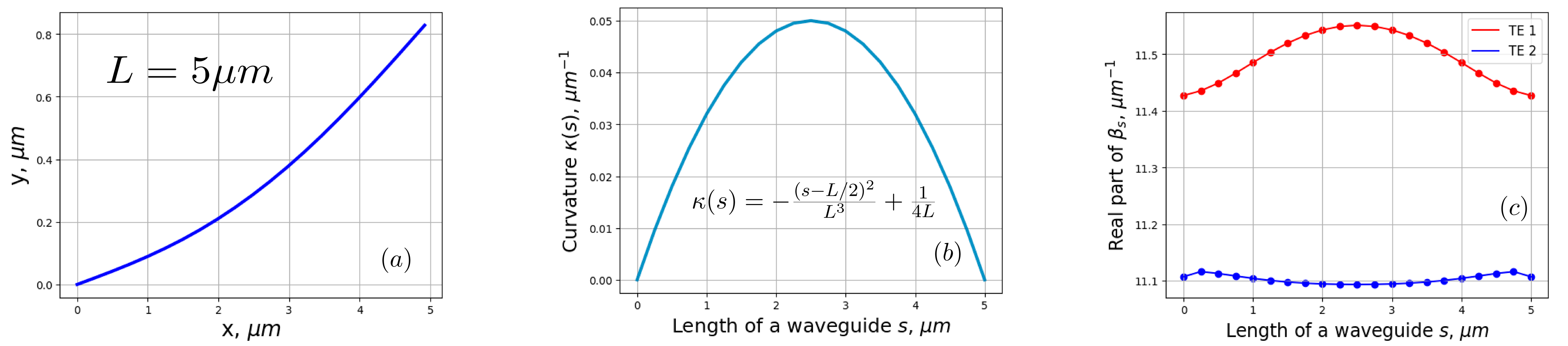

Fig. (3) (a) The guiding function $y(x)$ profile from the above view for $L = 5\ \mu m$ waveguide length; (b) The curvature function $\kappa(s)$, calculated for length $L = 5\ \mu m$; (c) The real parts of propogation constants $\beta_{1,2}(s)$ calculated by along length of the waveguide s with corresponding values of curvature $\kappa(s)$ (imaginary parts are approximately 0).

Prospective results

⊳ Nonlinear differential equation for the amplitude coefficients has a complex matrix functions $M^{(1)}(s)$, $M^{(2)}(s)$ as a coefficients of this equation. The approach for solving this equation is unknown and needs to be achieved firstly.

⊳ After the differential equation will be solved, the problem of determining the curvature function for predefined amplitude coefficients transformation T_0 is appearing. The solution of this problem will be useful for designing multimode microresonators and multimode waveguides bends, which perform transformations, based on the multimode mixing.