<u>Option Hedging</u>

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I. <u>Introduction</u>

This project aims at investigating the optimal option hedging strategy to hedge Apple stock options for 5 business days from November 30 to December 7, 2021. Hedging is a risk management strategy employed to offset losses in investments by taking an opposite position in a related asset. Our initial position is a portfolio consisting of 10 long calls on Apple Inc (AAPL). To offset the risk of our calls, we short sold the underlying stock - AAPL. The number of stocks to be shorted is calculated by multiplying the option delta, a ratio between the change in the price of an options contract and the corresponding movement of the underlying asset's value, with the number of options longed. There are two ways to calculate the option delta: using historical volatility and implied volatility. Historical volatility calculates volatility of stocks based on its past behavior whereas implied volatility shows what the market implies about the stock's volatility in the future. Implied volatility is a forward-looking metric that captures the market's view of the likelihood of changes in each security's price. The objective of this project is to determine which method is optimal for delta hedging.

II. <u>Findings</u>

A. Historical volatility was 21.487% which was significantly lower than the implied volatility from Yahoo Finance which was 28.36%. However, the implied volatility calculated using Black-Scholes Model and Excel's solver was 27.20% which is much closer to implied volatility from Yahoo Finance.

B. The option prices calculated using historical volatility for the first and last day were significantly different from the option prices from yahoo finance. On the contrary, the option prices calculated using implied volatility were close to the option prices from Yahoo Finance. Refer to the analysis section for data and analysis.

C. Using implied volatility obtained from Yahoo Finance caused less variability in net positional change for our delta hedging. We found that the standard deviation of net change using implied volatility and historical volatility were 305.39% and 328.87% respectively. This implies implied volatility is more effective for delta hedging than historical volatility.

III. <u>Discussion</u>

1. Method

Calculation of appropriate Inputs:

The first step to our project involved gathering the appropriate inputs for our pricing engines. The stock price, strike price, and dividend yield were collected from Yahoo Finance. The time to maturity was calculated by dividing the number of trading days until maturity by total trading days in a year. The risk-free interest rate was obtained from the US Treasury website. The risk-free interest rate used in our project is the same as the rate for a 1-year treasury bill. Finally, the volatility was calculated using the historical data. We downloaded the stock price for the past 20 trading days. After calculating the daily returns, we used the daily returns data to calculate the sample standard deviation. We annualized the standard deviation to find the volatility. We used the following formula to annualize the standard deviation:

$$\sigma = \frac{s}{\sqrt{\tau}}$$

where τ is the length of time intervals in years, and in this case, is equal to 1/252.

We used the volatility along with other inputs to establish our pricing engines.

Establishing the Pricing Engines:

We set up two pricing engines in our project: 5-step binomial trees model and Black-Scholes model. We used the binomial trees model and Black-Scholes model to calculate the option prices for American call and European call respectively. First, we calculated the up-factor(u), down-factor(d), the probability that the stock price will go up(p), and the probability that the stock price will go down(pnot). We used the following formulae to calculate each of these variables:

$$u = e^{\sigma * \sqrt{\delta t}}$$
$$d = e^{-\sigma * \sqrt{\delta t}}$$
$$p = \frac{(e^{r\delta t} - d)}{(u - d)}$$

$$pnot = 1 - p$$

where σ is the volatility, δt is the length of one period, and r is the risk-free interest rate.

We used MS-Excel to set up the binomial pricing model. We multiplied the initial stock price by the up factor and down factor to find the increased and decreased stock price after the first period. For each stock price the stock price could go up or down. So, multiplying each stock price by up factor and down factor we end up with all possibilities of stock prices. We repeated the process of multiplying each stock price by up factor and down factor to produce the possibilities of stock price until the time of maturity. We used 5 steps until maturity to find the stock prices. We then used the American call option payoff function to calculate the American call premium for each step. We started with the maturity date and went back to starting date to find option prices for each step. The option payoff in the maturity date is max (0, S(T)-K). Then, we used the following formula to find the option premium in each step:

$$Option\ Permium = \max\left((C_A(u) * p + C_A(d) * pnot) * e^{-r*\delta t}, S(t) - K\right)$$

where $C_A(u)$ and $C_A(d)$ are the option premiums of the future period if the stock price goes up and down respectively, S(t) is the stock price at that time and K is the strike price.

Similarly, we used Black-Scholes model and Excel's functions to establish pricing engines for the European Call Options. We used the following formulae for pricing the European Call Options.

$$d_1 = \frac{\left(\ln\left(\frac{S(0)}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right) * T\right)}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = S(0)e^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

Where c is the European call option price, $N(d_1)$ and $N(d_2)$ are the cumulative normal distributions for d1 and d2 respectively, and q is the dividend yield compounded continuously.

Calculation of deltas and Setting up Delta hedging Position:

After setting up the pricing engines, we calculated the option deltas using the binomial trees model. The formula for calculating the option deltas is:

$$\Delta = \frac{option\ payoff\ difference}{stock\ price\ difference}$$

We calculated option deltas for each step. We set up our delta hedging position starting from 11/30/2021. We bought 10 call options for appl. The option delta we got from the binomial pricing model was 0.40. So, we short sold 0.397 *1000 = 397 apple stocks to hedge our long position in the calls. The change in the value of the stock position was the price difference of the stocks multiplied by 397, and the change in the value of the long call options was calculated by multiplying the call premium difference by 1000. We calculated the net change in the overall position by adding the net change in the stock position and the net change in the options position.

Calculation of Implied Volatility using Black-Scholes Model and Excel Solver:

We used Excel's solver to find the implied volatility using the Black-Scholes model we established before. The option price was obtained from Yahoo Finance. The objective was to make the difference between the option price from Yahoo and option price from Black-Scholes model zero by changing the volatility used in our model. We compared the volatility from the solver with the implied volatility from Yahoo Finance. We included the analysis in the analysis section of this paper.

Delta Hedging Using Implied Volatility:

To achieve this goal, we used the implied volatility from the Yahoo Finance instead of volatility calculated using historical data to establish a delta hedging position. We obtained the implied volatility from Yahoo Finance and used our binomial pricing model to find the option deltas for each step. We used the new option delta to find the number of shares we need to short. Finally, we calculated the overall change in our position and compared the results with the previous results obtained using the historical data. We include the analysis of these results in the analysis section of this memo.

2. Main Analysis

Comparison of model call option prices with the market price

The European Call price and American Call price calculated using the Black-Scholes model and Binomial Trees model are summarized in the table below. The table shows that the model generated price and the actual price from Yahoo Finance are quite different. However, the European Call price obtained from Black-Scholes model is consistent with the American Call price obtained from the Binomial Trees model.

	Option Price		
	Binomial Trees	Black-Scholes Model	Yahoo Finance
First Day	3.85	3.74	5.05
Last Day	5.99	5.67	7.65

Figure 1 Using historical data to find option prices on the first day and last day

We went one step further and calculated options price using the implied volatility from Yahoo finance the pricing model. We summarize the results in table 2.

	Option Price		
	Binomial Trees	Black-Scholes Model	Yahoo Finance
First Day	5.99	5.67	5.05
Last Day	7.62	7.22	7.65

Figure 2 Using implied volatility from Yahoo Finance to calculate option prices on the first day and last day

We observed that the option prices calculated using implied volatility was closer to the option prices obtained from Yahoo Finance. This implies that the use of implied volatility to calculate option prices is more effective than using historical data to calculate volatility.

Comparison of volatility by Solver with the Implied volatility from Yahoo

We used solver and the Black-Scholes model to calculate the implied volatility. We got 27.20% as the implied volatility from the solver. The implied volatility from the Yahoo Finance on the day we started the project was 28.36%. The volatilities are very close to each other. This justifies our selection of the inputs for the pricing engines. However, we found that the historical volatility is significantly less than the implied volatility from Yahoo Finance. The implied volatility is calculated using the option prices determined by the market. Since the implied volatility is significantly greater than the historical volatility, we infer that the option prices might be overvalued. This can be confirmed by comparing the option prices obtained using historical volatility and the option prices from Yahoo Finance. Refer to Figure 1 above for data.

Hedging Results using historical data and implied volatility from Yahoo Finance

After 5 days, we find that the net change in our hedging position would be $638.82 if we used the historical data to find the option delta, and the net change would be $475.80 if we used the implied volatility from Yahoo Finance to calculate the option delta. We summarize the results below:

Using Historical Data			
Date	**Stock Price**	**Option Price**	**Net Change**
11/30/2021	166.03	5.05	
12/7/2021	170.97	7.65	638.82

Using Implied Volatility from Yahoo Finance			
Date	**Stock Price**	**Option Price**	**Net Change**
11/30/2021	166.03	5.05	
12/7/2021	170.97	7.65	475.80

The main objective of doing delta hedging is to minimize the losses that could occur from the option price changes. In our case, the options gained value instead of losing. However, the above table shows that using implied volatility causes smaller net change than using volatility from historical data. Thus, the hedging results suggest that use of implied volatility from Yahoo Finance is more effective to do delta hedging.

3. Limitations

The main limitation of our project was the length of time of hedging. We believe it will be erroneous to jump into conclusion just based on the 5 days of delta hedging. Since the hedging was done only for 5 days, we did not have to change the option deltas. But, when doing hedging for a longer period, the option deltas and position on stocks must be frequently adjusted to reflect the changes in the option price. These adjustments have transaction costs associated with them. The exclusion of transaction costs is another limitation to our project.

Since the implied volatility depends on the sentiment of the market, it could be overestimated or underestimated depending on the situations. Hence, it could be misleading to conclude that using implied volatility is more effective just because implied volatility gave good results for past 5 days. We believe more data and analysis is needed for a concrete conclusion.

IV. Conclusions

This project aimed at investigating the optimal option hedging strategy to hedge Apple stock options for 5 business days from November 30 to December 7, 2021, by employing two different hedging strategies: delta hedging and hedging with implied volatility. We found that using implied volatility to hedge our options yielded more accurate results than using historical volatilities. This is for several reasons. Firstly, the option prices calculated using implied volatility were close to the options prices from Yahoo Finance. Secondly, using implied volatility obtained from Yahoo Finance caused less variability in net positional change for our delta hedging. We found that the standard deviation of net change using implied volatility and historical volatility were 305.39% and 328.87% respectively. This implies implied volatility is more effective for delta hedging than historical volatility.

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