

Portfolio Allocation Memorandum

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DATE: October 1, 2021
SUBJECT: Portfolio Performance and Allocation Analysis

I. Background

This project aims at evaluating the portfolio performance of a portfolio consisting of 5 securities. We set up two optimization problems, aiming at adjusting the weights of our securities in a way that produces the maximum expected return of our portfolio and minimum risk. Securities in question are CAT, CSCO, and ANIK stocks, as well as corporate and treasury bonds, LQD and TLT.

II. Findings

1. CSCO stock is not only underperforming but it is also highly volatile which makes it the least attractive. It has the least expected return of 0.18% and it has a risk of 4.25%, which is greater than that of LQD and TLT.
2. CAT has the highest expected return of 0.52% followed by ANIK which has the expected return of 0.51%. However, they are also very volatile. ANIK has the largest volatility followed by CAT. LQD is the least risky followed by TLT.
3. Based on the risk and returns of the securities and the uncertainty of the future due to pandemic, we suggest the following weights for the portfolio:

CAT	CSCO	ANIK	LQD	TLT
37%	0%	15%	5%	43%

- Expected Return: 0.41%
- Risk: 2.31%

**Refer to the analysis section for more details.*

III. Discussion

1. Methods

For this project, we used weekly adjusted prices of five securities dated between January 1, 2019, and December 31, 2020. We gathered this data from finance.yahoo.com.

We identified an objective function with an aim to maximize the expected return as a function of weights of each of the five securities. Firstly, we found the efficient frontier given no constraints on weights, allowing them to range from 0% to 100%. Afterwards, we placed constraints on the weights, assigning a minimum of 5% and a maximum of 50% and calculated the second efficient frontier. We compared the two efficient frontiers to identify how each of the securities impacts risk and expected return.

Finally, we compared the 2-year data of our securities with their equivalent indices in the past 5, 10, 20, and 30 years to identify whether the trends in our portfolio match the general trends in those long-term series. These indices are:

- S&P 500 for Large Cap stocks (CAT and CSCO)
- Russel 2000 for Small Cap stocks (ANIK)
- US Corporate Investment Grade Bond Index (LQD)
- US Treasury Bond Index (TLT)

We set up two objective functions consisting of the weights of 4 indices. The aim of the first one is to maximize expected returns, while the second one aims at minimizing risk.

2. Analysis

Expected Return and Risk. Looking at the expected returns and risks in table 1, we can see that CAT boasts the highest expected return while CSCO yields the smallest one. Undoubtedly, the COVID19 pandemic has had a huge negative impact on the economy, and thus it impacted the portfolio in the same manner:

	CAT	CSCO	ANIK	LQD	TLT
Expected Return	0.52%	0.18%	0.51%	0.26%	0.31%
Standard Deviation	5.08%	4.25%	7.19%	1.54%	2.06%
Variance	0.26%	0.18%	0.52%	0.02%	0.04%

Table 1 Summary of Expected Returns and Risks associated with the securities

- **CAT** stocks sustained a large negative shock in Q1 of 2020 as their North America retail sales dropped by 20% (Singh). However, CAT muscled through the rest of 2020, hitting all time high prices by the end of 2020 despite lower revenue.
- **CSCO** also sustained heavy hits from the pandemic, but unlike CAT, CSCO suffered another heavy hit to its price in Q3. CSCO was left out of the tech stock rally that took advantage of increased demand for technological solutions, especially those for communications and cloud services. This lowers our confidence in this stock and, combined with indicators we discuss further in our analysis, lead us to believe that CSCO is too risky to invest into.

- **ANIK** has been performing especially after Anika Therapeutics announced on January 6th, that they are to acquire Parcus Medical and ArthroSurface (“Anika Therapeutics to Acquire Parcus Medical and ArthroSurface”). This would allow them to expand their portfolio in regenerative therapeutic products. After the initial shock in Q1, ANIK prices have been on an upward trend. We recognize their acquisition as a strong driver for the future, however the stock volatility is a considerable issue, and given the recent uncertainty shaped by the pandemic indicate that we ought to be weary of ANIK’s performance.
- **LQD and TLT** prices have increased during the pandemic, which is expected to happen as bond prices rise when stock prices fall.

Efficient Frontier Comparison. Comparing efficient frontiers in figure 1, we can determine the impact of constraints on weights. The unconstrained efficient frontier encompasses a wider range of risks and returns which is consistent with our findings in table 1. For example, looking at the lowest point on the unconstrained efficient frontier, it perfectly matches the expected return and risk of CSCO, and this is because for that point, CSCO weights are 100%. Similarly, the highest point on the frontier is where CAT weights are maximized, and it matches the risk and return of CAT stock seen in table 1. (See tables 1A and 2A in appendix A for weight allocation presented in both efficient frontiers.)

On the other hand, the constrained efficient frontier is less spread out, and for the most points, it yields a higher risk for a lower return. This is because the minimum weights we assigned are 5%, so at any point, all 5 securities have an impact on the risk and return. However, there are several things that are consistent between both frontiers. In each case, the expected return is maximized when CAT weights are maximized while CSCO is minimized. Therefore, we find that the efficient frontier is another piece of evidence that points us to minimize CSCO stocks in our portfolio.

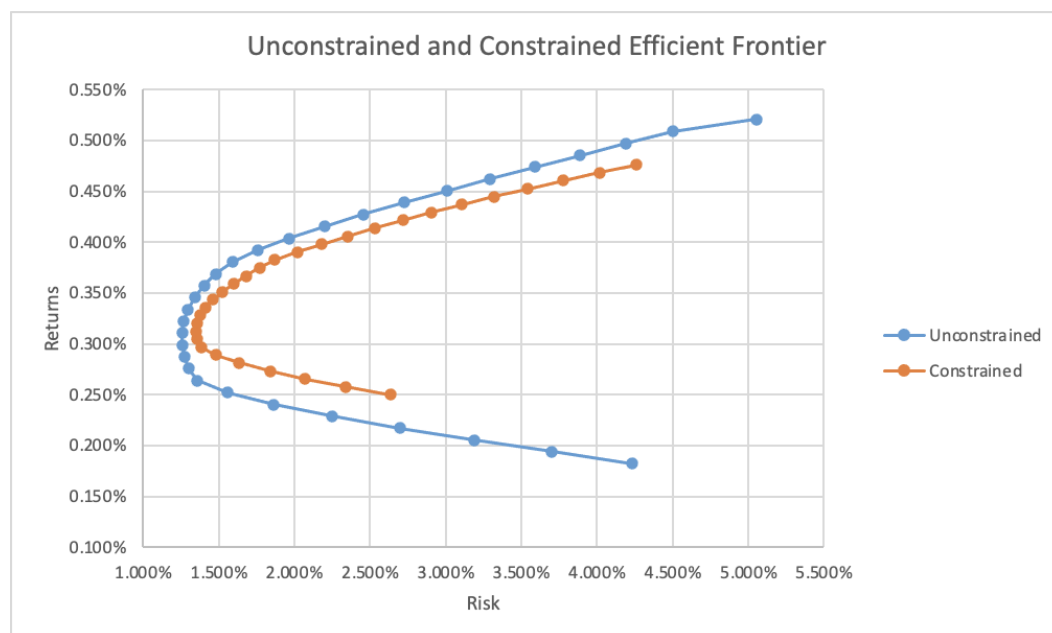


Figure 1 Comparison of Efficient Frontiers for Constrained and Unconstrained Optimization problems

Long-term series comparison. Comparison of our securities to the long-term performances of their respective indices yielded results that were mostly consistent with our previous findings. Four of our five securities are following long term trends. For example, table 1 shows the constrained weighted distribution that maximizes expected return. The findings for 5 and 10 years are consistent with our research, given that we found that the distribution of weights that maximizes profit for our portfolio is 55-35-5-5. However, CSCO is an outlier here as well. It does not follow the S&P 500 trend, and this is a third reason as to why we do not have confidence in this stock and recommend not to invest in it.

Portfolio Weights for Max Return				
Data Used	S&P 500	Russell 2000	U.S. Corp.	U.S. Treas.
5 Years	50%	40%	5%	5%
10 Years	50%	40%	5%	5%
20 Years	40%	50%	5%	5%
30 Years	40%	50%	5%	5%

Table 2 Portfolio Weights for Maximum Returns (with constraints applied)

Another thing to consider is that the COVID19 pandemic has created many insecurities across the globe and to minimize risk, we decided to construct a portfolio that follows a roughly 50% allocation to stocks and 50% allocation to bonds. This portfolio allocation allows us to retain 80% (see figure 2A in appendix A) of the maximum return at almost half risk which is 2.31%.

Finally, we opted to allocate most of our bond share of the portfolio to TLT. This is because Treasury Bonds historically have been the safest option. If corporations perform poorly, LQD might perform poorly as well or even default, hence, we find that an allocation of 5% to LQD and 43% to TLT is proper.

Portfolio Weights for Minimum Variance				
Data Used	S&P 500	Russell 2000	U.S. Corp.	U.S. Treas.
5 Years	5%	5%	40%	50%
10 Years	5%	5%	42%	48%
20 Years	5%	5%	50%	40%
30 Years	5%	5%	50%	40%

Table 3 Portfolio Weights for Minimum Variance (with constraints applied)

3. Limitations

It is important to consider the constraints of our research and methodology. We identified several limitations that should be considered:

- Firstly, **the data collected and used is purely quantitative.** While quantitative data is essential to providing concrete indicators of a stock performance, it does not tell the story of why this was the case. For example, looking at CSCO data, we can tell that there was a negative shock in Q3, but our mathematical conclusion does not weigh in whether this plunge in price reflected structural issues within the company, or some less severe issue.

- Secondly, the data we used in this study is heavily skewed by the effects of the pandemic. This makes it hard to get a true prediction of how the portfolio will perform in the future by only using asset prices. As such, the research may not be able to consider or predict how certain COVID-19 government response policies may affect the portfolio.

IV. Conclusions

Ultimately, we recommend the following portfolio allocation:

CAT	CSCO	ANIK	LQD	TLT
37%	0%	15%	5%	43%

This portfolio has an expected return of 0.41% and risk of 2.31%. We construct this portfolio for several reasons, namely:

- CAT has been consistent in its performance and has muscled through the pandemic with its stocks being on an increasing trend ever since the initial shock in Q1 of 2020.
- CSCO has been underperforming, showing a low rate of return combined with high volatility. Moreover, it is the only stock being out of line with long term trends for the S&P 500 index.
- The COVID19 pandemic has had a major negative impact on domestic markets and those are reflected in the performance of our portfolio with stocks exhibiting high volatility, and bonds showing higher returns because of a decrease in stock prices. Hence, a high bond participation ought to offset this volatility while retaining 80% of the maximum expected return for only 55% of its risk.

V. References

“Anika Therapeutics to Acquire Parcus Medical and ArthroSurface.” *Business Wire*, 6 Jan. 2020, [Link](#)

Singh, Rajesh Kumar. “How Deere, Caterpillar Kept Plants Running during the Coronavirus Outbreak.” *U.S.*, 8 May 2020, [Link](#)

Appendix A

Table 1A: Portfolio allocation with no constraints

CAT	CSCO	ANIK	LQD	TLT	Std Dev	Exp Ret
0.00%	100.00%	0.00%	0.00%	0.00%	4.23%	0.18%
0.00%	85.57%	0.00%	14.43%	0.00%	3.70%	0.19%
0.00%	71.14%	0.00%	28.86%	0.00%	3.19%	0.21%
0.00%	56.72%	0.00%	43.28%	0.00%	2.70%	0.22%
0.00%	42.29%	0.00%	57.71%	0.00%	2.25%	0.23%
0.00%	29.47%	0.00%	67.53%	3.00%	1.87%	0.24%
0.00%	21.60%	0.00%	63.17%	15.23%	1.56%	0.25%
0.00%	13.73%	0.00%	58.81%	27.46%	1.36%	0.26%
1.53%	8.43%	0.00%	54.65%	35.39%	1.30%	0.28%
4.78%	6.01%	0.00%	50.73%	38.47%	1.28%	0.29%
8.03%	3.60%	0.00%	46.81%	41.56%	1.26%	0.30%
11.29%	1.18%	0.00%	42.89%	44.64%	1.26%	0.31%
14.71%	0.00%	0.00%	36.27%	49.01%	1.27%	0.32%
18.30%	0.00%	0.00%	27.09%	54.61%	1.30%	0.33%
21.89%	0.00%	0.00%	17.90%	60.21%	1.34%	0.35%
25.45%	0.00%	0.03%	8.71%	65.81%	1.40%	0.36%
28.40%	0.00%	0.80%	0.00%	70.80%	1.48%	0.37%
32.29%	0.00%	2.45%	0.00%	65.26%	1.59%	0.38%
36.18%	0.00%	4.10%	0.00%	59.72%	1.76%	0.39%
40.07%	0.00%	5.75%	0.00%	54.19%	1.97%	0.40%
43.96%	0.00%	7.39%	0.00%	48.65%	2.20%	0.42%
47.85%	0.00%	9.04%	0.00%	43.11%	2.46%	0.43%
51.74%	0.00%	10.69%	0.00%	37.57%	2.73%	0.44%
55.63%	0.00%	12.33%	0.00%	32.04%	3.01%	0.45%
59.52%	0.00%	13.98%	0.00%	26.50%	3.30%	0.46%
63.41%	0.00%	15.63%	0.00%	20.96%	3.59%	0.47%
67.30%	0.00%	17.28%	0.00%	15.42%	3.89%	0.49%
71.19%	0.00%	18.92%	0.00%	9.89%	4.19%	0.50%
75.08%	0.00%	20.57%	0.00%	4.35%	4.50%	0.51%
100.00%	0.00%	0.00%	0.00%	0.00%	5.05%	0.52%

Table 2A: Portfolio allocation with constraints

CAT	CSCO	ANIK	LQD	TLT	Std Dev	Exp Ret
5.00%	50.00%	5.00%	35.00%	5.00%	2.63%	0.25%
5.00%	40.36%	5.00%	44.64%	5.00%	2.34%	0.26%
5.00%	32.21%	5.00%	50.00%	7.79%	2.07%	0.27%
5.00%	25.94%	5.00%	50.00%	14.06%	1.84%	0.27%
5.00%	19.67%	5.00%	50.00%	20.33%	1.64%	0.28%
5.00%	13.86%	5.00%	48.66%	27.48%	1.48%	0.29%
5.00%	8.60%	5.00%	45.75%	35.65%	1.38%	0.30%
5.69%	5.00%	5.00%	41.50%	42.81%	1.36%	0.30%
8.09%	5.00%	5.00%	35.36%	46.55%	1.35%	0.31%
10.53%	5.00%	5.00%	29.47%	50.00%	1.36%	0.32%
13.56%	5.00%	5.00%	26.44%	50.00%	1.38%	0.33%
16.59%	5.00%	5.00%	23.41%	50.00%	1.41%	0.34%
19.62%	5.00%	5.00%	20.38%	50.00%	1.46%	0.34%
22.64%	5.00%	5.00%	17.36%	50.00%	1.53%	0.35%
25.67%	5.00%	5.00%	14.33%	50.00%	1.60%	0.36%
28.70%	5.00%	5.00%	11.30%	50.00%	1.68%	0.37%
31.73%	5.00%	5.00%	8.27%	50.00%	1.77%	0.37%
34.49%	5.00%	5.28%	5.23%	50.00%	1.87%	0.38%
37.30%	5.00%	6.10%	5.00%	46.60%	2.02%	0.39%
39.90%	5.00%	7.20%	5.00%	42.90%	2.18%	0.40%
42.50%	5.00%	8.30%	5.00%	39.20%	2.35%	0.41%
45.10%	5.00%	9.40%	5.00%	35.50%	2.53%	0.41%
47.70%	5.00%	10.50%	5.00%	31.80%	2.72%	0.42%
50.00%	5.00%	11.92%	5.00%	28.08%	2.91%	0.43%
50.00%	5.00%	15.77%	5.00%	24.23%	3.11%	0.44%
50.00%	5.00%	19.61%	5.00%	20.39%	3.32%	0.44%
50.00%	5.00%	23.46%	5.00%	16.54%	3.54%	0.45%
50.00%	5.00%	27.31%	5.00%	12.69%	3.78%	0.46%
50.00%	5.00%	31.15%	5.00%	8.85%	4.02%	0.47%
50.00%	5.00%	35.00%	5.00%	5.00%	4.26%	0.48%