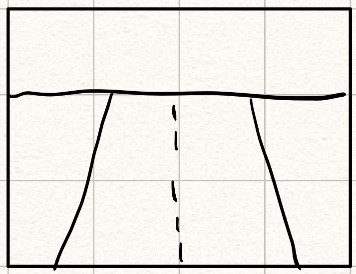
The starting point is an image and two camera matrices



Intrinsic parameters

$$K = \begin{bmatrix} f_x & 0 & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix},$$

Extrinsic parameters

Yaw	α	X
Pitch	β	, Y
Roll	γ	Z

Transforming (homogeneous) camera coordinates Q_c → Vehicle coordinates Q

Specifies the coordinate transform 'Camera → Vehicle'

$$\boxed{Q = R_{cam2veh} Q_c + T_{cam2veh}}$$

$$R_{cam2veh} = f_{euler\_rot}(\alpha, \beta, \gamma) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$T_{cam2veh} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

In order to transform vehicle coordinates Q → (homogeneous) camera coordinates Q_c

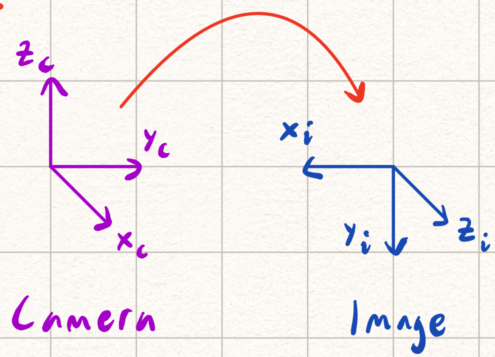
$$\boxed{Q_c = R_{veh2cam} Q + T_{veh2cam}}$$

$$R_{veh2cam} = R^{-1}_{cam2veh} = R^{T}_{cam2veh}$$ Property of rotation matrices

$$T_{veh2cam} = -T_{cam2veh}$$

To obtain image coordinates one must apply K and rotate axes to correspond to the image coordinate axes

$$C = K \, R_{img}$$



$$R_{img} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Camera		image
x	$\rightarrow$	$-y$
y	$\rightarrow$	$-z$
z	$\rightarrow$	x

Rotation matrix inductively derived to rotate camera frame coordinates $[x_c, y_c, z_c]^T$ to corresponding image frame coordinates $[x_i, y_i, z_i]^T$

Examples: $\bar{v}_{camera}^{T} R_{img} = \bar{v}_{image}$

$$x^T = [1, 0, 0] \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = [0, -1, 0] \equiv -y$$

$$y^T = [0, 1, 0] \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = [0, 0, -1] \equiv -z$$

$$z^T = [0, 0, 1] \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = [1, 0, 0] \equiv x$$

$$\boxed{q_h = C\,Q_c = C\left(R_{vch2cam}\,Q + T_{vch2cam}\right)}$$

Equally

$$P = C\,R_{vch2cam}$$
$$t = C\,T_{vch2cam}$$

$$\boxed{q_h = P\,R_{vch2cam} + t}$$

Remember to normalize homogeneous point q_h to get the actual pixel value

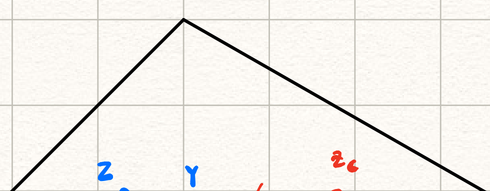
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} q_1/q_3 \\ q_2/q_3 \\ 1 \end{bmatrix}$$

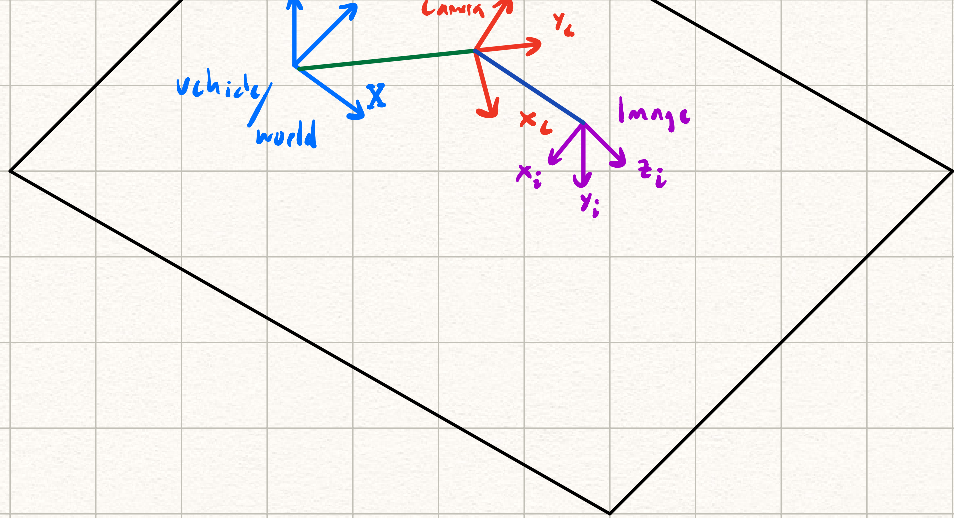
POINT: Assuming that the vehicle coordinate system correspond to the road plane (Z=0)

The inverse transformation from image coordinates q_h → Road plane (vehicle) coordinates Q (See Oliveira 2015)

$$\begin{bmatrix} X \\ Y \\ Z \\ w \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & -x \\ P_{21} & P_{22} & P_{23} & -y \\ P_{31} & P_{32} & P_{33} & -1 \\ a & b & c & 0 \end{bmatrix} \begin{bmatrix} -t_1 \\ -t_2 \\ -t_3 \\ -d \end{bmatrix}$$

where $(a, b, c, d) = (0, 0, 1, 0)$ plane $Z = 0$





Camera
y_c
vehicle/world
X
Image
x_c
x_i
z_i
y_i