Ratings & Reciprocity

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Abstract

Recent evidence suggests online ratings and reviews are intrinsically motivated, and reviewers reciprocate offers of sufficient value-for-money with good ratings. We incorporate reciprocity into a model of ratings: if firms offer a sufficiently high value for money, consumers perceive them as kind and rate them positively. To benefit from the reciprocal behavior of consumers, firms offer lower prices in exchange for a good rating. This allows firms to set higher prices in the future. Firms reinforce this mechanism and are more likely to induce good ratings when i) consumers exhibit a stronger sense of reciprocity or when ii) it is easier for consumers to leave ratings. We show that low-quality firms obtaining good ratings leads to less informative ratings. This way, reciprocity-motivated ratings provide a novel explanation for ratings inflation, the phenomenon where ratings are becoming less informative about product quality. However, less informative ratings may not harm consumers, as they benefit from lower prices. But even though consumers on average do not prefer perfectly-informative ratings, they prefer more-informative rating systems than average sellers. We discuss how common marketplace features affect reciprocity-motivated ratings and the informativeness of ratings. First, a two-sided platform might indeed choose to make it easier for consumers to rate, thereby encouraging more ratings, but also less-informative ratings. Second, many platforms screen sellers for quality, removing lower quality sellers. We characterize when such screening reduces or reinforces the informativeness of ratings. Third, competition between sellers leads to lower informativeness of ratings. Our results are robust to the ratings scale, time horizon, and distribution of firm quality.

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1 Introduction

Ratings feature prominently in the decisions we make each day. We often rely on the experience of others, through ratings and reviews, to make decisions. This is especially true for transactions that do not happen regularly. Examples include choosing a college, deciding on a holiday destination or buying a car. Everyday decisions, such as where to have a meal or which show to watch are also often influenced by ratings. But how far can we trust these ratings to inform us about the quality of a product?

Despite having an influential role on the way consumers make purchase decisions, evidence suggests that ratings are a poor signal of objective product quality (De Langhe et al., 2016; Siering et al., 2018). Starting with Li and Hitt (2010), empirical work highlights that prices influence ratings, and it is the value-for-money for which consumers provide ratings. While this suggests that prices affect ratings, other evidence shows that firms with better ratings have the ability to set higher prices (Luca & Reshef, 2021). Combined, these empirical works suggest a dual role between ratings and prices. Firms want to obtain good ratings, as this allows them to set higher prices. But prices are an important driver of ratings, giving firms an incentive to set lower prices to manipulate ratings.

While the evidence suggests that prices influence ratings, the large body of theoretical literature on reputation tends to assume that reputation is solely a consequence of quality. In doing so, these papers focus on only one direction of effects: how ratings can influence future prices. In this paper, we contribute to the theoretical literature by considering the dual role of prices in ratings, as an influence on the production of ratings and as an outcome of ratings. With this, we are able to address the question: "are value-for-money based ratings informative of product quality?".

To investigate the role that prices play in influencing the informativeness of ratings, we turn to a feature that is often suggested by empirical works, and widely studied in the behavioral economics literature: reciprocity (Bolton & Ockenfels, 2000; Dufwenberg & Kirchsteiger, 2004; Fradkin et al., 2020; Halliday & Lafky, 2019; Rabin, 1993). These papers suggest that firms may choose to lower prices or offer rebates, improving the value-for-money that consumers receive. Providing a sufficiently high value-for-money triggers an intrinsic reciprocity from consumers, who are then inclined to react in a positive manner, leaving a good rating. We model reciprocity based on Rabin (1993), which allows us to capture that ratings are a function of both observed quality and price. Thus we endogenize both the pricing decision of firms and the ratings decision of consumers.

Formally, we study a two period model with information asymmetry. Two actors participate in each period — a long lived firm and a short lived consumer. Firms are endowed with a product of high or low quality. Firms know the quality of their product. In each period, the firm sets a price to maximises lifetime profits. Initially, consumers only know the distribution of quality. Each consumer only participates in a single period. At the start of each period, consumers observe the historical ratings the firm received and the price set for the current period. Consumers only learn of the true quality of the product after consumption. After which, consumers may choose to leave a rating. This model features consumers who are uncertain about quality at the beginning of each period; but ratings may transmit some information about quality to future consumers.

To give ratings the best shot at being informative about quality, and in line with evidence that high-quality firms are more likely to get good ratings (Ananthakrishnan et al., 2019; Li et al., 2020), we focus on equilibria where high-quality firms set sufficiently low prices to obtain a good rating. Low-quality firms, however, face the following trade-off. First, in period 1 they can charge a very low price to trigger reciprocity. By doing so, the low-quality firm receives a good rating and earns larger profit in period 2, which is why we call this strategy 'ratings harvesting'. If low-quality firms harvest ratings, both firms get a good rating and future consumers cannot use ratings to distinguish firms. Second, in period 1 the low-quality firm could charge the same price as the high-quality firm and benefit from the fact that, ex-ante, consumers think this seller could be of a high quality. This strategy, however, leaves consumers with little value-for-money, which is why the low-quality firm will not get a good rating. Now only high-quality firms receive a good rating. This allows future consumers to distinguish between firms, and the low-quality firm can only set a low price in period 2. Because both firms charge the same price in period 1, we call this strategy 'price mimicking'.

In equilibrium, low-quality firms play a mixed strategy, and do both price mimicking and ratings harvesting with some positive probability. Intuitively, low-quality firms harvest ratings to get a good rating and free-ride on the reputation of high-quality sellers. This, however, undermines the value of a good rating until, in equilibrium, low-quality sellers are indifferent between ratings harvesting and price mimicking. This mixed strategy equilibrium has some very convenient features for our analysis. First, because ratings signal product quality only if low-quality firms mimic prices so that sellers get different ratings, the probability that low-quality firms mimic prices in equilibrium also measures how well ratings signal quality. Second, the mixed strategy captures the dual role of ratings and price. We leverage this trade-off between ratings harvesting and price mimicking to understand how firms respond in different ratings environments.

This model is representative of markets such as Amazon and Taobao, where ratings play the important role of developing trust between anonymous users. In these markets, new consumers are mostly unaware of the true quality of the sellers they are transacting with. They rely on information left by prior consumers—through ratings—to form beliefs over the quality of a product. Further, these websites only list historical ratings and current prices.[1] In these markets, firms build reputation in early periods by obtaining good ratings. Subsequently, they increase prices and harvest ratings (Cabral & Hortaçsu, 2010; Cabral & Li, 2015; Li et al., 2020).

Our first result shows that if consumers have a stronger sense of reciprocity, ratings become less informative and leading to ratings inflation.[2] When low-quality sellers harvest ratings, they reduce prices to trigger the reciprocal behavior of consumers, manipulating them into leaving good ratings. However, when consumers have a stronger sense of reciprocity, firms need only leave a smaller surplus to trigger consumer's reciprocal behavior. This means that firms can set a higher price and still receive a good rating, which means that the opportunity cost of a good rating is lower. Therefore, low-quality firms harvest more ratings and receive a good rating more often. An increase in ratings harvesting, however, leads to less informative ratings because low-quality firms get more good ratings, undermining, but not eliminating the ability of ratings to signal quality.

Even though ratings harvesting degrades how well ratings signal quality, our next result highlights how consumers benefit from ratings harvesting. Because firms need to set a sufficiently low price in order to trigger reciprocity, low-quality firms who harvest ratings provide some surplus to consumers, which we call the reciprocity rent. This rent increases consumer surplus. But since consumers only receive this surplus when firms harvest ratings, they benefit despite less-informative ratings. This has two key implications: (i) Consumers do not prefer fully informative ratings. If firms do not harvest ratings, ratings are perfectly informative, but consumers do not receive any reciprocity rent. (ii) Consumers receive material benefit prior to the decision to rate, and benefits arising from the informational content of ratings are an externality. These implications run counter to the conventional wisdom that consumers benefit from informative ratings.

Having addressed the question "are value-for-money based ratings informative of product

[1] While some services may exist to attempt to track historical prices, their validity cannot be ascertained and these services are unable to reflect the transaction price associated with each rating.

[2] Rating inflation here refers to the observation that ratings scores are improving over time, and most of the improvements cannot be attributed to product quality. Thus, leading to ratings becoming a less effective signal of quality. This observation is made in Filippas et al. (2022), suggesting that other attributes such as the cost of leaving ratings, kindness to seller and other forms of retaliation contributes to rating inflation.

quality?", we apply this mechanism to understand how different features of a marketplace could influence ratings. Specifically, we look at how the design of the ratings system and quality controls affect the informativeness of ratings and consumer surplus.

To study the design of a ratings system, we consider how a platform may choose to facilitate ratings. An example of this is Amazon's decision to transit to a one-click rating system, arguing that more ratings would "more accurately [...] reflect the experience of all purchasers".[3,4] This intuition follows directly from the law of large numbers. However, we show that this logic can be misleading as it ignores how sellers may adjust prices when it becomes less costly to rate. Making it easier to rate means that firms need only transfer a smaller surplus to consumers in order to invoke their reciprocal behavior. This reduces the opportunity cost of ratings harvesting, and leads to less informative ratings.

Easier ratings has a different impact on seller and consumer surplus. High-quality sellers unambiguously prefer a larger effort to rate, because this leads to more informative ratings and prevents other firms from free-riding on their reputation. However, on average, sellers prefer ratings that are easier and less informative, because less informative ratings goes hand-in-hand with lower reciprocity rent. Consumers benefit the most from these rents at an intermediate effort level for rating. This suggests that they prefer somewhat, but not fully informative ratings. Thus, consumers prefer more informative ratings than the average seller, but less informative ratings than high-quality sellers.

These results suggest that a platform can shift surplus between sellers and buyers by choosing to facilitate ratings, or making them more difficult to achieve. Using this insight, we show that a two-sided platform may facilitate ratings to encourage more, but less informative ratings.

Marketplaces may not just facilitate ratings, but also weed out low-quality sellers by restricting their access to the market. For example, Amazon suspends sellers who do not meet a minimum standard.[5] We show that improving the aggregate quality in the market can result in less ratings harvesting, leading to more informative ratings and less rating inflation. Thus, screening seller quality can complement the design of ratings systems and lead to an informative ratings environment. But we show that when aggregate quality is low, quality screening can also undermine the informativeness of the rating system and foster rating inflation instead.

[3] Previously, Amazon had required raters to leave a 100 word review along with their rating.

[4] Rey (2020), retrieved April 2021.

[5] "Info" (2018), retrieved April 2021; "Amazon to Boost Quality Control After Selling Toxic Kid Products" (2019), retrieved April 2021.

Finally, we study a range of extensions and robustness checks. We show that competition can further reinforce our previous results and lead to more ratings inflation. Indeed, evidence that platforms facilitate ratings over time suggest they engage in a race towards uninformative ratings. Additionally, our results are robust when consumers can leave negative ratings. In doing so, we discuss how ratings are extreme in equilibrium, that is ratings take the most positive and most negative ends of the ratings scale in equilibrium. Hence, we can also explain the existence of extreme ratings in practice as an outcome of reciprocity-based ratings. We also show that our results persist when we introduce a continuum of firms, and when we extend our model beyond 2 periods.

2 Related Literature

Our key results connect **evidence on ratings.** Ratings aid consumers by reducing uncertainty about the quality of the product. Much of the literature suggests that ratings do indeed signal quality on major platforms like eBay (Hui et al., 2018), Taobao (Zhang et al., 2012), and Airbnb (Proserpio et al., 2018). The broader empirical literature, however, suggests that other factors also influence ratings (Gao et al., 2018; Masterov et al., 2015; Nosko & Tadelis, 2015; Zervas et al., 2021).

Many studies have shown that prices have a significant influence on ratings. The evidence highlights two key patterns. First, for a given quality, lower prices induce higher ratings (Carnehl et al., 2021; Li & Hitt, 2010; Luca & Reshef, 2021; Neumann et al., 2018).[6] Second, firms with larger ratings charge larger prices in the future, a pattern frequently referred to as 'rating harvesting' (Cabral & Hortaçsu, 2010; Cabral & Li, 2015; Cai et al., 2014; Carnehl et al., 2021; Ert & Fleischer, 2019; Gutt & Herrmann, 2015; Jin & Kato, 2006; Jolivet et al., 2016; Lewis & Zervas, 2019; Li et al., 2020; Livingston, 2005; Luca & Reshef, 2021; McDonald & Slawson, 2002; Neumann et al., 2018; Proserpio et al., 2018). We can explain both of these patterns in a single framework, i.e. low-quality firms charge lower prices to induce consumers to reciprocate with a good rating. Firms harvest these good ratings by charging larger prices in the future.

We also contribute to the **literature on reciprocity in ratings**. A series of empirical articles argue that reciprocity is a key driver of rating behavior (Cabral & Li, 2015; Diekmann et al., 2014; Fradkin et al., 2020; Li & Xiao, 2014). Some experimental work directly identifies that reciprocity drives rating behavior (Bolton et al., 2013; Halliday & Lafky, 2019; Lafky,

[6] Chiles (2021) also finds this pattern for shrouded hotel fees, i.e. that surprise charges of hotels lead to lower ratings.

2014). Taken together, these experiments suggest: (i) reciprocity biases ratings upward in mutual-rating systems where buyers and sellers rate each other. But double-blind feedback strongly reduces this bias. Also following the work of Dellarocas and Wood (2008) and others on eBay, most online marketplaces adopted a double blind approach to feedback, which is why we do not look at mutual-rating systems. (ii) Also with one-sided or double-blind rating systems, sellers take advantage of their ability to influence ratings, and they use prices to do so. We contribute to this literature by modelling how sellers use prices to trigger reciprocity and thereby influence ratings. By doing so, we derive novel predictions on how reciprocity causes rating inflation, and how encouraging consumers to rate can backfire by making ratings less informative.

We also connect to the ongoing **debate on rating inflation**. Rating inflation describes the observation that ratings scores improve over time, and most of the improvements cannot be attributed to product quality (Filippas et al., 2022). They argue that raters evaluate ratings based on reference points, which causes rating inflation: for example, if consumers consider only above-average ratings as good ratings, and consumers are somewhat more prone to leave good than bad ratings, the average rating increases which raises the reference point. The larger reference point requires some consumers to increase their rating so that they remain above average and consumers still perceive them as good, which drives rating inflation. However, this does not seem to explain ratings inflation in binary ratings. By construction, only one rating is above average, so that a larger average rating cannot induce consumers to further raise their rating. Thus, reference-dependent ratings cannot easily explain rating inflation in binary ratings, while our reciprocity-based mechanism can. Indeed, Nosko and Tadelis, 2015 provide suggestive evidence for rating inflation in binary ratings.[7] We propose multiple channels through which reciprocity leads to rating inflation. For example, platforms increasingly encourage consumers to rate; we show that with reciprocating consumers, this encourages especially low-quality firms to harvest ratings, leading to rating inflation.[8]

We closely connect to the **theoretical literature on reputation** (e.g. Cabral (2000), Jullien and Park (2014), Stenzel et al. (2020), and Tadelis (1999); see also Bar-Isaac and Tadelis (2008) for a survey). In existing work on reputation usually (i) buyers do not endogenously choose if and how to rate, and (ii) ratings mostly reflect quality and are independent of price. While some papers relax some of these assumptions (e.g. in Stenzel

[7]In particular, Nosko and Tadelis, 2015 study the binary rating environment on eBay and find that more than 99% of ratings are positive.

[8]Research on information systems propose that social norm of good ratings increases over time (Qiu et al., 2012). Our mechanism can illustrate how social norms changed the way they did: platforms encourage consumers to rate (lowering e and/or raising Δ), which leads to a new equilibrium where reciprocity plays a more important role and ratings increase.

et al. (2020), ratings may depend on past prices, relaxing (ii)[9]), no article seems to feature that buyers choose if and how to rate strategically, and that prices influence ratings. We capture both of these features, which, in contrast to previous work, allows us to capture empirical patterns of prices and ratings.

We also connect to recent work by Rhodes and Wilson (2018) on **false advertising**. Low-quality firms falsely advertise high quality to free-ride on the reputation of high-quality firms. Also in our setting, low-quality firms free-ride on the reputation of others and undermine information transmission. But our mechanism is very different. In contrast to ads, firms in our setting do not choose their own ratings, but need to charge a low price to trigger reciprocity and get a good rating. This leads to novel and inherently dynamic effects like rating harvesting.

Overall, we use the mechanism of intrinsic reciprocity to develop a model of endogenous value-for-money ratings, and discuss the implications of such reciprocity-driven ratings. We explore a qualitatively novel trade-off between rating harvesting and price mimicking. This trade-off induces price dynamics like rating harvesting and price mimicking, which connect well to main findings in the empirical literature on ratings. Our key mechanism also drives new implications for the informativeness of ratings, payoffs and the design of rating systems.

3 Basic Model

We set up a 2 period model of incomplete information with a long lived monopoly seller and a unit mass of buyers in each period.

Firms

The seller can be of a high or low quality. A seller of type $j \in \{L, H\}$ has quality q^j, where $q^L < q^H$. The probability that the seller is of type q^H or q^L is common knowledge and given by $\gamma \in (0,1)$ and $(1-\gamma)$, respectively. The realized quality is private information to the seller and is constant between periods. In each period, the seller sets the price of its product to maximise its lifetime profit, $\sum_{t=1}^{T} p_t^j$, where p_t^j is the price of seller j in period t. We assume that the cost of production is zero regardless of quality. After selling in period 1, sellers may receive a rating R_t from consumers. If they do, this rating is made common

[9]Additionally, they focus on prices in long-run equilibria. Because we do not focus on long-run equilibria, we derive novel dynamics effects like rating harvesting in our framework.

knowledge to both the firm and consumers in subsequent periods.[10]

Consumers

Consumers participate in only one of the two periods, with a new unit mass of consumers arriving in each period. We normalize the value of their outside option to zero. When choosing to consume a product in period t, consumers observe the price on offer and past ratings, R_{t-1}. They do not observe the firm's quality or past prices. Consumers may choose to buy or not to buy. If they choose not to buy, they exit the market. If they consume, they observe the firm's true quality and decide on leaving a rating. For simplicity, in the main text, we focus on a simple rating system where consumers can choose between leaving a rating or not. More precisely, ratings, R_t, take the form $R_t \in \{0, 1\}$. The informational content of a rating will be determined in equilibrium, but we say that a rating is good if $R_t = 1$, and when $R_t = 0$, consumers choose not to provide any rating. We show below that our main results are qualitatively robust when consumers can leave bad ratings.[11]

To model reciprocity, we build on classic models of intrinsic reciprocity proposed by Rabin (1993) and Dufwenberg and Kirchsteiger (2004). By introducing reciprocity in our model, consumers are able to reward 'good' deals with a good rating. In equilibrium, good ratings reward sellers and increase their future profits. Given the true quality and price, the consumer decides if a seller made a sufficiently kind offer, measured in terms of value-for-money. If the consumer received a sufficiently large value-for-money, then it is able (and willing) to reciprocate the firm's kindness with an act of kindness of its own in the form of a good rating.

We distinguish between consumption utility and rating utility. This serves two purposes. First, we are able to capture the phenomenon that consumers do not factor the intention to rate into their purchase decision.[12] Second, this simplifies presentation of results. The consumption utility for consuming a product from seller j in period $t \in \{1, 2\}$ is given by $u_t = q^j - p_t^j$, where p_t^j represents the price that the firm sets in period t.

[10] We only require that low-quality firms face a smaller marginal cost than high-quality firms. However, without loss of substance, it simplifies the algebra to assume that both firms face zero cost of production. This is similar to allowing for costless signals in a cheaptalk game (see Kreps and Sobel (1994) and Crawford and Sobel (1982)). Additionally, our model features a concept of endogenous cost such as that in Martin and Shelegia (2019). In our setting, by harvesting ratings, low-quality firms reduce the benefit of ratings harvesting.

[11] We start with an environment where negative ratings do not exist to simplify exposition of results. We discuss how our results extend to a setting with negative ratings that punish bad deals in Section 7.

[12] This is highlighted by Cabral and Li (2015), who show that incentivizing consumers to rate does not change the number of bids or bid levels on eBay, indicating that consumers do not consider the rating incentives when making a purchase.

The rating utility for consumers in period t is given by $v_t = [\kappa q^j - p_t^j]\Delta - e$ if $R_t = 1$ and $v_t = 0$ if $R_t = 0$. $\kappa \in [0,1]$ represents the proportion of surplus which consumers think is equitable for firms to receive; $\Delta > 0$ represents the perceived kindness of a consumer leaving a rating to the firm; and $e \geq 0$ reflects the opportunity cost of providing a rating. This expression is based on the intrinsic reciprocity models in Dufwenberg and Kirchsteiger (2004) and Rabin (1993).[13]

The first term $[\kappa q^j - p_t^j]$ captures the consumer's perception of the firm's kindness. Consumers perceive a price equal κq^j as fair. Thus, they perceive any price below κq^j as a kindness, and $[\kappa q^j - p_t^j]$ is positive. Otherwise, if $[\kappa q^j - p_t^j]$ is weakly negative, firms keep more surplus than what consumers deem as equitable, and consumers perceive firms as unkind.

The second term Δ describes the consumers' sense of reciprocity and captures the warm glow that consumers receive from being kind to a firm by leaving a good rating. Finally, the cost of leaving a rating, e, reflects the value of a consumer's time, and also depends on the way platforms design their ratings systems, for example, one-click ratings, constant reminders, and purchase verification.

Timing of game

To summarize the timing of the game,

$t = 1$

Firm draws quality, q^j.

Firm sets price, p_1.

Consumers observe price and form expectations over consumption utility, u_1.

Consumers make purchase decision, if $u_1 \geq 0$.

Consumers that purchase, observe true quality.

Consumers evaluate rating utility, and rate if $v_1 \geq 0$.

Period 1 consumers exit the market.

$t = 2$

Period 2 consumers arrive.

Firm observes historical rating, R_1, and sets new price, p_2.

Consumers observe past ratings and current price, forming expected quality.

Consumers make purchase decision.

[13]Derivation can be found in Appendix B.

In the main body of the paper, we make some simplifying assumptions. Maybe most importantly, we only consider a 2-period game, and we consider that consumers are only able to leave positive ratings or no ratings at all. We show in Section 8), however, that results are qualitatively robust when we relax these assumptions and allow for longer time horizons and negative ratings, i.e. retaliation.[14]

Our results are also robust to N-periods (Section 8), firms with a continuum of quality types (Section 8), and different ratings scales (8).

This model is representative of markets such as Amazon, Taobao, eBay, AirBnB[15] and Google reviews. These platforms help to facilitate matches between consumers and firms. Consumers rely on ratings to form or update their expectations of product quality. Firms use ratings to differentiate themselves from lower quality firms, allowing them to build trust and gain patronage.

4 Equilibrium

We look for a perfect Bayesian equilibrium. We apply two additional restrictions as equilibrium selection assumptions.[16] With the first restriction we focus on equilibria where product quality is not diminishing in ratings.

Restriction 1. *We focus on the equilibria where high-quality firms obtain a rating of $R_t = 1$ with probability 1.*

This restriction ensures that high-quality firms receive a good rating. It reflects evidence that high-quality firms are more likely to receive good ratings (Ananthakrishnan et al., 2019; Li et al., 2020). Thus, the restriction allows us to focus on the economically more relevant case where high-quality firms are at least as likely as others to receive good ratings.

[14]Lewis and Zervas (2019) show that only the relative difference in stars affects the pricing decision of firms. This suggests that it is more important to consider the effect of a relative difference in ratings, which we already do with our simple binary-rating framework. Other papers find that negative ratings have a statistically insignificant impact on prices.(Bajari & Hortaçsu, 2003; Cabral & Hortaçsu, 2010; Livingston, 2005; Resnick et al., 2006) Hence, we believe that this simplification is well justified. Although, as we show in our extension, allowing for more flexibility in the ratings scale does not qualitatively change our result.

[15]Although we do not explicitly capture the two-sided nature of AirBnB's rating system, ratings on AirBnB are only revealed after both host and guest have provided a rating. This removes any threat of retaliation in response to a negative rating. Therefore, AirBnB, and other similar two-sided markets, has a rating system that resembles a one-sided market.

[16]These restrictions are similar in spirit to those used in Rhodes and Wilson (2018), who apply similar restrictions to study how advertisement can signal quality. We, however, apply them to study how ratings signal quality.

Restriction 1 focuses the analysis on the strategic decisions of low-quality firms.[17]

We make the second selection assumption to limit off-the-path beliefs.

Restriction 2. *For all prices such that low-quality firms obtain no rating, the expected quality is independent of prices.*[18]

Restriction 2 implies that ratings, rather than prices, serve as a signal for quality. The restriction captures that firms can mimic each others' prices, making prices a poor signal for quality. More precisely, because all firms have zero marginal cost, low-quality firms can always deviate to any price set by a high-quality firm. This makes it difficult for high-quality firms to use price signals to differentiate themselves from low-quality firms. By applying Restriction 2, we focus on equilibria where firms do not use prices to differentiate themselves, but ratings.

Together, these restrictions give ratings the best shot at being an informative signal for product quality. Restriction 1 implies high-type firms always receive a good rating, and Restriction 2 ensures that the information signal that future consumers receive comes from ratings alone.

The following proposition characterizes equilibria in this game.

Proposition 1. *All perfect Bayesian equilibria satisfy the following.*

1. *In period 1, high-quality firms charge* $\overline{p} \equiv \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)}$ *with probability one and receive a good rating.*

2. *In period 1, low-quality firms randomize their price.*

   a. *They charge* $\overline{p}$ *with probability* δ^* *and obtain no rating.*

   b. *They charge* $\underline{p} \equiv \kappa q^L - \frac{e}{\Delta}$ $(< \overline{p})$ *with probability* $1 - \delta^*$ *and obtain a good rating.*

3.

$$\delta^* = 1 \textit{ if and only if } (1-\gamma)(q^H - q^L) - (1-\kappa) q^L \leq \frac{e}{\Delta}, \textit{ and}$$
$$\delta^* \in (\frac{1}{2}, 1) \textit{ when } (1-\gamma)(q^H - q^L) - (1-\kappa) q^L > \frac{e}{\Delta} \quad (1)$$

[17] In particular, their decision to set a high price and fail to obtain a rating; or to set a sufficiently low price and obtain a good rating. Formally, equilibria where high-quality firms prefer to get a rating with probability one exist if κ is sufficiently large. Intuitively, when κ is large, the high-quality firm can charge a high price and still receive a good rating. This reduces their incentive to deviate to prices even closer to or at q^H, at which they receive no rating.

[18] Our results survive the D1 refinement, and we choose to use Restriction 2 instead, as this is a weaker and more intuitive than D1 in the applied setting.

4. In period 2, prices equal expected quality conditional on ratings.

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \textit{if } R_1 = 1 \\ q^L & \textit{if } R_1 = 0 \end{cases}.$$

The equilibrium is unique up to off-path beliefs and exists if $\kappa q^H - \frac{e}{\Delta} \geq \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$.[19]

Firms charge one of two prices in equilibrium. High-quality firms always set $\overline{p}$ to extract all expected surplus (conditional on observing $\overline{p}$).[20] If ratings would perfectly signal quality, this price would be equal to q^H. But low quality firms might get a good rating and free-ride on the reputation of high-quality firms. This, in equilibrium, can lower the conditionally expected consumer surplus below q^H. The low price $\underline{p}$ is just low enough so that consumers perceive low-quality firms as kind and give a good rating, i.e. $\underline{p}$ is such that $[\kappa q^L - \underline{p}]\Delta - e = 0$.

How do low-quality firms choose between these two prices? Low-quality firms benefit from ratings by earning higher prices in period 2 (point 4). But to get a rating, they have to induce positive reciprocity of consumers by charging a lower price $\underline{p}$ ($< \overline{p}$) in period 1, which is why we call this strategy 'ratings harvesting'. Alternatively, in period 1 low-quality firms can mimic the large price of high-quality firms $\overline{p}$, and get no ratings and lower profits in period 2. Because firms who follow this strategy copy the price of high-quality firms, we call it 'price mimicking'. Thus, low-quality firms trade-off ratings harvesting and price mimicking. The probability that low-quality firms mimic prices and do not receive a rating, δ^*, captures how firms resolve this trade-off in equilibrium. This δ^* also captures the informativeness of ratings: as δ^* increases, low-quality firms obtain a rating less often, and the ratings better help consumers distinguish between high-and low-quality firms.

Why may low-quality sellers set $\delta^* \in (\frac{1}{2}, 1)$? Intuitively, low-quality firms harvest ratings to free-ride in the reputation of high-quality sellers. This, however, undermines the expected quality associated with a good rating until—in equilibrium—low-quality sellers are indifferent between ratings harvesting and price mimicking.[21]

We now explain the impact of reciprocity on low-quality sellers' pricing and ratings. To begin, suppose that Δ is sufficiently small such that $\delta^* = 1$ in equilibrium. For a given quality of ratings δ^*, the discount low-quality firms need to offer to get a good rating $\overline{p} - \underline{p}$ decreases as Δ increases. Intuitively, when Δ is small, the rating utility $v_1 = [\kappa q^L - p_1^L]\Delta - e$

[19] Formal proofs can be found in Appendix A.

[20] Formally, Restriction 1 implies that this price extracts all conditional expected surplus. Without this restriction, there can be equilibria where the high price is lower.

[21] We can easily see why δ^* is bound below by $\frac{1}{2}$. When $\delta^* = \frac{1}{2}$, $\overline{p} = E[q_2|R_1 = 1] = \frac{\gamma q^H + \frac{1}{2}(1-\gamma)q^L}{\gamma + \frac{1}{2}(1-\gamma)}$. Therefore, low-quality firms receive a total profit of $\underline{p} + \overline{p}$ when obtaining a good rating, and $\overline{p} + q^L$ when obtaining no rating. Since $\underline{p} < q^L$, it is profitable to participate in less ratings harvesting, increasing δ^*.

is quite 'small' and consumers have a low level of intrinsic motivation to leave a good rating. To encourage consumers to rate and compensate them for the cost of ratings e, low-quality sellers need to offer a lower price to provide a larger value-for-money for consumers. When Δ is sufficiently small, low-quality firms find it too costly to obtain a rating, so they charge a large price $\overline{p}$ with probability $\delta^* = 1$. Because only high-quality firms obtain a rating, ratings perfectly signal quality.

Now suppose Δ increases. As Δ increases, consumers are more inclined to reciprocate a given value-for-money with a rating, which is why the price for which low-quality sellers can obtain a good rating $\underline{p}$ increases. This lowers the opportunity cost of obtaining a good rating. Low-quality firms set the low price of $\underline{p}$ more often, and obtain a good rating in equilibrium with positive probability.

These arguments explain why reciprocity reduces the quality of ratings: low-quality firms can offer a high value-for-money to trigger consumers' reciprocity and obtain good rating. Because low-quality firms start to harvest ratings, both high- and low- quality firms receive good ratings, which is why the signal that consumers receive from ratings is less informative of product quality. Thus, low-quality firms harvest ratings to trigger the consumers' kindness, but thereby undermine the quality of ratings. The following proposition summarizes this result.

Proposition 2. $\frac{\partial \delta^*}{\partial \Delta} < 0$ *when* (1) *holds. Hence, when low-quality firms play a mixed strategy, and consumers exhibit a larger sense of reciprocity, more ratings harvesting occurs.*

Two clear implications arise form this proposition. First, firms use prices to induce consumers to be kind. Second, when consumers are more reciprocal, ratings become less informative.

Growing evidence suggests that ratings are influenced not just by quality, but also by prices. More precisely, consumers seem to rate based on the value-for-money from a purchase. For example, studying marketplaces for digital cameras, Li and Hitt (2010) highlight that a 1% increase in price reduces ratings by 0.36 stars in 5-star ratings and 0.71 stars for 10-star ratings. On AirBnB, Gutt and Kundisch (2016) and Neumann et al. (2018) show that prices negatively impact ratings.[22] On Yelp, Luca and Reshef (2021) provides evidence that a 1% increase in prices leads to a 3-5% decrease in average rating. Abrate et al. (2021) suggests that a 1% increase in hotel prices leads to a decrease of 1 star (out of 10) in overall ratings. Because these articles control for product characteristics, they suggest that it is value-for-

[22]Interestingly, AirBnB asks guests to separately rate the overall transaction and its value, but both are negatively affected by prices.

money that influences ratings, and not quality alone. Our mechanism explains these findings through reciprocity: consumers perceive a larger value-for-money as a kindness, which they reciprocate with positive ratings. This way, our mechanism explains how both quality and prices affect ratings.

Our results also suggest that firms use lower prices strategically to influence their ratings. This connects our results to evidence on rating inflation. Rating inflation is well documented in the literature (Filippas et al., 2022), but not well understood. We offer novel explanations for this phenomenon based on the idea that low-quality firms disproportionately benefit from exploiting consumer reciprocity to boost their ratings: First, low-quality firms disproportionately use lower prices to trigger consumer reciprocity and good ratings. Second, we find that only the profits of low-quality firms—i.e. the firms that exploit reciprocity in equilibrium—increase in consumer kindness Δ. This suggests that such firms can also adopt non-price strategies to encourage consumers reciprocal behaviour. Firms may increase consumers reciprocal behaviour by actively asking consumers to leave a rating, engaging with and sharing customer reviews. These practices are commonplace, and often suggested by the review platforms themselves.[23]

We have shown how reciprocity can impact the informativeness of ratings. First, we provide a model that highlights how reciprocity based ratings can capture both quality and price, leading to value-for-money ratings. Second, we are able to establish the link between value-for-money ratings and ratings inflation, showing that when firms focus on obtaining ratings, this lowers the ability of ratings to signal quality. We now explore how the design of ratings systems can impact the informativeness of ratings, and ask how beneficial ratings are for consumers.

We restrict the remainder of our analysis to situations where the low-quality firm plays a mixed-strategy.[24] Doing so allows us to study directly how changes in the rating environment impact the informativeness of ratings, captured by δ^*.

5 Designing ratings environments

In this section, we discuss how common features of a platform, such as the cost to consumers for leaving ratings and the overall product quality on a marketplace, influence the

[23] Grimes (2012), "Get Reviews on Google - Google My Business Help" (2021), "Ratings, Reviews, and Responses - App Store" (2021), retrieved April 2021. "The Secret To Getting Five-Star Reviews" (2021), retrieved June 2021.

[24] $\kappa q^L - \frac{e}{\Delta} + q^H > \gamma q^H + (1-\gamma)q^L + q^L$, is required for the mixed strategy to exist.

informativeness of ratings.

Facilitating ratings

The design of ratings systems can make it easier, or more difficult, to rate. For example, raters may have to complete a verification process, asked to rate along multiple dimensions, or receive monetary rebates and reminders to rate. These design features influence the time and cognitive effort it takes to evaluate a product, and therefore the cost of leaving a rating. Intuitively, the law of large number would suggest that collecting more ratings would lead to more-informative ratings. From this perspective, reducing the cost of leaving a rating seems like a good idea. We show that this intuition is misleading as it ignores how firms and consumers respond when ratings become easier.

In our setting, less-costly ratings lead to less informative ratings. The reason is closely related to reciprocity: as it becomes easier for consumers to rate (i.e. e decreases), firms need to leave less surplus to induce consumers to reciprocate with a good rating.[25] In equilibrium, this encourages especially low-quality firms to harvest ratings: they set $\underline{p}$ with a higher probability $(1-\delta^*)$. Consumers who observe a good rating, know that the product is now less likely to be of high quality, which worsens the informativeness of ratings.

Corollary 1. $\frac{\partial \delta^*}{\partial e} > 0$ *when* (1) *holds. Hence, when low-quality firms play a mixed strategy, and the cost of providing a rating increases, firms harvest less ratings.*

The result suggests that making ratings less costly for consumers, even though it induces more ratings in equilibrium, encourages low-quality firms to harvest ratings and thereby undermines the informativeness of ratings. This provides another channel through which reciprocity-based ratings induce rating inflation: easier ratings encourage especially low-quality firms to harvest ratings. In turn, this means that making ratings costly can make them more informative.

Our finding closely mirrors evidence. In line with our result that lower costs of ratings encourage especially low-quality firms to harvest ratings, Cabral and Li (2015) find that higher rebates for ratings decrease the proportion of negative ratings among low-quality products. Since rebates offset the cost of rating, their result highlights — as in our model — that reducing the cost of rating could worsen the informativeness of ratings. Lafky (2014) shows that when it becomes more costly to rate, ratings become more extreme. This follows our prediction that it is easier to differentiate between products of relative quality when ratings are costly.

[25] Formally, $\underline{p}$ increases and the difference between $\overline{p}$ and $\underline{p}$ shrinks.

Anecdotal evidence suggests that the cost to leave a rating is decreasing on many ratings platforms. On google reviews, users are provided with constant reminders to leave ratings and are allowed to leave one-click ratings on their smartphone. This trend of fuss-free ratings is also gaining traction on Amazon. Prior to 2020, Amazon required customers to write a review in order to leave a rating, subsequently removing this requirement and allowing for one-click ratings.[26] Publicly, they stated that more feedback would lead to more accurate ratings, relying on the law of large numbers to drown out fake reviews.[27] Our results suggest that Amazon's effort to encourage ratings in hopes of raising the informativeness of ratings may backfire. Although more 'real' customers may be rating, which could solve the problem of fake ratings on Amazon, this does not consider the strategic effects from firms. By reducing the cost to leave a rating, this makes it easier for firms to trigger reciprocity and obtain a good rating, ultimately still leading to less informative ratings.

Screening quality

We have shown that (i) a stronger sense of reciprocity (Δ), and (ii) lower cost to leave a rating (e) encourage low-quality firms to harvest ratings, triggering rating inflation. In both cases, rating inflation is the result of less informative ratings. In practice, however, rating inflation could happen simply because the average product quality improves. If this was the case, rating inflation might not be problematic at all. We now shed light on this scenario and explore how changes in the aggregate quality of sellers in the market affect the informativeness of ratings.

The aggregate quality on a marketplace may change for a variety of reasons. First, low-quality firms may invest in better quality (Klein et al., 2016) or leave the market (Cabral & Hortaçsu, 2010; Nosko & Tadelis, 2015). Second, platforms may pre-screen and weed out low-quality sellers to control the quality of firms (Casner, 2020; Nosko & Tadelis, 2015; Wang, 2021). Both channels can improve the product quality on the market and ultimately lead to better average ratings. If sellers have larger average quality, however, the incentives of low-quality sellers to harvest ratings also changes. This way, larger seller quality may render ratings more or less informative.

We show that when the aggregate quality improves (i.e. γ increases), the remaining low-quality firms may harvest more or less ratings, depending on the aggregate level of quality in the market.

The starting point to understand this result is that low-quality firms only harvest ratings,

[26] Masters (2021), retrieved April 2021.
[27] "Amazon Tests a One-Tap Review System for Product Feedback" (2019), retrieved April 2021.

when ratings are useful to distinguish seller quality. Intuitively, low-quality firms can only free-ride on the reputation of high-quality firms, when high-quality sellers indeed have a reputation because ratings are somewhat useful to distinguish sellers.

Let us first consider the case where the aggregate quality in the market is low. For simplicity, suppose γ is close to zero. Almost all firms are of low quality, so ratings are somewhat useless to help consumers distinguish sellers. But as γ increases, ratings become more useful, which encourages low-quality firms to harvest more ratings. Thus, for 'small' γ, an increase in the aggregate quality leads to less-informative ratings.

Now consider the opposite scenario where the aggregate quality is high. For simplicity, suppose γ is close to 1. Again, ratings are not useful to distinguish sellers. But as γ increases, they become even less useful to distinguish sellers so that low-quality firms harvest ratings less. This is why for 'large' γ, an increase in the aggregate quality leads to more informative ratings.

The following proposition summarizes this result.

Proposition 3. $\frac{\partial \delta^*}{\partial \gamma} \leq 0 \iff \gamma \leq \bar{\gamma}$, *and* $\frac{\partial \delta^*}{\partial \gamma} = 0$ *only if* $\gamma = \bar{\gamma}$.

The proposition works out how changes in aggregate quality interact with the informativeness of ratings. When aggregate quality is low, quality increases make ratings less informative. But when aggregate quality is high, improvements in quality have a double dividend: aggregate quality increases and ratings become more informative.

Our results help understand when quality-control measures improve the informativeness of ratings. For example, Amazon actively enforces seller quality, suspending sellers who do not meet a minimum standard. [28] Also Uber has announced that it will remove both riders and drivers with consistently poor ratings,[29] and their subsidiary Uber Eats suspends delivery drivers who fall below a minimum rating.[30] Booking.com suspends properties for quality control purposes;[31] Airbnb bans hosts based on a combination of factors, including being in the bottom 1% of overall ratings and guest feedback.[32]

Our results help shed light on these practices. First, even though good ratings may not be

[28] Amazon makes this decision through a combination of customer reviews, feedback and other measures."Info" (2018), retrieved April 2021; "Amazon to Boost Quality Control After Selling Toxic Kid Products" (2019), retrieved April 2021.

[29] "Uber Will Start Deactivating Riders with Low Ratings — TechCrunch" (2019), retrieved April 2021.

[30] "UberEat's Deactivation Policy for Delivery Drivers/Couriers" (2017), retrieved April 2021.

[31] "Closing for Quality Control — Booking.Com for Partners" (2020), retrieved April 2021.

[32] "Why was my listing paused or suspended?" (n.d.), retrieved April 2021, "What to Do If Your Airbnb Listing Is Suspended — Zodiak Airbnb Management" (2021), retrieved April 2021.

fully informative about underlying quality, no ratings or—as we show in an extension—bad ratings are informative about low quality. Thus, even in scenarios when good ratings are not informative, platforms can indeed use bad or no ratings to weed out low-quality sellers. Second, when screening quality makes ratings less informative, our previous results suggest that platforms can counter such adverse affects by adjusting the effort to rate accordingly. Second, our results are in line with the view that improvements in aggregate quality may well cause rating inflation; but even such quality-driven rating inflation may go hand in hand with less informative ratings.

6 Surplus Analysis

So far, we studied how reciprocity affects the informativeness of ratings. Conventional wisdom suggests that more-informative ratings help consumers make more-informed purchase decisions and ultimately benefit consumers. We now explore the link between the informativeness of ratings and consumer surplus more carefully. The key insight is that consumer-optimal rating systems are often somewhat, but never fully, informative.

To start, we investigate how ratings harvesting affects consumer surplus. In order to induce consumers to reciprocate, firms need to set a price below the consumers' ex-post willingness to pay. This is why sellers, even though they are monopolists, leave a rent to consumers. To see this rent, we write down the expected consumer surplus

$$CS = (1-\gamma)(1-\delta^*)(q^L(1-\kappa) + \frac{e}{\Delta}). \tag{2}$$

In period 2, and in period 1 when consumers face a large price $\overline{p}$, firms set prices to extract all conditionally expected consumer surplus. But in period 1 when consumers observe a low price, which happens with probability $(1-\gamma)(1-\delta^*)$, they get the surplus $q^L - \underline{p} = (q^L(1-\kappa) + \frac{e}{\Delta})$. Low-quality sellers are willing to leave this surplus so that consumers reciprocate with a good rating, allowing low-quality sellers to free-ride on the reputation of high-quality firms and charge a larger price in period 2.

We have shown above that low-quality firms' ratings harvesting undermines the informativeness of ratings. But in order to harvest ratings, low-quality firms need to offer low prices. This is why rating harvesting benefits consumers. This, however, does not imply that consumers prefer uninformative ratings. If ratings were completely uninformative, low-quality firms would not harvest ratings, and consumers would earn no surplus. We now discuss implications of this result more carefully for the impact of effort e on consumer surplus.

Cost of ratings

We know from Corollary 1 that lower cost of ratings for consumers (e) makes ratings less informative in equilibrium. But what is the impact on consumer surplus? A lower cost of ratings e has two opposing effects on consumer surplus. First, and following directly from Corollary 1, low-quality firms harvest ratings and charge the low price $\underline{p}$ more often; this tends to increase consumer surplus. Second, however, the level of the low price $\underline{p}$ increases: when consumers can rate more easily, they are more inclined to reciprocate kindness. This is why low-quality firms who harvest ratings charge a larger price $\underline{p}$ and still get a good rating, which tends to reduce consumer surplus. Overall, because easier ratings encourage low-quality firms to harvest ratings, they harvest ratings more often, which tends to benefit consumers, but do so with a larger price, which tends to harm consumers.

We show that these two opposing effects pin down a positive level of rating costs that maximizes consumer surplus.[33]

Proposition 4. *There exists a level of effort $e^{CS} > 0$ that maximizes consumer surplus. At this level of effort, $\delta^* \in (\frac{1}{2}, 1)$. This is true if*

$$(1-\gamma)^2\gamma(q^H - q^L)^2 \geq (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2. \tag{3}$$

Otherwise, consumers prefer $e^{CS} = 0$.

The proposition characterizes when the conventional wisdom that more-informative ratings benefit consumers is true. When e is sufficiently small ($e < e^{CS}$), making it more costly to rate leads to more-informative ratings (Corollary 1), and increases consumer surplus. In this case, the price effect on $\underline{p}$ dominates and more-informative ratings put pressure on prices for low-quality firms.

More surprisingly, when e is sufficiently large ($e \geq e^{CS}$), more-informative ratings harm consumers. Intuitively, when e is large, low-quality firms charge a very low price $\underline{p}$ to harvest ratings. This is why, as e increases further, low-quality firms get discouraged from harvesting ratings, which reduces consumer surplus.

[33]Note that if we would incorporate rating utility into consumer surplus, consumers would get an additional benefit and cost from rating, so the consumer optimal level of effort could be larger or smaller. But crucially, it can still be positive.

The key implication of the Proposition is that even though consumers benefit when low-quality firms harvest ratings and undermine the informativeness of ratings, consumers still prefer a somewhat informative rating system. The reason is that low-quality firms are only willing to harvest ratings if they can free ride on the good reputation of high-quality firms; but this requires ratings to be somewhat informative.

By condition (3) consumers prefer somewhat informative ratings if the difference between low- and high-quality firms is sufficiently large. This is rather intuitive: if the condition is violated and the difference in quality is small, then the price difference in period 2 is also small so that low-quality firms have little incentive to harvest ratings and hardly ever do so. Because low-quality firms harvest ratings so rarely, consumers want firms to harvest ratings more often and prefer $e^{CS} = 0$. This result, however, seems economically less relevant, since it only applies when the quality differences are small so that ratings are less relevant in the first place.

We now explore the impact of rating costs on seller surplus. While buyers prefer somewhat informative ratings, sellers on average prefer uninformative ratings. Intuitively, seller surplus is largest when sellers leave no reciprocity rent to buyers. This is true when $e = 0$ and ratings are uninformative, because low-quality sellers have no incentive to harvest ratings and free-ride on the reputation of high-quality sellers.

While sellers on average prefer less-informative ratings, high-quality sellers' profit increases in e and they prefer perfectly-informative ratings: informative ratings allow high-quality sellers to distinguish themselves from low-quality sellers who try to free-ride on their reputation. As e increases, this free-riding becomes more costly, which leads to more-informative ratings and allows high-quality sellers to extract more of the surplus they generate. The following corollary summarizes these results.

Corollary 2. *When* (3) *holds, average seller surplus is maximal at* $e^S = 0$ $(< e^{CS})$. *When* (1) *additionally holds, profits of high-quality firms increases in* e*, and profits of low-quality firms decreases in* e.

This Corollary implies that only high-quality sellers unambiguously prefer perfectly-informative ratings, because this limits free-riding on their reputation. Neither buyers nor sellers on average prefer perfectly informative ratings. But buyers prefer more informative ratings than sellers. The reason is that somewhat informative ratings push sellers to harvest ratings, which puts pressure on prices. This suggests that the aforementioned efforts of Google reviews and Amazon to facilitate and encourage ratings might not just lead to less-informative ratings, but also harm consumers through larger prices. They do, however, benefit sellers.

Quality

We now discuss the effect of aggregate product quality on consumer surplus. Our first insight is that improvements in aggregate quality can make consumers worse off.

The intuition has two steps. First, we know from Proposition 3 that low-quality firms harvest ratings when they can free-ride on the reputation of high-quality sellers. Thus, an individual low-quality firm harvests ratings more and benefits consumers when γ is large and there are more high-quality firms to free-ride on. But on the other hand, only low-quality firms harvest ratings and leave a rent to consumers, suggesting benefits to consumers when γ is sufficiently small. As a result of these opposing forces, consumer surplus is concave in γ and an intermediate $\gamma^{CS} \in (0, 1)$ maximizes consumer surplus.

The second key insight resembles a previous one on the cost of ratings: consumers prefer quality levels that lead to somewhat informative ratings ($\gamma^{CS} < \overline{\gamma}$), but not fully informative ratings ($\gamma^{CS} \to 1$). The intuition is familiar from above: with somewhat informative ratings, low-quality sellers harvest ratings, which puts pressure on prices.

The next proposition summarizes these results.

Proposition 5. *Equilibrium consumer surplus is strictly concave in γ. There exists an aggregate quality level, denoted by γ^{CS}, that maximises consumer surplus, where $\overline{\gamma} > \gamma^{CS}$ and $\gamma^{CS} > 0$.*

Proposition 5 implies that, even when larger aggregate quality leads to more-informative ratings, consumers can be worse off. This is the case when $\gamma \notin [\gamma^{CS}, \overline{\gamma}]$. For $\gamma < \gamma^{CS}$, larger quality encourages low-quality sellers to harvest ratings, which benefits consumers but undermines the informativeness of ratings. For $\gamma > \overline{\gamma}$, an increase in aggregate quality makes ratings more informative; but as low-quality firms participate in less ratings harvesting, consumers surplus diminishes. Thus, to evaluate a rating system, observing that ratings reflect quality more closely is not enough to conclude that consumers benefit.

Sellers unambiguously benefit if their average quality increases. High-quality sellers are able set larger prices in both periods. Low-quality sellers benefit either from being able to set a larger price in the first period when they mimic prices, or from setting a larger price in the second period when they harvest ratings.

Lemma 1. *Firms unambiguously benefit from screening, and seller profits are maximised at $\gamma^S = 1$*

Because all firms benefit from a larger average quality, this also implies that (remaining) firms prefer a larger level of screening than consumers. Together with Proposition 3, Lemma 1 implies that sellers prefer ratings that are uninformative. Taking Proposition 5 into account, Lemma 1 suggests that firms prefer ratings that are less informative than what consumers prefer.

Additionally, when considering Proposition 5, Lemma 1 suggests that (i) firms prefer ratings that are less informative than what consumers prefer; (ii) platforms looking to maximise total surplus should engage in screening which is at least as stringent as γ^{CS}. To see the first implication, firms prefer uninformative ratings, and consumers prefer somewhat informative ratings. Hence, firms prefer ratings that are less informative than consumers. To see the second implication, consider the consumer problem, and notice that any deviation from γ^{CS} reduces consumer surplus. Next, consider the firm problem, and notice that higher levels of γ improves firm surplus. Hence, to maximise surplus, γ must be at least as large as γ^{CS}, any $\gamma < \gamma^{CS}$ reduces both consumer and firm surplus, while any $\gamma > \gamma^{CS}$ increases firm surplus but reduces consumer surplus.

7 Extensions

Designing a profit maximizing ratings system

So far we studied how different features of a market environment affect the informativeness of ratings and surplus. In practice, rating systems are typically designed by online platforms, and many platforms constantly tweak their ratings system. We now investigate how a platform would actually design its ratings system.

To do so, we introduce some modifications to our base model.

Suppose there is a unit mass of consumers who are heterogeneous only in their outside option to joining the platform. This outside option is uniformly distributed between $[0, 1]$. We denote the mass of consumers who join the platform n_b. Further, we call the per transaction consumer benefit as u_b. Consumers choose to join the platform and are then randomly assigned (with equal probability) to the first period or the second period. This captures that consumers join the platform when there are more or less ratings available to guide their choices.

We suppose that there exists a unit mass of sellers that are monopolists in a product category. Sellers are ex-ante homogeneous and face an outside option of $\bar{v}_s$. We denote the mass of sellers who join the platform by n_s and call π^s the per transaction revenue of the seller. We assume that sellers face some additional marginal cost of selling on the platform, t, like shipment costs.[34] We say that sellers join the platform prior to period 1 of our main model and therefore do not have information over their true quality when deciding to join the platform.[35]

To maximize profits, the platform makes two choices. First, the platform sets a royalty, r, which it charges sellers. This gives the platform a share r of the sellers' revenue. Second, the platform chooses how easy it is for buyers to leave a rating, i.e. the platform sets e. We assume that the platform can choose any $e \in [0, \bar{e}]$. To simplify exposition, we assume $\bar{e} \leq e^{CS}$. This implies that a larger effort always increases u_b and reduces π_s. The platform makes these decisions prior to period 1 in the base model.

The effort choice e captures that the platform designs a ratings system to make it easier or more difficult for consumers to leave a rating. For example, to facilitate ratings, a platform can introduce a one-click ratings system or provide users a link to go directly to the ratings page, automated ratings[36], etc. Conversely, a platform can make ratings more effort for consumers if it introduces additional authentication and verification steps — such as proof of identity, proof of purchase[37], multiple ratings components — such as the use of multi-dimensional ratings (Schneider et al., 2021), or requiring a written review along with the rating[38].

Proposition 6. *Platforms design a ratings environment that favours sellers ($e^* = e^S = 0$) if and only if $\pi^s - t < u_b$.*[39]

The proposition characterizes when the platform wants to make it easier or harder to rate. We show that platforms want to make it easier to rate whenever this has a stronger impact on per-transaction profit of firms (revenue less cost of transacting on the platform) than per-transaction consumer surplus. This captures the common finding that platforms want to balance surplus between buyers and sellers to get both sides on the platform(Armstrong, 2006; Caillaud & Jullien, 2003; Rochet & Tirole, 2003). Combining this result with Proposition 4 suggests that when per-transaction consumer surplus exceeds the profits of the firm, platforms have an incentive to encourage ratings and undermines the informativeness of rat-

[34] This cost captures the difference in cost of selling on a platform rather than direct to consumers. This reflects cost in addition to the platforms ad valorem fees. For instance, on Amazon, in addition to the ad valorem fees, there are additional charges for fulfilled by amazon and per item transaction fees.

[35] This assumption captures the importance of relative quality of products on a marketplace, and sellers learning their true relative quality after joining the marketplace. This assumption also allows us to abstract away from seller selection by the platform and focus on the role that the platform plays in influencing ratings.

[36] "Taobao Item Review Guide" (2021), retrieved June 2021.

[37] "Amazon.Com Help: About Amazon Verified Purchase Reviews" (2021), retrieved June 2021.

[38] "Introducing Steam Reviews" (2021), retrieved June 2021.

[39] Platforms may favour consumers when $\pi^s - t > u_b$.

ings. By doing so, platforms reduce reciprocity rent and shifts surplus per interaction from buyers to sellers.

These results support concerns of regulators that platforms design insufficiently informative ratings environments for consumers, and that minimum standards of ratings may help to protect consumers (Competition and Markets Authority (UK), 2017).

This analysis makes the simplifying assumptions that sellers are ex-ante identical and do not yet know their relative quality level when entering the platform. Corollary 2 suggests that relaxing this assumption could lead to a novel selection effect. To raise profits, platforms may still prefer to lower e in order to raise average seller surplus. This would discourage high-quality sellers from joining the platform and lower the average quality. This way, facilitating ratings can undermine the very purpose of a ratings system.

Competitive environment

Our basic model focuses on monopoly sellers. We now introduce competition to the model. To do so in a simple way, we assume that there exists a competitive fringe of non-strategic firms that offers a product of quality q at a price equal to their marginal cost c, where $q \geq 0$. Intuitively, we capture competition by changing the outside option of purchasing in the market to $q - c \geq 0$. This could capture (i) established firms for which consumers do not need ratings to evaluate the quality of their products; (ii) brick-and-mortar stores; or (iii) the expected utility a consumer gets when participating in another ratings environment like another marketplace.

We now study how the presence of the competitive fringe affects the equilibrium strategy of our previous strategic sellers.

To start, suppose $q - c = 0$. Then the competitive fringe offers the same value as our previous outside option and the equilibrium is the same as in Proposition 1. Now suppose c decreases marginally and the competitive fringe offers a small but strictly positive surplus $q - c > 0$. This encourages sellers to harvest more ratings and reduces the informativeness of ratings. Intuitively, in period 1 firms that charge the large price $\overline{p}$ extract the consumers' conditional expected value. Thus, firms reduce $\overline{p}$ to remain more beneficial to consumers than the fringe. The low-quality sellers that charge $\underline{p}$, however, already leave a rent to consumers 2, which is why the low price $\underline{p}$ does not decrease. Thus, the fringe puts more pressure on large prices $\overline{p}$ than on small prices $\underline{p}$, which is why low-quality firms harvest more ratings, leading to a less informative ratings in equilibrium (lower δ^*).

As c decreases further, $q - c$ becomes so large that it puts equal pressure on large prices $\overline{p}$

and low prices $\underline{p}$, so that the competitive fringe no longer affects the incentives to harvest ratings and the informativeness of ratings δ^*.

The following proposition summarizes this result.

Proposition 7. $\frac{\partial \delta^*}{\partial c} > 0$ *if* $q^L - (q - c) > \kappa q^L - \frac{e}{\Delta}$. *Otherwise,* $\frac{\partial \delta^*}{\partial c} = 0$

The proposition shows that competition puts disproportionate pressure on firms that extract more surplus from consumers, i.e. firms that do not harvest ratings. This is why competition encourages firms to harvest ratings and makes ratings less informative. Crucially, however, ratings will not become uninformative: as the fringe becomes more competitive, it will put equal pressure on all prices and no longer effect incentives to harvest ratings, leaving the informativeness of ratings constant at some $\delta^* > 0$.

Even though competition makes ratings less informative, it unambiguously benefits consumers through the following two channels. First, competition exerts pressure on the higher price and lowers $\overline{p}$. Second, competition encourages low-quality firms to harvest ratings and charge low prices $\underline{p}$, for which consumers receive a higher surplus. Thus, our results suggest that making consumers aware of alternative sellers can make ratings less informative, but benefits consumers nonetheless.

8 Robustness

Negative ratings

We made the simplifying assumption in the main text that we only observe positive and no ratings. To relax this assumption, we allow for potentially negative ratings. In a setup that is identical to the main model with the exception that $R_t \in \{1, -1, 0\}$, representing a good, bad and no rating respectively. In equilibrium, bad ratings arise from retaliation (negative reciprocity). When firms leave a sufficiently small surplus to consumers, consumers perceive this as unkind and respond with an unkindness, retaliating with a bad rating. Hence, instead of a positive warm glow of Δ, consumers receive a negative warm glow of $-\Delta$. For details, see A.11.

In equilibrium, there exists a range of prices for which all ratings are possible. However, low-quality firms prefer bad over no ratings, for as long as consumers continue to buy at the price that induces a bad rating. Intuitively, both no ratings and bad ratings have the same informational content, but bad ratings allow low-quality firms to charge a larger price in period 1. Qualitatively, the results are the same as in the main setting with binary ratings.

But the presence of negative ratings leads to an equilibrium with extreme ratings. Moreover, when consumers exhibit biased reciprocity, that is the warm glow they receive from leaving a good rating exceeds the warm glow they receive from leaving a bad rating, our result provides theoretical support for Dellarocas and Wood (2008) suggestion that reciprocity bias leads to a J-shaped ratings distribution. For details, see A.11.

Continuum of Firms

We now show that our results extend to firms with a continuum of quality types. To do so, we modify the monopolist to draw quality from a uniform distribution, $U[0, 1]$.[40]

In this setting, we find a pure strategy equilibrium where firms are divided into 3 groups: (i) top firms that set the highest possible price that consumers are willing to pay, these firms receive a good rating in equilibrium; (ii) middle firms choose to set sufficiently low prices that triggers reciprocity and allows them to harvest ratings; (iii) bottom firms that choose to set the highest possible price, mimicking the price of the top firms, and forgoing ratings.

This equilibrium has results that are qualitatively identical to our basic setting. In this setting, the middle firms choose to use prices to influence ratings, lowering the information content of ratings, and allowing them to harvest ratings. Further, as ratings become easier, the size of this group increases, leading to less informative ratings.

Long Horizon Model

We show that our results are robust to more than two periods by looking at a three period model. We find equilibri that are similar to those described in our base model, and show that low-quality firms may choose to play either ratings harvesting or price mimicking in every period.[41] For details, see A.13.

Additionally, we argue that our model does not suffer from the terminal period effect. We show that there exist equilibria where the low-quality firm chooses to play a mixed strategy in period 1, obtaining a good rating with some positive probability, and a pure strategy of price mimicking in period 2. This means that the choice to harvest ratings in the non-terminal period is an endogenous one, and not a result of the terminal period.

[40] We also modify Restrictions 1, 2 to the new setting. For details, see Appendix A.12.

[41] The result differs from the base setting only in that a stricter set of restrictions are equired for a unique mixed strategy to exist in every non-terminal period. Therefore, our mixed strategy equilibrium is qualitatively robust to an N-period setting.

9 Conclusion

Ratings are an essential element of the online economy, building trust between strangers. But ratings are only able to build trust if they are informative about the underlying products and services. This is why ratings inflation is seen as a potential threat for the functioning of digital marketplaces. In this paper, we study why and how consumers choose to rate, and provide insight on how firms may influence the informativeness of ratings through prices. We show that firms prefer uninformative ratings, while consumers prefer ratings that are at least somewhat informative but not fully informative. These results are fundamentally driven by ratings based on value-for-money, and we use reciprocity to motivate why people choose to rate at all.

We argue that our reciprocity-based mechanism reflects the reality of how and why consumers rate by discussing carefully how it is in line with the evidence on value-for-money drives ratings (Li et al., 2020). Allowing us to bridge the gap between the traditional theoretical literature on reputation, and the evidence.

We find that uninformative ratings on average benefit sellers. However, this is driven by low-quality sellers. In contrast, high-quality sellers prefer informative ratings. This suggests to us that uninformative ratings systems can lead to a selection effect, that only lower quality firms join the platform when platforms lean in favour of facilitating ratings.

We also suggest that sellers prefer the highest level of screening. However, because we have a monopolist, we do not consider competitive effects. If screening becomes too high, sellers may face only competition only from other high-quality firms. This may lead to price competition and as a result less profits. Conversely, if a platform engages in a high level of screening, this may deter sellers, who do not know their ex-ante quality, from entering the market in the first place, reducing competition.

Moreover, while we explain how making it easier to obtain ratings induces firms to get good ratings, we do not touch on the topic of fake ratings — where firms buy ratings outright. We speculate that if it is cheaper to obtain fake ratings, this corresponds to a lower opportunity cost of ratings harvesting, and this leads to less informative ratings. However, because consumer surplus is driven by low-quality firms transferring reciprocity rent to consumers, this would not occur in a model of fake ratings, and consumers would be strictly worse off in the presence of fake ratings.

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Appendix A Proofs

A.1 Proof of Proposition 1

We proceed as follows. First, we pin down equilibrium prices in period 1 in Lemma 2 and equilibrium beliefs in Lemma 3. Afterwards, we use these lemmas to prove the remaining statements in Proposition 1.

Lemma 2. *In equilibrium, firm j plays the price p^j_{t,R_t} in period t, in order to receive the rating R_t.*

In the first period, firms place the following equilibrium prices with positive probability.

- *High type firm:* $p^H_{1,1} = min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p^H_{1,1}]\}$
- *Low type firm:*

$$p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$$

$$p^L_{1,0} = E[q_1|R_0, p^H_{1,1}]$$

Proof of Lemma 2.
We proceed in three steps. First, we consider the pricing strategy of the high-quality firms. Second, we look at the pricing strategy of low-quality firms receiving a good rating. Third, we focus on prices of low-quality firms obtaining no rating.

To start, we look at the pricing strategy of high-quality firms. We show that their pricing strategy is unique and $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p^H_{1,1}]\}$ with probability 1.

Given Restriction 1, we focus on equilibria where the high-quality firm sets a price which allows it to obtain a good rating, $R_1 = 1$. Therefore, in equilibrium, the firm does not consider the pricing strategy that obtains no rating.

We show that high-quality firms set a unique price in period 1 with probability 1. Suppose towards a contradiction that high-quality firms set more than one price with positive probability.

Without loss of generality, suppose that the high-quality firm sets a distribution of prices, $p \in [p', p'']$ such that $p'' > p'$, and the firm receives a rating $R_1 = 1$ with probability 1 for all $p \in [p', p'']$. Therefore, at all $p \in [p', p'']$, consumers purchase products with probability 1.

Notice that for any price $\hat{p} \in [p', p'']$ such that $\hat{p} > p$, we have $\pi^H(\hat{p}) > \pi^H(p)$. To see this, observe that both prices induce the same demand in period 1, but $\hat{p}$ induces a larger margin and therefore strictly larger profits in period 1. Further, both prices induce $R_1 = 1$ with probability 1, and therefore the same expected profits in period 2. As a result, $\pi^H(\hat{p}) > \pi^H(p)$. Shifting the probability mass of the entire price distribution in period 1 to one mass point, p'', strictly increases profits for the high-quality firm, contradicting that the firm sets more than one price with positive probability. Essentially, the same argument implies that the high-quality firm does not set finitely many prices more than one with strictly positive probability. We conclude that the high-quality firm sets a unique price in period 1 with probability 1.

Next, we prove that there exist an upper bound on prices, $\overline{p_t^j}$ for $j \in \{L, H\}$ such that a firm j receives a good rating. In order for a firm to induce a good rating, the rating utility must be weakly positive, i.e. $v_t \geq 0$. Therefore,

$$[\kappa q^j - p_{t,1}^j]\Delta - e \geq 0 \Leftrightarrow p_{t,1}^j \leq \overline{p_t^j} \equiv \kappa q^j - \frac{e}{\Delta}.$$

Therefore, the upper bound on prices such that the high-quality firm receives a positive rating is $\overline{p_1^H} = \kappa q^H - \frac{e}{\Delta}$.

Finally, consider that this upper bound is restricted by consumer's beliefs, $E[q_1|R_0, p_{1,1}^H] < \overline{p_1^H}$. Under such scenarios, by Restriction 1, high type firms prefer obtaining a good rating. This can only be achieved if consumers buy. Therefore, $p_{1,1}^H$ has an upper bound of $E[q_1|R_0, p_{1,1}^H]$.

We show next that $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_{1,1}^H]\}$ with probability 1. To see this, note first that $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta}$ is the cut-off price above which the low-quality firm receives no rating. See also that $\overline{p_1^H} > \overline{p_1^L}$ and $q_L > \overline{p_1^L}$. Thus, because $E[q_1|R_0, p_{1,1}^H] \geq q_L$, the high-quality firm sets its equilibrium price in period 1 strictly above $\overline{p_1^L}$, i.e. $p_{1,1}^H > \overline{p_1^L}$. By Restriction 2, consumers have the same beliefs for all prices strictly above $\overline{p_1^L}$, and since $p_{1,1}^H > \overline{p_1^L}$, these beliefs are the correct equilibrium beliefs $E[q_1|R_0, p_{1,1}^H]$. Because consumers have the same beliefs $E[q_1|R_0, p_{1,1}^H]$ for all prices above $\overline{p_1^L}$, the high-quality firm optimally sets the largest price for which consumers purchase and rate with probability 1, which is $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_{1,1}^H]\}$.

We conclude that high-type firms set a unique price $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_{1,1}^H]\}$ with probability 1.

We now proceed to the second step and characterize the pricing strategy of low-quality firms who receive a good rating. First, we show that the price which it sets is unique. Then, that

$p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$.

Essentially the same argument as used for high-type firms implies that—conditionally on obtaining a good rating—the low-type firm sets a single price with probability 1.

By definition of $\overline{p^L_1}$, $\overline{p^L_1} = \kappa q^L - \frac{e}{\Delta}$. Since this is strictly less than q^L, and since consumers beliefs must be weakly above q_L, consumers are always willing to buy at any price weakly below $\overline{p^L_1}$. Since demand and ratings are the same for all prices weakly below $\overline{p^L_1}$, a low-quality firm that obtains a good rating must optimally set $\overline{p^L_1}$ with probability 1. We conclude that conditional on obtaining a good rating—the low-quality firm sets $\overline{p^L_1}$ with probability 1.

We now proceed to the third step of the proof and determine prices of low-quality firms who obtain no rating. We show that low-quality firms who obtain no rating optimally set $E[q_1|R_0, p^H_{1,1}]$. We have shown in step 2 that all prices above $\overline{p^L_1}$ induce the same beliefs $E[q_1|R_0, p^H_{1,1}]$. Thus, low-quality firms who obtain a no rating optimally set the largest possible price, $E[q_1|R_0, p^H_{1,1}]$ with probability 1.

This concludes the proof.

□

Lemma 3. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 > \overline{p^L_1} \\ q^L & \text{if } p_1 \leq \overline{p^L_1}, \end{cases}$$

and in the second period,

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

Proof of Lemma 3.
We prove this Lemma by constructing expected quality using Bayes rule. We start by considering the second period, followed by the first period.

Before we start, note that by Lemma 2, high-quality firms obtain a rating with probability 1 and low-quality firms obtain a rating if and only if they charge a low price $p^L_{1,1}$. We denote the probability δ as the probability with which low-quality firms charge $p^L_{1,0}$. Thus, $(1-\delta^*)$ is the probability which the low-quality firm obtains a good rating.

Now consider the second period. Given the consumer's information set in the second period, they are aware of historical ratings R_1, and current prices p_2. We now show that expected quality in period 2 is independent of second period prices p_2. Since period 2 is the final period, no firm obtains a rating, which, by Restriction 2, implies that the expected quality in period 2 is independent of second period prices. We conclude that expected quality in period 2 only depends on past ratings R_1.

Next, we pin down consumers' expectations in period 2. In equilibrium, consumers observe $R_1 = 1$ from all high-quality firms and low-quality firms with low prices, i.e. with probability $\gamma + (1-\delta^*)(1-\gamma)$. Because only low-quality firms get no rating, the expected quality after observing $R_1 = 0$ is q^L. Thus, applying Bayes rule leads to

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

We now consider the first period. Note first that there are no informative past ratings in period 1, which is why expected ratings in the first period are independent of R_0.

We distinguish two cases, (i) $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = E[q_1|R_0, p^H_{1,1}]$ and (ii) $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = \kappa q^H - \frac{e}{\Delta}$.

We start with case (i) and suppose $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = E[q_1|R_0, p^H_{1,1}]$. Then by Lemma 2, we have $p^L_{1,0} = p^H_{1,1}$ and $p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$. Applying Bayes rule leads to

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 = p^H_{1,1} \\ q^L & \text{if } p_1 = p^L_{1,1}. \end{cases}$$

Now consider case (ii) and suppose $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = \kappa q^H - \frac{e}{\Delta}$. Then $p^H_{1,1} \neq p^L_{1,0}$, and Bayes rule implies $E[q_1|R_0, p^H_{1,1}] = q^H$. This is only consistent with the finding in Lemma 2 that $p^L_{1,0} = E[q_1|R_0, p^H_{1,1}] = q^H$ if $\delta^* = 0$, i.e. the low-quality firm sets $p^L_{1,0}$ with probability zero. Thus, beliefs are given again by

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 = p_{1,1}^H \\ q^L & \text{if } p_1 = p_{1,1}^L, \end{cases}$$

applied at $\delta^* = 0$. We conclude from cases (i) and (ii) that for equilibrium prices in period 1, beliefs are given by

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 = p_{1,1}^H \\ q^L & \text{if } p_1 = p_{1,1}^L. \end{cases}$$

Because $p_{1,1}^H > \overline{p_1^L}$, $p_{1,0}^L > \overline{p_1^L}$, and $p_{1,1}^L \leq \overline{p_1^L}$, this concludes the proof.

$\square$

We prove a slightly more general statement than **Proposition 1**.

Proposition 8. *All perfect Bayesian equilibria satisfy the following. In period 1:*

1. *High-quality firms receive a good rating with probability one and charge $\overline{p} \equiv E[q_1|R_0, p_{1,1}^H]$.*
2. *Low-quality firms randomize their strategy.*
   a. *They charge $\overline{p}$ and obtain no rating with probability δ^*.*
   b. *They charge $\underline{p} \equiv \kappa q^L - \frac{e}{\Delta}$ and obtain a good rating with probability $1 - \delta^*$.*
3. *Consumers beliefs of equilibrium prices are given by* $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta} \end{cases}.$

In period 2:

4. *Prices are equal to expected quality conditional on ratings.*
5. *Consumer beliefs are given by* $E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \end{cases}.$

The equilibrium is unique up to off-path beliefs. $\delta^* = 1$ if and only if $\kappa q^L - \frac{e}{\Delta} + q^H \leq \gamma q^H + (1-\gamma) q^L + q^L$, and $\delta^* \in (\frac{1}{2}, 1)$ otherwise. The equilibrium exists if $\kappa q^H - \frac{e}{\Delta} \geq \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)}$.

Proposition 1 in the main text is obtained as a special case.

Proof of Proposition 8.
From Lemma 2 and 3, we have shown statements 2, 3 and 5. Hence, what remains is to prove statement 1 and 4, as well as existence and uniqueness up to off-equilibrium-path beliefs.

To prove statement 1, note that by Lemma 2, we know $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p_{1,1}^H]\}$, and it remains to show that $\min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p_{1,1}^H]\} = E[q_1|R_0, p_{1,1}^H]$. Towards a contradiction suppose otherwise, i.e. that $\min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p_{1,1}^H]\} = \kappa q^H - \frac{e}{\Delta}$. As we argued in the proof of Lemma 3, we then have $p_1^H \neq p_{1,0}^L$, and $E[q_1|R_0, p_{1,1}^H] = q_H$, and $p_{1,0}^L = E[q_1|R_0, p_{1,1}^H] = q^H$ is played with probability $\delta^* = 0$. Because the low-quality firm charges $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$ with probability one, all firms get a good rating with probability one. By Lemma 3, low-quality firms earn up to $\kappa q^L - \frac{e}{\Delta} + \gamma q^H + (1-\gamma)q^L$. Low-quality firms can strictly increase profits by charging a first period price of q^H. By Restriction 2, consumers believe $E[q_1|R_0, p_1] = q^H$ for all prices $p_1 \geq \kappa q^L - \frac{e}{\Delta}$, so they purchase in period 1. In period 2, the deviation earns q^L. Overall, the deviation earns $q^H + q^L$. Since $q^H \geq \gamma q^H + (1-\gamma)q^L$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, this deviation profitable for low-quality firms, a contradiction.

We conclude that $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p_{1,1}^H]\} = E[q_1|R_0, p_{1,1}^H]$, which proves statement 1. Because we have shown that $p_{1,1}^H = E[q_1|R_0, p_{1,1}^H]$, and this is the same as $p_{1,0}^L$, to simplify notation, we state that $\overline{p} = p_{1,1}^H = p_{1,0}^L = E[q_1|R_0, \overline{p}]$. To further simplify notation, we label $\underline{p} = p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$.

We now prove statement 4. We have shown in the the Lemma 3 that in period 2, firms are no longer incentivized by future ratings. We have also shown that consumers' beliefs only depend on past ratings. Thus, firms optimally charge prices equal to the expected profits conditional on the past ratings they received. We conclude that in period 2, prices equal expected quality conditional on ratings, which proofs statement 4.

We conclude that statements 1 - 5 hold.

We now show that equilibria are unique up to off-equilibrium-path beliefs. To show uniqueness of equilibrium, consider that for some δ^*, low-quality firms are indifferent between getting good and no ratings. From the proof of statement 1, we know that in equilibrium we must have $\delta^* < 1$ and $\kappa q^H - \frac{e}{\Delta} > E[q_1|p_{1,1}^H]$, implying that $\overline{p} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$.

Charging $\underline{p} = q^L - \frac{e}{\Delta}$ induces a good rating and, given correct equilibrium beliefs, the following total profits for low-quality firm:

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}. \tag{4}$$

This is strictly increasing in δ^* for all $\gamma \in (0,1)$ and $q^H > q^L$.

When the low-quality firm charges $\overline{p} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$ in period 1, it obtains no rating and earns

$$\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L, \tag{5}$$

which strictly decreases in δ^* for all $\gamma \in (0,1)$ and $q^H > q^L$.

To start we show that $\delta^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists. Suppose $\delta^* = 1$. Then (4) and (5) become $\kappa q^L - \frac{e}{\Delta} + q^H$ and $\gamma q^H + (1-\gamma)q^L + q^L$, respectively. For $\delta^* = 1$ to be optimal, it must be the case that

$$\kappa q^L - \frac{e}{\Delta} + q^H \leq \gamma q^H + (1-\gamma)q^L + q^L. \tag{6}$$

Since (4) strictly increases in δ^* and (5) strictly decreases in δ^*, this means whenever $\delta^* = 1$ is an equilibrium, we cannot have a mixed-strategy equilibrium. We conclude that $\delta^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists.

We now show that $\delta^* = 0$ cannot be an equilibrium. Towards a contradiction, suppose $\delta^* = 0$. Then (4) and (5) become $\kappa q^L - \frac{e}{\Delta} + \gamma q^H + (1-\gamma)q^L$ and $q^H + q^L$, respectively. But since $q^H > q^H + (1-\gamma)q^L$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, low-quality firms optimally set $\overline{p} = q^H$ when consumers believe they set this price with probability zero in period 1, a contradiction. We conclude that $\delta^* = 0$ cannot be an equilibrium.

We now characterize the mixed-strategy equilibrium. To have a mixed-strategy equilibrium, consumers must have beliefs such that (4) = (5) and low-quality firms must play some δ^* such that these beliefs are correct. Thus, in a mixed strategy equilibrium, we have

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L. \tag{7}$$

We have two candidates that solve this equation:

$$\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} \pm \frac{(4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})}$$

Recall that as a probability, δ^* is bounded by 0 and 1.

Consider the scenario where the last term is subtracted.

$$\begin{aligned}
&\frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{(4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} \\
&< \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{((1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} \\
&< \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{(1+\gamma)}{2(1-\gamma)} \\
&< \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{(1-\gamma)}{2(1-\gamma)} = -\frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} < 0
\end{aligned}$$

.

Therefore, we conclude that

$$\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} + \frac{(4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})}.$$

Further, notice that the optimal δ^* is lies strictly between $\frac{1}{2}$ and 1.

We see from (4) and (5) that $\delta^* < 1$ if $\kappa q^L - \frac{e}{\Delta} + q^H > \gamma q^H + (1-\gamma)q^L + q^L$. Also note that

$$\delta^* > \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} + \frac{(4\gamma^2(q^H - q^L)^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} = \frac{1}{2}.$$

We now show the equilibrium is unique up to off-equilibrium-path beliefs. This follows immediately from having shown that we cannot have $\delta^* = 0$, and that $\delta^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists. Additionally, if a $\delta^* \in (0, 1)$ exists such that (7) holds, it must be unique, because (4) strictly increases, and (5) strictly decreases in δ^*. We conclude that if mixed-strategy equlibria exists, it is a unique mixed strategy, $\delta^* \in (0,1)$ Thus, we either have a unique pure-strategy equilibrium or a unique mixed-strategy equilibrium, but not both. We conclude that the equilibrium is unique up to off-equilibrium-path beliefs.

Therefore, we can conclude that $\delta^* < 1$ if

$$(1-\gamma)(q^H - q^L) > q^L(1-\kappa) + \frac{e}{\Delta}. \tag{8}$$

And this equilibrium is a unique interior solution where $\delta^* \in (\frac{1}{2}, 1)$ up to off-path beliefs. Otherwise, there is a unique corner solution at $\delta^* = 1$ up to off-path beliefs.

$$\delta^* = \begin{cases} \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} + \frac{(4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} & \text{if (8) holds} \\ 1 & \text{otherwise} \end{cases} \tag{9}$$

We now show that these equilibria exist.

To start, consider the case where (6) holds. In the candidate equilibrium in period 1, the high-quality firm sets $\overline{p} = \gamma q^H + (1-\gamma)q^L$ with probability 1 and obtains a good rating, and the low-quality firm charges $\overline{p}$ with probability 1 and gets no rating. In period 2, the high-quality firm charges a price equal q^H, and the low-quality firm charges q^L. Consumers' beliefs are as follows. In period 1, they believe

$$E[q_1|p_1] = \begin{cases} \gamma q^H + (1-\gamma)q^L & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}, \end{cases}$$

in the second period, beliefs are independent of prices and are

$$E[q_2|R_1] = \begin{cases} q^H & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

These beliefs follow Bayes rule on the path of play. The candidate equilibrium is also consistent with our restrictions. Because high-quality firms obtain a rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Because consumers have the same beliefs for all second period prices, and the same beliefs for all first period prices where the low-quality firm obtains no rating, the candidate equilibrium is consistent with Restriction 2.

We now show that firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns $\gamma q^H + (1-\gamma)q^L + q^H$. Deviations in period 2 to a higher price would induce zero demand, and deviations to lower prices would reduce margins without increasing demand. There are no profitable deviations in period 2. In period 1, deviations to a higher price reduces demand to zero and earns a maximal total profit of $0 + q^L$. Deviations to a lower price in period 1 reduce margins without increasing demand or increasing ratings. Therefore, there is no profitable deviation in period 1. We conclude that high-quality firms have no profitable deviations.

We now show that low-quality firms have no profitable deviations. In the candidate equilibrium they earn $\gamma q^H + (1-\gamma)q^L + q^L$. Deviations in period 2 to a higher price would induce zero demand, and deviations to lower prices would reduce margins without increasing demand. There are no profitable deviations in period 2. In period 1, deviations to a higher price reduces demand to zero and earns a maximal total profit of $0 + q^L$, which is not a profitable deviation. In period 1, deviations to a lower price above $\kappa q^L - \frac{e}{\Delta}$ does not improve the rating and only reduces margins without raising demand, this is not a profitable deviation. Deviations to lower prices below $\kappa q^L - \frac{e}{\Delta}$ leads to profits weakly below $\kappa q^L - \frac{e}{\Delta} + q^H$, which is not a profitable deviation since (6) holds.

We conclude that if (6) holds, no profitable deviations exist for either type of firm.

Finally, we have shown above that $\overline{p} = E[q_1|\overline{p}]$, which requires $\gamma q^H + (1-\gamma)q^L \leq \kappa q^H - \frac{e}{\Delta}$.

We conclude that if $(1-\gamma)(q^H - q^L) \leq q^L(1-\kappa) + \frac{e}{\Delta}$ and $\gamma q^H + (1-\gamma)q^L \leq \kappa q^H - \frac{e}{\Delta}$, the candidate equilibrium exists. As we have shown above, it must be the unique equilibrium up to off-equilibrium beliefs.

Now consider the case where (8) holds. We have shown above that no pure-strategy equilibrium exists in this case. In the candidate equilibrium in period 1, the high-quality firm sets $\overline{p} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$ with probability 1 and obtains a good rating. The low-quality firm charges $\overline{p}$ with probability δ^* and gets no rating, and sets $\underline{p} = \kappa q^L - \frac{e}{\Delta}$ with probability $1-\delta^*$ and gets a rating. In period 2, all firms with a good rating charge $\frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}$, and firms without a rating charge q^L. Consumers' beliefs are as follows. In period 1, they believe

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}. \end{cases}$$

in the second period, beliefs are independent of prices and are

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

These beliefs follow Bayes rule on the path of play. The candidate equilibrium is also consistent with our restrictions. Because high-quality firms obtain a rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Because second period beliefs are independent of prices, and consumers have the same beliefs for all first period prices where the low-quality firm obtains no rating, the candidate equilibrium is consistent with Restriction 2.

We now show that firms have no profitable deviations.

We start with low-quality firms, who earn total profits $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)}$. The firm is indifferent between charging $\overline{p}$ and $\underline{p}$, where $\overline{p} > \underline{p}$. If the firm deviates to a price above $\overline{p}$, demand drops to zero and total profits are weakly below $0 + q^L$, this is not a profitable deviation. Deviations to a price $p_1 \in (\underline{p}, \overline{p})$, for which the firm gets the same rating as charging $\overline{p}$ and therefore earns the same profit in period 2, but the firm earns a lower margin than when it charges $\overline{p}$ without increasing demand in period 1, this is not a profitable deviation. Deviations to a price below $\underline{p}$ lead to the same rating as when charging $\underline{p}$ and therefore the same continuation profits, but decrease margins in period 1 without increasing demand, this is not a profitable deviation. In period 2, the low-quality firm extracts expected total surplus conditional on the rating, and cannot strictly increase profits. We conclude that low-quality firms have no profitable deviation.

We now show that high-quality firms have no profitable deviation. In the candidate equilibrium, high-quality firms earn $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)}$. In the second period, high.quality firms extract total surplus conditional on their good rating and therefore cannot profitably deviate. In the first period, a larger price reduces demand to zero and earns profits weakly below $0 + q^L$, which is not a profitable deviation. Deviating to a lower price does not improve the rating and therefore does not increase continuation profits, but reduces margins in period 1 without increasing demand, this is also not a profitable deviation. We conclude that high-quality firms have no profitable deviation.

We conclude that no firm has a profitable deviation.

Finally, we need to check $\overline{p} = \min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|\overline{p}]\} = E[q_1|\overline{p}]$, which requires $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} \leq \kappa q^H - \frac{e}{\Delta}$.

We conclude that if $(1-\gamma)(q^H - q^L) > q^L(1-\kappa) + \frac{e}{\Delta}$ and $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} \leq \kappa q^H - \frac{e}{\Delta}$, the candidate equilibrium exists. As we have shown above, it must be the unique equilibrium up to off-equilibrium beliefs.

This concludes the proof.

□

A.2 Proof of Proposition 2

Proof of Proposition 2.
In this proof, we show that $\frac{\partial \delta^*}{\partial \Delta} < 0$.

We know from (9) that $\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} + \frac{(4\gamma^2(q^H-q^L)^2 + (1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})}$, and that $\delta^* \in (\frac{1}{2}, 1)$ when $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}$. We can then calculate the derivative with respect to Δ, which leads to:

$$\frac{\partial \delta^*}{\partial \Delta} = \frac{e}{\Delta^2} \frac{\gamma(q^H - q^L)[2\gamma(q^H - q^L) - ((2\gamma(q^H - q^L))^2 + ((1+\gamma)(q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}}]}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})^2((2\gamma(q^H - q^L))^2 + ((1+\gamma)(q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}}} < 0$$

Thus we have shown that when low-quality firms would play a mixed strategy, as consumers kindness increases, low-quality firms are more likely to participate in ratings harvesting.

This concludes the proof.

□

A.3 Proof of Corollary 1

Proof of Corollary 1.
In this proof, we show that $\frac{\partial \delta^*}{\partial e} > 0$.

We know from (9) that $\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} + \frac{(4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})}$, and that $\delta^* \in (\frac{1}{2}, 1)$ when $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}$. We can then calculate the derivative with respect to e, which leads to:

$$\frac{\partial \delta^*}{\partial e} = \frac{\gamma(q^H - q^L)(((2\gamma(q^H - q^L))^2 + ((1+\gamma)(q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}} - 2\gamma(q^H - q^L))}{\Delta(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})^2((2\gamma(q^H - q^L))^2 + ((1+\gamma)(q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}}} > 0$$

Thus we have shown that when low-quality firms would play a mixed strategy, as the opportunity cost of rating increases, low-quality firms are less likely to participate in ratings harvesting.

This concludes the proof.

□

A.4 Proof of Proposition 3

Proof of Proposition 3.
We show $\frac{\partial \delta^*}{\partial \gamma} \leq 0$ when γ is sufficiently small and $\frac{\partial \delta^*}{\partial \gamma} > 0$ when γ is sufficiently large. We then characterise this switching point, $\overline{\gamma}$, and show its uniqueness and existence.

To start, note from (7), δ^* is such that low-quality firms are indifferent between charging low prices and large prices,

$$\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L = \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}.$$

Taking the derivative of the right-hand side leads to

$$\frac{\partial RHS}{\partial \gamma} = \frac{(q^H - q^L)((1-\delta^*) + (1-\gamma)\gamma\frac{\partial \delta^*}{\partial \gamma})}{(\gamma + (1-\delta^*)(1-\gamma))^2},$$

and for the left-hand side

$$\frac{\partial LHS}{\partial \gamma} = \frac{(q^H - q^L)(\delta^* - (1-\gamma)\gamma\frac{\partial \delta^*}{\partial \gamma})}{(\gamma + (1-\gamma)\delta^*)^2}.$$

Since both derivatives must be equal in equilibrium, we get

$$\frac{\partial \delta^*}{\partial \gamma} = \frac{(2\delta^* - 1)(\gamma^2 + \delta^{*2}(1-\gamma)^2 - \delta^*(1-\gamma)^2)}{\gamma(1-\gamma)((\gamma + (1-\delta^*)(1-\gamma))^2 + (\gamma + (1-\gamma)\delta^*)^2)}$$

In particular, we show that the sign of $\frac{\partial \delta^*}{\partial \gamma}$ switches at some point $\overline{\gamma}$. Since the denominator is strictly positive, the expression is negative if and only if the numerator is negative, i.e.

$$(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) < 0.$$

And positive when

$$(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) > 0,$$

and 0 if

$$(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) = 0.$$

Since $\delta^* > 0.5$, the when evaluating the sign of $(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta))$, we only need consider $(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta))$

We show that there is only one γ such that $(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) = 0$, therefore there is a unique switching point.

$$\overline{\gamma} = \frac{(q^H - q^L)^2 - (q^L(1-\kappa) + \frac{e}{\Delta})^2}{3(q^H - q^L) + (1-\kappa)^2 q^L + \frac{e}{\Delta}(2q^L(1-\kappa) + \frac{e}{\Delta})}$$

Finally, we show that if $\gamma < \overline{\gamma}$, then $\frac{\partial \delta^*}{\partial \gamma} < 0$, and $\frac{\partial \delta^*}{\partial \gamma} > 0$ when $\gamma \in (\overline{\gamma}, 1)$.

Notice that when $\gamma = 0$, this implies that $\delta^* = 1$. Thus $\delta^*(1-\delta^*) = 0$ and $\frac{\gamma^2}{(1-\gamma)^2} = 0$. And when $\gamma = 1$, $\delta^* = 1$. Further, when $\gamma = 1$, $\frac{\gamma^2}{(1-\gamma)^2} = \infty$.

We show that $\delta^*(1-\delta^*) > \frac{\gamma^2}{(1-\gamma)^2}$ when γ is sufficiently small. Consider that $\frac{\gamma^2}{(1-\gamma)^2}$ is strictly convex, with $\frac{\partial \frac{\gamma^2}{(1-\gamma)^2}}{\partial \gamma} = \frac{2\gamma}{(1-\gamma)^3} > 0$ and $\frac{\partial^2 \frac{\gamma^2}{(1-\gamma)^2}}{\partial \gamma^2} = \frac{2+4\gamma}{(1-\gamma)^4} > 0$. Evaluated at $\gamma = 0$,

$\frac{\partial \frac{\gamma^2}{(1-\gamma)^2}}{\partial \gamma} = \frac{2\gamma}{(1-\gamma)^3} = 0$. Further, consider that $\frac{\partial \delta^*(1-\delta^*)}{\partial \gamma} = \frac{\partial \delta^*}{\partial \gamma}(1-2\delta^*)$. Evaluated at $\gamma = 0$, $\delta^* = 1$ and $\frac{\partial \delta^*}{\partial \gamma} < 0$. Then, $\frac{\partial \delta^*(1-\delta^*)}{\partial \gamma} = \frac{\partial \delta^*}{\partial \gamma}(1-2\delta^*) > 0$. Hence, there exists a range of $\gamma \in (0,1)$ where $\delta^*(1-\delta^*) > \frac{\gamma^2}{(1-\gamma)^2}$ is satisfied. We label the first upper boundary (closest to 0) of this range as γ', such that $\gamma' \in [0,1]$ and when $\gamma \in (0,\gamma')$, $\delta^*(1-\delta^*) > \frac{\gamma^2}{(1-\gamma)^2}$ is satisfied and $\frac{\partial \delta^*}{\partial \gamma} < 0$.

Additionally, when $\gamma = 1$, $\delta^* = 1$ and $\frac{\gamma^2}{(1-\gamma)^2} = \infty$. This implies that $\frac{\gamma^2}{(1-\gamma)^2} > \delta^*(1-\delta^*)$ for some range of γ and $\frac{\partial \delta^*}{\partial \gamma} > 0$. This implies that at $\gamma = 1$, $\frac{\partial \delta^*(1-\delta^*)}{\partial \gamma} = \frac{\partial \delta^*}{\partial \gamma}(1-2\delta^*) < 0$. Therefore, we can conclude that there exists a range of $\gamma \in (0,1)$ where $\frac{\gamma^2}{(1-\gamma)^2} > \delta^*(1-\delta^*)$ is satisfied. We label the first lower boundary (closest to 1) of this range as γ'', such that $\gamma'' \in (0,1)$ and when $\gamma \in (\gamma'',1)$, $\frac{\gamma^2}{(1-\gamma)^2} > \delta^*(1-\delta^*)$ is satisfied and $\frac{\partial \delta^*}{\partial \gamma} > 0$.

Notice that by definition $\gamma'' \geq \gamma'$. We have shown before that there is a unique switching point, therefore $\overline{\gamma} = \gamma'' = \gamma'$.

We conclude that there is a unique switching point $\overline{\gamma}$, below which $\frac{\partial \delta^*}{\partial \gamma} < 0$, and above which $\frac{\partial \delta^*}{\partial \gamma} > 0$. Where $\frac{\partial \delta^*}{\partial \gamma} = 0$ only if $\gamma = \overline{\gamma}$.

□

A.5 Proof of Proposition 4

Proof of Proposition 4.
In this proof, we show that if a social planner concerned with the welfare of consumers would set an optimal e, e^{CS}.

First, we find consumer surplus. This is the sum of the difference between actual price and quality that consumers receive in each period.

$$\begin{aligned}
CS_1 =& \gamma[q^H - \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}] + (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}] + \\
& \delta^*(1-\gamma)[q^L - \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}] = (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}] \\
CS_2 =& \gamma[q^H - \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + (1-\delta^*)(1-\gamma)[q^L - \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \\
& \delta^*(1-\gamma)[q^L - q^L] = 0.
\end{aligned}$$

Therefore, consumer surplus arises only from the low-quality firm's attempt to receive a good rating, and total consumer surplus in our model is given by

$$CS = (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}]. \tag{10}$$

From (10) we can evaluate the effects of changes to effort cost.

First, we show that consumer surplus is concave in e.

$$\frac{\partial CS}{\partial e} = \frac{1}{2}[1-\gamma - \frac{(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})}{\sqrt{4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2}}]$$

This is positive if and only if

$$(1-\gamma)^2\gamma(q^H-q^L)^2 > (1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2$$

, and $\frac{\partial CS}{\partial e} = 0$ with equality. This condition is (3).

We now look at the second derivative.

$$\frac{\partial^2 CS}{\partial e^2} = -\frac{2\gamma^2(1+\gamma)^2(q^H-q^L)^2}{\Delta^2((2\gamma(q^H-q^L))^2+((1+\gamma)(q^L(1-\kappa)+\frac{e}{\Delta}))^2)^{\frac{3}{2}}} < 0$$

Second, we solve for the optimal level of effort required to leave a rating.

$$e = -(1-\kappa)q^L\Delta \pm \frac{\Delta(q^H-q^L)(1-\gamma)\sqrt{\gamma}}{1+\gamma}$$

We may reject the negative as we assume that $e \geq 0$. Therefore,

$$e^{CS} = -(1-\kappa)q^L\Delta + \frac{\Delta(q^H-q^L)(1-\gamma)\sqrt{\gamma}}{1+\gamma}$$

where e^{CS} is the level of effort cost that maximises consumer surplus. This e^{CS} is indeed positive when (3) holds. Which is the restriction required for $\frac{\partial CS}{\partial e} > 0$ at $e = 0$.

Therefore, for a planner maximizing the welfare of consumers, $e^{CS} = -(1-\kappa)q^L\Delta + \frac{\Delta(q^H - q^L)(1-\gamma)\sqrt{\gamma}}{1+\gamma}$.

This concludes the proof.

□

A.6 Proof of Corollary 2

Proof of Corollary 2.

In this proof, we show that in expectation, sellers prefer $e = 0$, and thus completely uninformative ratings. Because high-quality sellers prefer more informative ratings, we argue that the expected sellers' preference for uninformative ratings is driven by low-quality sellers.

First, we look at the average profit function of the firm,

$$\begin{aligned}
\pi = \gamma[&\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}]+ \\
&(1-\gamma)[(1-\delta^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \delta^*[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L]] \\
&= 2[\gamma q^H + (1-\gamma)q^L] - (1-\delta^*)(q^L(1-\kappa) + \frac{e}{\Delta})
\end{aligned}$$

Taking the derivative to e,

$$\begin{aligned}
\frac{\partial \pi}{\partial e} &= \frac{\partial \delta^*}{\partial e}[(1-\kappa)q^L + \frac{e}{\Delta}] - \frac{(1-\gamma)(1-\delta^*)}{\Delta} \\
&= -\frac{1}{2} + \frac{(1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})}{2(1-\gamma)\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2}}
\end{aligned}$$

and this is negative when (3) holds.

Therefore, on average firms prefer the smallest level of e, and the level of effort cost that maximises firm's profit is $e^s = 0$.

Second, we show that high-quality sellers prefer informative ratings.

$$
\begin{aligned}
\pi^H &= \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} \\
\frac{\partial \pi^H}{\partial e} &= -\frac{(1-\gamma)^2\gamma(1+\gamma)(q^H - q^L)(1-2\delta^*)\frac{\partial \delta^*}{\partial e}}{(\gamma + (1-\gamma)(1-\delta^*))^2(\gamma + (1-\gamma)\delta^*)^2}
\end{aligned}
$$

Recall that $\delta^* \in (\frac{1}{2}, 1)$, and $\frac{\partial \delta^*}{\partial e} > 0$ when (1) holds. Therefore, $\frac{\partial \pi^H}{\partial e} > 0$.

This means that when low-quality firms would play a mixed strategy, more informative ratings benefits high-quality firms.

We conclude that more informative ratings leads to an decrease in the average profits of the firm. This decrease in profits is driven by low-quality firms, and high-quality firms benefit from more informative ratings environments.

□

A.7 Proof of Proposition 5

Proof of Proposition 5.
We show that there is some γ^{CS} that maximises consumer surplus. To start, recall from Proposition 4 that total consumer surplus reduces to (10), i.e. $CS = (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}]$. We take the derivative of (10) with respect to γ and show that there is some $\gamma^{CS} < \overline{\gamma}$ that maximises consumer surplus.

$$
\frac{\partial CS}{\partial \gamma} = -(1 - \delta^* + (1-\gamma)\frac{\partial \delta^*}{\partial \gamma})(q^L(1-\kappa) + \frac{e}{\Delta})
$$

Since $q^L(1-\kappa) + \frac{e}{\Delta} > 0$, therefore, the sign of $\frac{\partial CS}{\partial \gamma}$ depends on $-(1 - \delta^* + (1-\gamma)\frac{\partial \delta^*}{\partial \gamma})$.

We solve for γ^{CS}, the optimal choice of γ that maximises consumer surplus.

$$
\gamma^{CS} = \frac{-(q^H - q^L)(q^L(1-\kappa) + \frac{e}{\Delta})^2 \pm ((q^H - q^L)^3(q^L(1-\kappa) + \frac{e}{\Delta})(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{1/2}}{(q^H - q^L)(4(q^H - q^L)^2 + (q^L(1-\kappa) + \frac{e}{\Delta})^2)}
$$

Because the denominator is positive, we reject the negative, as $\gamma \in (0,1)$. Therefore, we show that $\gamma^{CS} \in (0,1)$ exists if $(q^H - q^L)(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2 \geq (q^L(1-\kappa) + \frac{e}{\Delta})^3$.

We now show that $(q^H - q^L)(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2 \geq (q^L(1-\kappa) + \frac{e}{\Delta})^3$ always holds.

By applying $(q^H - q^L) > \gamma q^H + (1-\gamma)q^L - \kappa q^L + \frac{e}{\Delta} > q^L(1-\kappa) + \frac{e}{\Delta}$,

$$\begin{aligned}(q^H - q^L)(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2 &\geq (q^L(1-\kappa) + \frac{e}{\Delta})^3 \\ (q^H - q^L)(q^L(1-\kappa) + \frac{e}{\Delta})^2 &> (q^L(1-\kappa) + \frac{e}{\Delta})^3 \\ (q^H - q^L) &> (q^L(1-\kappa) + \frac{e}{\Delta})\end{aligned}$$

We now show that $\frac{\partial CS}{\partial \gamma} < 0$ when $\gamma > \gamma^{CS}$ and $\frac{\partial CS}{\partial \gamma} > 0$ when $\gamma < \gamma^{CS}$ and $\frac{\partial CS}{\partial \gamma} = 0$, $\gamma = \gamma^{CS}$. Otherwise, $\frac{\partial CS}{\partial \gamma} < 0$ for all γ.

First, we shall argue that $\gamma^{CS} < \overline{\gamma}$. Since $\overline{\gamma}$ solves $\frac{\partial \delta^*}{\partial \gamma} = 0$, and $\delta^* \in (1/2, 1)$. Therefore, $\frac{\partial CS}{\partial \gamma} < 0$. Since $(1-\delta^*) > 0$ for all $\gamma > 0$, at γ^{CS}, $\frac{\partial \delta^*}{\partial \gamma} < 0$. This implies that $\gamma^{CS} < \overline{\gamma}$.

Second, we show that when $\gamma > \gamma^{CS}$, $\frac{\partial CS}{\partial \gamma} < 0$. When $\gamma > \overline{\gamma}$, $(1-\delta^*) > 0$ and $(1-\gamma)\frac{\partial \delta^*}{\partial \gamma} > 0$. This implies that $\frac{\partial CS}{\partial \gamma} < 0$. Since the sign of $\frac{\partial CS}{\partial \gamma}$ changes at γ^{CS}, for any $\gamma > \overline{\gamma}$, $\frac{\partial CS}{\partial \gamma} < 0$.

This concludes the proof and we have shown that there exist some $\gamma^{CS} \in (0, \overline{\gamma})$ that maximises consumer surplus.

□

A.8 Proof of Lemma 1

Proof of Lemma 1.

To show that sellers of all types benefit from screening, we look at the lifetime profits of both high- and low- quality sellers independently.

We begin with low-quality sellers.

$$\pi^L = \delta^*(\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L) + (1-\delta^*)(\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)})$$

We know that δ^* is such that the low-quality firm is indifferent between $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L$ and $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}$.

Therefore,

$$\pi^L = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L$$
$$\frac{\partial \pi^L}{\partial \gamma} = \frac{(q^H - q^L)(\delta^* - \gamma(1-\gamma)\frac{\partial \delta^*}{\partial \gamma})}{(\gamma + \delta^*(1-\gamma))^2}, > 0 \text{ when } \gamma < \gamma^{CS}.$$

And we conclude that low-quality sellers benefit when platforms screen for quality.

Moreover, we show that high-quality firms also benefit from screening.

$$\pi^H = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}$$
$$\frac{\partial \pi^H}{\partial \gamma} = \frac{32\gamma^2(q^H - q^L)^3(q^L(1-\kappa) + \frac{e}{\Delta})^2(\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2} - 2\gamma(q^H - q^L))}{(\gamma + \delta^*(1-\gamma))^2(\gamma + (1-\delta^*)(1-\gamma))^2}$$

Because $\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2} - 2\gamma(q^H - q^L) > 0$, it is immediate that $\frac{\partial \pi^H}{\partial \gamma} > 0$.

Therefore, both high- and low- quality firms benefit when platforms screen for seller quality.

This concludes the proof.

□

A.9 Proof of Proposition 6

Proof of Proposition 6.
To proof Proposition 6, we show that $\frac{\partial \pi^p}{\partial e} < 0$.

To begin we characterize the actions of users on either side of the platform. Afterwards, we look at the strategy of the profit-maximizing platform.

To begin we characterize when consumers join the platform. The consumers' value from joining the platform is $n_s u_b$, i.e. the consumer gets expected surplus u_b from each interaction with a seller, and there are n_s sellers in total. Consumers' outside option is uniformly distributed on $[0, 1]$, which is why buyer demand is given by $n_b^* = n_s^* u_b$.

Next, we consider the firms. Since firms are ex-ante homogeneous, they have the same ex-ante expected revenue per transaction, π^s. Firms also face the same commission fee, r, which is set by the platform. Additionally, all firms face a marginal cost t of selling on the platform. Therefore the per-transaction profit for firms is $\pi^s(1-r)-t$. Since all firms face the same cost of entry $\bar{v}_s$, then any firm whose total profits are weakly above the outside option joins the platform. This means that $n_b^*(\pi^s(1-r)-t) \geq \bar{v}_s$. Because all firms face the same decision, the number of firms to join the platform is either $n_s^* = 0$ or $n_s^* = 1$. If $n_s^* = 0$, there is no activity on the platform and the platform earns zero profits, which is not optimal. We conclude that $n_s^* = 1$. Note that this implies $n_b^* = u_b$.

We now turn our attention to the platform. Because sellers are homogeneous and $n_s^* = 1$, the profit-maximizing platform extracts the highest possible benefit from the royalty fee subject to sellers participating. This implies that the platform sets the optimal r^* such that

$$n_b^*(\pi^s(1-r^*)-t) = \bar{v}_s \Leftrightarrow r^* = 1 - \frac{\bar{v}_s}{n_b^*\pi^s} - \frac{t}{\pi^s}.$$

Using this, we can simplify the platform's profits to

$$\pi^p = n_b^*\pi^s r^* = n_b^*\pi^s - \bar{v}_s - tn_b^* = n_b^*(\pi^s - t) - \bar{v}_s.$$

Now, we consider the effects of the ratings environment on the profits of the platform. To see this, we need to understand how the platform's profits are affected by changes to effort cost,

$$\frac{\partial \pi^p}{\partial e} = \frac{\partial u_b}{\partial e}(\pi^s - t) + \frac{\partial \pi^s}{\partial e} u_b. \tag{11}$$

To understand the platform's strategy, we evaluate $\frac{\partial \pi^p}{\partial e}$. To do so, we first show that $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi^s}{\partial e}$.

Consider first u_b. This is the transaction benefit of each consumer. Because consumers purchase first or second with equal probability, their ex-ante expected benefit per transaction is $u_b = \frac{1}{2}CS$, where we know from (10) that $CS = (1-\delta^*)(1-\gamma)[(1-\kappa)q^L + \frac{e}{\Delta}]$. Thus,

$$\frac{\partial u_b}{\partial e} = \frac{1}{2}\frac{(1-\gamma)(1-\delta^*)}{\Delta} - \frac{1}{2}\frac{\partial \delta^*}{\partial e}[(1-\kappa)q^L + \frac{e}{\Delta}].$$

Next, consider π^s. This is the per transaction profit of firms before taking into account the commission fee of the platform. Ex-ante, this is equivalent to the expected revenue that the firms receives per consumer, i.e.

$$\begin{aligned}\pi^s =&\frac{1}{2}\gamma[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}]+ \\ &\frac{1}{2}(1-\gamma)[(1-\delta^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \delta^*[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L]] \\ =&[\gamma q^H + (1-\gamma)q^L] - \frac{1}{2}(1-\delta^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})\end{aligned}$$

Taking the derivative,

$$\frac{\partial \pi^s}{\partial e} = \frac{1}{2}\frac{\partial \delta^*}{\partial e}[(1-\kappa)q^L + \frac{e}{\Delta}] - \frac{1}{2}\frac{(1-\gamma)(1-\delta^*)}{\Delta}.$$

Thus we conclude that $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi^s}{\partial e}$.

Returning to equation (11), if $\frac{\partial \pi^p}{\partial e} < 0$, the platform designs a ratings system such that it minimizes the effort cost associated with rating.

$$\frac{\partial \pi^p}{\partial e} < 0 \Leftrightarrow \frac{\partial u_b}{\partial e}(\pi^s - t) + \frac{\partial \pi^s}{\partial e}u_b < 0 \Leftrightarrow \frac{\frac{\partial u_b}{\partial e}}{u_b} < -\frac{\frac{\partial \pi^s}{\partial e}}{(\pi^s - t)}.$$

From this formulation, we see that the platform's design decision depends on the relative elasticity of consumer transaction surplus and firms transaction revenue. In particular, since $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi^s}{\partial e}$,

$$\frac{\frac{\partial u_b}{\partial e}}{u_b} < \frac{\frac{\partial u_b}{\partial e}}{(\pi^s - t)} \iff (\pi^s - t) < u_b$$

implies that $\frac{\partial \pi^p}{\partial e} < 0$.

We now show when this condition can be satisfied. First, we show that $\frac{\partial u_b}{\partial e}$ may be either positive or negative. Second, depending the sign of $\frac{\partial u_b}{\partial e}$, we provide the conditions for which

$\frac{\frac{\partial u_b}{\partial e}}{u_b} < \frac{\frac{\partial u_b}{\partial e}}{(\pi^s - t)}$ is satisfied.

Recall equation (7) and take the derivative with respect to e,

$$\frac{\partial u_b}{\partial e} = \frac{1}{4}[1 - \gamma - \frac{(1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})}{\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2}}]$$

From equation (3), this is positive if and only if

$$(1-\gamma)^2\gamma(q^H - q^L)^2 > (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2$$

and this is true whenever $e \in [0, e^{CS})$.

We now provide the conditions for which $(\pi^s - t) < u_b$ is satisfied.

Assuming that $\frac{\partial u_b}{\partial e} \geq 0$ ((3) holds weakly), then

$$\pi^s - t < u_b \Leftrightarrow \frac{\gamma q^H + (1-\gamma)q^L - t}{(1-\kappa)q^L + \frac{e}{\Delta}} < (1-\delta^*)(1-\gamma). \tag{12}$$

We conclude that when (12) holds platforms will minimize the effort costs required to leave a rating.

We have provided the condition for which platforms are incentivised to minimize effort costs associated with rating. This concludes the proof.

□

A.10 Proof of Proposition 7

To proof Proposition 7, we first need to show some lemmas. First, Lemma 4 guides when the competitive fringe may be considered to be active in the market. Second, Lemma 5 shows the adjusted pricing strategies of strategic firms. Finally, Lemma 6 characterizes the adjusted consumer's beliefs in the presence of the competitive fringe. We then use these results to prove Proposition 7.

Lemma 4. *The competitive fringe is only relevant whenever $c < q^j$.*

Proof of Lemma 4.
Whenever $c > q^j$, consumers receive negative utility from the competitive fringe. Whenever $c = q^j$, consumers receive 0 utility. Since the strategic firms provide at least non-negative utility, by assumption they would receive the full consumer demand. Therefore, the fringe only captures consumers if $c < q^j$.

This concludes the proof.

$\square$

Lemma 5. *Pricing strategy of the high-quality firm:*

- *Period 1:* $p_1'^H = E[q_1|R_0, p_1^H] - (q^j - c)$
- *Period 2:* $p_2'^H = E[q_2|R_1] - (q^j - c)$

Pricing strategy of the low-quality firm:

- *Period 1:*
  - $R_1 = 1$, $p_{1,1}'^L = min\{q^L - (q^j - c), \kappa q^L - \frac{e}{\Delta}\}$
  - $R_1 = 0$, $p_{1,0}'^L = p_1'^H$
- *Period 2:* $p_2'^L = E[q_2|R_1] - (q^j - c)$

Proof of Lemma 5.
To prove this, we first discuss the tie breaker rule. Second, we consider the second period as this is straight forward. Then we consider the high-quality in the first period, followed by the low-quality in the first period. The proofs are similar to that of Lemma 2.

Our tie breaker rule is such that whenever both the strategic firm and the fringe provide the same level of consumer surplus, consumers choose to purchase from the strategic firm. Here, we shall argue why this tie breaker rule is a simplification that yields virtually identical results to a tie breaker rule where consumers randomise between the two firms.

By construction, the fringe sets some fixed price c, while the strategic firm selects prices. Therefore, for any situation where the strategic firm and fringe provide the same level of consumer surplus, the strategic firm can always choose to set prices some small $\epsilon > 0$ lower, such that they obtain the full demand. Therefore, by assuming this tie breaker rule, we are able to simplify our discussion without considering the need for some small ϵ deviation.

We now turn our attention to the discussion of equilibrium prices in the second period.

In the second period, consumers are aware that consumption from the fringe will leave them a surplus of $q^j - c$. This forms the outside option for consumers. Strategic firms therefore have to provide consumers with a surplus of at least $q^j - c$, doing so will shift all the demand towards the strategic firm. We conclude that the firm will not set prices lower than $E[q_2|R_1] - (q^L - c)$.

We now turn our attention to the high-quality firm in the first period. As in Lemma 2, a high-quality firm wishing to obtain a positive rating must set prices no higher than $\kappa q^H - \frac{e}{\Delta}$. As in the proof for prices in the second period, consumers are aware that consumption of the outside good will provide $q^j - c$ level of utility. Therefore, firms must set prices no higher than $E[q_1|R_0, p_1] - (q^j - c)$. Because we show in the proof of Proposition 8 that we must have $min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_1^H]\} = E[q_1|R_0, p_1^H]$ in equilibrium, high-quality firms will continue to receive a good rating at $E[q_1|R_0, p_1] - (q^j - c)$. It follows that the equilibrium price is $p_1'^H = E[q_1|R_0, p_1^H] - (q^j - c)$.

We now look at the low-quality firm in period 1. When the low-quality firm prefers to obtain a good rating, it has to set a price no higher than $\kappa q^L - \frac{e}{\Delta}$. With the presence of the competitive fringe, consumers are aware that they would be able to receive at least $q^j - c$ of surplus. Therefore, in order for the low-quality firm to capture demand and get a good rating, they are unable to command prices higher than $q^L - (q^j - c)$. Thus, the equilibrium price is $p_{1,1}'^L = min\{c, \kappa q^L - \frac{e}{\Delta}\}$.

For the low-quality firm receiving no rating, it sets prices above $\kappa q^L - \frac{e}{\Delta}$. The profit maximizing firm will set the highest possible price at which consumers buy. For the same argument as in the proof of Lemma 2, low-type firms do not set prices higher than the high-quality firm. Therefore, when low-quality firms receive no rating, it sets prices equal to that of the high-quality firm.

This concludes the proof.

□

Lemma 6. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \textit{if } p_1 > \overline{p_1^L} \\ q^L & \textit{if } p_1 \leq \overline{p_1^L}, \end{cases}$$

and in the second period,

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \textit{if } R_1 = 1 \\ q^L & \textit{if } R_1 = 0 \end{cases}$$

Proof of Lemma 6.
The proof is identical to that of Lemma 3. □

Proof of Proposition 7.
We now proof Proposition 7.

First, we discuss the possible cases for how the competitive fringe might affect the behavior of strategic firms. Then we consider these cases one at a time and find the effect that changes in c have on the mixed strategy of the low-quality firm.

Since we know that the second period prices are equally affected by the competitive fringe in all cases, we turn our attention to the first period. Notice that in period 1, the high-quality firm sets a single price, and the low-quality firm sets two out of three possible prices, i.e. $q^L - (q^j - c)$, $\kappa q^L - \frac{e}{\Delta}$, or $p_1'^H$. Further, notice that $\overline{p_1^H} > \overline{p_1^L}$ and $E[q_1|p_1'^H] - (q^j - c) > q^L - (q^j - c)$.

We know from the proof of Proposition 8 that in equilibrium we have $\overline{p_1^H} > E[q_1|p_1'^H]$, this leaves us with the following possible scenarios:

$$1. \overline{p_1^H} > E[q_1|p_1'^H] - (q^j - c) > \overline{p_1^L} > q^L - (q^j - c)$$
$$2. \overline{p_1^H} > E[q_1|p_1'^H] - (q^j - c) > q^L - (q^j - c) \geq \overline{p_1^L}$$

We consider the scenarios individually.

1. $\overline{p_1^H} > E[q_1|p_1'^H] - (q^j - c) > \overline{p_1^L} > q^L - (q^j - c)$

Because $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta} > c$, we know from Lemma 5 that the low-quality firm is indifferent between setting prices $p_1'^H$ and $q^L - (q^j - c)$. When charging $p_1'^H$, the firm gets no rating and charges $q^L - (q^j - c)$ in period 2. When charging $q^L - (q^j - c)$, the firm gets a good rating and earns the expected quality of a firm with a good rating in period 2 minus $(q^j - c)$. This leads to the following condition.

$$
\begin{aligned}
&E[q_1|p_1'^H] - (q^j - c) + q^L - (q^j - c) = q^L - (q^j - c) + E[q_2|R_1] - (q^j - c) \\
\Leftrightarrow &\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} = \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} \\
\Leftrightarrow &\delta^* = 0.5,
\end{aligned}
$$

which is independent of c.

This concludes case 1.

2. $\overline{p_1^H} > E[q_1|p_1'^H] - (q^j - c) > q^L - (q^j - c) \geq \overline{p_1^L}$

Because $q^L - (q^j - c) \geq \overline{p_1^L}$, and we know from Lemma 5 that the low-quality firm is indifferent between setting prices $p_1'^H$ and $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta}$. To be indifferent between these prices, the following condition must hold. The left-hand side is as in the previous case. On the right-hand side, the firm charges $\kappa q^L - \frac{e}{\Delta}$ in period 1 and obtains a good rating. In period 2, it earns the expected quality of a firm with good rating in period 2 minus $(q^j - c)$. This leads to the following condition.

$$
\begin{aligned}
&E[q_1|p_1'^H] - (q^j - c) + q^L - (q^j - c) = \overline{p_1^L} + E[q_2|R_1] - (q^j - c) \\
\Leftrightarrow &\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} - q^j + c = \kappa q^L - \frac{e}{\Delta} + \frac{\gamma(q^H - q^L)}{\gamma + (1-\delta^*)(1-\gamma)}.
\end{aligned}
$$

Using the implicit-function theorem then leads to

$$
\frac{\partial \delta^*}{\partial c} = \frac{(\gamma + (1-\delta^*)(1-\gamma))^2(\gamma + \delta^*(1-\gamma))^2}{\gamma(1-\gamma)(q^H - q^L)[(\gamma + (1-\delta^*)(1-\gamma))^2 + (\gamma + \delta^*(1-\gamma))^2]} > 0.
$$

This concludes case 2.

We conclude that $\frac{\partial \delta^*}{\partial c} > 0$ if $q^L - (q^j - c) \geq \kappa q^L - \frac{e}{\Delta}$.

□

A.11 Negative Ratings

Proposition 9. *There exists a range of prices for which good, no and bad ratings may occur. In equilibrium, it is always beneficial for low-quality firms to obtain a bad rating over no rating, if consumers continue to buy at that price.*

We can proof this proposition using the following two Lemmas

Lemma 7. *Consumers obtain the same signal from a negative rating and no rating.*

Proof of Lemma 7.
This proof holds directly from Restriction 1. Consumers' belief is such that high-quality firms always get a good rating. Hence, on observing no or bad ratings, they would believe this to be obtained by low-quality firms.

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \\ q^L & \text{if } R_1 = -1 \end{cases}$$

where δ^* continues to represent the probability that low-quality firms get no rating.

This concludes the proof.

□

Lemma 8. *When low-quality firms choose not to receive a good rating, it prefers to receive a bad rating over no rating, if consumers continue to buy at the price level that induces a bad rating.*

Proof of Lemma 8.
First, we discuss the prices set in the second period. Then we look at consumer's beliefs in the first period and the prices set in the first period. We then show that in equilibrium, there exist some price for which low-quality firms receive no rating, and another price where they receive bad ratings.

When looking at the prices set in the second period, notice first that firms of all types will set the highest possible price in the second period. This is equivalent to consumer's expectation in the second period. Since this is only dependent on ratings, on receiving a good rating in the first period, firms set $\frac{\gamma q^H+(1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)}$; on receiving no rating, they set a price of q^L; on receiving a bad rating, they also set a price of q^L.

Now, consider the possible prices in the first period. In the first period, high-quality firms will set a price $p_1^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_1^H]\}$ and receives a good rating. The proof of this is identical to the proof of high-quality firm's prices in Lemma 2.

Low-quality firm's do one of the following: set a price such that it receives a good rating, no rating or a bad rating.

When a low-quality firm receives a good rating, it sets a unique price and this is $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$. This proof is identical to that of Lemma 2 when low-quality firms receive a good rating.

We turn our attention to the situation when low-quality firms receive a bad rating. When a low-quality firm receives a bad rating, this only occurs if $[\kappa q^L - p_1^L]\Delta' - e \geq 0$. Further, bad ratings imply that consumers are exhibiting negative reciprocity. Therefore, we define $\Delta' = -\Delta$. And consumer's leave a bad rating whenever $p_1^L \geq \kappa q^L + \frac{e}{\Delta}$, gaining a positive rating utility from doing so. From Restriction 2, we know that since this price is larger than $\overline{p_1^L}$, consumers expectations are fixed and from Lemma 2, we know that the firm set the highest possible price. Therefore, when obtaining a bad rating, the low-quality firm sets the price $p_1^L = E[q_1|R_0, p_1^H]$.

Now we turn our attention to the situation when low-quality firms receive no rating. This occurs when $[\kappa q^L - p_1^L]\Delta' - e < 0$. That is the rating utility from leaving a good or bad rating is negative. Recall that the utility from giving no rating is 0.

Suppose that the consumer considers between leaving a good rating and no rating. By reciprocity, the consumer acts with a kindness of their own, implying that $\Delta' = \Delta > 0$. Therefore, no rating only occurs if $[\kappa q^L - p_1^L] < \frac{e}{\Delta}$. In other words, $p_1^L > \kappa q^L - \frac{e}{\Delta}$.

Now suppose that the consumer considers between leaving a bad rating and no rating. By reciprocity, the consumer acts negatively, implying that $\Delta' = -\Delta < 0$. Therefore, no rating only occurs if $p_1^L < \kappa q^L + \frac{e}{\Delta}$ and $p_1^L > \kappa q^L - \frac{e}{\Delta}$ hold together.

Notice that $\kappa q^L - \frac{e}{\Delta} < \kappa q^L + \frac{e}{\Delta}$. This implies that there exist a range of prices, $p_1^L \in (\kappa q^L - \frac{e}{\Delta}, \kappa q^L + \frac{e}{\Delta})$ such that consumers maximising their utility provide no ratings to low-quality firms.

In equilibrium, the total profit of a low-quality firm obtaining a bad rating is $\frac{\gamma q^H+\delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)} + q^L$, while the total profit of obtaining no rating is strictly below $\kappa q^L + \frac{e}{\Delta} + q^L$.

We can now show that low-quality firms set some unique price in equilibrium, when they choose not to obtain a good rating. In equilibrium, when consumers observe price of p_1^H, they anticipate a utility of $\frac{\gamma q^H+\delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)}$. If this is above $\kappa q^L + \frac{e}{\Delta}$, low-quality firms maximise their profits by extracting the full surplus from consumers, and obtaining a bad rating. Alternatively, if the anticipated utility is below $\kappa q^L + \frac{e}{\Delta}$, it is not profitable for firms to set $\kappa q^L + \frac{e}{\Delta}$ as this would lead to zero demand. Instead, the low-quality firm sets p_1^H and obtains no rating. Therefore, when choosing not to obtain a good rating, low-quality firms always sets the unique price mimicking price of p_1^H.

We conclude that when we allow for negative ratings, there exists a range of prices for which good, bad, and no ratings may occur. However, in equilibrium, a profit maximising low-quality firm mixes between setting a price that allows it to harvest ratings and price mimicking. This allows it to obtain the extreme ratings of no, or negative rating depending on the cost of leaving a rating.

This concludes the proof.

□

Together, Lemma 7 and Lemma 8 show that in the equilibrium, negative ratings will replace no ratings when going from a system of 2 ratings options to 3 ratings options, and consumers continue to buy at the price level that induces a negative rating.

For completeness, suppose that consumers face a different warm glow when leaving a good rating (Δ_g) and a bad rating (Δ_b). Moreover, assume that $\Delta_g > \Delta_b$ such that the warm glow from leaving a good rating is larger than that of leaving a bad rating. This is consistent with the argument made by Dellarocas and Wood (2008), and allows us to explain the J-shaped ratings found in the empirical literature (Hu et al., 2009).

Lemma 9. *When consumers are more likely to exhibit positive reciprocity than negative reciprocity, firms are more likely to obtain good ratings, and less likely to obtain bad ratings.*

Proof of Lemma 9.
To complete this proof, first consider the possible prices played by the high-quality firm. This is unchanged and follows directly from the Proof of Lemma 8. Second, we now consider the possible prices played by the low-quality firm.

When receiving a good rating, the low-quality firm sets a unique price and this is given by $p_{1,1}^L = \overline{p_1^L} = \kappa q^L - \frac{e}{\Delta_g}$. This proof is identical to that of Lemma 2 when the low-quality firm receives a good rating.

Next, we turn our attention to the low-quality firm receiving a bad rating. This only occurs if $[\kappa q^L - p_1^L](-\Delta_b) - e > 0$. As in Lemma 8 consumers leaving a bad rating exhibit negative reciprocity, this is why the warm glow is negative, and negative ratings occur if $p_1^L > \kappa q^L + \frac{e}{\Delta_b}$.

Notice that applying the same logic as in Lemma A.11, there exists a range of prices such that no rating occurs, and this is $p_1^L \in (\kappa q^L - \frac{e}{\Delta_g}, \kappa q^l + \frac{e}{\Delta_b})$.

This has two implications. First, as Δ_g increases, and consumers exhibit more positive reciprocity, then low-quality sellers may set higher prices and still continue to receive a good rating. I.e. the range of price for which low-quality firms may receive good ratings is larger.

Second, as Δ_b decreases, and consumers exhibit less negative reciprocity, then low-quality sellers only receive a bad rating if they set a higher price. However, as this runs up against the upper bound of prices that they may set, prices must be less than consumers willingness to pay, firms are unlikely to receive bad ratings.

This concludes the proof. □

A.12 Continuum of quality

Proposition 10. *Firms with quality $q \in [\hat{q}, 1]$ obtain a good rating in equilibrium, without manipulating prices. Firms with quality $q \in [q'', \hat{q})$ choose to participate in ratings harvesting in equilibrium. Firms with quality $q \in [0, q'')$ do not obtain a rating in equilibrium.*

Restriction 3. *The highest quality firm receives a good rating with probability* 1

Proof of Proposition 10.
To begin, we first look at the second period. In period 2, conditional on observing a good rating, consumers form expectations and are willing to pay

$$\overline{p}_2 = \int_0^1 (1 - \delta^*(q))q dq.$$

Here, we define δ^* as the probability that a firm receives no rating.

And conditional on observing no rating, consumers are willing to pay

$$\underline{p}_2 = \int_0^1 \delta^*(q) q dq.$$

Since period 2 is a terminal period there is no more strategic pricing and firms have no incentive to obtain a good rating in period 2. Therefore, in period 2, firms set prices equal to consumers expected quality and consumers choose to buy. Firms are able to extract the full surplus from consumers.

We now turn our attention to period 1. First we show that no firm can set a price larger than the highest quality firm. Second, we show that there exist a group of high quality firms with quality larger than $\hat{q}$ that receive a good rating with probability 1. Third, we show that there exist a group of low quality firms with quality lower than $\overline{q}$ that receive a good rating with probability 1.

To show the highest possible price, we first look at the price that the highest quality firm chooses to set. From the modified restriction 1, Restriction 3, the highest quality firm, with $q = 1$, sets a price weakly below $\kappa - \frac{e}{\Delta}$. At this price, consumers are willing to pay, and the firm receives a good rating with probability 1.

We now show that no firm sets a price above the price that the highest quality firm sets, $p(q = 1)$. Suppose there is some price $p > p(q = 1)$, and all firms setting the price p receive no rating. Firms that receive no rating receive a payoff of $\underline{p}_2$ in period 2. Firms may choose to get a good rating with some probability, and to do so, they set a price $\kappa q - \frac{e}{\Delta}$ and obtain a good rating, allowing them to receive a payoff of $\overline{p}_2$ in period 2. The firm that benefits the most from receiving a good rating is the lowest quality firm of $q = 0$. Supposing that only the highest quality firm receives a good rating, for the firm $q = 0$, they choose to receive no rating if $p + 0.5 \geq -\frac{e}{\Delta} + 1$ and because $p > p(q = 1)$, $p + 0.5 > p(q = 1) + 0.5 \geq -\frac{e}{\Delta} + 1$. Therefore, for $p(q = 1) + \frac{e}{\Delta} \geq 0.5$, the lowest quality firm chooses to set the higher price p and obtain no rating. $p(q = 1) + \frac{e}{\Delta} \geq 0.5$ being the sufficient condition for the lowest quality firm to prefer no rating. Supposing $p(q = 1) + \frac{e}{\Delta} \geq 0.5$ (note that this means $\kappa > 0.5$ is a sufficient condition) and applying the D1 criterion, consumers believe that any firm that sets p is of quality $q = 0$, and choose not to buy the product. Hence, demand at any $p > p(q = 1)$ is zero. Knowing this, no firm sets p. We conclude that the highest price that can be set is the price that the price that the highest quality firm sets.

We now show that there exists a group of high quality firms that sets the price $p(q = 1)$ and obtains a good rating with probability 1. Suppose that the highest quality firm is the only firm that receives a good rating. This means that $p(q = 1) = \kappa - \frac{e}{\Delta}$. However, all other firms can choose to set $\kappa - \frac{e}{\Delta}$ in the first period and obtain $\underline{p}_2 = \int_0^1 \delta^*(q)qdq$ in the second period. Alternatively, they set the price $\kappa q - \frac{e}{\Delta}$ in period 1 and receive $\overline{p}_2 = \int_0^1 (1 - \delta^*(q))qdq$ in period 2. Therefore, for all firms where $\kappa(1 - q) \geq \int_0^1 (1 - 2\delta^*(q))qdq$, these firms choose to set $\kappa - \frac{e}{\Delta}$ with some positive probability. This means that in expectation, consumers choose not to purchase a product at $\kappa - \frac{e}{\Delta}$. Therefore, in order to receive a positive demand and obtain a positive rating, highest quality firm lower the price to some $p(q=1) < \kappa - \frac{e}{\Delta}$. Recall that no firm can set a price larger than the highest quality firm. Thus, for some firm $\hat{q}$, $p(q=1) = \kappa\hat{q} - \frac{e}{\Delta}$, and all firms with quality larger than $\hat{q}$, 'naturally' receive a good rating. This means, in equilibrium, they do not manipulate prices in order to obtain a good rating. We call these firms the top firms.

Therefore, we now know that in equilibrium, top firms with quality $q \geq \hat{q}$ play $\delta^*(q) = 0$. And $\overline{p}_2 = \int_{\hat{q}}^1 qdq + \int_0^{\hat{q}}(1-\delta^*(q))qdq$ and $\underline{p}_2 = \int_0^{\hat{q}} \delta^*(q)qdq$. We now know that $\hat{q}$ is the lowest quality firm that 'naturally' receives a good rating with probability one. This implies that $\kappa\hat{q} - \frac{e}{\Delta} = q'$, where q' is the highest price for which consumers are willing to buy. This means that $q' = \int_{\hat{q}}^1 qdq + \int_0^{\hat{q}} \delta^*(q)qdq$.

We now look at what non-top firms do.

For all firms with quality $q < \hat{q}$, they choose $\delta^*(q)$ such that they maximise

$$\pi(q) = \delta(q)[q' + \underline{p}_2] + (1-\delta(q))[\kappa q - \frac{e}{\Delta} + \overline{p}_2]$$

When firms choose $\delta(q)$, and considering that $q' = \int_{\hat{q}}^1 qdq + \int_0^{\hat{q}} \delta^*(q)qdq = \kappa\hat{q} - \frac{e}{\Delta}$,

$$\begin{aligned}
\frac{\partial\pi(q)}{\partial\delta(q)} &= q' + \underline{p}_2 - \kappa q + \frac{e}{\Delta} - \overline{p}_2 \\
&= \kappa\hat{q} - \frac{e}{\Delta} - \kappa q + \frac{e}{\Delta} + \int_0^{\hat{q}} \delta^*(q)qdq - \int_{\hat{q}}^1 qdq - \int_0^{\hat{q}}(1-\delta^*(q))qdq \\
&= \kappa(\hat{q} - q) - \frac{1}{2} + 2\int_0^{\hat{q}} \delta^*(q)qdq \\
&= \kappa(3\hat{q} - q) - \frac{3}{2} - 2\frac{e}{\Delta} + \hat{q}^2
\end{aligned}$$

Setting $\frac{\partial\pi(q)}{\partial\delta^*(q)} = 0$, we obtain the cutoff quality level above which firm's strictly prefer $\delta^*(q) = 0$, and below which $\delta^*(q) = 1$. We label this cutoff as q'', and $q'' = 3\hat{q} + \frac{\hat{q}^2 - \frac{3}{2} - \frac{2e}{\Delta}}{\kappa}$. Therefore, we conclude that for any firm with quality $q \in [q'', \hat{q}]$, firms choose to manipulate prices to obtain a good rating, and for any firm with quality $q \in [0, q'')$, they obtain no rating in equilibrium.

This implies that for any firm of a sufficiently low quality, they choose to play q' and receive no rating. We call these firms the bottom firms. Further, any firm $q \in [q'', \hat{q}]$, they choose to play $\kappa q - \frac{e}{\Delta}$ and obtain a good rating. We call these firms the middle firms.

We can now show that comparative statics of e are qualitatively the same. Reformulating $q' = \kappa\hat{q} - \frac{e}{\Delta}$, we get $\hat{q} = \frac{q'+\frac{e}{\Delta}}{\kappa}$. Using the envelope theorem, since $q' = \int_{\hat{q}}^{1} qdq + \int_{0}^{\hat{q}} \delta^*(q)qdq$, we do not need to consider the indirect effects. Therefore, we need only consider the direct effect that changes to e has on $\hat{q}$. $\frac{\partial\hat{q}}{\partial e} = \frac{1}{\Delta\kappa}$. Therefore, when it becomes easier to leave a rating, this causes the number of top firms to increase by $\frac{1}{\Delta\kappa}$. This means that the highest quality middle firms now become a top firm.

Next, consider what happens to q'' when e changes. Reformulating $q'' = 3\hat{q} + \frac{\hat{q}^2 - \frac{3}{2} - \frac{2e}{\Delta}}{\kappa}$, we get $q'' = \frac{3q'}{\kappa} - \frac{3}{2\kappa} + \frac{e}{\Delta\kappa} + \frac{(q'+\frac{e}{\Delta})^2}{\kappa^3}$. Again, applying the envelope theorem, we find that $\frac{\partial q''}{\partial e} = \frac{1}{\Delta\kappa} + \frac{2(q'+\frac{e}{\Delta})}{\Delta\kappa^3}$. This means that the highest quality bottom firms now join the group of middle firms.

We have shown that making ratings easier to leave, leads to a higher likelihood of a firm being in the group of middle firms. This is because the proportion of firms that obtain a good rating (sum of top and middle firms) increases by more than the proportion of firms that naturally obtain a good rating (top firms). This difference is $\frac{2(q'+\frac{e}{\Delta})}{\Delta\kappa^3}$. There is an increase in the number of firms that manipulate ratings. This leads to less informative ratings, and ratings inflation.

This concludes the proof, and we have shown that in a setting with a continuum of firm quality types and 2 ratings, top firms naturally obtain a good rating with probability 1, middle firms manipulate ratings through prices and obtain a good rating with probability 1, and bottom firms choose not to participate in ratings harvesting. We also show that our main effects regarding the cost of leaving ratings pass through in this setting. This is true when the sufficient condition of $\kappa > 0.5$ holds.

□

A.13 Multi-period model

Proposition 11. *A unique mixed strategy equilibrium with properties similar to the base model exists in a 3 period model.*

To show Proposition 11, we prove a more general proposition, Proposition 12.

Proposition 12. *All perfect Bayesian equilibria satisfy the following.*

1. *High-quality firms receive a good rating with probability 1 and charge* $p_t^H = E[q_t|R_{t-1}, p_t^H]$, $t \in \{1, 2\}$.

2. *Low-quality firms randomize their strategy in period* $t \in \{1, 2\}$.

a. *They charge* $\overline{p_t^L} = \kappa q^L - \frac{e}{\Delta}$ *and obtain a good rating with probability* $1 - \delta_t^*$.

b. *They charge* p_t^H *and obtain no rating with probability* δ_t^*.

3. *Firms set prices equal to expected quality conditional on ratings in the last period.*

4. *Consumer beliefs given equilibrium prices are given by:*

a. *In period 1,* $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L} \end{cases}$

b. *In period 2,* $E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \overline{p_2^L} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases}$

c. *In period 3,* $E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}$

Furthermore, we show that $\delta_1^* \in (0,1)$ *and* $\delta_2^* \in (0,1)$ *is an equilibrium if* $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$*, and* $\delta_1^* \in (0,1)$ *and* $\delta_2^* = 1$ *is an equilibrium if* $(1-\kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$*. These equilibria exist if* $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\gamma)(q^H - q^L)}{2 - \delta_2^*}$*,* $\gamma > \frac{1}{3}$*, and* $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$.

We proceed as follows. Before proving Proposition 12, we show two Lemmas. First, we use our restrictions to characterize firms' pricing strategies. Second, we pin down equilibrium beliefs. Finally, we use these results to show Proposition 12. Most of the arguments used in proof are similar to the ones from Proposition 1, which is why we briefly sketch them here.

We begin by pinning down the equilibrium prices in period 1 and 2 in Lemma 10.

Lemma 10. *In equilibrium, firms play the following prices in period 1 and 2 with weakly positive probability.*

- *High-quality firm:* $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$
- *Low-quality firm:*

$$p_{1,1}^L = p_{2,11}^L = \kappa q^L - \frac{e}{\Delta}$$

$$p_{1,0}^L = p_{2,10}^L = E[q_t|R_{t-1}, p_t^H]$$

$$p_{2,00}^L = p_{2,01}^L = q^L$$

Proof of Lemma 10.
The proof of Lemma 10 is similar to the proof of Lemma 2.

The key difference lies in the possible rating histories. Given the extension to 3 periods, the possible rating histories are now $R_0 \in \{\}, R_1 \in \{0, 1\}, R_2 \in \{00, 01, 10, 11\}$. As in the proof of Lemma 2, we will consider the pricing strategy of the high-quality firm followed by low-quality firm.

From Restriction 1, a high-quality firm wants to set prices which allows it to get a good rating and therefore in equilibrium her rating's history are $R_0 = \{\}, R_1 = \{1\}$ and $R_2 = \{11\}$. We show that on the equilibrium path $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$ for $t \in \{1, 2\}$.

To show that the high-quality firm sets a unique price in each period $t \in \{1, 2\}$ with probability 1, we follow the proof set out in Lemma 2. Suppose towards a contradiction that the high-quality firm sets more than one price with positive probability. Without loss of generality, suppose that the high-quality firm sets a continuous distribution of prices in either period, i.e. it charges prices in some interval $p_t \in [p'_t, p''_t]$ such that $p''_t > p'_t$, and the high-quality firm receives a good rating with probability 1 for all $p_t \in [p'_t, p''_t]$. Note that when the high-quality firm gets a rating for all $p_t \in [p'_t, p''_t]$, consumers purchase products with probability 1. Notice that for any price $\hat{p}_t > p_t$ such that $\hat{p}_t \in [p'_t, p''_t]$, we have $\pi_t^H(\hat{p}_t) > \pi_t^H(p_t)$. The reason is that both prices induce the same demand in period t, and the same rating and therefore the same continuation profits. Thus, the firm can strictly increase profits by shifting all probability mass from $[p'_t, p''_t]$ to p''_t. This contradicts the assumption that the high-quality firm sets a uncountably infinite prices with positive probability. Similarly, the firm will not set countably finite or infinite prices that induce the same rating. Therefore, we conclude that the high-quality firm sets a unique price in each period t on the equilibrium path with probability 1. We denote this as p_t^H.

Next, we show that there exist an upper bound on prices, $\overline{p_t^j}$ for $j \in \{L, H\}$ such that j-quality firm receives a positive rating. In order for a firm to induce a positive rating, the rating utility must be positive. Therefore,

$$[\kappa q^j - p_t^j]\Delta - e \geq 0 \iff p_t^j \leq \overline{p_t^j} \equiv \kappa q^j - \frac{e}{\Delta}$$

Finally, consider that prices are bound by consumer's beliefs, $E[q_t|R_{t-1}, p_t^H] < \overline{p_t^H}$. Under such scenarios, by Restriction 1, high-quality firms prefer obtaining a good rating. This can only be achieved if consumers buy. Therefore, p_t^H is bound by $E[q_t|R_{t-1}, p_t^H]$.

We now show that $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$ with probability 1. As with Lemma

2, note that $\overline{p_t^H} > \overline{p_t^L}$ and $q^L > \overline{p_t^L}$. Thus, because for equilibrium expectations we have $E[q_t|R_{t-1}, p_t^H] > q^L$, the high-quality firm sets equilibrium prices strictly larger than $\overline{p_t^L}$. Applying Restriction, 2, consumers have the same beliefs for all prices strictly above $\overline{p_t^L}$ in each period t. Since $p_t^H > \overline{p_t^L}$, these beliefs are given by $E[q_t|R_{t-1}, p_t^H]$, the correct equilibrium beliefs. Further, since consumers have the same beliefs for all prices above $\overline{p_t^L}$ in each period, the high-quality firm optimally sets the largest possible price at which consumers purchase and rate with probability 1. Hence, $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$. We conclude that high-quality firms set this unique price, $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$, with probability 1.

We turn our attention to low-quality firms. First, we show that the price which the low-quality firm sets when it receive a good rating in any period, $t \in \{1, 2\}$, is unique. And this price is $p_{1,1}^L = p_{1,11}^L = \kappa q^L - \frac{e}{\Delta}$. The argument here is the same as the argument used for when high-quality firms receive a good rating. The low-quality firm receives a good rating at any price weakly below $\overline{p_t^L} = \kappa q^L - \frac{e}{\Delta}$, and $\overline{p_t^L} < q^L$. As consumers beliefs are weakly above q^L, they buy at any price weakly below $\overline{p_t^L}$. Since demand and ratings are the same for all prices weakly below $\overline{p_t^L}$, a low-quality firm obtaining a positive rating optimally sets $\overline{p_t^L}$ with probability 1. We can conclude that low-quality firms who obtain a rating in period t set $\overline{p_t^L}$ with probability 1.

Next, we consider the situation where the low-quality firm receives no rating for the first time. The related ratings history are $R_1 = \{0\}, R_2 = \{10\}$. We show that $p_{1,0}^L = E[q_1|R_0, p_1^H]$ and $p_{2,10}^L = E[q_2|R_1, p_2^H]$. We have shown above that all prices above $\overline{p_t^L}$ induces the belief $E[q_t|R_{t-1}, p_t^H]$ - as a result of Restriction 2. Therefore, a low-quality firm receiving no rating optimally sets the largest possible price, $E[q_t|R_{t-1}, p_t^H]$ with probability 1.

Finally, consider the situation where the low-quality firm already has a history of receiving no rating. The related ratings history is $R_2 = \{00, 01\}$. Since by Restriction 1 only low-quality firms receive no rating on the equilibrium path, consumer's belief on observing any history of no rating is that the firm is of a low-quality. Hence $p_{2,00}^L = p_{2,01}^L = E[q_2|R_1 = \{0\}] = q^L$.

This concludes the proof.

$\square$

Lemma 11. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L} \end{cases},$$

in the second period,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \overline{p_2^L} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases},$$

and in the third period,

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}.$$

Proof of Lemma 11.
As with Lemma 3, we prove this lemma by constructing expected quality using Bayes rule. We start by considering the third period, followed by the second then the first.

We begin with the third period. In the third period, consumers are aware of historical ratings, R_2 and current prices p_3. Given that this is the final period of the game, new ratings are not useful for firms - as there is no future period to signal to. By Restriction 2, this implies that the expected quality in period 3 is independent of prices. Thus, the expected quality in period 3 is independent of prices and only depends on the rating history R_2. As a result, firms set the highest possible price and extract the full consumer surplus.

Note that the possible ratings history are $R_2 \in \{00, 01, 10, 11\}$. When consumers observe a ratings history of $\{00, 01, 10\}$, they expect that the firm is a low-quality firm (Restriction 1).

Using Bayes rule, we pin down consumers' expectations on observing $R_2 = \{11\}$ in period 3. In equilibrium, low-quality firms receive no rating with some probability δ_t^* in period $t \in \{1, 2\}$. Hence, consumers observe a good rating with probability $\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)$ - and know that the probability of a high-quality firm is γ. Hence, $E[q_3|R_2 = 11] = \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$.

We conclude that

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}.$$

Next, we consider the second period. In the second period, consumers observe $R_1 \in \{0, 1\}$ and the price p_2. As with the third period, on observing $R_1 = 0$ Restriction 1 implies that the firm is of a low-quality.

Using Bayes rule, we pin down consumers' expectations in equilibrium in period 2, conditional on $R_1 = 1$. We distinguish between two cases: the low-quality firm choosing between a good or no rating. When the low-quality firm receives no rating, it sets $p_2^L = E[q_2|R_1 = 1, p_2^H]$ and when it receives a good rating, $p_2^L = \overline{p_2^L} = \kappa q^L - \frac{e}{\Delta}$. Note that $p_2^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_2|R_1, p_2^H]\}$.

We now start with the first case, i.e. the low-quality firm sets $p_2^L = \overline{p_2^L} = \kappa q^L - \frac{e}{\Delta}$ and gets a good rating. Since $\kappa q^H - \frac{e}{\Delta} > \kappa q^L - \frac{e}{\Delta}$ and $E[q_2|R_1 = 1, p_2^H] \geq q^L > \kappa q^L - \frac{e}{\Delta}$, in the equilibrium, consumers who observe $\kappa q^L - \frac{e}{\Delta}$ believe the firm is a low-quality firm, i.e. $E[q_2|R_1 = 1, p_2 = \kappa q^L - \frac{e}{\Delta}] = q^L$.

Next, consider the second case where the low-quality firm sets, by Lemma 10, $p_2^L = E[q_2|R_1 = 1, p_2^H]$ and gets no rating. We distinguish two scenarios. First, suppose $p_2^H = min\{E[q_2|R_1 = 1, p_2^H], \kappa q^H - \frac{e}{\Delta}\} = E[q_2|R_1 = 1, p_2^H]$. By Lemma 10, $p_{2,10}^L = E[q_2|R_1 = 1, p_2^H] = p_2^H$. On observing this price level, Bayes rule implies $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma) q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ in equilibrium. Second, consider the scenario where $p_2^H = min\{E[q_2|R_1 = 1, p_2^H], \kappa q^H - \frac{e}{\Delta}\} = \kappa q^H - \frac{e}{\Delta}$. Since $p_2^L = E[q_2|R_1, p_2^H] \neq \kappa q^H - \frac{e}{\Delta} = p_2^H$, consumers who observe p_2^H believe $E[q_2|R_1, p_2^H] = q^H$. Because $p_2^L = E[q_2|R_1, p_2^H] = q_H > \overline{p_2^L}$ and $p_2^H = \kappa q^H - \frac{e}{\Delta} > \overline{p_2^L}$, Restriction 2 implies that both p_2^L and p_2^H induce the same equilibrium beliefs. But then we must have $\delta_2^* = 1$ in equilibrium. Note that beliefs $E[q_2|R_1, p_2^H] = q_H$ are then a special case of $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma) q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ for $\delta_2^* = 1$.

This concludes the second case.

We conclude that $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma) q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$.

The final stage is to consider period 1. The proof for this is identical to proof in Lemma 11.

This concludes the proof.

$\square$

With Lemma 10 and 11, we can now prove Proposition 12.

Proof of Proposition 12.

We show that all equilibria satisfying our equilibrium restrictions exhibits similar characteristics as those in the main section of the paper. To do so, we show that a perfect Bayesian equilibrium exists, and that all equilibria satisfying our equilibrium restrictions satisfy similar properties as those in the main body. Specifically,

1. High-quality firms receive a good rating with probability 1 and charge $p_t^H = E[q_t|R_{t-1}, p_t^H]$, $t \in \{1, 2\}$.
2. Low-quality firms randomize their strategy in period $t \in \{1, 2\}$.
   a. They charge $\overline{p_t^L} = \kappa q^L - \frac{e}{\Delta}$ and obtain a good rating with probability $1 - \delta_t^*$.
   b. They charge p_t^H and obtain no rating with probability δ_t^*.
3. Firms set prices equal to expected quality conditional on ratings in the last period.
4. Consumer beliefs given equilibrium prices are given by:
   a. In period 1, $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L} \end{cases}$
   b. In period 2, $E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \overline{p_2^L} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases}$
   c. In period 3, $E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}$

Furthermore, we show that $\delta_1^* \in (0, 1)$ and $\delta_2^* \in (0, 1)$ is an equilibrium if $(1 - \kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, and $\delta_1^* \in (0, 1)$ and $\delta_2^* = 1$ is an equilibrium if $(1 - \kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$. These equilibria exist if $(1 - \kappa)q^L + \frac{e}{\Delta} < \frac{(1-\gamma)(q^H - q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$, and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$.

From Lemmas 10 and 11, we have shown statement 4 and the low-quality firm's prices in statement 2. What remains, is to show statements 1 and 3, the mixed strategy in statement 2, and to show the existence of the equilibrium.

We prove statement 3. In period 3, firms are no longer incentivized by future ratings. We have also shown in Lemma 10 that by Restriction 2, in the final period consumers' beliefs are only dependent on past ratings and therefore independent of the price they observe in period 3. Firms set prices in period 3 equal to the expected quality conditional on past ratings. This concludes the proof of statement 3.

Next, we prove statement 1. From Lemma 10, we have shown that $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$. To show that $min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\} = E[q_t|R_{t-1}, p_t^H]$, suppose towards a contradiction that $min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\} = \kappa q^H - \frac{e}{\Delta}$. We do this in two parts, for period 2 then period 1.

Note first that in period 2, the low-quality firm with no rating in period 1 always sets the price q^L in periods 2 and 3. Thus, we only consider histories after which the low-quality firm received a good rating in period 1. We know from Lemma 10 that $p_{2,10}^L = E[q_2|R_1, p_2^H]$. Thus, in the candidate equilibrium we have $p_2^H \neq p_{2,10}^L$ and only high-quality firms set $p_2^H = \kappa q^H - \frac{e}{\Delta}$, which is why consumers believe that $E[q_2|R_1 = 1, p_2^H] = q^H$. Given Restriction 2, for any price above $\kappa q^L - \frac{e}{\Delta}$ consumers beliefs are the same. Hence $p_{2,10}^L = E[q_2|R_1 = 1, p_2^H] = q^H$. These beliefs are only correct in equilibrium if $\delta_2^* = 0$. But we cannot have $\delta_2^* = 1$ in equilibrium. To see why, note that in period 2 in the candidate equilibrium the low-quality seller charges a low price and receives a good rating, earning in periods 2 and 3 together $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. Deviating by setting a high price q_H gives no rating, but earns in periods 2 and 3 the profit $q^H + q^L$. Since the firm earns the same profits in either case in period 1, the deviation is profitable if it increases profits in periods 2 and 3. Because $q^L > \kappa q^L - \frac{e}{\Delta}$ and $q^H > \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, this deviation is profitable, contradicting $\delta_2^* = 0$ and that $p_2^H = \kappa q^H - \frac{e}{\Delta}$.

We conclude that $p_2^H = E[q_2|R_1, p_2^H]$.

In period 1, we know from Lemma 10 that $p_{1,0}^L = E[q_1|R_0, p_1^H]$. In the candidate equilibrium, we have $p_1^H \neq p_{1,0}^L$ and only high-quality firms charge $p_1^H = \kappa q^H - \frac{e}{\Delta}$, which is why consumers believe that $E[q_1|R_0, p_1^H] = q^H$. Given Restriction 2, for any price above $\kappa q^L - \frac{e}{\Delta}$ consumers beliefs are the same. Hence $p_{1,0}^L = E[q_1|R_0, p_1^H] = q^H$. These beliefs are only correct in equilibrium if $\delta_1^* = 0$. We now show that we cannot have $\delta_1^* = 0$ in equilibrium. To see why, note that in period 1 in the candidate equilibrium, the low-quality firm sets a low price, receives a good rating and earns total expected profits $\kappa q^L - \frac{e}{\Delta} + (1 - \delta_2^*)(\kappa q^L - \frac{e}{\Delta} + E[q_3|R_2 = 11]) + \delta_2^*(E[q_2|R_1 = 1, p_2^H] + q^L)$. If the firm deviates in period 1 to price q_H and charges q_L in subsequent periods, it will earn $q^H + q^L + q^L$. Because $q^L > \kappa q^L - \frac{e}{\Delta}$, $q^L > (1 - \delta_2^*)(\kappa q^L - \frac{e}{\Delta}) + \delta_2^*(q^L)$, and $q^H > (1 - \delta_2^*)(E[q_3|R_2 = 11]) + \delta_2^*(E[q_2|R_1 = 1, p_2^H])$, this deviation is profitable, contradicting $\delta_1^* = 0$ and that $p_1^H = \kappa q^H - \frac{e}{\Delta}$.

We conclude that $p_1^H = E[q_1|R_0, p_1^H]$.

Overall, this prove statement 1, i.e. that $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\} = E[q_t|R_{t-1}, p_t^H]$.

We have now shown that statements 1,3 and 4 hold in equilibria that satisfy our restrictions. We continue to show statement 2 by characterizing the (mixed) strategies of low-quality firms.

We now characterize the low-quality firms' mixed strategy in equilibrium in period 2, i.e. δ_2^*. Recall that δ_2^* is the probability of setting a high price of p_2^H that leads to a no rating. Reversely, $1-\delta_2^*$ is the probability of setting a low price of $\kappa q^L - \frac{e}{\Delta}$ that leads to a good rating. Note that low-quality firms only mix prices after histories where they received a good rating in period 1. This is because consumers' beliefs are such that on observing a period of no rating, they believe the firm to be a low-quality firm.

After a history of a good rating in period 1, when the low-quality firm sets a price of $\kappa q^L - \frac{e}{\Delta}$ in period 2, it obtains a good rating and earns in periods 2 and 3

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}. \tag{13}$$

This is strictly increasing in δ_2^*.

Alternatively, if the low-quality firm sets the high price p_2^H it obtains no rating in period 2 and earns in periods 2 and 3

$$\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L. \tag{14}$$

This is strictly decreasing in δ_2^*.

We now show that $\delta_2^* = 1$ is only possible if no mixed strategy exists in period 2. First, consider that $\delta_2^* = 1$. Then (13) and (14) become $\kappa q^L - \frac{e}{\Delta} + q^H$ and $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$ respectively. In order for $\delta_2^* = 1$ to be optimal, we must have that at $\delta_2^* = 1$, (14) is weakly larger than (13), i.e.

$$\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L \geq \kappa q^L - \frac{e}{\Delta} + q^H \iff (1-\kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}. \tag{15}$$

Observe that (13) is strictly increasing in δ_2^* and (14) is strictly decreasing in δ_2^*, which is why (15) implies that (14) is larger than (13) for all $\delta_2^* \leq 1$. Therefore, if (15) is met, $\delta_2^* = 1$ is an equilibrium and no mixed-strategy equilibrium exists in period 2.

We now show that $\delta_2^* = 0$ is not a possible equilibrium in period 2. Suppose towards a contradiction that $\delta_2^* = 0$. Then (13) and (14) become $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and $q^H + q^L$ respectively. Since $q^H > \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, the firm is strictly better of by deviating, setting a high price and receiving no rating, which contradicts $\delta_2^* = 0$. We conclude that $\delta_2^* = 0$ cannot be an equilibrium.

We now show that for

$$(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}, \tag{16}$$

there is a unique $\delta_2^* \in (0,1)$ that characterizes the low-quality firms mixed strategy in period 2.

Recall that a mixed-strategy equilibrium only exists when (13) and (14) are equal. Observe that for $\delta_2^* = 1$, (14) is strictly below (13) if and only if (16) holds. Next, observe that for $\delta_2^* = 0$, (14) is strictly above (13) because $q^L + q^H > \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$.

Since (13) is strictly increasing and (14) is strictly decreasing in δ_2^*, there is a unique $\delta_2^* \in (0,1)$ such that (13) equals (14) if and only if (16). Otherwise, i.e. if and only if (15), we have $\delta_2^* = 1$.

We now pin down δ_2^*. Note that if $\delta_2^* < 1$, it is determined by (13) equal (14), i.e.

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$$

We have two candidates that solve this equation:

$$\delta_2^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} \pm \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma) + 2\gamma)^2((1-\kappa)q^L + \frac{e}{\Delta})^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})}$$

Recall that $\delta_2^* \in (0,1)$. In the subtractive case, $1 - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa q^L) + \frac{e}{\Delta})} < 0$. Therefore, we conclude that

$$\delta_2^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} + \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma) + 2\gamma)^2((1-\kappa)q^L + \frac{e}{\Delta})^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})}$$

Further, $\delta_2^* > \frac{1}{2} + \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} - \frac{2\gamma(q^H - q^L)}{2(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} = \frac{1}{2}$.

Therefore, $\delta_2^* \in (\frac{1}{2}, 1)$ if and only if $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, and $\delta_2^* = 1$ otherwise.

We now turn our attention to period 1 and characterize δ_1^*. We first show that δ_1^* is unique. Recall that in period 1, the low-quality firm charges a low price $\kappa q^L - \frac{e}{\Delta}$ with probability $1 - \delta_1^*$ and obtains a rating, and it charges a high price $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$ with probability δ_1^* and obtains no rating in period 1.

When the low-quality firm charges $\kappa q^L - \frac{e}{\Delta}$, the total continuation profit of the low-quality firm is

$$\begin{aligned}\pi_1^L(R_1 = 1) =& \kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}] + \\ & \delta_2^*[\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L],\end{aligned} \tag{17}$$

where the first term are the profits in period 1, the second term are the continuation profits if the firm charges a low price and obtains a good rating in period 2, and the third term are the continuation profits if the firm charges a large price and obtains no rating in period 2.

We show that (17) is strictly increasing in δ_1^* if $\gamma > \frac{1}{3}$.

To show that (17) is strictly increasing in δ_1^*, we first show that $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$.

Taking the derivative of (13) and (14) and solving for $\frac{\partial \delta_2^*}{\partial \delta_1^*}$. We find that

$$\frac{\partial \delta_2^*}{\partial \delta_1^*} = \frac{\delta_2^*(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 - (1-\delta_2^*)(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2}{(1-\delta_1^*)((\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2)}.$$

Notice that the denominator is positive. The numerator simplifies to $(2\delta_2^* - 1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2)$. We show that if $\gamma > \frac{1}{3}$, we have $(2\delta_2^* - 1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2) > 0$. Since $\delta_2^* \in (\frac{1}{2}, 1]$, $2\delta_2^* - 1 > 0$. We know that $\delta_1^* \in (0, 1]$ and that $\delta_2^*(1-\delta_2^*)$ is maximum at $\delta_2^* = 0.5$. Therefore, $\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2 > \gamma^2 - 0.25(1-\gamma)^2$ and $\gamma^2 - 0.25(1-\gamma)^2 > 0$ whenever $\gamma > \frac{1}{3}$. Thus, if $\gamma > \frac{1}{3}$, we have $(2\delta_2^* - 1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-$

$\delta_2^*)(1-\gamma)^2) > 0$ and $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$. $\gamma > \frac{1}{3}$ is a sufficient condition.

Next, we show that total derivative of (17) with respect to δ_1^* is greater than 0.

$$\frac{\partial \pi_1^L(R_1=1)}{\partial \delta_1^*} = \frac{\partial \delta_2^*}{\partial \delta_1^*}[q^L(1-\kappa) + \frac{e}{\Delta} - \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}] + \frac{\gamma(1-\gamma)(1-\delta_2^*)(q^H-q^L)(\frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*) + (1-\delta_2^*))}{(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2} + \frac{\gamma(1-\gamma)\delta_2^*(q^H-q^L)(\delta_2^* - \frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*))}{(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2}$$

We show that $\frac{\partial \pi_2^L}{\partial \delta_1^*} > 0$ in three parts.

Consider first that $\frac{\partial \delta_2^*}{\partial \delta_1^*}[q^L(1-\kappa) + \frac{e}{\Delta}]$ and show that this is greater than 0. Since $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$ when $\gamma > \frac{1}{3}$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, we can conclude that $\frac{\partial \delta_2^*}{\partial \delta_1^*}[\kappa q^L - \frac{e}{\Delta} - q^L] > 0$.

Second, we reformulate $\frac{\partial \pi_2^L}{\partial \delta_1^*}$.

To do so, consider $\frac{\gamma(1-\gamma)(1-\delta_2^*)(q^H-q^L)(\frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*)+(1-\delta_2^*))}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2} + \frac{\gamma(1-\gamma)\delta_2^*(q^H-q^L)(\delta_2^* - \frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*))}{(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$. Recall that $\frac{\partial \delta_2^*}{\partial \delta_1^*} = \frac{\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 - (1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}{(1-\delta_1^*)((\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}$ and substitute this into the former. This simplifies to $\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$. This leaves us with the simplified equation of

$$\frac{\partial \pi_1^L(R_1=1)}{\partial \delta_1^*} = \frac{\partial \delta_2^*}{\partial \delta_1^*}[q^L(1-\kappa) + \frac{e}{\Delta} - \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}] + \frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2}.$$

Finally, consider $\frac{\partial \delta_2^*}{\partial \delta_1^*}[\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} - \frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}] + \frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$ and substituting $\frac{\partial \delta_2^*}{\partial \delta_1^*}$ into the former, we get

$$\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2}[1 + \frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 - (1-\delta_2^*)(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))}]$$

We check that this is positive. Notice that $\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2} > 0$. Therefore, what remains is to show $1+\frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2-(1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))} > 0$

$$
\begin{aligned}
&1+\frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2-(1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))} > 0 \\
&\iff (1-\delta_1^*)(1-2\delta_2^*)^2[\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2] \\
&< \gamma^2+\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*).
\end{aligned}
$$

We can verify that this is always true. Since $\delta_1^* \in [0,1]$ and $\delta_2^* \in [\frac{1}{2},1]$, $\delta_1^*(1-2\delta_2^*)^2 \in [0,1]$. Therefore, $(1-\delta_1^*)(1-2\delta_2^*)^2[\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2] < \gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2$. It is easy to see that $\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2 < \gamma^2+\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)$. For $-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2 < \gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)$, the left hand side is negative and the right hand side is positive.

We conclude that $\frac{\partial \pi_1^L(R_1=1)}{\partial \delta_1^*} > 0$ when $\gamma > \frac{1}{3}$. Note that $\gamma > \frac{1}{3}$ is a sufficient but not necessary condition.

Next consider the situation when the low-quality firm charges a high price $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ with probability δ_1^* and obtains no rating in period 1. The firm then charges q^L in subsequent periods. Thus, the total continuation profits are given by

$$
\pi_1^L(R_1=0) = \frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + q^L + q^L \tag{18}
$$

This is strictly decreasing in δ_1^*.

We now show that $\delta_1^* = 1$ is only possible if no mixed-strategy equilibrium in period 1 exists. Suppose that $\delta_1^* = 1$. Then (17) and (18) become $\kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta} + q^H] + \delta_2^*[q^H + q^L]$ and $\gamma q^H + (1-\gamma)q^L + q^L + q^L$ respectively. For $\delta_1^* = 1$ to be optimal, we must have (17) lower that (18) for $\delta_1^* = 1$,

$$
\begin{aligned}
\kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta}] + \delta_2^*[q^L] + q^H &\le \gamma q^H + (1-\gamma)q^L + q^L + q^L \\
\iff (2-\delta_2^*)((1-\kappa)q^L + \frac{e}{\Delta}) &\ge (1-\gamma)(q^H - q^L).
\end{aligned}
$$

Further, when $\gamma > \frac{1}{3}$, (17) increases in δ_1^* and (18) decreases in δ_1^*. Hence, when $(2-\delta_2^*)((1-$

$\kappa)q^L + \frac{e}{\Delta}) > (1-\gamma)(q^H - q^L)$, (18) is larger than (17) for all $\delta_1^* \leq 1$. This indicates that $\delta_1^* = 1$ is an equilibrium and no mixed strategy equilibrium exists in period 1. We conclude that $\delta_1^* = 1$ can only be an equilibrium if no mixed strategy equilibrium exists.

We now show that $\delta_1^* = 0$ is not an equilibrium. Suppose towards a contradiction that $\delta_1^* = 0$. Then (17) and (18) become $\kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_2^*)(1-\gamma)}] + \delta_2^*[\frac{\gamma q^H + \delta_2^*(1-\gamma)q^L}{\gamma + \delta_2^*(1-\gamma)} + q^L]$ and $q^H + q^L + q^L$ respectively. Since $\kappa q^L - \frac{e}{\Delta} < q^L$, $\frac{\gamma q^H + (1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_2^*)(1-\gamma)} < q^H$, and $\frac{\gamma q^H + \delta_2^*(1-\gamma)q^L}{\gamma + \delta_2^*(1-\gamma)} < q^H$ then (17) $<$ (18). Therefore, if $\delta_1^* = 0$ and the low-quality firm participates in ratings harvesting with probability 1, deviating to the high price and obtaining no ratings is profitable. This contradicts $\delta_1^* = 0$. We conclude that $\delta_1^* = 0$ cannot be an equilibrium.

We now show that for

$$(2-\delta_2^*)((1-\kappa)q^L + \frac{e}{\Delta}) < (1-\gamma)(q^H - q^L), \tag{19}$$

there is a unique $\delta_1^* \in (0,1)$ that characterises the low-quality firms mixed strategy in period 1. Recall that a mixed-strategy equilibrium only exists when (17) and (18) are equal. First, observe that for $\delta_1^* = 1$, (18) is strictly below (17). Next, observe that $\delta_1^* = 0$, (18) is strictly above (17). Since (17) is strictly increasing in δ_1^* when $\gamma > \frac{1}{3}$ and (18) is strictly decreasing in δ_1^*, there is a unique $\delta_1^* \in (0,1)$ such that (17) and (18) are equal if (19) and $\gamma > \frac{1}{3}$ hold.

We have shown that we cannot have an equilibrium where $\delta_1^* = 0$ or $\delta_2^* = 0$, and that $\delta_2^* = 1$ can be an equilibrium if no mixed strategy equilibrium exists. Additionally, if $\gamma > \frac{1}{3}$, then $\delta_1^* = 1$ can be an equilibrium if no mixed strategy equilibrium exists and if a mixed strategy exists, $\delta_1^* \in (0,1)$ exists such that (17) and (18) are equal, it is unique if $\gamma > \frac{1}{3}$. And, if $\delta_2^* \in (0,1)$ exists such that (13) and (14) are equal, it must be unique. Therefore, we either have a unique pure strategy in equilibrium or a unique mixed strategy in equilibrium in each period when $\gamma > \frac{1}{3}$. We have also characterised the mixed strategy equilibrium for period 2 and shown that $\delta_2^* \in (\frac{1}{2}, 1]$. We conclude that statement 2 holds, and therefore conclude that statements 1-4 hold in equilibrium.

We now show that equilibria satisfying statements 1-4 indeed exist.

We begin by considering the scenario where (16), (19), $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$ and $\gamma > \frac{1}{3}$ hold together.

Consumers' beliefs are as follows. In period 1,

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}, \end{cases}$$

and in period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \kappa q^L - \frac{e}{\Delta} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination,} \end{cases}$$

and in period 3,

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2. \end{cases}$$

These beliefs follow Bayes rule on the candidate equilibrium path of play. The candidate equilibrium is consistent with our restrictions. Because high-quality firms obtain a good rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Further, whenever low-quality firms obtain no rating, consumers' beliefs are independent of prices (in both periods 1 and 2), and in period 3, beliefs are the same for all period 3 prices and they only depend on the history of ratings. Therefore, the candidate equilibrium is consistent with Restriction 2.

The candidate equilibrium is such that high-quality firms always play $p_t^H = E[q_t|R_{t-1}, p_t^H]$ for all t and get a good rating. Low-quality firms mix between playing a low price $\kappa q_L - \frac{e}{\Delta}$ and getting a good rating with probability δ_t^*, and setting a high price p_t^H and getting no rating with probability $(1-\delta_t^*)$ in each period $t \in \{1, 2\}$.

More precisely, in the candidate equilibrium in period 1, the high-quality firm sets a price $p_{1,1}^H = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains a good rating with probability 1. In period 2, conditional on receiving a good rating in period 1, the high-quality firm sets a price $p_{2,11}^H = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ and obtains a good rating with probability 1. If the high-quality firm did not receive a good rating in period 1, consumers believe the firm is of a low quality and thus the maximum price that the high-quality firm can set is q^L and receives a good rating. To see that the firm receives a good rating, note that by assumption $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$ and both of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and $\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ are larger than q^L. Therefore, when consumers pay q^L for a high quality product, they receive a sufficient amount of excess surplus and leave a good rating. In the third period of the candidate equilibrium, having received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, the high-quality firm sets a price of $\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. If the firm did not receive a continuation of good ratings, i.e. a rating history of $R_2 \in \{00, 01, 10\}$, then it sets a maximum price of q^L.

In period 1 of the candidate equilibrium, the low-quality firm sets $p_{1,0}^L = \frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains no rating with some probability δ_1^*, and $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$ and obtains a good rating with some probability $1-\delta_1^*$. In period 2, conditional on having obtained a good rating in period 1, the low-quality firm sets a price $p_{2,11}^L = \kappa q^L - \frac{e}{\Delta}$ with some probability $1-\delta_2^*$ and obtains a good rating, and a price $p_{2,10}^L = \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ with some probability δ_2^* and obtains no rating. If the low-quality firm obtained no rating in period 1, then it sets a price q^L in period 2 and obtains no rating. In period 3, if the low-quality firm received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, it sets a price of $\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, and when it receives a rating history of $R_2 = \{00, 10, 01\}$, it sets a price of q^L. This fully characterizes the firms' prices and consumers' beliefs.

We now show that the firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns a total profit of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} + \frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$.

In period 3, deviations to a higher price would reduce demand to zero and earn a total maximum profit of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} + 0$, and deviations to a lower price would reduce profit margins in the third period without increasing demand. Neither deviation increases profits.

In period 2, deviations to a higher price would reduce demand to zero and result in no rating, this leads to a total maximum profit of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + 0 + q^L$, and deviations to a lower price would reduce profit margins in the second period without increasing demand. Neither deviation increases profits.

In period 1, deviations to a higher price would reduce demand to zero and result in no rating, this leads to a total maximum profit of $0 + q^L + q^L$, and deviations to a lower price would reduce profit margins in the first period without increasing demand. Neither deviation increases profits.

Therefore, there are no profitable deviations for the high-quality firm in any period.

Next, we show that low-quality firms have no profitable deviation.

In the candidate equilibrium, the low-quality firm earns a total profit of $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. For the equilibrium $\delta_1^* \in (0,1)$ and $\delta_2^* \in (0,1)$, the low-quality firm is indifferent between setting the lower price that obtains good rating and setting a higher price that obtains no rating in all periods. Hence, the firm is indifferent between the total profits of $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$. Recall that (16) implies $\delta_2^* \in (0,1)$ and (19) implies $\delta_1^* \in (0,1)$, and $\gamma > \frac{1}{3}$ implies $\delta_1^* \in (0,1)$ is unique. Showing that there exists no profitable deviation from any of these cases shows that there is no profitable deviation for the low-quality firm.

In period 3, deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ would result in zero demand and a maximum total profit of $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + 0$, and a deviation towards a lower price results in smaller profit margins without increasing demand. Hence, there is no profitable deviation in period 3.

In period 2, the low-quality firm is indifferentrent between setting the price $\kappa q^L - \frac{e}{\Delta}$ and $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$, where $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} > \kappa q^L - \frac{e}{\Delta}$ and her total profits are $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ and $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$ respectively. Deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ leads to zero demand in period 2 and the maximal profits of $\kappa q^L - \frac{e}{\Delta} + 0 + q^L$, which is not a profitable deviation. Deviations towards a price $p_2 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)})$, leads to the same rating as when the firm sets $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$; hence, any deviation to p_2 lowers margin without improving demand or future profit, and are therefore not profitable. Deviations towards a price below $\kappa q^L - \frac{e}{\Delta}$ decreases profits in period 2 but does not increase demand or change the rating the firm receives when setting a price of $\kappa q^L - \frac{e}{\Delta}$, which is why this is also not a profitable deviation. Therefore, in period 2, there is no profitable deviation for the firm.

In period 1, the low-quality firm is indifferent between setting the price $\kappa q^L - \frac{e}{\Delta}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$, where its total profits are $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$ respectively. If it deviates to a price above $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$, demand falls to zero and it makes a total profit of $0 + q^L + q^L$, which is not a profitable deviation. If it deviates to a price $p_1 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)})$, then it receives the same rating as when it sets the price of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$; hence such a deviation reduces margins in the first period without improving demand or future profit and is not profitable. When deviating to a price below $\kappa q^L - \frac{e}{\Delta}$, the firm receives a good rating; however, the deviation does not increase demand and her margins are lower than when setting the price of $\kappa q^L - \frac{e}{\Delta}$, which is why the deviation is not profitable.

We conclude that there are no profitable deviations for either high- or low-quality firms from the candidate equilibrium.

We conclude that if $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H-q^L)}{\gamma+(1-\delta_1^*)(1-\gamma)}$, $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\gamma)(q^H-q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$ and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$, the candidate equilibrium exists.

Consider next the case where (15), (19), $\gamma > \frac{1}{3}$ and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$. In this scenario, $\delta_2^* = 1$. The last three inequalities are identical to the previous case and play the same role in this case.

In the candidate equilibrium, consumers' beliefs are as follows. In period 1,

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}, \end{cases}$$

and in period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H+(1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} & \text{if } p_2 > \kappa q^L - \frac{e}{\Delta} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination,} \end{cases}$$

and in period 3,

$$E[q_3|R_2] = \begin{cases} q^H & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2. \end{cases}$$

These beliefs follow Bayes rule on the candidate equilibrium path of play. The candidate equilibrium is consistent with our restrictions. Because high-quality firms obtain a good rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Further, whenever low-quality firms obtain no rating, consumers' beliefs are independent of prices (in both periods 1 and 2), and in period 3, beliefs are the same for all prices. Therefore, the candidate equilibrium is consistent with Restriction 2.

In the candidate equilibrium in period 1, the high-quality firm sets a price $p_{1,1}^H = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains a good rating with probability 1. In period 2, conditional on receiving a good rating in period 1, the high-quality firm sets a price $p_{2,11}^H = \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}$ and obtains a good rating with probability 1. If the high-quality firm did not receive a rating in period 1, consumers believe the firm is of a low quality and thus the maximum price that the high-quality firm can set is q^L and the firm receives a good rating. To see that the firm receives a good rating, note that by assumption $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}\}$ and both of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}$ are larger than q^L. In the third period of the candidate equilibrium, having received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, the high-quality firm sets a price of q^H. If the firm did not receive a continuation of good ratings, i.e. a rating history of $R_2 \in \{00, 01, 10\}$, then it sets a maximum price of q^L.

In period 1 of the candidate equilibrium, the low-quality firm sets a price $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains no rating with some probability δ_1^* and $\kappa q^L - \frac{e}{\Delta}$ and obtains a good rating with some probability $1 - \delta_1^*$. In period 2, the low-quality firm sets a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}$ with probability 1 and receives no rating. In period 3, if the low-quality firm received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, it sets a price of q^H, and when it receives a rating history of $R_2 = \{00, 10, 01\}$, it sets a price of q^L.

We now show that the firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns a total profit of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} + q^H$. In period 3, deviations to a higher price reduces demand to zero, and deviations to a lower price reduces profit margins without increasing demand, thus there is no profitable deviation in period 3. In period 2, deviations to a higher price reduces demand to zero, which induces no rating and also reduces continuation profits; deviations to a lower price reduces profit margins without increasing demand. Thus, neither deviation is profitable in period 2. In period 1, deviations to a higher price reduces demand to zero, induce no rating and therefore reduces continuation profits in all future periods to q_L, and deviations to a lower price reduces profit margins without increasing demand. Neither deviation is profitable in period 1. Therefore, there are no profitable deviations for the high-quality firm.

In the candidate equilibrium, the low-quality firm earns a total profit of $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} + q^L$. For the equilibrium $\delta_1^* \in (0, 1)$, the low-quality firm is indifferent between setting a lower price that obtains a good rating and setting a higher price that obtains no rating in period 1, and for $\delta_2^* = 1$, the low-quality firm always sets a high price and obtains no rating in period 2. The low-quality firm is therefore indifferent between the total profits of

$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$. Recall that (15) implies $\delta_2^* = 1$, (19) implies $\delta_1^* \in (0,1)$, and $\gamma > \frac{1}{3}$ implies $\delta_1^* \in (0,1)$ is unique. In period 3, deviations towards a higher price, above q^L, reduces demand to zero, and deviations towards a lower price results in a lower profit margin without improving demand, which is why there is no profitable deviation in period 3.

In period 2, conditional on receiving a good rating in period 1, deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ leads to zero demand and results in the same rating (and hence future profit) as setting a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$, which is why it is not a profitable deviations. Further, deviations towards a price of $p_2 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)})$ results in the same rating (and hence future profit) as setting a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$, therefore this reduces margins without providing any additional continuation profit, and is not a profitable deviation. Deviations towards a price $p_2 \leq \kappa q^L - \frac{e}{\Delta}$ leads to a maximal total profit of $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + q^H$, but since (15) holds, this is not a profitable deviation. Therefore, there are no profitable deviations in period 2 conditional on receiving a good rating in period 1. In period 2, conditional on having received no rating in period 1, deviations towards a price above q^L leads to zero demand and does not change rating and continuation profits, which is why this is not a profitable deviation. Further, since consumers beliefs on observing a single period of no rating is that the firm is of a low-quality, setting a price lower than q^L only reduce the margins in period 2 without increasing demand or future profits, thus this is not a profitable deviation. Therefore, there are no profitable deviations in period 2 conditional on not having received a rating in period 1.

In period 1, the low-quality firm is indifferent between setting the price $\kappa q^L - \frac{e}{\Delta}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)}$. If it deviates to a price above $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)}$, demand falls to zero, it gets no rating and it makes a total profit of $0 + q^L + q^L$, which is not a profitable deviation. If it deviates to a price $p_1 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)})$, then it receives the same rating as when it sets the price of $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)}$; hence such a deviation reduces margins in the first period without improving demand or future profit and is not a profitable deviation. When deviating to a price below $\kappa q^L - \frac{e}{\Delta}$, the firm receives a good rating; however, the deviation does not increase demand and her margins are lower than when setting the price of $\kappa q^L - \frac{e}{\Delta}$. Therefore, we conclude that there is no profitable deviation in period 1.

We conclude that there are no profitable deviations for either the high- or low- quality firm from the candidate equilibrium.

We conclude that if $(1-\kappa) q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, $(1-\kappa) q^L + \frac{e}{\Delta} < (1-\gamma)(q^H - q^L)$, $\gamma > \frac{1}{3}$

and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}\}$, the candidate equilibrium exists. This concludes the proof.

□

Appendix B Rating Utility

We derive the rating utility function from the utility function proposed in Rabin (1993). In his paper, Rabin proposes a utility function which incorporates a reciprocity term in addition to a consumption utility. This reciprocity term depends on the additional surplus that some player i is allowed to obtain given the actions of another player j relative to a predefined equity point.

$$U_i = \pi_i + [\frac{\pi_i - \pi_i^e}{\pi_i^H - \pi_i^{min}}][1 + \frac{\pi_j - \pi_j^e}{\pi_j^H - \pi_j^{min}}]$$

- π_h is the consumption utility.
- $\pi_h^m in$ is the lowest possible payoff to player h.
- π_h^H is the highest possible pareto efficient payoff to player h.
- $\pi_h^e = \frac{\pi_h^H + \pi_h^L}{2}$, where π_h^L denotes the lowest possible pareto efficient payoff to player h. π_h^e is the equitable reference point.
- For all $h \in \{i, j\}$.

At this junction, allow us to provide some intuition. Suppose that player i is the consumer. Then this function takes into account the consumption utility and some additional reciprocity term. The additional term is what we consider the rating utility. Suppose a firm sets a low price, such that $\pi_i - \pi_i^e > 0$, the consumer would believe that the firm is treating him kindly. In response, consumers will receive a higher overall utility if he is kind to the firm, $\pi_j - \pi_j^e > 0$. In our context, a positive rating will result in π_j being higher and therefore by leaving a positive rating, consumers would be being kind to the firms.

On the contrary, a firm charging a high price such that pay off for the consumer is below the equitable point would result in consumers punishing the firm by lowering their profits in the future periods - perhaps through a negative rating. For simplicity, we remove the ability of consumers to punish a firm and assume that consumers are only able to provide positive or no ratings. Specifically, in our model, we show that a good rating results in a better future pay off and hence satisfies this feature.

In what follows, I show how we adapt this framework to better fit our context. Firstly, we have assumed that providing a rating is costly for consumers. This seems to be an intuitive feature of our model and follows from the literature of costly provision of reviews and ratings (Avery et al., 1999; Miller et al., 2005). Secondly, we make some simplifications that make the framework more tractable in our setup. We remove the normalization terms in the denominator and remove the "1".[42] Thirdly, we consider that the rating component of the utility only comes into effect when a positive rating is provided. This leaves us with the following function:

$$U_i = \pi_i + \mathbb{1}_{\{R_t=1\}}\{[\pi_i - \pi_i^e][\pi_j - \pi_j^e] - e\}$$

Since $pi_i = q^j - p_1^j$, we substitute the following:

- $\pi_i^e = \frac{(q^j - q^j)+(q^j-0)}{2}$. The highest possible pareto payoff to consumers being $q^j - 0$ where sellers set a price of 0 and the lowest possible being 0, where sellers set a price of q^j.
- $\pi_j^e = \frac{(p_1+q^H)+(p_1+q^L)}{2}$. The seller, setting some price p_1 in period 1, is able to get a maximum benefit of q^H and a minimum benefit of q^L in period 2.
- $\pi_j = p_1 + p_2$, profits of the firm being the sum of profits in two periods, given 0 marginal cost, profits is the sum of prices in both periods.

$$U_i = q^j - p_1 + \mathbb{1}_{\{R_t=1\}}\{[\frac{q^j}{2} - p_1][p_2 - \frac{q^H + q^L}{2}] - e\}$$

Next, we replace $\frac{q^j}{2}$ with κq^j, where $\kappa \in [0,1]$ and $[p_2 - \frac{q^H+q^L}{2}]$ with Δ. This reflects the notion that an equitable payoff may not be one of equal split, allowing us to generalise the equitable point. Hence, when κ is sufficiently high, firms are able to charge some price slightly below quality and still receive a positive rating if e is sufficiently small. We do not make any assumptions over Δ, except that it is positive. This allows us to capture kindness from consumers by enabling firms to set higher prices in subsequent periods.

Finally, we split the consumption utility and the rating utility. This allows for more compatible purchase decision across periods as consumer's purchase decision does not depend on whether they anticipate giving a good rating.

[42] Rabin notes that doing so does not affect the behavior of the utility function.