Ratings & Reciprocity

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Abstract

Evidence suggests online ratings and reviews are intrinsically motivated, and reviewers reciprocate offers of sufficient value-for-money with good ratings. Incorporating reciprocity into a model of ratings, we explore how firms use prices to impact their own ratings. We show that firms harvest ratings: they offer lower prices in early periods to trigger consumers' reciprocal behaviour and obtain a good rating and larger profits in the future. Because also low-quality firms harvest ratings, reciprocity makes ratings less-informative about quality. Based on this mechanism, (i) we argue that reciprocity-based ratings cause rating inflation; (ii) we show that a marketplace that facilitates ratings (e.g. through reminders, one-click ratings etc) may get more ratings, but also less-informative ratings; (iii) a marketplace that screens the quality of sellers (e.g. via a minimum average rating or quality controls) makes ratings less-informative if the screening is insufficient. We also explore the impact of reciprocity-based ratings on surplus. We show that even as ratings become less-informative, consumers can benefit from lower prices. Nonetheless consumers prefer more-informative ratings than average sellers. We apply these results to characterise when a two-sided platform wants to facilitate ratings, and thereby undermines the informativeness of ratings and harms consumers. We connect our results to evidence and a wide range of applications and provide robustness checks.

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1 Introduction

Ratings feature prominently in the decisions we make each day. We often rely on the experience of others, through ratings and reviews, to make decisions. This is especially true for transactions that do not happen regularly. Examples include choosing a college, deciding on a holiday destination or buying a car. Everyday decisions, such as where to have a meal or which show to watch are also often influenced by ratings. But how far can we trust these ratings to inform us about the quality of a product?

Despite having an influential role on the way consumers make purchase decisions, evidence suggests that ratings can be a poor signal of objective product quality (De Langhe et al., 2016; Siering et al., 2018). Starting with Li and Hitt (2010), empirical work highlights that prices influence ratings, and it is the value-for-money for which consumers provide ratings. While this suggests that prices affect ratings, other evidence shows that firms with better ratings have the ability to set higher prices (Luca & Reshef, 2021). Combined, this evidence suggests a dual role between prices and ratings. Firms use lower prices to obtain good ratings, as this allows them to set higher prices in the future.

While the evidence suggests that prices influence ratings, the body of theoretical literature on reputation tends to assume that reputation is solely a consequence of quality. In doing so, these papers focus on only one direction of effects: how ratings can influence future prices. In this paper, we contribute to the theoretical literature by considering the dual role of prices in ratings, as an influence on the production of ratings and as an outcome of ratings. We study this dual role of ratings to address the question: "are value-for-money based ratings informative of product quality?"

To develop a framework where prices influence ratings, we build on the well-established concept of reciprocity (Bolton & Ockenfels, 2000; Dufwenberg & Kirchsteiger, 2004; Rabin, 1993). Evidence on reciprocity suggests that firms may choose to lower prices or provide rebates to offer a higher value-for-money to consumers. These consumers interpret a sufficiently high value-for-money as a kindness, which they reciprocate with a good rating (Fradkin et al., 2021; Halliday & Lafky, 2019; Li & Hitt, 2010). We model reciprocity-based on Dufwenberg and Kirchsteiger (2004) and Rabin (1993), which allows us to capture that ratings are a function of both observed quality and price. Thus, we endogenize the rating decision of consumers, and enable firms to use their prices to influence ratings.

Formally, we study a two period model with asymmetric information. Two actors participate in each period—a long lived monopolistic firm and a short lived consumer. The firm is endowed with a product of high or low quality. The firm knows the quality of its product, and sets prices in each period to maximise lifetime profits. Initially, consumers only know the distribution of quality. Each consumer only participates in a single period. At the start of each period, consumers observe the historical ratings of the firm and its price in the current period. Consumers only learn of the true quality of the product after consumption, and may then choose to leave a rating. Thus, consumers are uncertain about quality at the beginning of each period; but they may use ratings to transmit some information about quality to future consumers.

This model is representative of markets such as Amazon and Taobao, where ratings play the important role of developing trust between anonymous users. In these markets, new consumers are mostly unaware of the true quality of sellers they are transacting with. They rely on information left by prior consumers—through ratings—to form beliefs over the quality of a product. Further, these websites only list historical ratings and current prices.[1]

In line with evidence that high-quality firms are more likely to get good ratings (Ananthakrishnan et al., 2019; Li et al., 2020), we focus on equilibria where high-quality firms set sufficiently low prices to obtain a good rating.

Low-quality firms, however, face the following trade-off. First, in period 1 they can set a price which is sufficiently low to trigger reciprocity. By doing so, the low-quality firm receives a good rating and earns larger profit in period 2. We call this strategy 'ratings harvesting'. Second, in period 1 the low-quality firm could charge the same price as the high-quality firm and benefit from the fact that, ex-ante, consumers think this seller could be of a high quality. Because both firms charge the same price in period 1, we call this strategy 'price mimicking'.

These strategies of low-quality firms influence how well consumers can infer quality from ratings. If low-quality firms harvest ratings, both types of firms get a good rating, and future consumers cannot use ratings to distinguish firms. But if low-quality firms mimic prices, only high-quality firms offer a sufficiently high value-for-money and obtain a good rating, allowing future consumers to distinguish firms.

In equilibrium, low-quality firms play a mixed-strategy, and do both price mimicking and ratings harvesting with positive probability. Intuitively, low-quality firms harvest ratings to get a good rating and free-ride on the reputation of high-quality firms. This, however, undermines the value of a good rating until, in equilibrium, low-quality firms are indifferent between ratings harvesting and price mimicking. This mixed-strategy equilibrium has some

[1] While some services may exist to attempt to track historical prices, their validity cannot be ascertained and these services are unable to reflect the transaction price associated with each rating.

very convenient features for our analysis. First, ratings signal product quality only if low-quality firms mimic prices and the two types of firms get different ratings. This is why the probability that low-quality firms mimic prices in equilibrium also measures how well ratings signal quality. Second, the mixed-strategy captures the dual role of ratings and price. We leverage this trade-off between ratings harvesting and price mimicking to understand how changes in the ratings environment impact the informativeness of ratings.

The dynamic pricing in our equilibrium closely resembles evidence on rating harvesting: in online marketplaces, firms build reputation in early periods by obtaining good ratings. Subsequently, they increase prices (Cabral & Hortaçsu, 2010; Cabral & Li, 2015; Li et al., 2020).

Our first result shows that reciprocity makes ratings less-informative and causes rating inflation.[2] Intuitively, more reciprocity lowers the opportunity cost of a good rating: when consumers have a stronger sense of reciprocity, firms need only leave a smaller surplus to trigger consumer's reciprocal behavior. This means that firms can set a higher price and still receive a good rating. Therefore, low-quality firms harvest ratings more often and receive better ratings. But if low-quality firms get better ratings, ratings become less-informative about quality.

Even though ratings harvesting makes ratings less-informative, consumers may still benefit: low-quality firms who harvest ratings lower prices and provide some surplus to consumers. We call this surplus 'reciprocity rent'. Since consumers only receive this surplus when firms harvest ratings, they benefit despite less-informative ratings. This has two key implications: (i) Consumers do not prefer fully-informative ratings. If low-quality firms do not harvest ratings, ratings are perfectly-informative, but consumers do not receive any reciprocity rent. (ii) Counter to the conventional wisdom that consumers mainly benefit from the information transmitted by ratings, we show that consumers may benefit from lower prices, even before they rate.

Next, we explore how different features of a marketplace influence ratings, namely (i) how easy it is to rate and (ii) the extent of quality controls.

Many platforms try to encourage and facilitate ratings by lowering the effort it takes for consumers to leave a rating. For example, Amazon transitioned to a one-click rating system, arguing that—in the spirit of the law of large numbers—more ratings "more accurately [...]

[2] Rating inflation here refers to the observation that ratings scores are improving over time, and most of the improvements cannot be attributed to product quality. Thus, leading to ratings becoming a less effective signal of quality. This observation is made in Filippas et al. (2022), suggesting that other attributes such as the cost of leaving ratings, kindness to seller and other forms of retaliation contributes to rating inflation.

reflect the experience of all purchasers".[3,4] We show that this logic ignores how a lower effort to rate impact how sellers harvest ratings. Making it easier to rate means that firms need only transfer a smaller reciprocity rent to consumers to encourage a good rating. Thus, low-quality firms can harvest ratings with higher prices and will do so more often in equilibrium. Even though this leads to more ratings in equilibrium, ratings are also less-informative. In line with this prediction, Li and Xiao (2014) find that paying consumers to leave any rating leads to higher average ratings. More generally, together with evidence that platforms facilitate ratings over time, this result suggests platforms engage in a race towards uninformative ratings and reinforce rating inflation.

Easier ratings affect buyer- and seller-surplus differently. High-quality firms unambiguously prefer a higher effort to rate, because this leads to more-informative ratings and prevents other firms from free-riding on their reputation. But because a higher effort to rate reduces reciprocity rents, average sellers prefer easier ratings, while consumers prefer an intermediate effort level that leads to somewhat-informative ratings. Thus, consumers prefer more-informative ratings than the average seller, but less-informative ratings than high-quality firms.

These results suggest that a platform can facilitate or discourage ratings to shift surplus between sellers and buyers. Using this insight, we show when a two-sided platform may facilitate ratings to encourage more ratings, but less-informative ratings.

Marketplaces do not just facilitate ratings, but also employ quality controls and weed out low-quality firms. For example, Amazon suspends sellers who do not meet a minimum standard.[5] We show that improving the aggregate quality in the market can discourage ratings harvesting, leading to more-informative ratings and less rating inflation. But when aggregate quality is low, quality improvements can also foster rating inflation instead.

Finally, we study a range of extensions and robustness checks. We show that competition between sellers encourages ratings harvesting and leads to more rating inflation. Additionally, our results are robust when consumers can leave negative ratings. In this extension, we show that reciprocity can explain why we observe extreme (very positive or negative) ratings in practice. In an extension beyond two periods, we capture the evidence that firms harvest ratings, but that lower-quality firms are less likely to maintain good ratings (Cabral & Hortaçsu, 2010; Jin & Kato, 2006). We also show that our results persist when we introduce

[3] Quote from article Rey (2020) on vox.com.

[4] Prior to 2019, Amazon had required raters to leave a 20-word review along with their rating. Documented by Amazon reviews and forums (Amazon Customer, 2012; crebel, 2017).

[5] See information disseminated to sellers by Amazon (Rushdie, 2018) and a Bloomberg news article (Soper, 2019).

a continuum of firms.

We introduce the basic model in Section 2, and discuss the equilibrium in Section 3. Section 4 shows how various features in the rating system influence how well ratings reflect quality. We then discuss implications on surplus in Section 5. We present extension in Section 6, and robustness in Section 7. Section 8 connects our results to the literature, and Section 9 concludes.

2 Basic Model

We set up a 2 period model of incomplete information with a long lived monopoly firm and a unit mass of buyers in each period.

Firms. The firm can be of a high or low quality. A firm of type $j \in \{L, H\}$ has quality q^j, where $q^L < q^H$. The probability that the firm is of type q^H or q^L is common knowledge and given by $\gamma \in (0, 1)$ and $(1 - \gamma)$, respectively. The realized quality is private information to the firm and is constant between periods. In each period, the firm sets the price of its product to maximise its lifetime profit, $\sum_{t=1}^{T} p_t^j$, where p_t^j is the price of firm j in period t. We assume that the cost of production is zero regardless of quality.[6] After selling in period 1, sellers may receive a rating R_1 from consumers. If they do, this rating is made common knowledge to both the firm and consumers in subsequent periods.

Consumers. Consumers participate in only one of the two periods, with a new unit mass of consumers arriving in each period. We normalize the value of their outside option to zero. When choosing to consume a product in period t, consumers observe the price on offer and past ratings, R_{t-1}. They do not observe the firm's quality or past prices. Consumers may choose to buy or not to buy. If they choose not to buy, they exit the market. If they consume, they observe the firm's true quality and decide on leaving a rating. For simplicity, in the main model, we focus on a binary rating system where consumers can choose between leaving a rating or not. More precisely, ratings take the form $R_t \in \{0, 1\}$. The informational content of a rating will be determined in equilibrium, but we say that a rating is good if $R_t = 1$, and when $R_t = 0$, consumers choose not to provide any rating. We focus on positive ratings to simplify exposition and we show below that our main results are qualitatively

[6] We only require that low-quality firms face a smaller marginal cost than high-quality firms. Without loss of substance, it simplifies the analysis to assume that both firms face zero cost of production. This is similar to allowing for costless signals in a cheaptalk game (see Kreps and Sobel (1994) and Crawford and Sobel (1982)). Additionally, our model features a concept of endogenous cost such as that in Martin and Shelegia (2021). In our setting, by harvesting ratings, low-quality firms reduce the benefit of ratings harvesting.

robust when consumers are able to leave bad ratings.[7] Without loss of generality, we say $R_0 = 0$, i.e. firms have no previous ratings.[8]

To model reciprocity, we build on classic models of intrinsic reciprocity proposed by Dufwenberg and Kirchsteiger (2004) and Rabin (1993). By introducing reciprocity in our model, consumers are able to reward 'good' deals with a good rating. In equilibrium, good ratings reward sellers by increasing their future profits. Given the true quality and price, the consumer decides if a seller made a sufficiently kind offer, measured in terms of value-for-money. If the consumer receives a sufficiently high value-for-money, then it is able (and willing) to reciprocate the firm's kindness with an act of kindness of its own in the form of a good rating. We distinguish between consumption utility and rating utility. This serves two purposes. First, we are able to capture the phenomenon that consumers do not factor the intention to rate into their purchase decision.[9] Second, this simplifies presentation of results. The consumption utility for consuming a product from firm j in period $t \in \{1, 2\}$ is given by $u_t = q^j - p_t^j$, where p_t^j represents the price that the firm sets in period t.

The rating utility for consumers in period t is given by $v_t = [\kappa q^j - p_t^j]\Delta - e$ if $R_t = 1$ and $v_t = 0$ if $R_t = 0$. $\kappa \in [0, 1]$ represents the proportion of surplus which consumers think is equitable for firms to receive; $\Delta > 0$ represents the perceived kindness of a consumer leaving a rating to the firm; and $e \geq 0$ reflects the opportunity cost of providing a rating. This expression is based on the intrinsic reciprocity models in Dufwenberg and Kirchsteiger (2004) and Rabin (1993).[10]

The first term $[\kappa q^j - p_t^j]$ captures the consumer's perception of the firm's kindness. Consumers perceive a price equal κq^j as fair. Thus, they perceive any price below κq^j as a kindness, and $[\kappa q^j - p_t^j]$ is positive. Otherwise, if $[\kappa q^j - p_t^j]$ is weakly negative, firms keep more surplus than what consumers deem as equitable, and consumers perceive firms as unkind.

The second term Δ describes the consumers' sense of reciprocity and captures the warm glow that consumers receive from being kind to a firm by leaving a good rating. Finally, the consumer faces some costs when leaving a rating e, such as time, effort, attention etc. Below, we consider that e may depend on the design of a ratings systems, for example, one-click ratings, constant reminders, and purchase verification.

[7] We start with an environment where negative ratings do not exist to simplify exposition of results. We discuss how our results extend to a setting with negative ratings that punish bad deals in Section 6.

[8] This captures cases where the firm is new in the market. More generally, our model captures cases where two firms of different quality have the same rating at the beginning of period 1.

[9] This is highlighted by Cabral and Li (2015), who show that incentivizing consumers to rate does not change the number of bids or bid levels on eBay, indicating that consumers do not consider the rating incentives when making a purchase.

[10] Derivation can be found in Appendix B.

Timing of game. To summarize the timing of the game,

$t = 1$

- Firm draws quality, q^j.
- Firm sets price, p_1.
- Consumers observe price and form expectations over consumption utility, u_1.
- Consumers make purchase decision, if $u_1 \geq 0$.
- Consumers that purchase observe true quality.
- Consumers evaluate rating utility, and rate if $v_1 \geq 0$.
- Period 1 consumers exit the market.

$t = 2$

- Period 2 consumers arrive.
- Firm observes historical rating, R_1, and sets new price, p_2.
- Consumers observe past ratings and current price, forming expected quality.
- Consumers make purchase decision.

In the main model, we make some simplifying assumptions. We discuss in Section 7 how results are robust to various extensions, namely a game with more than 2 periods, a continuum of quality types of the firm, and a rating system with positive, and negative ratings.[11]

This model is representative of markets such as Amazon, Taobao, eBay, AirBnB[12] and Google reviews. These platforms help to facilitate matches between consumers and firms. Consumers rely on ratings to form or update their expectations of product quality. Firms use ratings to differentiate themselves from lower quality firms, allowing them to build trust and gain patronage.

[11] Lewis and Zervas (2019) show that only the relative difference in stars affects the pricing decision of firms. This suggests that it is more important to consider the effect of a relative difference in ratings, which we already do with our simple binary-rating framework. Other papers find that negative ratings have a statistically insignificant impact on prices (Bajari & Hortaçsu, 2003; Cabral & Hortaçsu, 2010; Livingston, 2005; Resnick et al., 2006). Hence, we believe that this simplification is well justified. Although, as we show in our extension, allowing for more flexibility in the ratings scale does not qualitatively change our result.

[12] Although we do not explicitly capture the mutual nature of AirBnB's rating system, ratings on AirBnB are only revealed after both host and guest have provided a rating. This removes the threat of retaliation rating-for-rating in response to a negative rating, which is why AirBnB and other similar mutual rating system resemble our one-directional rating system.

3 Equilibrium

We look for a perfect Bayesian equilibrium. We apply two additional restrictions as equilibrium selection assumptions.[13] With the first restriction we focus on equilibria where product quality is not diminishing in ratings.

Restriction 1. *We focus on the equilibria where high-quality firms obtain a rating of* $R_1 = 1$ *with probability 1.*

We focus on equilibria in line with evidence that high-quality firms are more likely to receive good ratings (Ananthakrishnan et al., 2019; Li et al., 2020).[14] Importantly, Restriction 1 allows us to focus on the strategic decisions of low-quality firms.

We make a second selection assumption to limit off-the-path beliefs.

Restriction 2. *For all prices such that low-quality firms obtain no rating, the expected quality is independent of prices.*[15]

Restriction 2 implies that ratings, rather than prices, serve as a signal of quality. The restriction captures that firms can mimic each others' prices, making prices a poor signal of quality. More precisely, because all firms have zero marginal cost, low-quality firms can always deviate to any price set by a high-quality firm. This makes it difficult for high-quality firms to use price signals to differentiate themselves from low-quality firms.

Together, these restrictions give ratings the best shot at being an informative signal for product quality. Restriction 1 focuses on equilibria where high-type firms always receive a good rating, and Restriction 2 ensures that the information signal that future consumers receive comes from ratings alone.

The following proposition characterizes equilibria in this game.

Proposition 1. *All perfect Bayesian equilibria satisfy the following.*

1. *In period 1, high-quality firms charge* $\overline{p} \equiv \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$ *with probability 1 and receive a good rating.*

[13] These restrictions are similar in spirit to those used in Rhodes and Wilson (2018), who study how advertisement can signal quality.

[14] Formally, equilibria where high-quality firms prefer to get a rating with probability 1 exist if κ is sufficiently large. Intuitively, when κ is large, the high-quality firm can charge a high price and still receive a good rating. This reduces their incentive to deviate to prices even closer to or at q^H, at which they receive no rating.

[15] Alternatively, we could use the D1 refinement, but we choose to use Restriction 2 instead, as it is weaker restriction and more intuitive (Cho & Sobel, 1990).

2. In period 1, low-quality firms randomize their price.

a. They charge $\overline{p}$ with probability δ^* and obtain no rating.

b. They charge $\underline{p} \equiv \kappa q^L - \frac{e}{\Delta}$ ($< \overline{p}$) with probability $1 - \delta^*$ and obtain a good rating.

c. $\delta^* \in (\frac{1}{2}, 1)$ if and only if

$$(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}, \tag{1}$$

and $\delta^* = 1$ otherwise.

4. In period 2, prices equal expected quality conditional on ratings.

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \end{cases}.$$

The equilibrium is unique up to off-equilibrium-path beliefs and exists if $\kappa q^H - \frac{e}{\Delta} \geq \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$.[16]

Firms charge one of two prices in equilibrium. High-quality firms always set $\overline{p}$ to extract all expected surplus (conditional on observing $\overline{p}$).[17] If consumers could distinguish high- and low-quality firms, this price would be equal to q^H. But because in period 1 consumers cannot distinguish firms, low quality firms can mimic the high-quality firm and charge $\overline{p}$. This, in equilibrium, can lower the conditionally expected surplus below q^H. The low price $\underline{p}$ is just low enough so that consumers perceive low-quality firms as sufficiently kind to give them a good rating, i.e. $\underline{p}$ is such that $[\kappa q^L - \underline{p}]\Delta - e = 0$.

How do low-quality firms choose between these two prices? Low-quality firms benefit from ratings by earning higher prices in period 2 (point 4). But to get a rating, they have to charge a low price $\underline{p}$ ($< \overline{p}$) in period 1 so that consumers reciprocate with a good rating, which is why we call this strategy 'ratings harvesting'. Alternatively, in period 1 low-quality firms can mimic the high price of high-quality firms $\overline{p}$, and get no ratings and lower profits in period 2. Because firms who follow this strategy copy the price of high-quality firms, we call it 'price mimicking'. Thus, low-quality firms trade-off ratings harvesting and price mimicking. The probability that low-quality firms mimic prices and do not receive a rating, δ^*, captures how firms resolve this trade-off in equilibrium. This δ^* also captures the informativeness of ratings: as δ^* increases, low-quality firms obtain a rating less often, and the ratings better help consumers distinguish between high-and low-quality firms.

[16] Formal proofs can be found in Appendix C.

[17] Formally, Restriction 2 implies that $\overline{p}$ extracts all conditional expected surplus. Without this restriction, there can be equilibria where the high price is lower than the conditional expected surplus.

Why may low-quality firms set $\delta^* \in (\frac{1}{2}, 1)$? Intuitively, low-quality firms harvest ratings to free-ride in the reputation of high-quality firms. This, however, undermines the expected quality associated with a good rating until—in equilibrium—low-quality firms are indifferent between ratings harvesting and price mimicking.

We now explain the impact of reciprocity on low-quality firms' pricing and ratings. To begin, suppose that Δ is sufficiently small such that $\delta^* = 1$ in equilibrium. For a given quality of ratings δ^*, low-quality firms need to offer a discount $\overline{p} - \underline{p}$ to get a good rating; this discount $\overline{p} - \underline{p}$ decreases as Δ increases. Intuitively, when Δ is small, the rating utility $v_1 = [\kappa q^L - p_1^L]\Delta - e$ is quite 'small' and consumers have a low level of intrinsic motivation to leave a good rating. To encourage consumers to rate and compensate them for the cost of ratings e, low-quality firms need to offer a lower price to provide a higher value-for-money for consumers. When Δ is sufficiently small, low-quality firms find it too costly to obtain a rating, so they charge the higher price of $\overline{p}$ with probability $\delta^* = 1$. Because only high-quality firms obtain a rating, ratings perfectly signal quality.

Now suppose Δ increases. As Δ increases, consumers are more inclined to reciprocate a given value-for-money with a rating, which is why the price for which low-quality firms can obtain a good rating, $\underline{p}$, increases. In other words, more kindness (Δ) lowers the opportunity cost of obtaining a good rating. Low-quality firms set the lower price of $\underline{p}$ more often, and obtain a good rating in equilibrium with positive probability.

These arguments explain why reciprocity reduces the quality of ratings: low-quality firms can offer a higher value-for-money to trigger consumers' reciprocity and obtain good ratings. But when low-quality firms start to harvest ratings, both high- and low-quality firms receive good ratings with strictly positive probability, which makes ratings less-informative about product quality. Thus, low-quality firms harvest ratings to trigger the consumers' kindness, but thereby undermine the quality of ratings. The following proposition summarizes this result.

Proposition 2. $\frac{\partial \delta^*}{\partial \Delta} < 0$ *when* (1) *holds, and* $\frac{\partial \delta^*}{\partial \Delta} = 0$ *otherwise.*

When consumers exhibit a larger sense of reciprocity, low-quality firms harvest ratings more frequently. Two clear implications arise form this proposition. First, firms use prices to induce consumers to be kind. Second, when consumers are more reciprocal, ratings become less-informative.

Growing evidence suggests that ratings are influenced not just by quality, but also by prices. More precisely, consumers seem to rate based on the value-for-money they obtain from a purchase. For example, studying marketplaces for digital cameras, Li and Hitt (2010) highlight that a 1% increase in price reduces ratings by 0.36 stars in 5-star ratings and 0.71 stars for 10-star ratings. On AirBnB, Gutt and Kundisch (2016) and Neumann et al. (2018) show that prices negatively impact ratings. On Yelp, Luca and Reshef (2021) provides evidence that a 1% increase in prices leads to a 3-5% decrease in average rating. Abrate et al. (2021) suggests that a 1% increase in hotel prices leads to a decrease of 1 star (out of 10) in overall ratings. Because these articles control for product characteristics, they suggest that it is value-for-money that influences ratings, and not quality alone. Our mechanism explains how both quality and prices affect ratings through reciprocity: consumers perceive a higher value-for-money as a kindness, which they reciprocate with positive ratings.

Our results connect to evidence on rating inflation. Rating inflation is well documented in the literature (Filippas et al., 2022), but not well understood. We offer novel explanations for this phenomenon: low-quality firms lower prices to offer a larger value-for-money and boost their ratings. This benefits these firms, but, in equilibrium, makes ratings less-informative about product quality.

We have shown how reciprocity can impact the informativeness of ratings. We now explore how the design of ratings systems can impact the informativeness of ratings, and ask how beneficial ratings are for consumers.

We restrict the remainder of our analysis to situations where the low-quality firm plays a mixed-strategy, i.e. where (1) holds. First, the condition is satisfied when the difference in quality is sufficiently large, i.e. when ratings are more relevant in the first place. Second, doing so allows us to directly study how changes to the ratings environment impacts the informativeness of ratings, captured by δ^*.

4 Designing ratings environments

In this section, we discuss how common features of a platform, such as the effort needed to leave a rating and the aggregate product quality on a marketplace, influence the informativeness of ratings.

Facilitating ratings

The design of ratings systems can make it easier, or more difficult, to rate. For example, raters may have to complete a verification process, they may be asked to rate along multiple dimensions, or raters may receive monetary rebates and reminders to rate. These design features influence the time and cognitive effort it takes to evaluate a product, and therefore the cost of leaving a rating.

Intuitively, the law of large numbers would suggest that collecting more ratings would lead to more precise and more-informative ratings. From this perspective, reducing the cost of leaving a rating seems to be a good idea. We show that this intuition is misleading as it ignores how firms and consumers respond when ratings become easier.

In our setting, less-costly ratings lead to less-informative ratings. The reason is closely related to reciprocity: if a kind action is less effort, consumers are more inclined to do be kind. Thus, as becomes easier for consumers to rate (i.e. e decreases), firms need to leave less surplus to induce consumers to reciprocate with a good rating.[18] In equilibrium, this encourages especially low-quality firms to harvest ratings: they set $\underline{p}$ with a higher probability $(1-\delta^*)$, which makes ratings less-informative about quality.

Corollary 1. *If* (1) *holds, then* $\frac{\partial \delta^*}{\partial e} > 0$.

The result suggests that making ratings less costly for consumers, even though it induces more ratings in equilibrium, encourages low-quality firms to harvest ratings and makes ratings less-informative. This provides another channel through which reciprocity-based ratings induces rating inflation: easier ratings encourage especially low-quality firms to harvest ratings. In turn, this means that making ratings costly can make them more-informative.

This result connects well to evidence. Cabral and Li (2015) measure quality using shipping speed. They find that for low-quality products, higher rebates for ratings decrease the proportion of negative ratings. Since rebates offset the cost of rating, their result highlights—as in our model—that lower cost of rating makes ratings less-informative about quality. Lafky (2014) shows that when it becomes more costly to rate, ratings become more extreme. This follows our prediction that it is easier to differentiate between firms when ratings are costly.

Anecdotal evidence suggests that the cost to leave a rating has decreased over time on many ratings platforms. Yelp and google encourage ratings with various perks, such as invitations to exclusive events and discount codes.[19] On google reviews, users receive constant reminders to leave ratings and can leave one-click ratings on their smartphone. This trend of fuss-free ratings is also gaining traction on Amazon. Prior to 2020, Amazon required customers to write a review in order to leave a rating, subsequently removing this requirement and

[18]Formally, $\underline{p}$ increases and the difference between $\overline{p}$ and $\underline{p}$ shrinks.

[19]Yelp Elite Squad and Google Local Guides perks as described by Yelp (Yelp, 2022) and the Harvard Business Review article Donaker et al. (2019).

allowing for one-click ratings.[20] Publicly, they stated that more feedback would lead to more accurate ratings, relying on the law of large numbers to drown out fake reviews.[21] Our results suggest that Amazon's effort to encourage ratings to make them more-informative may backfire. Although more 'real' customers may be rating, which could solve the problem of fake ratings on Amazon, easier ratings also affect pricing incentives of firms. By reducing the cost to leave a rating, low-quality firms firms find it easier to trigger reciprocity and obtain a good rating, which can ultimately lead to less-informative ratings.

Quality control

We have shown that (i) a stronger sense of reciprocity (Δ), and (ii) lower cost to leave a rating (e) encourage low-quality firms to harvest ratings, triggering rating inflation. In both cases, rating inflation is the result of less-informative ratings. In practice, however, an increase in the average product quality also improves ratings. We now shed light on this scenario and explore how changes in the aggregate quality of sellers in the market affect the informativeness of ratings.

The aggregate quality on a marketplace may change for a variety of reasons. First, low-quality firms may invest in better quality (Klein et al., 2016) or leave the market (Cabral & Hortaçsu, 2010; Nosko & Tadelis, 2015). Second, platforms may pre-screen and weed out low-quality firms to control the quality of firms (Casner, 2020; Nosko & Tadelis, 2015; Wang, 2021). Both channels can improve the aggregate quality on the market and ultimately lead to better average ratings. If firms have higher average quality, however, the incentives of low-quality firms to harvest ratings also changes. This way, better average seller quality may render ratings more- or less-informative.

We show that when the aggregate quality improves (i.e. γ increases), the remaining low-quality firms may harvest more or less ratings, depending on the aggregate quality level of the market.

The starting point to understand this result is that low-quality firms only harvest ratings, when ratings are useful to distinguish seller quality. Intuitively, low-quality firms can only free-ride on the reputation of high-quality firms, when ratings are somewhat useful to distinguish sellers so that high-quality firms indeed have a reputation.

Let us first consider the case where the aggregate quality in the market is low. For simplicity, suppose γ is close to zero. Almost all firms are of low quality, so ratings are somewhat useless

[20] A timeline of Amazon's rating system as documented by Forbes (Masters, 2021).
[21] As reported by TechCrunch (Perez, 2019).

to help consumers distinguish sellers. But as γ increases, ratings become more useful, which encourages low-quality firms to harvest more ratings. Thus, for 'small' γ, an increase in the aggregate quality leads to less-informative ratings.

Now consider the opposite scenario where the aggregate quality is high. For simplicity, suppose γ is close to 1. Again, ratings are not useful to distinguish sellers. But as γ increases, they become even less useful to distinguish sellers so that low-quality firms harvest ratings less. This is why for 'large' γ, an increase in the aggregate quality leads to more-informative ratings.

The following proposition summarizes this result.

Proposition 3. *There exists a unique* $\overline{\gamma} \in (0,1)$ *such that* $\frac{\partial \delta^*}{\partial \gamma} < 0 \iff \gamma < \overline{\gamma}$*, and* $\frac{\partial \delta^*}{\partial \gamma} > 0 \iff \gamma > \overline{\gamma}$*.*[22]

The proposition works out how changes in aggregate quality interacts with the informativeness of ratings. When aggregate quality is low, quality increases make ratings less-informative. But when aggregate quality is high, improvements in quality have a double dividend: aggregate quality increases and ratings become more-informative.

Our result connects with the observation of quality-controls in practice. For example, Amazon actively enforces seller quality, suspending sellers who do not meet a minimum standard. [23] Also Uber has announced that it will remove both riders and drivers with consistently poor ratings,[24] and both Uber and their subsidiary Uber Eats suspend drivers who fall below a minimum rating.[25] Booking.com suspends properties for quality control purposes;[26] Airbnb bans hosts based on a combination of factors, including being in the bottom 1% of overall ratings and guest feedback.[27]

We explain how such measures may affect the informativeness of ratings. First, even though good ratings may not be fully-informative about underlying quality, no ratings or—as we show in an extension—bad ratings are quite informative about low quality.[28] Thus, even

[22] $\overline{\gamma}$ is given by (11).

[23] Amazon makes this decision through a combination of customer reviews, feedback and other measures (Rushdie, 2018; Soper, 2019).

[24] Uber announces that it will begin deactivating riders with poor ratings (Dickey, 2019).

[25] Leaked documents from Uber suggest drivers risk deactivation if they fall below 4.6 stars as reported by TechCrunch (Dickey, 2019). Uber Eats communication with couriers as posted on a forum for drivers (UberLyftDriver, 2017).

[26] Partners report that Booking.com closes their apartments on Booking.com Partner Hub (Prodius, 2020).

[27] Information from Airbnb help center information (Airbnb, 2022) and a third party Airbnb management service (Zodiak, 2021).

[28] In practice, consumers may not rate for various reasons, so that products without ratings are not necessarily low quality. This is why we check that the result also holds in the setting with negative ratings which are

in scenarios when good ratings are uninformative, platforms can indeed use bad ratings to weed out low-quality firms. Second, when quality controls make ratings less-informative, that platforms can counter such adverse affects by adjusting the effort to rate accordingly.

5 Surplus Analysis

So far, we studied how reciprocity affects the informativeness of ratings. Conventional wisdom suggests that more-informative ratings help consumers make more-informed purchase decisions and ultimately benefit consumers. We now explore the link between the informativeness of ratings and consumer surplus more carefully. The key insight is that consumer-optimal rating systems are often somewhat, but never fully, informative.

To start, we investigate how ratings harvesting affects consumer surplus. In order to induce consumers to reciprocate, firms need to set a price below the consumers' ex-post willingness to pay. This is why sellers, even though they are monopolists, leave a rent to consumers. To see this rent, we write down the expected consumer surplus

$$CS = (1-\gamma)(1-\delta^*)(q^L(1-\kappa) + \frac{e}{\Delta}). \tag{2}$$

In period 2, and in period 1 when consumers face a high price of $\overline{p}$, firms set prices to extract all conditionally expected consumer surplus. But in period 1 when consumers observe a low price, which happens with probability $(1-\gamma)(1-\delta^*)$, they get the surplus $q^L - \underline{p} = (q^L(1-\kappa) + \frac{e}{\Delta})$. Low-quality firms leave this surplus so that consumers reciprocate with a good rating, allowing low-quality firms to free-ride on the reputation of high-quality firms and charge a higher price in period 2.

We have shown above that low-quality firms harvest ratings and make ratings less-informative. But in order to harvest ratings, low-quality firms need to offer low prices. This is why rating harvesting benefits consumers. This, however, does not imply that consumers prefer uninformative ratings. If ratings were completely uninformative, low-quality firms would not harvest ratings, and consumers would earn no surplus. We now discuss implications of this result more carefully for the impact of rating effort on consumer surplus.

Cost of ratings

A lower cost of ratings (e) has two opposing effects on consumer surplus. First, and following directly from Corollary 1, low-quality firms harvest ratings and charge the low price $\underline{p}$ more

informative about low quality.

often; this tends to increase consumer surplus. Second, however, the level of the low price, $\underline{p}$, increases: when consumers can rate more easily, they are more inclined to reciprocate kindness. This is why low-quality firms who harvest ratings can now charge a higher $\underline{p}$ and still receive a good rating, reducing consumer surplus. Overall, because easier ratings encourage low-quality firms to harvest ratings, they harvest ratings more often, which tends to benefit consumers. But low-quality firms do so with a higher price, which tends to harm consumers.

We show that these two opposing effects pin down a positive level of ratings effort that maximizes consumer surplus.[29]

Proposition 4. *There exists a level of effort $e^{cs} > 0$ that maximizes consumer surplus. At this level of effort, $\delta^* \in (\frac{1}{2}, 1)$. This is true if*

$$(1-\gamma)^2\gamma(q^H - q^L)^2 \geq (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2. \tag{3}$$

Otherwise, consumers prefer $e^{cs} = 0$.

The proposition characterizes when the conventional wisdom that more-informative ratings benefit consumers is true. When e is sufficiently small ($e < e^{cs}$), making it more costly to rate leads to more-informative ratings (Corollary 1), and increases consumer surplus. In this case, the price effect on $\underline{p}$ dominates and more-informative ratings put pressure on prices for low-quality firms.

More surprisingly, when e is sufficiently large ($e \geq e^{cs}$), more-informative ratings harm consumers. Intuitively, when e is large, low-quality firms charge a very low $\underline{p}$ to harvest ratings. This is why, as e increases further, low-quality firms get discouraged from harvesting ratings, which reduces consumer surplus.

A key implication of the Proposition is that even though consumers benefit when low-quality firms harvest ratings, consumers still prefer a somewhat-informative rating system. The reason is that low-quality firms are only willing to harvest ratings if they can free-ride on the good reputation of high-quality firms; but this requires ratings to be somewhat-informative.

By condition (3) consumers prefer somewhat-informative ratings if the difference between high- and low-quality firms is sufficiently large. This is similar in spirit to (1) and rather

[29] Note that if we would incorporate rating utility into consumer surplus, consumers would get an additional benefit and cost from rating, so the consumer optimal level of effort could be higher or lower. But crucially, it can still be positive.

intuitive: if the condition is violated and the difference in quality is small, then the price difference in period 2 is also small. Low-quality firms have little incentive to harvest ratings and hardly ever do so. Because low-quality firms harvest ratings so rarely, consumers want firms to harvest ratings more often and prefer $e^{cs} = 0$. This result, however, seems economically less relevant, since it only applies when the quality differences are small so that ratings are less relevant in the first place.

We now explore the impact of costly ratings on seller surplus. While buyers prefer somewhat-informative ratings, sellers on average prefer uninformative ratings. Intuitively, seller surplus is largest when sellers leave the smallest reciprocity rent to buyers. This is true when $e = 0$ and ratings are least-informative. In this case. low-quality firms have the highest incentive to harvest ratings, but face the smallest opportunity cost for doing so.

While sellers on average prefer less-informative ratings, high-quality firms prefer a large e and perfectly-informative ratings: informative ratings allow high-quality firms to distinguish themselves from low-quality firms who try to free-ride on their reputation. As e increases, this free-riding becomes more costly, which leads to more-informative ratings and allows high-quality firms to extract more of the surplus they generate. The following corollary summarizes these results.

Corollary 2. *When* (3) *holds, average seller surplus is maximal at* $e^s = 0$ $(< e^{cs})$. *Moreover, profits of high-quality firms increases in* e*, and profits of low-quality firms decreases in* e.

The corollary implies that only high-quality firms unambiguously prefer perfectly-informative ratings, because this limits free-riding on their reputation. Neither buyers nor sellers, on average, prefer perfectly-informative ratings. But buyers prefer more-informative ratings than the average seller. The reason is that somewhat-informative ratings push sellers to harvest ratings, which puts pressure on prices. This suggests that the aforementioned efforts of Google reviews and Amazon to facilitate and encourage ratings might not just lead to less-informative ratings, but also harm consumers through higher prices. They do, however, benefit sellers.

Quality controls

We now discuss the effect of aggregate product quality on consumer surplus. Our first insight is that improvements in aggregate quality can make consumers worse off.

The intuition has two steps. First, we know from Proposition 3 that low-quality firms harvest ratings when they can free-ride on the reputation of high-quality firms. Thus, when γ is large and there are more high-quality firms to free-ride on, an individual low-quality firm harvests ratings more and benefits consumers. On the other hand, only low-quality firms harvest ratings and leave reciprocity rent to consumers, suggesting consumers benefit when γ is sufficiently small. As a result of these opposing forces, consumer surplus is concave in γ and an intermediate $\gamma^{cs} \in (0,1)$ maximizes consumer surplus.

The second key insight resembles a previous one on the cost of ratings: consumers prefer quality levels that lead to somewhat-informative ratings ($\gamma^{cs} < \overline{\gamma}$), but not fully-informative ratings ($\gamma^{cs} \to 1$). The intuition is familiar from above: with somewhat-informative ratings, low-quality firms harvest ratings, which puts pressure on prices.

The next proposition summarizes these results.

Proposition 5. *Equilibrium consumer surplus is strictly concave in γ. There exists an aggregate quality level, denoted by γ^{cs}, that maximises consumer surplus, where $\overline{\gamma} > \gamma^{cs}$ and $\gamma^{cs} > 0$.*

Proposition 5 implies that, even when larger aggregate quality leads to more-informative ratings, consumers can be worse off. This is the case when $\gamma \notin [\gamma^{cs}, \overline{\gamma}]$. For $\gamma < \gamma^{cs}$, better quality encourages low-quality firms to harvest ratings, which benefits consumers but undermines the informativeness of ratings. For $\gamma > \overline{\gamma}$, an increase in aggregate quality makes ratings more-informative; but as low-quality firms participate in less ratings harvesting, consumers surplus diminishes. Thus, to evaluate a rating system, observing that ratings reflect quality more closely is not enough to conclude that consumers benefit.

Sellers unambiguously benefit if their average quality increases. High-quality firms are able set higher prices in both periods. Low-quality firms benefit either from being able to set a higher price in the first period when they mimic prices, or from setting a higher price in the second period when they harvest ratings.

Lemma 1. *The profits of high- and low-quality firms increases in γ. Seller surplus is maximised at $\gamma^s \to 1$*

Because all firms benefit from a higher average quality, this also implies that (remaining) firms prefer a higher level of quality controls than consumers. Together with Proposition 3, Lemma 1 implies that sellers prefer ratings that are uninformative. Additionally, when considering Proposition 5, Lemma 1 suggests that firms prefer ratings that are less-informative than what consumers prefer. The intuition is familiar from above: firms on average prefer uninformative ratings, and consumers prefer somewhat-informative ratings. Hence, firms prefer ratings that are less-informative than consumers.

6 Extensions

Designing a profit maximizing ratings system

So far we studied how different features of a marketplace environment affects the informativeness of ratings and surplus. In practice, rating systems are typically designed by online platforms, and many platforms constantly tweak their ratings system. We now investigate how a platform would actually design its ratings system. To do so, we introduce some modifications to our base model.

Suppose there is a unit mass of consumers who are heterogeneous only in their outside option to joining the platform. This outside option is uniformly distributed between $[0, 1]$. We denote the mass of consumers who join the platform as n_b. Further, we call the per transaction consumer benefit u_b. Consumers choose to join the platform and are then randomly assigned (with equal probability) to the first period or the second period. This captures that consumers use the platform both when there are more or less ratings available to guide their choices.

We suppose that there exists a unit mass of sellers that are monopolists in a product category. Sellers are ex-ante homogeneous and face an outside option of $\bar{v}_s$. We denote the mass of sellers who join the platform by n_s and call π_s the per transaction revenue of the seller. We assume that sellers face some additional platform specific marginal cost of selling on the platform, t.[30] To simplify illustration, we suppose that sellers join the platform prior to period 1 of our main model and therefore do not have information over their true quality when deciding to join the platform.[31]

To maximize profits, the platform makes two choices. First, the platform sets a royalty, r, which it charges sellers. This gives the platform a share r of the sellers' revenue. Second, the platform chooses how easy it is for buyers to leave a rating, i.e. the platform sets e. We assume that the platform can choose any $e \in [0, \bar{e}]$. To simplify exposition, we assume $\bar{e} \leq e^{cs}$. This implies that a larger effort always increases u_b and reduces π_s. The platform makes these decisions prior to period 1 in the base model.

The effort choice e captures that the platform designs a ratings system to make it easier or

[30] This cost captures the difference in cost of selling on a platform rather than direct to consumers. Reflecting costs in addition to the platforms ad valorem fees. For instance, on Amazon, in addition to the ad valorem fees, there are additional charges for fulfilled by amazon and other transaction fees.

[31] This assumption captures the importance of relative quality of products on a marketplace, and sellers learning their true relative quality after joining the marketplace. This assumption also allows us to abstract away from seller selection by the platform and focus on the role that the platform plays in influencing ratings.

more difficult for consumers to leave a rating. For example, to facilitate ratings, a platform can introduce a one-click ratings system or provide users a link to go directly to the ratings page, automated ratings[32], etc. Conversely, a platform can make rating more costly for consumers if it introduces additional authentication and verification steps—such as proof of identity, proof of purchase[33], multiple ratings components—such as the use of multi-dimensional ratings (Schneider et al., 2021), or requiring a written review along with the rating[34].

Proposition 6. *Platforms design a ratings environment that favours sellers ($e^* = e^s = 0$) if and only if $\pi_s - t < u_b$ at $e = 0$.*

The proposition characterizes when the platform wants to make it easier to rate. We show that platforms want to make it easier to rate whenever this has a stronger impact on per-transaction profit of sellers ($\pi_s - t$) than per-transaction consumer surplus (u_b). This captures the common finding that platforms want to balance surplus between buyers and sellers to generate value on the platform (Armstrong, 2006; Caillaud & Jullien, 2003; Rochet & Tirole, 2003). Combining this result with Proposition 4, we find that when $\pi_s - t < u_b$, the platform encourages ratings. Intuitively, a lower e raises profits per seller but also lowers the number of buyers who join the platform. $\pi_s - t < u_b$ implies that the former effect dominates. Thus, the platform facilitates ratings to lower reciprocity rents and shift surplus to sellers, but thereby undermines the informativeness of ratings.

These results support concerns of regulators that platforms design insufficiently informative ratings environments for consumers, and that minimum standards of ratings may help to protect consumers (Competition and Markets Authority (UK), 2017).

We made the simplifying assumptions that sellers are ex-ante identical and do not yet know their relative quality level when entering the platform. Corollary 2 suggests that relaxing this assumption could lead to a novel selection effect. Platforms may still prefer to lower e in order to raise average seller surplus. But this discourages high-quality firms from joining the platform and leads to lower average quality on the platform. This way, facilitating ratings can undermine a key purpose of a ratings system.

Competitive environment

Our basic model focuses on monopoly sellers. We now introduce competition to the model. To do so in a simple way, we assume that there exists a competitive fringe of non-strategic

[32] Documented by a website which guides non-mandarin speakers the use of Taobao (TaobaoTranslate, 2021).
[33] As explained by Amazon (Amazon, 2021).
[34] As is required by Steam ("Introducing Steam Reviews", 2021).

firms that offers a product of quality q at a price equal to their marginal cost c, where $q \geq 0$. Intuitively, we capture competition by changing the outside option of purchasing in the market to $q - c \geq 0$. This could capture (i) established firms for which consumers do not need ratings to evaluate the quality of their products; (ii) brick-and-mortar stores; or (iii) the expected utility a consumer gets when participating in another ratings environment such as another marketplace.

We now study how the presence of the competitive fringe affects the equilibrium strategy of our previous strategic sellers.

To start, suppose $q - c = 0$. Then the competitive fringe offers the same value as our previous outside option and the equilibrium is the same as in Proposition 1. Now suppose c decreases marginally and the competitive fringe offers a small but strictly positive surplus $q - c > 0$. This encourages sellers to harvest more ratings and reduces the informativeness of ratings. Intuitively, in period 1 firms that charge $\overline{p}$ extract consumers' conditional expected value. As c decreases, firms can no longer extract all surplus and reduce $\overline{p}$ to remain more beneficial to consumers than the fringe. The low-quality firms that charge $\underline{p}$, however, already leave a rent to consumers, which is why $\underline{p}$ does not decrease. Thus, the fringe puts more pressure on $\overline{p}$ than on $\underline{p}$, which is why low-quality firms harvest more ratings, leading to less-informative ratings in equilibrium (lower δ^*).

As c decreases further, $q - c$ becomes so large that it puts equal pressure on $\overline{p}$ and $\underline{p}$, so that the competitive fringe no longer affects the incentives to harvest ratings and the informativeness of ratings, δ^*.

The following proposition summarizes this result.

Proposition 7. $\frac{\partial \delta^*}{\partial c} > 0$ *if* $q^L - (q - c) \geq \kappa q^L - \frac{e}{\Delta}$. *Otherwise,* $\frac{\partial \delta^*}{\partial c} = 0$

The proposition shows that competition puts disproportionate pressure on firms that extract more surplus from consumers, i.e. firms that do not harvest ratings. This is why competition encourages firms to harvest ratings and makes ratings less-informative. Crucially, however, ratings will not become uninformative: as the fringe becomes more competitive ($q - c$ increases), it will put equal pressure on all prices and no longer affect incentives to harvest ratings, leaving the informativeness of ratings constant at $\delta^* = \frac{1}{2}$.

Even though competition makes ratings less-informative, it unambiguously benefits consumers through the following two channels. First, competition exerts pressure on the higher price and lowers $\overline{p}$. Second, competition encourages low-quality firms to harvest ratings and charge $\underline{p}$, for which consumers receive a higher surplus. Thus, our results suggest that making consumers aware of alternative sellers can make ratings less-informative, but benefits consumers nonetheless.

7 Robustness

Negative ratings. We relax our simplifying assumption and allow for negative ratings—in addition to positive ratings and no ratings. In equilibrium, negative ratings arise from retaliation (negative reciprocity). When firms leave a sufficiently small surplus to consumers, consumers perceive this as unkind and respond with an unkindness, retaliating with a bad rating. In equilibrium, only low-quality firms receive negative ratings, which is why negative ratings transmit the same information as no ratings in the main model, and our results are qualitatively robust.[35] But—different from the main model—ratings are more extreme.[36] For details, see A.1.

Continuum of Firms. We also show that our results extend beyond firms with two types and study a continuum of firms with different quality types. We find a pure-strategy equilibrium where (i) an interval of highest quality firms gets a good rating; (ii) an interval of lowest quality firms gets a bad rating, and (iii) a middle interval of firms harvests ratings and gets a good rating as well. In this equilibrium, middle-quality firms harvest ratings and make ratings less-informative. Additionally, as ratings become easier (e decreases),the middle interval of firms that harvest ratings grows, leading to less-informative ratings as in the main model. For details, see Appendix A.2.

Longer Horizon Model We show that our results are robust to more than two periods by looking at a three period model. We find equilibria that are similar to those described in our base model, and show that low-quality firms harvest ratings and mimic prices with strictly positive probability in every non-terminal period. Interestingly, and in line with evidence by Cabral and Hortaçsu (2010) and Jin and Kato (2006), low-quality firms are less likely to sustain a good rating. For details, see A.3.

[35] Fradkin and Holtz (2022) show that paying guests of hosts without ratings to rate leads to more negative reviews. In line with this, consumers who do not rate are more likely to have had a low-quality product, so encouraging them to rate when negative ratings are possible leads to more negative ratings.

[36] When consumers exhibit biased reciprocity, that is the warm glow they receive from leaving a good rating exceeds the warm glow they receive from leaving a bad rating, our result provides theoretical support for suggestions by Dellarocas and Wood (2008) and Filippas and Horton (2022) that reciprocity bias leads to a J-shaped ratings distribution, i.e. extreme ratings where negative ratings are less common than good ratings.

8 Related Literature

Our key results connect **evidence on ratings.** Ratings aid consumers by reducing uncertainty about the quality of the product. Much of the literature suggests that ratings do indeed signal quality on major platforms like eBay (Hui et al., 2021), Taobao (Zhang et al., 2012), and Airbnb (Proserpio et al., 2018). The broader empirical literature, however, suggests that other factors also influence ratings (Gao et al., 2018; Masterov et al., 2015; Nosko & Tadelis, 2015; Zervas et al., 2021).

Many studies have shown that prices have a significant influence on ratings. The evidence highlights two key patterns. First, for a given quality, lower prices induce better ratings (Cai et al., 2014; Carnehl et al., 2021; Li & Hitt, 2010; Luca & Reshef, 2021; Neumann et al., 2018). Second, firms with better ratings charge higher prices in the future, a pattern frequently referred to as 'rating harvesting' (Cabral & Hortaçsu, 2010; Cabral & Li, 2015; Cai et al., 2014; Carnehl et al., 2021; Ert & Fleischer, 2019; Gutt & Herrmann, 2015; Jin & Kato, 2006; Jolivet et al., 2016; Lewis & Zervas, 2019; Li et al., 2020; Livingston, 2005; Luca & Reshef, 2021; McDonald & Slawson, 2002; Neumann et al., 2018; Proserpio et al., 2018). We can explain both of these patterns in a single framework, i.e. low-quality firms charge lower prices to induce consumers to reciprocate with a good rating. Firms harvest these good ratings by charging higher prices in the future.

We also contribute to the **literature on reciprocity in ratings**. A series of empirical articles argue that reciprocity is a key driver of rating behavior, (Cabral & Li, 2015; Diekmann et al., 2014; Fradkin et al., 2021; Li & Xiao, 2014; Zervas et al., 2021). Some experimental work directly identifies that reciprocity drives rating behavior (Bolton et al., 2013; Halliday & Lafky, 2019; Lafky, 2014). Taken together, these experiments suggest: (i) reciprocity biases ratings upward in mutual-rating systems where buyers and sellers rate each other. But double-blind feedback strongly reduces this bias. Also following the work of Dellarocas and Wood (2008) and others on eBay, most online marketplaces adopted a double-blind approach to feedback, which is why we do not look at mutual-rating systems. (ii) Also with one-sided or double-blind rating systems, sellers take advantage of their ability to influence ratings, and they use prices to do so. We contribute to this literature by modelling how sellers use prices to trigger reciprocity and thereby influence ratings. By doing so, we derive novel predictions on how reciprocity causes rating inflation, namely that reciprocity induces ratings harvesting, reciprocity makes ratings less-informative, and that encouraging consumers to rate (e.g. through one-click ratings or reminders to rate) can backfire by making ratings less-informative.

We also connect to the ongoing **debate on rating inflation**. Rating inflation describes the observation that rating scores improve over time, and most of the improvements cannot be attributed to product quality (Filippas & Horton, 2022; Filippas et al., 2022; Nosko & Tadelis, 2015; Zervas et al., 2021).[37] We propose multiple channels through which reciprocity leads to more rating harvesting and therefore rating inflation: (i) platforms encouraging consumers to rate, (ii) increased quality controls by platforms, and (iii) increased competition between sellers.[38]

We connect to the wider theoretical literature on **trust and information transmission in the digital economy**. Platforms may recommend products (Hagiu & Jullien, 2011; Peitz & Sobolev, 2022), shroud additional fees and features of third-party sellers (Johnen & Somogyi, 2021), and marketplaces may have fake reviews (He et al., 2022). We contribute to this literature by studying information transmission via ratings and study how firms can use prices to affect their own ratings.

We closely connect to the **theoretical literature on reputation** (e.g. Cabral (2000), Jullien and Park (2014), Stenzel et al. (2020), and Tadelis (1999); see also Bar-Isaac and Tadelis (2008) for a survey). In existing work on reputation usually (i) buyers do not endogenously choose if and how to rate, and (ii) ratings mostly reflect quality and are independent of price. While some papers relax some of these assumptions (e.g. in Stenzel et al. (2020), ratings may depend on past prices, relaxing (ii)[39]), no article seems to feature that buyers choose if and how to rate strategically, and prices influence ratings. We capture both of these features, which, in contrast to previous work, allows us to capture empirical patterns of prices and ratings.

Some researchers argue that consumers should be **paid to rate**. One argument is that sellers should be allowed to pay for feedback, because only high-quality firms are willing to pay for feedback, making this a credible signal (Halliday & Lafky, 2019; Kihlstrom & Riordan, 1984; Milgrom & Roberts, 1986; Nelson, 1974). Others argue that feedback is like a public good

[37] In particular, Nosko and Tadelis (2015) study the binary rating environment on eBay and find that more than 99% of ratings are positive. Filippas and Horton (2022) study a marketplace that kept ratings unpublished at first and then started to publish ratings. Once ratings were published, they got better. This is in line with our mechanism: consumers can only reciprocate a good value-for-money once ratings are published, so our mechanism predicts more rating harvesting and therefore rating inflation once ratings are published.

[38] Research on information systems propose that social norm of good ratings increases over time (Qiu et al., 2012). Our mechanism can illustrate how social norms changed the way they did: platforms encourage consumers to rate (lowering e and/or raising Δ), which leads to a new equilibrium where reciprocity plays a more important role and ratings increase.

[39] Additionally, they focus on prices in the long-run equilibria. Because we do not focus on the long-run equilibria, we derive novel dynamics effects like rating harvesting in our framework.

that is underprovided (Avery et al., 1999; Bolton et al., 2004; Chen et al., 2010). In contrast, we show that encouraging ratings from all consumers (e.g. when marketplaces pay consumers to rate, introduce simpler one-click ratings, or reminding consumers to rate) encourages low-quality firms to harvest ratings, leading possibly to more ratings, but also less-informative ratings. This is in line with evidence by Cabral and Li (2015) and Lafky (2014) that we discussed above.

We also connect to recent work by Rhodes and Wilson (2018) on **false advertising**. Low-quality firms falsely advertise high quality to free-ride on the reputation of high-quality firms. Also in our setting, low-quality firms free-ride on the reputation of others and undermine information transmission. But our mechanism is very different. In contrast to ads, firms in our setting do not choose their own ratings, but need to charge a low price to trigger reciprocity and get a good rating. This leads to novel and inherently dynamic effects like rating harvesting.

9 Conclusion

Ratings are an essential element of the online economy, building trust between strangers. But ratings are only able to build trust if they are informative about the underlying products and services. In this paper, we use a model of consumer reciprocity to study how firms use prices to influence their own ratings. We explore a qualitatively novel trade-off between rating harvesting and price mimicking, which connects well to evidence on the dynamic interaction between prices and ratings. We identify several factors that encourage rating harvesting and lead to less-informative ratings. We also explore implications for buyer and seller surplus.

The key feature of our analysis is that consumers who obtain a higher value-for-money are more likely to leave a good rating. But consumers may rate for other reasons, e.g. to help other consumers by signalling product quality through ratings, or because they are intrinsically motivated to reveal the true quality of the product. In these motivations, however, ratings are not affected by price. Thus, even if some consumers have such other motivations, we would still expect price dynamics resembling rating harvesting, as long as at least some consumers rate based on their value-for-money.

In practice, fake ratings also undermine how informative ratings are. If low-quality firms are somewhat more inclined to acquire fake ratings, fake ratings will also make ratings less-informative of quality. In contrast, firms in our setting use lower prices to get better reviews, which benefits consumers and can explain evidence that lower prices induce better ratings.

Many platforms give consumers easy access to past ratings, but do not connect them to the prices that the raters payed.[40] This feature of many rating environments is a key reason, in our setting, why consumers cannot distinguish whether a given rating is the result of high quality, or a low price. If ratings, however, would reflect purchase prices, consumers may be better able to identify high-quality firms, which could discourage rating harvesting. We do, however, leave this and other questions for future research.

[40] For example, platforms such as Amazon do not reveal past prices to consumers on their listings. While there exist third party websites such as https://camelcamelcamel.com/ and https://keepa.com/ that track past prices on Amazon, such websites do not link prices to actual sales and reviews.

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Appendix A Robustness

A.1 Negative Ratings

For exposition we made the simplifying assumption of only good and no ratings. In this section, we relax this assumption to allow for more than 2 ratings, allowing consumers to rate positively, negatively, or not at all. In doing so, we show that if consumers rate value for money, only extreme ratings are played in equilibrium.

Formally, we setup a model that is identical to the base model, with the exception that $R_t \in \{1, 0, -1\}$, representing good, bad or no ratings respectively. Consumers provide a negative rating when they receive what they perceive to be unfair, and want to respond with negative reciprocity, potentially harming the firm. Hence, instead of obtaining a positive warm glow of Δ, we say consumers exhibit a negative warm glow of $-\Delta$ when leaving a bad rating. Finally, consumers only rate (bad or good) if they have a weakly positive rating utility.

In this setting, we show that there exist a range of prices for which each rating is possible: (i) when consumers receive a sufficiently high value-for-money, they choose to leave a good rating; (ii) when consumers receive a sufficiently low value-for-money, they choose to leave a bad rating; (iii) and for middle levels of value-for-money, consumers choose not to leave a rating.

However, there exist equilibria where low-quality firms choose between setting the prices of $\overline{p}$ and $\underline{p}$. This is because with Restriction 1, we search for equilibria where consumers' beliefs are such that both $R_t \in \{0, -1\}$ are a reflection of a low-quality firm. This means that, conditional on receiving no or a bad rating, firms receive the same pay off in the second period. Hence, a profit maximising firm sets the highest possible price of $\overline{p}$ if it chooses not to receive a good rating.

Proposition 8 summarises the implication of this.

Proposition 8. *There exists a range of prices for which good, no and bad ratings may occur. In equilibrium, it is always beneficial for low-quality firms to obtain a bad rating over no rating, if consumers continue to buy at the price that induces a bad rating.*

For completeness, we conduct a minor extension to show that our model connects well with suggestions in the empirical literature that consumers find it more difficult to leave a bad rating than they do a good rating (Cabral & Hortaçsu, 2010; Dellarocas & Wood, 2008; Filippas & Horton, 2022; Filippas et al., 2022). To do so, we modify the model with negative ratings to study the situation where consumers face different costs to leaving good and bad ratings. We say that consumers face a cost of e_b when leaving a bad rating, and a cost of e_g when leaving a good rating, where $e_b > e_g$.

Lemma 2. *When it is more difficult to leave a bad rating than a good rating, firms are more likely to obtain good ratings, and less likely to obtain bad ratings.*

Lemma 2 shows that when e_b is larger, firms are able to set a much higher price before they obtain a bad rating. This result implies that when e_b is sufficiently large, low-quality firms may not receive any bad ratings in equilibrium. Moreover, because e_g is relatively smaller, combined with Corollary 1, low-quality firms are likely to participate in ratings harvesting and receive a good rating.

We show that extreme ratings by the low-quality firm is an equilibrium result in a model of more than 2 ratings, and our model does not lose any information by considering a binary ratings system. Further, when considering the difference in cost needed to provide a rating, we provide theoretical foundation for the J-shape distribution of ratings found in the empirical literature, suggesting that it is an equilibrium result of reciprocity-based ratings (Dellarocas & Wood, 2008; Filippas et al., 2022; Hu et al., 2009).

A.2 Continuum of quality

For exposition, we discussed a binary setting of firms with only high or low quality. In this section, We shall show that our results are robust to firms with a continuum of quality types.

Formally, we setup a model that is identical to the base model, with the following modifications: (i) we have firms with a continuum of type, uniformly distributed along the interval [0,1]; (ii) we also introduced a modified version of our restrictions, replacing Restriction 1 and 2 with Restriction 3 and the D1 refinement respectively.

Restriction 3. *The highest quality firm receives a good rating with probability* 1

We show that in this setting, firms are divided into 3 groups: (i) higher quality firms which set the highest possible price and receive a good rating in equilibrium; (ii) middle quality firms which choose to set lower prices, depending on their quality, to participate in ratings harvesting; (iii) lower quality firms that choose to set the highest possible price and forgo ratings.

Proposition 9. *Firms with quality $q \in [\hat{q}, 1]$ obtain a good rating in equilibrium, without influencing ratings. Firms with quality $q \in [q'', \hat{q})$ choose to participate in ratings harvesting* *in equilibrium. Firms with quality $q \in [0, q'')$ do not obtain a rating in equilibrium.*

To show that this connects qualitatively with results on effort cost, we show that as it becomes easier to rate, the middle group increases and thus more ratings harvesting occurs. This occurs because more lower quality firms choose to join the middle group than middle quality firms join the top group. Thus leading to less-informative ratings. Although, this suggests that ratings become less-informative, it is not on account of the lowest quality firms receiving a good rating. Suggesting that ratings are probably still somewhat useful for signalling quality.

A.3 Multi-period model

We check for robustness to time horizon by looking at a three period model. In this model, we find that equilibria similar to those described in our main model exists.

Notice that in a three period model, ratings take the following possible histories:

- $R_0 = \{0\}$
- $R_1 = \{01, 00\}$
- $R_2 = \{011, 010, 001, 000\}$

To save on notation, we omit the 0 in the initial period, capturing that all sellers start with the same reputation. We also extend Restriction 1 to reflect the additional period.

Restriction 4. *We focus on the equilibria where high-quality firms obtain a rating of $R_t = 1$, $t \in \{1, 2\}$ with probability 1.*

Further, in each period low-quality firms may decide to randomize with different probabilities, δ_t. Aside from the addition of a third period, the model remains the same as that of the main body of this paper.

Our intention is to uncover equilibrium strategies similar to the main body of the paper and not to consider all possible equilibria. We show that our findings continue to exist in a 3 period model. That is, we show that low-quality firms may choose to play a mixed strategy in every period. The result differs from our base setting in that a stricter set of restrictions is required for a unique mixed strategy to exist in every period.

We show that there exist equilibria where the low-quality firm chooses to play a mixed strategy in period 1 but a pure strategy in period 2. This means that low-quality firms choose to obtain good ratings in the first period even if they are unable to sustain this ratings outcome. As a result, we believe that our results are not driven by the end game effect of the final period. Instead, this effect is continuous and trading off pay offs in earlier periods for a big pay off in latter periods is a consideration made independent of the terminal period.

This is summarised in the following proposition

Proposition 10. *A unique mixed strategy equilibrium with properties similar to the base model exists in a 3 period model.*

Therefore, we conclude that the main results of our paper are robust to multiple periods and limiting our analysis to 2 periods in the main text helps to focus the discussion on the direct implications of reciprocity-based ratings.

Appendix B Rating Utility

We derive the rating utility function from the utility function proposed in Rabin (1993). In his paper, Rabin proposes a utility function which incorporates a reciprocity term in addition to a consumption utility. This reciprocity term depends on the additional surplus that some player i is allowed to obtain given the actions of another player j relative to a predefined equity point.

$$U_i = \pi_i + [\frac{\pi_i - \pi_i^e}{\pi_i^H - \pi_i^{min}}][1 + \frac{\pi_j - \pi_j^e}{\pi_j^H - \pi_j^{min}}]$$

- For all $h \in \{i, j\}$.
- π_h is the utility.
- π_h^{min} is the lowest possible payoff to player h.
- π_h^H is the highest possible pareto efficient payoff to player h.
- $\pi_h^e = \frac{\pi_h^H + \pi_h^L}{2}$, where π_h^L denotes the lowest possible pareto efficient payoff to player h. π_h^e is the equitable reference point.

At this junction, allow us to provide some intuition. Suppose that player i is the consumer. Then this function takes into account the consumption utility and some additional reciprocity term. The additional term is what we consider the rating utility. Suppose a firm sets a low price, such that $\pi_i - \pi_i^e > 0$, the consumer would believe that the firm is treating him kindly. In response, consumers will receive a higher overall utility if he is kind to the firm, $\pi_j - \pi_j^e > 0$. In our context, a good rating will result in π_j being higher and therefore by leaving a good rating, consumers would be being kind to the firms.

On the contrary, a firm charging a high price such that pay off for the consumer is below the equitable point would result in consumers punishing the firm by lowering their profits in the future periods - perhaps through a negative rating. For simplicity, we remove the ability of consumers to punish a firm and assume that consumers are only able to provide good or no ratings. Specifically, in our model, we show that a good rating results in a better future pay off and hence satisfies this feature.

In what follows, I show how we adapt this framework to better fit our context. Firstly, we have assumed that providing a rating is costly for consumers. This seems to be an intuitive feature of our model and follows from the literature of costly provision of reviews and ratings (Avery et al., 1999; Miller et al., 2005). Secondly, we make some simplifications that make the framework more tractable in our setup. We remove the normalization terms in the denominator and remove the "1".[41] Thirdly, we consider that the rating component of the utility only comes into effect when a good rating is provided. This leaves us with the following function:

$$U_i = \pi_i + \mathbb{1}_{\{R_t=1\}}\{[\pi_i - \pi_i^e][\pi_j - \pi_j^e] - e\}$$

Since $\pi_i = q^j - p_1^j$, $\pi_i^e = \frac{(q^j - q^j)+(q^j - 0)}{2}$. The highest possible pareto payoff to consumers being $q^j - 0$ where sellers set a price of 0 and the lowest possible being 0, where sellers set a price of q^j.

Moreover, $\pi_j = p_1 + p_2$, profits of the firm being the sum of profits in two periods, given 0 marginal cost, profits is the sum of prices in both periods. And $\pi_j^e = \frac{(p_1+q^H)+(p_1+q^L)}{2}$. The seller, setting some price p_1 in period 1, is able to get a maximum benefit of q^H and a minimum benefit of q^L in period 2.

This leaves us with:

$$U_i = q^j - p_1 + \mathbb{1}_{\{R_t=1\}}\{[\frac{q^j}{2} - p_1][p_2 - \frac{q^H + q^L}{2}] - e\}$$

Next, we replace $\frac{q^j}{2}$ with κq^j, where $\kappa \in [0,1]$ and $[p_2 - \frac{q^H+q^L}{2}]$ with Δ. This reflects the notion that an equitable payoff may not be one of equal split, allowing us to generalise the equitable point. Hence, when κ is sufficiently high, firms are able to charge some price slightly below quality and still receive a positive rating if e is sufficiently small. We do not make any assumptions over Δ, except that it is positive. This allows us to capture that consumers may not fully understand how firms benefit from ratings, only that a good rating is beneficial for a firm, and a bad rating can harm the firm. Thus, capturing kindness from consumers which enables firms to gain some benefits in subsequent periods.

Finally, we split the consumption utility and the rating utility. This allows for more compatible purchase decision across periods as consumer's purchase decision does not depend on whether they anticipate giving a good rating.

[41]Rabin notes that doing so does not affect the behavior of the utility function.

Appendix C Proofs

Proof of Proposition 1

We proceed as follows. First, we pin down equilibrium prices in period 1 in Lemma 3 and equilibrium beliefs in Lemma 4. Afterwards, we use these lemmas to prove the remaining statements in Proposition 1.

Lemma 3. *In equilibrium, firm j plays the price p^j_{t,R_t} in period t, in order to receive the rating R_t.*

In the first period, firms play the following equilibrium prices with positive probability.

- *High-quality firm:* $p^H_{1,1} = min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p^H_{1,1}]\}$
- *Low-quality firm:*

$$p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$$

$$p^L_{1,0} = E[q_1|R_0, p^H_{1,1}]$$

Proof of Lemma 3.
We proceed in three steps. First, we consider the pricing strategy of the high-quality firms. Second, we look at the pricing strategy of low-quality firms receiving a rating. Third, we focus on prices of low-quality firms obtaining no rating.

To start, we look at the pricing strategy of high-quality firms. We show that their pricing strategy is unique and $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta},\, E[q_1|R_0, p^H_{1,1}]\}$ with probability 1.

Given Restriction 1, we focus on equilibria where the high-quality firm sets a price which allows it to obtain a rating, $R_1 = 1$. Therefore, in equilibrium, the firm does not consider the pricing strategy that obtains no rating.

We show that high-quality firms set a unique price in period 1 with probability 1. Suppose towards a contradiction that high-quality firms set more than one price with positive probability.

Without loss of generality, suppose that the high-quality firm sets a distribution of prices, $p \in [p', p'']$ such that $p'' > p'$, and the firm receives a rating $R_1 = 1$ with probability 1 for all $p \in [p', p'']$. Therefore, at all $p \in [p', p'']$, consumers purchase products with probability 1.

Notice that for any price $\hat{p} \in [p', p'']$ such that $\hat{p} > p$, we have $\pi^H(\hat{p}) > \pi^H(p)$. To see this, observe that both prices induce the same demand in period 1, but $\hat{p}$ induces a larger margin and therefore strictly larger profits in period 1. Further, both prices induce $R_1 = 1$ with probability 1, and therefore the same expected profits in period 2. As a result, $\pi^H(\hat{p}) > \pi^H(p)$. Shifting the probability mass of the entire price distribution in period 1 to one mass point, p'', strictly increases profits for the high-quality firm, contradicting that the firm sets more than one price with positive probability. Essentially, the same argument implies that the high-quality firm does not set more than one price with strictly positive probability. We conclude that the high-quality firm sets a unique price in period 1 with probability 1.

Next, we prove that there exist an upper bound on prices, $\overline{p_t^j}$ for $j \in \{L, H\}$ such that a firm j receives a rating. In order for a firm to induce a rating, the rating utility must be weakly positive, i.e. $v_t \geq 0$. Therefore,

$$[\kappa q^j - p_{t,1}^j]\Delta - e \geq 0 \Leftrightarrow p_{t,1}^j \leq \overline{p_t^j} \equiv \kappa q^j - \frac{e}{\Delta}.$$

Therefore, the upper bound on prices such that the high-quality firm receives a positive rating is $\overline{p_1^H} = \kappa q^H - \frac{e}{\Delta}$.

Finally, consider that this upper bound is restricted by consumer's beliefs, $E[q_1|R_0, p_{1,1}^H] < \overline{p_1^H}$. Under such scenarios, by Restriction 1, high-quality firms prefer obtaining a rating. This can only be achieved if consumers buy. Therefore, $p_{1,1}^H$ has an upper bound of $E[q_1|R_0, p_{1,1}^H]$.

We next show that $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_{1,1}^H]\}$ with probability 1. To see this, note that $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta}$ is the cut-off price above which the low-quality firm receives no rating. See also that $\overline{p_1^H} > \overline{p_1^L}$ and $q_L > \overline{p_1^L}$. Thus, because $E[q_1|R_0, p_{1,1}^H] \geq q_L$, the high-quality firm sets its equilibrium price in period 1 strictly above $\overline{p_1^L}$, i.e. $p_{1,1}^H > \overline{p_1^L}$. By Restriction 2, consumers have the same beliefs for all prices strictly above $\overline{p_1^L}$, and since $p_{1,1}^H > \overline{p_1^L}$, these beliefs are the correct equilibrium beliefs $E[q_1|R_0, p_{1,1}^H]$. Because consumers have the same beliefs $E[q_1|R_0, p_{1,1}^H]$ for all prices above $\overline{p_1^L}$, the high-quality firm optimally sets the largest price for which consumers purchase and rate with probability 1, which is $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_{1,1}^H]\}$.

We conclude that high-quality firms set a unique price $p_{1,1}^H = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_{1,1}^H]\}$ with probability 1.

We now proceed to the second step and characterize the pricing strategy of low-quality firms who receive a rating. First, we show that the price which it sets and receives a rating is unique. Then, that $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$.

Essentially the same argument as used for high-quality firms implies that—conditional on obtaining a rating—the low-quality firm sets a single price with probability 1.

By definition of $\overline{p_1^L}$, $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta}$. Since this is strictly less than q^L, and since consumers beliefs must be weakly above q^L, consumers are always willing to buy at any price weakly below $\overline{p_1^L}$. Since demand and ratings are the same for all prices weakly below $\overline{p_1^L}$, a low-quality firm that obtains a rating must optimally set $\overline{p_1^L}$ with probability 1. We conclude that conditional on obtaining a rating—the low-quality firm sets $\overline{p_1^L}$ with probability 1.

We now proceed to the third step of the proof and determine prices of low-quality firms who obtain no rating. We show that low-quality firms who obtain no rating optimally set $E[q_1|R_0, p^H_{1,1}]$. We have shown in step 2 that all prices above $\overline{p_1^L}$ induce the same beliefs $E[q_1|R_0, p^H_{1,1}]$. Thus, low-quality firms who obtain a no rating optimally set the highest possible price, $E[q_1|R_0, p^H_{1,1}]$ with probability 1.

This concludes the proof. $\square$

Lemma 4. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \textit{if } p_1 > \overline{p_1^L} \\ q^L & \textit{if } p_1 \leq \overline{p_1^L}, \end{cases}$$

and in the second period,

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \textit{if } R_1 = 1 \\ q^L & \textit{if } R_1 = 0. \end{cases}$$

Proof of Lemma 4.
We prove this Lemma by constructing expected quality using Bayes rule. We start by considering the second period, followed by the first period.

Before we begin, note that by Lemma 3, high-quality firms charge $p^H_{1,1}$ and obtain a rating with probability 1, and low-quality firms obtain a rating if and only if they charge a low price $p^L_{1,1}$. We denote the probability δ as the probability with which low-quality firms charge $p^L_{1,0}$. Thus, $(1 - \delta^*)$ is the probability which the low-quality firm obtains a good rating.

Now consider the second period. Given the consumer's information set in the second period, they are aware of historical ratings R_1, and current prices, p_2. We now show that expected quality in period 2 is independent of second period prices, p_2. Since period 2 is the final period, no firm obtains a rating, which, by Restriction 2, implies that the expected quality in period 2 is independent of second period prices. We conclude that expected quality in period 2 only depends on past ratings, R_1.

Next, we pin down consumers' expectations in period 2. In equilibrium, consumers observe $R_1 = 1$ from all high-quality firms and low-quality firms with low prices, i.e. with probability $\gamma + (1-\delta^*)(1-\gamma)$. Because only low-quality firms get no rating, the expected quality after observing $R_1 = 0$ is q^L. Thus, applying Bayes rule leads to

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

We now consider the first period. First, recall that $R_0 = 0$.

We distinguish two cases, (i) $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = E[q_1|R_0, p^H_{1,1}]$ and (ii) $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = \kappa q^H - \frac{e}{\Delta}$.

We start with case (i) and suppose $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = E[q_1|R_0, p^H_{1,1}]$. Then by Lemma 3, we have $p^L_{1,0} = p^H_{1,1}$ and $p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$. Applying Bayes rule leads to

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 = p^H_{1,1} \\ q^L & \text{if } p_1 = p^L_{1,1}. \end{cases}$$

Now consider case (ii) and suppose $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = \kappa q^H - \frac{e}{\Delta}$. Then $p^H_{1,1} \neq p^L_{1,0}$, and Bayes rule implies $E[q_1|R_0, p^H_{1,1}] = q^H$. This is only consistent with the finding in Lemma 3 that $p^L_{1,0} = E[q_1|R_0, p^H_{1,1}] = q^H$ if $\delta^* = 0$, i.e. the low-quality firm sets $p^L_{1,0}$ with probability zero. Thus, beliefs are given again by

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 = p^H_{1,1} \\ q^L & \text{if } p_1 = p^L_{1,1}, \end{cases}$$

applied at $\delta^* = 0$. We conclude from cases (i) and (ii) that for equilibrium prices in period 1, beliefs are given by

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 = p^H_{1,1} \\ q^L & \text{if } p_1 = p^L_{1,1}. \end{cases}$$

Because $p^H_{1,1} > \overline{p^L_1}$, $p^L_{1,0} > \overline{p^L_1}$, and $p^L_{1,1} \leq \overline{p^L_1}$, this concludes the proof. □

We prove a slightly more general statement than **Proposition 1**.

Proposition 11. *All perfect Bayesian equilibria satisfy the following. In period 1:*

1. *High-quality firms receive a good rating with probability 1 and charge* $\overline{p} \equiv E[q_1|R_0, p^H_{1,1}]$.

2. *Low-quality firms randomize their strategy.*

   a. *They charge* $\overline{p}$ *and obtain no rating with probability* δ^*.

   b. *They charge* $\underline{p} \equiv \kappa q^L - \frac{e}{\Delta}$ *and obtain a good rating with probability* $1 - \delta^*$.

3. *Consumers beliefs of equilibrium prices are given by*
$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta} \end{cases}.$$

In period 2:

4. *Prices are equal to expected quality conditional on ratings.*

5. *Consumer beliefs are given by* $E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \end{cases}.$

The equilibrium is unique up to off-equilibrium-path beliefs. $\delta^* = 1$ *if and only if* $\kappa q^L - \frac{e}{\Delta} + q^H \leq \gamma q^H + (1-\gamma)q^L + q^L$, *and* $\delta^* \in (\frac{1}{2}, 1)$ *otherwise. The equilibrium exists if* $\kappa q^H - \frac{e}{\Delta} \geq \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$.

Proposition 1 in the main text is obtained as a special case.

Proof of Proposition 11.
From Lemma 3 and 4, we have shown statements 2, 3 and 5. Hence, what remains is to prove statement 1 and 4, as well as existence and uniqueness up to off-equilibrium-path beliefs.

To prove statement 1, note that by Lemma 3, we know $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\}$, and it remains to show that $\min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = E[q_1|R_0, p^H_{1,1}]$. Suppose towards a contradiction that $\min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = \kappa q^H - \frac{e}{\Delta}$. As we argued in the proof of Lemma 4, we then have $p^H_1 \neq p^L_{1,0}$, and $E[q_1|R_0, p^H_{1,1}] = q^H$, and $p^L_{1,0} = E[q_1|R_0, p^H_{1,1}] = q^H$ is played with probability $\delta^* = 0$. Because the low-quality firm charges $p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$ with probability 1, all firms get a rating with probability 1. By Lemma 4, low-quality firms earn up to $\kappa q^L - \frac{e}{\Delta} + \gamma q^H + (1-\gamma)q^L$. Low-quality firms can strictly increase profits by charging a first period price of q^H. By Restriction 2, consumers believe $E[q_1|R_0, p_1] = q^H$ for all prices $p_1 \geq \kappa q^L - \frac{e}{\Delta}$, so they purchase in period 1. In period 2, the deviation earns q^L. Overall, the deviation earns $q^H + q^L$. Since $q^H \geq \gamma q^H + (1-\gamma)q^L$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, this deviation profitable for low-quality firms, a contradiction.

We conclude that $p^H_{1,1} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p^H_{1,1}]\} = E[q_1|R_0, p^H_{1,1}]$, which proves statement 1. Because we have shown that $p^H_{1,1} = E[q_1|R_0, p^H_{1,1}]$, and this is the same as $p^L_{1,0}$, to simplify notation, we state that $\overline{p} = p^H_{1,1} = p^L_{1,0} = E[q_1|R_0, \overline{p}]$. To further simplify notation, we label $\underline{p} = p^L_{1,1} = \kappa q^L - \frac{e}{\Delta}$.

We now prove statement 4. We have shown in Lemma 4 that in period 2, firms are no longer incentivized by future ratings. We have also shown that consumers' beliefs only depend on past ratings. Thus, firms optimally charge prices equal to the expected profits conditional on the past ratings they received. We conclude that in period 2, prices equal expected quality conditional on ratings, which proofs statement 4.

We conclude that statements 1 - 5 hold.

We now show that equilibria are unique up to off-equilibrium-path beliefs. To show uniqueness of equilibrium, consider that for some δ^*, low-quality firms are indifferent between getting a rating and no rating. From the proof of statement 1, we know that in equilibrium we must have $\delta^* < 1$ and $\kappa q^H - \frac{e}{\Delta} > E[q_1|p^H_{1,1}]$, implying that $\overline{p} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$.

Charging $\underline{p} = q^L - \frac{e}{\Delta}$ induces a rating and, given correct equilibrium beliefs, the following is the total profits for the low-quality firm:

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}. \tag{4}$$

This is strictly increasing in δ^* for all $\gamma \in (0,1)$ and $q^H > q^L$.

When the low-quality firm charges $\overline{p} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$ in period 1, it obtains no rating and earns

$$\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L, \tag{5}$$

which strictly decreases in δ^* for all $\gamma \in (0,1)$ and $q^H > q^L$.

To start we show that $\delta^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists. Suppose $\delta^* = 1$. Then (4) and (5) become $\kappa q^L - \frac{e}{\Delta} + q^H$ and $\gamma q^H + (1-\gamma)q^L + q^L$, respectively. For $\delta^* = 1$ to be optimal, it must be the case that

$$\kappa q^L - \frac{e}{\Delta} + q^H \leq \gamma q^H + (1-\gamma)q^L + q^L. \tag{6}$$

Since (4) strictly increases in δ^* and (5) strictly decreases in δ^*, this means whenever $\delta^* = 1$ is an equilibrium, we cannot have a mixed-strategy equilibrium. We conclude that $\delta^* = 1$

can only be an equilibrium if no mixed-strategy equilibrium exists.

We now show that $\delta^* = 0$ cannot be an equilibrium. Suppose towards a contradiction that $\delta^* = 0$. Then (4) and (5) become $\kappa q^L - \frac{e}{\Delta} + \gamma q^H + (1-\gamma)q^L$ and $q^H + q^L$, respectively. But since $q^H > q^H + (1-\gamma)q^L$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, low-quality firms optimally set $\overline{p} = q^H$ when consumers believe they set this price with probability zero in period 1, a contradiction. We conclude that $\delta^* = 0$ cannot be an equilibrium.

We now characterize the mixed-strategy equilibrium. To have a mixed-strategy equilibrium, consumers must have beliefs such that (4) = (5) and low-quality firms must play some δ^* such that these beliefs are correct. Thus, in a mixed-strategy equilibrium, we have

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} = \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L. \tag{7}$$

We have two candidates that solve this equation:

$$\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} \pm \frac{(4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})}$$

Recall that as a probability, δ^* is bound by 0 and 1.

Consider the scenario where the last term is subtracted.

$$\begin{aligned}
&\frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{(4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} \\
&< \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{((1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} \\
&< \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{(1+\gamma)}{2(1-\gamma)} \\
&< \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} - \frac{(1-\gamma)}{2(1-\gamma)} = -\frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} < 0
\end{aligned}$$

. Therefore, we conclude that

$$\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} + \frac{(4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})}.$$

Further, notice that the optimal δ^* is lies strictly between $\frac{1}{2}$ and 1.

We see from (4) and (5) that $\delta^* < 1$ if $\kappa q^L - \frac{e}{\Delta} + q^H > \gamma q^H + (1-\gamma)q^L + q^L$. Also note that

$$\delta^* > \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} + \frac{(4\gamma^2(q^H - q^L)^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})} = \frac{1}{2}.$$

We now show the equilibrium is unique up to off-equilibrium-path beliefs. This follows immediately from having shown that we cannot have $\delta^* = 0$, and that $\delta^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists. Additionally, if a $\delta^* \in (0,1)$ exists such that (7) holds, it must be unique, because (4) strictly increases, and (5) strictly decreases in δ^*. We conclude that if mixed-strategy equlibria exists, it is a unique mixed-strategy, $\delta^* \in (0,1)$ Thus, we either have a unique pure-strategy equilibrium or a unique mixed-strategy equilibrium, but not both. We conclude that the equilibrium is unique up to off-equilibrium-path beliefs.

Therefore, we can conclude that $\delta^* < 1$ if

$$(1-\gamma)(q^H - q^L) > q^L(1-\kappa) + \frac{e}{\Delta}. \tag{8}$$

And this equilibrium is a unique interior solution where $\delta^* \in (\frac{1}{2}, 1)$ up to off-equilibrium-path beliefs. Otherwise, there is a unique corner solution at $\delta^* = 1$ up to off-equilibrium-path beliefs.

$$\delta^* = \begin{cases} \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} + \frac{(4\gamma^2(q^H-q^L)^2 + (1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} & \text{if (8) holds} \\ 1 & \text{otherwise} \end{cases} \tag{9}$$

We now show that these equilibria exist.

To start, consider the case where (6) holds. In the candidate equilibrium in period 1, the high-quality firm sets $\overline{p} = \gamma q^H + (1-\gamma)q^L$ with probability 1 and obtains a rating, and the low-quality firm charges $\overline{p}$ with probability 1 and gets no rating. In period 2, the high-quality firm charges a price equal q^H, and the low-quality firm charges q^L. Consumers' beliefs are as follows. In period 1, they believe

$$E[q_1|p_1] = \begin{cases} \gamma q^H + (1-\gamma)q^L & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}, \end{cases}$$

in the second period, beliefs are independent of prices and are

$$E[q_2|R_1] = \begin{cases} q^H & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

These beliefs follow Bayes rule on the path of play. The candidate equilibrium is also consistent with our restrictions. Because high-quality firms obtain a rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Because consumers have the same beliefs for all second period prices, and the same beliefs for all first period prices where the low-quality firm obtains no rating, the candidate equilibrium is consistent with Restriction 2.

We now show that firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns $\gamma q^H + (1-\gamma)q^L + q^H$. Deviations in period 2 to a higher price would induce zero demand, and deviations to lower prices would reduce margins without increasing demand. There are no profitable deviations in period 2. In period 1, deviations to a higher price reduces demand to zero and earns a maximal total profit of $0 + q^L$. Deviations to a lower price in period 1 reduce margins without increasing demand or increasing ratings. Therefore, there is no profitable deviation in period 1. We conclude that high-quality firms have no profitable deviations.

We now show that low-quality firms have no profitable deviations. In the candidate equilibrium they earn $\gamma q^H + (1-\gamma)q^L + q^L$. Deviations in period 2 to a higher price would induce zero demand, and deviations to lower prices would reduce margins without increasing demand. There are no profitable deviations in period 2. In period 1, deviations to a higher price reduces demand to zero and earns a maximal total profit of $0 + q^L$, which is not a profitable deviation. In period 1, deviations to a lower price above $\kappa q^L - \frac{e}{\Delta}$ does not improve the rating and only reduces margins without raising demand, this is not a profitable deviation. Deviations to lower prices below $\kappa q^L - \frac{e}{\Delta}$ leads to profits weakly below $\kappa q^L - \frac{e}{\Delta} + q^H$, which is not a profitable deviation since (6) holds.

We conclude that if (6) holds, no profitable deviations exist for either type of firm.

Finally, we have shown above that $\overline{p} = E[q_1|\overline{p}]$, which requires $\gamma q^H + (1-\gamma)q^L \leq \kappa q^H - \frac{e}{\Delta}$.

We conclude that if $(1-\gamma)(q^H - q^L) \leq q^L(1-\kappa) + \frac{e}{\Delta}$ and $\gamma q^H + (1-\gamma)q^L \leq \kappa q^H - \frac{e}{\Delta}$, the candidate equilibrium exists. As we have shown above, it must be the unique equilibrium up to off-equilibrium beliefs.

Now consider the case where (8) holds. We have shown above that no pure-strategy equilibrium exists in this case. In the candidate equilibrium in period 1, the high-quality firm sets $\overline{p} = \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)}$ with probability 1 and obtains a rating. The low-quality firm charges $\overline{p}$ with probability δ^* and gets no rating, and sets $\underline{p} = \kappa q^L - \frac{e}{\Delta}$ with probability $1 - \delta^*$ and gets a rating. In period 2, all firms with a good rating charge $\frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)}$, and firms without a rating charge q^L. Consumers' beliefs are as follows. In period 1, they believe

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}. \end{cases}$$

in the second period, beliefs are independent of prices and are

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0. \end{cases}$$

These beliefs follow Bayes rule on the path of play. The candidate equilibrium is also consistent with our restrictions. Because high-quality firms obtain a rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Because second period beliefs are independent of prices, and consumers have the same beliefs for all first period prices where the low-quality firm obtains no rating, the candidate equilibrium is consistent with Restriction 2.

We now show that firms have no profitable deviations.

We start with low-quality firms, who earn total profits $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)}$. The firm is indifferent between charging $\overline{p}$ and $\underline{p}$, where $\overline{p} > \underline{p}$. If the firm deviates to a price above $\overline{p}$, demand drops to zero and total profits are weakly below $0 + q^L$, this is not a profitable deviation. Deviations to a price $p_1 \in (\underline{p}, \overline{p})$, for which the firm gets the same rating as charging $\overline{p}$ and therefore earns the same profit in period 2, but the firm earns a lower margin than when it charges $\overline{p}$ without increasing demand in period 1, this is not a profitable deviation. Deviations to a price below $\underline{p}$ lead to the same rating as when charging $\underline{p}$ and therefore the same continuation profits, but decrease margins in period 1 without increasing demand, this is not a profitable deviation. In period 2, the low-quality firm extracts expected total surplus conditional on the rating, and cannot strictly increase profits. We conclude that low-quality firms have no profitable deviation.

We now show that high-quality firms have no profitable deviation. In the candidate equilibrium, high-quality firms earn $\frac{\gamma q^H + \delta^*(1-\gamma) q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)}$. In the second period, high.quality firms extract total surplus conditional on their good rating and therefore cannot profitably deviate. In the first period, a higher price reduces demand to zero and earns profits weakly below $0 + q^L$, which is not a profitable deviation. Deviating to a lower price does not improve the rating and therefore does not increase continuation profits, but reduces margins in period 1 without increasing demand, this is also not a profitable deviation. We conclude that high-quality firms have no profitable deviation.

We conclude that no firm has a profitable deviation.

Finally, we need to check $\overline{p} = \min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|\overline{p}]\} = E[q_1|\overline{p}]$, which requires $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} \leq \kappa q^H - \frac{e}{\Delta}$.

We conclude that if $(1-\gamma)(q^H - q^L) > q^L(1-\kappa) + \frac{e}{\Delta}$ and $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} \leq \kappa q^H - \frac{e}{\Delta}$, the candidate equilibrium exists. As we have shown above, it must be the unique equilibrium up to off-equilibrium beliefs.

This concludes the proof. □

Proof of Proposition 2

Proof of Proposition 2.
In this proof, we show that $\frac{\partial \delta^*}{\partial \Delta} < 0$ if $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}$ and argue it is 0 otherwise.

We know from Proposition 1 that $\delta^* = \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} + \frac{(4\gamma^2(q^H-q^L)^2 + (1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})}$, and that $\delta^* \in (\frac{1}{2}, 1)$ when $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}$. We can then calculate the derivative with respect to Δ, which leads to:

$$\frac{\partial \delta^*}{\partial \Delta} = \frac{e}{\Delta^2} \frac{\gamma(q^H - q^L)[2\gamma(q^H - q^L) - ((2\gamma(q^H - q^L))^2 + ((1+\gamma)(q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}}]}{(1-\gamma)(q^L(1-\kappa) + \frac{e}{\Delta})^2((2\gamma(q^H - q^L))^2 + ((1+\gamma)(q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}}} < 0$$

When $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L \leq \frac{e}{\Delta}$, $\delta^* = 1$, and further increases to Δ cannot increase δ^* as it is is a probability and is weakly bound by 0 and 1.

Thus we have shown that when low-quality firms would play a mixed-strategy, as consumers kindness increases, low-quality firms are more likely to participate in ratings harvesting. Otherwise, low-quality firms play a pure strategy of price mimicking.

This concludes the proof. □

Proof of Corollary 1

Proof of Corollary 1.
In this proof, we show that $\frac{\partial\delta^*}{\partial e} > 0$ if $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}$ holds.

We know from Proposition 1 that $\delta^* = \frac{1}{2} - \frac{\gamma(q^H-q^L)}{(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})} + \frac{(4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2)^{\frac{1}{2}}}{2(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})}$, and that $\delta^* \in (\frac{1}{2}, 1)$ when $(1-\gamma)(q^H - q^L) - (1-\kappa)q^L > \frac{e}{\Delta}$. We can then calculate the derivative with respect to e, which leads to:

$$\frac{\partial\delta^*}{\partial e} = \frac{\gamma(q^H-q^L)(((2\gamma(q^H-q^L))^2 + ((1+\gamma)(q^L(1-\kappa)+\frac{e}{\Delta}))^2)^{\frac{1}{2}} - 2\gamma(q^H-q^L))}{\Delta(1-\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})^2((2\gamma(q^H-q^L))^2 + ((1+\gamma)(q^L(1-\kappa)+\frac{e}{\Delta}))^2)^{\frac{1}{2}}} > 0$$

Thus we have shown that when low-quality firms would play a mixed-strategy, as the opportunity cost of rating increases, low-quality firms are less likely to participate in ratings harvesting.

This concludes the proof. □

Proof of Proposition 3

Proof of Proposition 3.
We show $\frac{\partial\delta^*}{\partial\gamma} \leq 0$ when γ is sufficiently small and $\frac{\partial\delta^*}{\partial\gamma} > 0$ when γ is sufficiently large. We then characterise this switching point, $\overline{\gamma}$, and show its uniqueness and existence.

To start, note from (7), δ^* is such that low-quality firms are indifferent between charging low prices and high prices,

$$\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L = \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}.$$

Taking the derivative of the right-hand side leads to

$$\frac{\partial RHS}{\partial\gamma} = \frac{(q^H-q^L)((1-\delta^*) + (1-\gamma)\gamma\frac{\partial\delta^*}{\partial\gamma})}{(\gamma + (1-\delta^*)(1-\gamma))^2},$$

and for the left-hand side

$$\frac{\partial LHS}{\partial\gamma} = \frac{(q^H-q^L)(\delta^* - (1-\gamma)\gamma\frac{\partial\delta^*}{\partial\gamma})}{(\gamma + (1-\gamma)\delta^*)^2}.$$

Since both derivatives must be equal in equilibrium, we get

$$\frac{\partial \delta^*}{\partial \gamma} = \frac{(2\delta^* - 1)(\gamma^2 + \delta^{*2}(1-\gamma)^2 - \delta^*(1-\gamma)^2)}{\gamma(1-\gamma)((\gamma + (1-\delta^*)(1-\gamma))^2 + (\gamma + (1-\gamma)\delta^*)^2)} \tag{10}$$

In particular, we show that the sign of $\frac{\partial \delta^*}{\partial \gamma}$ switches at some point $\overline{\gamma}$. Since the denominator is strictly positive, the expression is negative if and only if the numerator is negative, i.e.

$$(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) < 0.$$

And positive when

$$(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) > 0,$$

and 0 if

$$(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) = 0.$$

Since $\delta^* > 0.5$, the when evaluating the sign of $(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta))$, we only need consider $(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta))$

We show that there is only one γ such that $(2\delta^* - 1)(\gamma^2 - (1-\gamma)^2\delta^*(1-\delta)) = 0$, therefore there is a unique switching point.

$$\overline{\gamma} = \frac{(q^H - q^L)^2 - (q^L(1-\kappa) + \frac{e}{\Delta})^2}{3(q^H - q^L)^2 + (q^L(1-\kappa) + \frac{e}{\Delta})^2} \tag{11}$$

We show that $\overline{\gamma} \in (0,1)$. The denominator is strictly positive. From (1), the numerator is strictly positive. Hence, $\overline{\gamma} > 0$. Moreover, because the denominator is always strictly larger than the numerator, $\overline{\gamma} < 1$.

Finally, we show that if $\gamma < \overline{\gamma}$, then $\frac{\partial \delta^*}{\partial \gamma} < 0$, and $\frac{\partial \delta^*}{\partial \gamma} > 0$ when $\gamma \in (\overline{\gamma}, 1)$.

Notice that when $\gamma \to 0$, and from (9) $\delta^* \to 1$. Thus $\delta^*(1-\delta^*) \to 0$ and $\frac{\gamma^2}{(1-\gamma)^2} \to 0$. And when $\gamma \to 1$, $\delta^* = 1$. Further, when $\gamma = 1$, $\frac{\gamma^2}{(1-\gamma)^2} = \infty$.

We show that $\delta^*(1-\delta^*) > \frac{\gamma^2}{(1-\gamma)^2}$ when γ is sufficiently small. Consider that $\frac{\gamma^2}{(1-\gamma)^2}$ is strictly convex, with $\frac{\partial \frac{\gamma^2}{(1-\gamma)^2}}{\partial \gamma} = \frac{2\gamma}{(1-\gamma)^3} > 0$ and $\frac{\partial^2 \frac{\gamma^2}{(1-\gamma)^2}}{\partial \gamma^2} = \frac{2+4\gamma}{(1-\gamma)^4} > 0$. Evaluated at $\gamma \to 0$, $\frac{\partial \frac{\gamma^2}{(1-\gamma)^2}}{\partial \gamma} = \frac{2\gamma}{(1-\gamma)^3} \to 0$. Further, consider that $\frac{\partial \delta^*(1-\delta^*)}{\partial \gamma} = \frac{\partial \delta^*}{\partial \gamma}(1-2\delta^*)$. Evaluated at $\gamma \to 0$, $\delta^* \to 1$ and $\frac{\partial \delta^*}{\partial \gamma} < 0$. Then, $\frac{\partial \delta^*(1-\delta^*)}{\partial \gamma} = \frac{\partial \delta^*}{\partial \gamma}(1-2\delta^*) > 0$. Hence, there exists a range of

$\gamma \in (0,1)$ where $\delta^*(1-\delta^*) > \frac{\gamma^2}{(1-\gamma)^2}$ is satisfied. We label the first upper boundary (closest to 0) of this range as γ', such that $\gamma' \in (0,1)$ and when $\gamma \in (0,\gamma')$, $\delta^*(1-\delta^*) > \frac{\gamma^2}{(1-\gamma)^2}$ is satisfied and $\frac{\partial\delta^*}{\partial\gamma} < 0$.

Additionally, when $\gamma \to 1$, $\delta^* \to 1$ and $\frac{\gamma^2}{(1-\gamma)^2} \to \infty$. This implies that $\frac{\gamma^2}{(1-\gamma)^2} > \delta^*(1-\delta^*)$ for some range of γ and $\frac{\partial\delta^*}{\partial\gamma} > 0$. This implies that at $\gamma \to 1$, $\frac{\partial\delta^*(1-\delta^*)}{\partial\gamma} = \frac{\partial\delta^*}{\partial\gamma}(1-2\delta^*) < 0$. Therefore, we can conclude that there exists a range of $\gamma \in (0,1)$ where $\frac{\gamma^2}{(1-\gamma)^2} > \delta^*(1-\delta^*)$ is satisfied. We label the first lower boundary (closest to 1) of this range as γ'', such that $\gamma'' \in (0,1)$ and when $\gamma \in (\gamma'',1)$, $\frac{\gamma^2}{(1-\gamma)^2} > \delta^*(1-\delta^*)$ is satisfied and $\frac{\partial\delta^*}{\partial\gamma} > 0$.

Notice that by definition $\gamma'' \geq \gamma'$. We have shown before that there is a unique switching point, therefore $\overline{\gamma} = \gamma'' = \gamma'$.

We conclude that there is a unique switching point $\overline{\gamma}$, below which $\frac{\partial\delta^*}{\partial\gamma} < 0$, and above which $\frac{\partial\delta^*}{\partial\gamma} > 0$. Where $\frac{\partial\delta^*}{\partial\gamma} = 0$ only if $\gamma = \overline{\gamma}$, and $\overline{\gamma} \in (0,1)$. □

Proof of Proposition 4

Proof of Proposition 4.
In this proof, we show that if a social planner concerned with the welfare of consumers would set an optimal e, e^{cs}.

First, we find consumer surplus. This is the sum of the difference between actual price and quality that consumers receive in each period.

$$\begin{aligned}
CS_1 =& \gamma[q^H - \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}] + (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}] + \\
& \delta^*(1-\gamma)[q^L - \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}] = (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}] \\
CS_2 =& \gamma[q^H - \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + (1-\delta^*)(1-\gamma)[q^L - \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \\
& \delta^*(1-\gamma)[q^L - q^L] = 0.
\end{aligned}$$

Therefore, consumer surplus arises only from the low-quality firm's attempt to receive a good rating, and total consumer surplus in our model is given by

$$CS = (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}]. \tag{12}$$

This is (2). From (2) we can evaluate the effects of changes to effort cost.

First, we show that consumer surplus is concave in e.

$$\frac{\partial CS}{\partial e} = \frac{1}{2}[1-\gamma - \frac{(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})}{\sqrt{4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2}}]$$

This is positive if and only if

$$(1-\gamma)^2\gamma(q^H-q^L)^2 > (1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2$$

, and $\frac{\partial CS}{\partial e}=0$ with equality. This condition is (3).

We now look at the second derivative.

$$\frac{\partial^2 CS}{\partial e^2} = -\frac{2\gamma^2(1+\gamma)^2(q^H-q^L)^2}{\Delta^2((2\gamma(q^H-q^L))^2+((1+\gamma)(q^L(1-\kappa)+\frac{e}{\Delta}))^2)^{\frac{3}{2}}} < 0$$

Second, we solve for the optimal level of effort required to leave a rating.

$$e = -(1-\kappa)q^L\Delta \pm \frac{\Delta(q^H-q^L)(1-\gamma)\sqrt{\gamma}}{1+\gamma}$$

We may reject the negative as we assume that $e \geq 0$. Therefore,

$$e^{cs} = -(1-\kappa)q^L\Delta + \frac{\Delta(q^H-q^L)(1-\gamma)\sqrt{\gamma}}{1+\gamma}$$

where e^{cs} is the level of effort cost that maximises consumer surplus. This e^{cs} is indeed positive when (3) holds. Which is the restriction required for $\frac{\partial CS}{\partial e} > 0$ at $e=0$.

When (3) does not hold, then $\frac{\partial CS}{\partial e} < 0$ and this implies that $e^{cs}=0$, the lower bound.

Therefore, for a planner maximizing the welfare of consumers,
$e^{cs} = -(1-\kappa)q^L\Delta + \frac{\Delta(q^H-q^L)(1-\gamma)\sqrt{\gamma}}{1+\gamma}$ when (3) holds, and 0 otherwise.

This concludes the proof. □

Proof of Corollary 2

Proof of Corollary 2.
In this proof, we show that in expectation, sellers prefer $e=0$, and thus completely uninformative ratings. Because high-quality firms prefer more-informative ratings, we argue that the expected sellers' preference for uninformative ratings is driven by low-quality firms.

First, we look at the average profit function of the firm,

$$
\begin{aligned}
\pi = & \gamma[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \\
& (1-\gamma)[(1-\delta^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \delta^*[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L]] \\
& = 2[\gamma q^H + (1-\gamma)q^L] - (1-\delta^*)(q^L(1-\kappa) + \frac{e}{\Delta})
\end{aligned}
$$

Taking the derivative to e,

$$
\begin{aligned}
\frac{\partial \pi}{\partial e} &= \frac{\partial \delta^*}{\partial e}[(1-\kappa)q^L + \frac{e}{\Delta}] - \frac{(1-\gamma)(1-\delta^*)}{\Delta} \\
&= -\frac{1}{2} + \frac{(1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})}{2(1-\gamma)\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2}}
\end{aligned}
$$

and this is negative when (3) holds.

Therefore, on average firms prefer the smallest level of e, and the level of effort cost that maximises firm's profit is $e^s = 0$.

Second, we show that high-quality firms prefer informative ratings.

$$
\begin{aligned}
\pi^H &= \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} \\
\frac{\partial \pi^H}{\partial e} &= -\frac{(1-\gamma)^2\gamma(1+\gamma)(q^H - q^L)(1-2\delta^*)\frac{\partial \delta^*}{\partial e}}{(\gamma + (1-\gamma)(1-\delta^*))^2(\gamma + (1-\gamma)\delta^*)^2}
\end{aligned}
$$

Recall that $\delta^* \in (\frac{1}{2}, 1)$, and $\frac{\partial \delta^*}{\partial e} > 0$ when (1) holds. Therefore, $\frac{\partial \pi^H}{\partial e} > 0$.

This means that when low-quality firms would play a mixed-strategy, more-informative ratings benefits high-quality firms.

We can show that (3) is a stricter condition than (1).

From (3),

$$
\begin{aligned}
(1-\gamma)^2\gamma(q^H - q^L)^2 &\geq (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2 \\
\frac{\gamma}{(1+\gamma)^2} &\geq \frac{(q^L(1-\kappa) + \frac{e}{\Delta})^2}{(1-\gamma)^2(q^H - q^L)^2}
\end{aligned}
$$

And from (1),

$$
\begin{aligned}
(1-\gamma)(q^H-q^L)-(1-\kappa)q^L &> \frac{e}{\Delta} \\
(1-\gamma)(q^H-q^L) &> q^L(1-\kappa)+\frac{e}{\Delta} \text{ since both sides are positive,} \\
(1-\gamma)^2(q^H-q^L)^2 &> (q^L(1-\kappa)+\frac{e}{\Delta})^2 \\
1 &> \frac{(q^L(1-\kappa)+\frac{e}{\Delta})^2}{(1-\gamma)^2(q^H-q^L)^2}
\end{aligned}
$$

Finally, notice that $\gamma \in (0,1)$ and thus $\frac{\gamma}{(1+\gamma)^2} < 1$. Therefore, (3) is a stricter condition than (1).

We conclude that more-informative ratings leads to an decrease in the average profits of the firm. This decrease in profits is driven by low-quality firms, and high-quality firms benefit from more-informative ratings environments. □

Proof of Proposition 5

Proof of Proposition 5.
We show that there is some γ^{cs} that maximises consumer surplus, and that consumer surplus is concave in γ.

To start, recall from Proposition 4 that total consumer surplus reduces to (2), i.e. $CS = (1-\delta^*)(1-\gamma)[q^L - \kappa q^L + \frac{e}{\Delta}]$. We take the derivative of (2) with respect to γ and show that there is some $\gamma^{cs} < \overline{\gamma}$ that maximises consumer surplus.

$$
\frac{\partial CS}{\partial \gamma} = -(1-\delta^* + (1-\gamma)\frac{\partial \delta^*}{\partial \gamma})(q^L(1-\kappa)+\frac{e}{\Delta}) \tag{13}
$$

Since $q^L(1-\kappa)+\frac{e}{\Delta} > 0$, therefore, the sign of $\frac{\partial CS}{\partial \gamma}$ depends on $-(1-\delta^* + (1-\gamma)\frac{\partial \delta^*}{\partial \gamma})$.

We solve for γ^{cs}, the optimal choice of γ that maximises consumer surplus.

$$
\begin{aligned}
\gamma^{cs} =& \frac{-(q^H-q^L)(q^L(1-\kappa)+\frac{e}{\Delta})^2}{(q^H-q^L)(4(q^H-q^L)^2+(q^L(1-\kappa)+\frac{e}{\Delta})^2)} \pm \\
& \frac{((q^H-q^L)^3(q^L(1-\kappa)+\frac{e}{\Delta})(2(q^H-q^L)-(q^L(1-\kappa)+\frac{e}{\Delta}))^2)^{\frac{1}{2}}}{(q^H-q^L)(4(q^H-q^L)^2+(q^L(1-\kappa)+\frac{e}{\Delta})^2)}
\end{aligned}
$$

Because the denominator is positive, we reject the negative, as $\gamma \in (0,1)$. Therefore, we show that $\gamma^{cs} \in (0,1)$ exists if $(q^H-q^L)(2(q^H-q^L)-(q^L(1-\kappa)+\frac{e}{\Delta}))^2 \geq (q^L(1-\kappa)+\frac{e}{\Delta})^3$.

We now show that $(q^H - q^L)(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2 \geq (q^L(1-\kappa) + \frac{e}{\Delta})^3$ always holds. By applying $(q^H - q^L) > \gamma q^H + (1-\gamma)q^L - \kappa q^L + \frac{e}{\Delta} > q^L(1-\kappa) + \frac{e}{\Delta}$,

$$\begin{aligned}(q^H - q^L)(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2 &\geq (q^L(1-\kappa) + \frac{e}{\Delta})^3 \\ (q^H - q^L)(q^L(1-\kappa) + \frac{e}{\Delta})^2 &> (q^L(1-\kappa) + \frac{e}{\Delta})^3 \\ (q^H - q^L) &> (q^L(1-\kappa) + \frac{e}{\Delta})\end{aligned}$$

This is a weaker condition than (1).

Thus we conclude that γ^{cs} exists when low-quality firms play a mixed-strategy.

We now show that $\frac{\partial CS}{\partial \gamma}$ is strictly concave. First, we shall argue that $\gamma^{cs} < \overline{\gamma}$. Second, we show that when $\gamma > \gamma^{cs}$, $\frac{\partial CS}{\partial \gamma} < 0$. Finally, we show that $\frac{\partial CS}{\partial \gamma} > 0$ when $\gamma < \gamma^{cs}$.

Since $\overline{\gamma}$ solves $\frac{\partial \delta^*}{\partial \gamma}$ implies $\frac{\partial \delta^*}{\partial \gamma} = 0$ at $\overline{\gamma}$. Further, $\delta^* \in (\frac{1}{2}, 1)$ for $\gamma > 0$, implying that $(1-\delta^*) > 0$. Therefore, from (13), at $\overline{\gamma}$, $\frac{\partial CS}{\partial \gamma} < 0$. This implies that $\gamma^{cs} < \overline{\gamma}$.

Second, we show that when $\gamma > \gamma^{cs}$, $\frac{\partial CS}{\partial \gamma} < 0$. When $\gamma > \overline{\gamma}$, we know that $(1-\delta^*) > 0$, $(1-\gamma) > 0$, and from Proposition 3 $\frac{\partial \delta^*}{\partial \gamma} > 0$. This implies that $\frac{\partial CS}{\partial \gamma} < 0$ when $\gamma > \gamma^{CS}$.

Finally, we show that when $\gamma < \gamma^{cs}$, $\frac{\partial CS}{\partial \gamma} > 0$. Notice that $(1-\delta^* + (1-\gamma)\frac{\partial \delta^*}{\partial \gamma}) \to 1 - \frac{q^H - q^L}{q^L(1-\kappa) + \frac{e}{\Delta}}$ when evaluated at $\gamma \to 0$. From (1) we know that this is strictly negative. Therefore, $\frac{\partial CS}{\partial \gamma} > 0$ at $\gamma \to 0$. And we have shown that there is only one point where the sign changes when $\gamma > 0$. Therefore, when $\gamma < \gamma^{cs}$, $\frac{\partial CS}{\partial \gamma} > 0$

This concludes the proof and we have shown that there exist some $\gamma^{cs} \in (0, \overline{\gamma})$ that maximises consumer surplus,

$$\begin{aligned}\gamma^{cs} = &\frac{-(q^H - q^L)(q^L(1-\kappa) + \frac{e}{\Delta})^2}{(q^H - q^L)(4(q^H - q^L)^2 + (q^L(1-\kappa) + \frac{e}{\Delta})^2)} + \\ &\frac{((q^H - q^L)^3(q^L(1-\kappa) + \frac{e}{\Delta})(2(q^H - q^L) - (q^L(1-\kappa) + \frac{e}{\Delta}))^2)^{\frac{1}{2}}}{(q^H - q^L)(4(q^H - q^L)^2 + (q^L(1-\kappa) + \frac{e}{\Delta})^2)}\end{aligned}$$

and consumer surplus is strictly concave in γ.

This concludes the proof. □

Proof of Lemma 1

Proof of Lemma 1.

To show that sellers of all types benefit from quality controls, we look at the lifetime profits of both high- and low-quality firms independently.

We begin with low-quality firms.

$$\pi^L = \delta^*(\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L) + (1-\delta^*)(\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)})$$

We know that δ^* is such that the low-quality firm is indifferent between $\frac{\gamma q^H+\delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)} + q^L$ and $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H+(1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)}$.

Therefore,

$$\begin{aligned}
\pi^L &= \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L \\
\frac{\partial \pi^L}{\partial \gamma} &= \frac{(q^H - q^L)(\delta^* - \gamma(1-\gamma)\frac{\partial \delta^*}{\partial \gamma})}{(\gamma + \delta^*(1-\gamma))^2}
\end{aligned}$$

Since the denominator is positive, and $q^H - q^L > 0$, we evaluate $\delta^* - \gamma(1-\gamma)\frac{\partial \delta^*}{\partial \gamma}$. $\delta^* - \gamma(1-\gamma)\frac{\partial \delta^*}{\partial \gamma} = \frac{(1+\gamma)(q^L(1-\kappa)+\frac{e}{\Delta})}{2\sqrt{4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa)+\frac{e}{\Delta})^2}} + \frac{1}{2} > 0$ Therefore, profits of low-quality firms benefits from quality controls.

Moreover, we show that high-quality firms also benefit from quality controls.

$$\begin{aligned}
\pi^H =& \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} \\
\frac{\partial \pi^H}{\partial \gamma} =& \frac{32\gamma^2(q^H - q^L)^3(q^L(1-\kappa) + \frac{e}{\Delta})^2\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2}}{(\gamma + \delta^*(1-\gamma))^2(\gamma + (1-\delta^*)(1-\gamma))^2} - \\
& \frac{32\gamma^2(q^H - q^L)^3(q^L(1-\kappa) + \frac{e}{\Delta})^2(2\gamma(q^H - q^L))}{(\gamma + \delta^*(1-\gamma))^2(\gamma + (1-\delta^*)(1-\gamma))^2}
\end{aligned}$$

Because $\sqrt{4\gamma^2(q^H - q^L)^2 + (1+\gamma)^2(q^L(1-\kappa) + \frac{e}{\Delta})^2} - 2\gamma(q^H - q^L) > 0$, it is immediate that $\frac{\partial \pi^H}{\partial \gamma} > 0$.

Therefore, both high- and low-quality firms benefit when platforms implements quality controls.

This concludes the proof. □

Proof of Proposition 6

Proof of Proposition 6.

To proof Proposition 6, we show that $\frac{\partial \pi_p}{\partial e} < 0$.

To begin we characterize the actions of users on either side of the platform. Afterwards, we look at the strategy of the profit-maximizing platform.

To begin we characterize when consumers join the platform. The consumers' value from joining the platform is $n_s u_b$, i.e. the consumer gets expected surplus u_b from each interaction with a seller, and there are n_s sellers in total. Consumers' outside option is uniformly distributed on $[0, 1]$, which is why buyer demand is given by $n_b^* = n_s^* u_b$.

Next, we consider the firms. Since firms are ex-ante homogeneous, they have the same ex-ante expected revenue per transaction, π_s. Firms also face the same commission fee, r, which is set by the platform. Additionally, all firms face a marginal cost t of selling on the platform. Therefore the per-transaction profit for firms is $\pi_s(1-r)-t$. Since all firms face the same cost of entry $\bar{v}_s$, then any firm whose total profits are weakly above the outside option joins the platform. This means that $n_b^*(\pi_s(1-r)-t) \geq \bar{v}_s$. Because all firms face the same decision, the number of firms to join the platform is either $n_s^* = 0$ or $n_s^* = 1$. If $n_s^* = 0$, there is no activity on the platform and the platform earns zero profits, which is not optimal. We conclude that $n_s^* = 1$. Note that this implies $n_b^* = u_b$.

We now turn our attention to the platform. Because sellers are homogeneous and $n_s^* = 1$, the profit-maximizing platform extracts the highest possible benefit from the royalty fee subject to sellers participating. This implies that the platform sets the optimal r^* such that

$$n_b^*(\pi_s(1-r^*)-t) = \bar{v}_s \Leftrightarrow r^* = 1 - \frac{\bar{v}_s}{n_b^* \pi_s} - \frac{t}{\pi_s}.$$

Using this, we can simplify the platform's profits to

$$\pi_p = n_b^* \pi_s r^* = n_b^* \pi_s - \bar{v}_s - t n_b^* = n_b^*(\pi_s - t) - \bar{v}_s.$$

Now, we consider the effects of the ratings environment on the profits of the platform. To see this, we need to understand how the platform's profits are affected by changes to effort cost,

$$\frac{\partial \pi_p}{\partial e} = \frac{\partial u_b}{\partial e}(\pi_s - t) + \frac{\partial \pi_s}{\partial e} u_b. \tag{14}$$

To understand the platform's strategy, we evaluate $\frac{\partial \pi_p}{\partial e}$. To do so, we first show that $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi_s}{\partial e}$.

Consider first u_b. This is the transaction benefit of each consumer. Because consumers purchase first or second with equal probability, their ex-ante expected benefit per transaction is $u_b = \frac{1}{2}CS$, where we know from (2) that $CS = (1-\delta^*)(1-\gamma)[(1-\kappa)q^L + \frac{e}{\Delta}]$. Thus,

$$\frac{\partial u_b}{\partial e} = \frac{1}{2}\frac{(1-\gamma)(1-\delta^*)}{\Delta} - \frac{1}{2}\frac{\partial \delta^*}{\partial e}[(1-\kappa)q^L + \frac{e}{\Delta}].$$

Next, consider π_s. This is the per transaction profit of firms before taking into account the commission fee of the platform. Ex-ante, this is equivalent to the expected revenue that the firms receives per consumer, i.e.

$$\begin{aligned}\pi_s =& \frac{1}{2}\gamma[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \\ & \frac{1}{2}(1-\gamma)[(1-\delta^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \delta^*[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L]] \\ =& [\gamma q^H + (1-\gamma)q^L] - \frac{1}{2}(1-\delta^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})\end{aligned}$$

Taking the derivative,

$$\frac{\partial \pi_s}{\partial e} = \frac{1}{2}\frac{\partial \delta^*}{\partial e}[(1-\kappa)q^L + \frac{e}{\Delta}] - \frac{1}{2}\frac{(1-\gamma)(1-\delta^*)}{\Delta}.$$

Thus we conclude that $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi_s}{\partial e}$.

Returning to equation (14), if $\frac{\partial \pi_p}{\partial e} < 0$, the platform designs a ratings system such that it minimizes the effort cost associated with rating.

$$\frac{\partial \pi_p}{\partial e} < 0 \Leftrightarrow \frac{\partial u_b}{\partial e}(\pi_s - t) + \frac{\partial \pi_s}{\partial e}u_b < 0 \Leftrightarrow \frac{\frac{\partial u_b}{\partial e}}{u_b} < -\frac{\frac{\partial \pi_s}{\partial e}}{(\pi_s - t)}.$$

From this formulation, we see that the platform's design decision depends on the relative elasticity of consumer transaction surplus and firms transaction revenue. In particular, since $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi_s}{\partial e}$,

$$\frac{\frac{\partial u_b}{\partial e}}{u_b} < \frac{\frac{\partial u_b}{\partial e}}{(\pi_s - t)} \iff (\pi_s - t) < u_b$$

implies that $\frac{\partial \pi_p}{\partial e} < 0$.

We now show when this condition can be satisfied. First, note that $e < e^{cs}$, so $\frac{\partial u_b}{\partial e} > 0$.

We now provide the conditions for which $\frac{\frac{\partial u_b}{\partial e}}{u_b} < \frac{\frac{\partial u_b}{\partial e}}{(\pi_s - t)}$ is satisfied.

$$\pi_s - t < u_b \Leftrightarrow \frac{\gamma q^H + (1-\gamma)q^L - t}{(1-\kappa)q^L + \frac{e}{\Delta}} < (1-\delta^*)(1-\gamma). \tag{15}$$

Therefore, when (15) holds, we show that platforms favour sellers and minimise the effort cost required to leave a rating.

We conclude that when (15) holds platforms will minimize the effort costs required to leave a rating.

Specifically, when $t \geq \frac{q^L((1+\gamma)+\kappa(1-\gamma))\sqrt{4\gamma^2(q^H-q^L)^2+(1+\gamma)^2(q^L(1-\kappa))^2}}{2}$, $e^* = 0$. Otherwise,

$$e = \frac{q^L(-1+\gamma(2\kappa-1)) + t(1-\gamma) \pm \sqrt{(q^L-t)^2(1+2\gamma+\gamma^2) - 4\gamma^3(q^H-q^L)^2}}{2\gamma}$$

Since $e^* > 0$, we reject the negative and

$$e^* = \frac{q^L(-1+\gamma(2\kappa-1)) + t(1-\gamma) + \sqrt{(q^L-t)^2(1+2\gamma+\gamma^2) - 4\gamma^3(q^H-q^L)^2}}{2\gamma}$$

We have provided the condition for which platforms are incentivised to minimize effort costs associated with rating. And we have also shown more generally the level of effort that maximises platform's profit when (15) holds.

This concludes the proof. $\square$

Proof of Proposition 7

To proof Proposition 7, we first need to show some lemmas. First, Lemma 5 guides when the competitive fringe may be considered to be active in the market. Second, Lemma 6 shows the adjusted pricing strategies of strategic firms. Finally, Lemma 7 characterizes the adjusted consumer's beliefs in the presence of the competitive fringe. We then use these results to prove Proposition 7.

Lemma 5. *The competitive fringe is only relevant whenever $c < q$.*

Proof of Lemma 5.
Whenever $c > q$, consumers receive negative utility from the competitive fringe. Whenever $c = q$, consumers receive 0 utility. Since the strategic firms provide at least non-negative

utility, by assumption they would receive the full consumer demand. Therefore, the fringe only captures consumers if $c < q$.

This concludes the proof. □

Lemma 6. *Pricing strategy of the high-quality firm:*

- *Period 1:* $p_1'^H = E[q_1|R_0, p_1^H] - (q - c)$
- *Period 2:* $p_2'^H = E[q_2|R_1] - (q - c)$

Pricing strategy of the low-quality firm:

- *Period 1:*
  - $R_1 = 1$, $p_{1,1}'^L = min\{q^L - (q - c), \kappa q^L - \frac{e}{\Delta}\}$
  - $R_1 = 0$, $p_{1,0}'^L = p_1'^H$
- *Period 2:* $p_2'^L = E[q_2|R_1] - (q - c)$

Proof of Lemma 6.
To prove this, we first discuss the tie breaker rule. Second, we consider the second period as this is straight forward. Then we consider the high-quality firm in the first period, followed by the low-quality firm in the first period. The proofs are similar to that of Lemma 3.

Our tie breaker rule is such that whenever both the strategic firm and the fringe provide the same level of consumer surplus, consumers choose to purchase from the strategic firm. Here, we shall argue why this tie breaker rule is a simplification that yields virtually identical results to a tie breaker rule where consumers randomise between the two firms.

By construction, the fringe sets some fixed price c, while the strategic firm selects prices. Therefore, for any situation where the strategic firm and fringe provide the same level of consumer surplus, the strategic firm can always choose to set prices some small $\epsilon > 0$ lower, such that they obtain the full demand. Therefore, by assuming this tie breaker rule, we are able to simplify our discussion without considering the need for some small ϵ deviation.

We now turn our attention to the discussion of equilibrium prices in the second period. In the second period, consumers are aware that consumption from the fringe will leave them a surplus of $q - c$. This forms the outside option for consumers. Strategic firms therefore have to provide consumers with a surplus of at least $q-c$, doing so will shift all the demand towards the strategic firm. We conclude that the firm will not set prices lower than $E[q_2|R_1]-(q^L-c)$.

We now turn our attention to the high-quality firm in the first period. As in Lemma 3, a high-quality firm wishing to obtain a positive rating must set prices no higher than $\kappa q^H - \frac{e}{\Delta}$. As in the proof of Proposition 11, consumers are aware that consumption of the outside good will provide $q - c$ level of utility. Therefore, firms must set prices no higher than $E[q_1|R_0, p_1] - (q - c)$. Because we show in the proof of Proposition 11 that we must have $min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_1^H]\} = E[q_1|R_0, p_1^H]$ in equilibrium, high-quality firms will continue to receive a good rating at $E[q_1|R_0, p_1] - (q - c)$. It follows that the equilibrium price is $p_1'^H = E[q_1|R_0, p_1^H] - (q - c)$.

We now look at the low-quality firm in period 1. When the low-quality firm prefers to obtain a good rating, it has to set a price no higher than $\kappa q^L - \frac{e}{\Delta}$. With the presence of the competitive fringe, consumers are aware that they would be able to receive at least $q - c$ of surplus. Therefore, in order for the low-quality firm to capture demand and get a good rating, they are unable to command prices higher than $q^L - (q - c)$. Thus, the equilibrium price is $p_{1,1}'^L = min\{c, \kappa q^L - \frac{e}{\Delta}\}$.

For the low-quality firm receiving no rating, it sets prices above $\kappa q^L - \frac{e}{\Delta}$. The profit maximizing firm will set the highest possible price at which consumers buy. For the same argument as in the proof of Lemma 3, low-type firms do not set prices higher than the high-quality firm. Therefore, when low-quality firms receive no rating, it sets prices equal to that of the high-quality firm.

This concludes the proof. $\square$

Lemma 7. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L}, \end{cases}$$

and in the second period,

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \end{cases}$$

Proof of Lemma 7.
The proof is identical to that of Lemma 4. $\square$

Proof of Proposition 7.
We now proof Proposition 7.

First, we discuss the possible cases for how the competitive fringe might affect the behavior of strategic firms. Then we consider these cases one at a time and find the effect that changes in c have on the mixed-strategy of the low-quality firm.

Since we know that the second period prices are equally affected by the competitive fringe in all cases, we turn our attention to the first period. Notice that in period 1, the high-quality firm sets a single price, and the low-quality firm sets two out of three possible prices, i.e. $q^L-(q-c)$, $\kappa q^L-\frac{e}{\Delta}$, or $p_1'^H$. Further, notice that $\overline{p_1^H} > \overline{p_1^L}$ and $E[q_1|p_1'^H]-(q-c) > q^L-(q-c)$.

We know from the proof of Proposition 11 that in equilibrium we have $\overline{p_1^H} > E[q_1|p_1'^H]$, this leaves us with the following possible scenarios:

$$
\begin{aligned}
&1.\overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > \overline{p_1^L} > q^L - (q-c)\\
&2.\overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > q^L - (q-c) \geq \overline{p_1^L}
\end{aligned}
$$

We consider the scenarios individually.

1. $\overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > \overline{p_1^L} > q^L - (q-c)$

Because $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta} > q^L - (q-c)$, we know from Lemma 6 that the low-quality firm is indifferent between setting prices $p_1'^H$ and $q^L-(q-c)$. When charging $p_1'^H$, the firm gets no rating and charges $q^L-(q-c)$ in period 2. When charging $q^L-(q-c)$, the firm gets a good rating and earns the expected quality of a firm with a good rating in period 2 minus $(q-c)$. This leads to the following condition.

$$
\begin{aligned}
&E[q_1|p_1'^H] - (q-c) + q^L - (q-c) = q^L - (q-c) + E[q_2|R_1] - (q-c)\\
\Leftrightarrow &\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} = \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}\\
\Leftrightarrow &\delta^* = 0.5,
\end{aligned}
$$

which is independent of c.

This concludes case 1.

2. $\overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > q^L - (q-c) \geq \overline{p_1^L}$

Because $q^L-(q-c) \geq \overline{p_1^L}$, and we know from Lemma 6 that the low-quality firm is indifferent between setting prices $p_1'^H$ and $\overline{p_1^L} = \kappa q^L - \frac{e}{\Delta}$. To be indifferent between these prices, the following condition must hold. The left-hand side is as in the previous case. On the right-hand side, the firm charges $\kappa q^L - \frac{e}{\Delta}$ in period 1 and obtains a good rating. In period 2, it earns the expected quality of a firm with good rating in period 2 minus $(q-c)$. This leads to the following condition.

$$
\begin{aligned}
&E[q_1|p_1'^H] - (q-c) + q^L - (q-c) = \overline{p_1^L} + E[q_2|R_1] - (q-c) \\
\Leftrightarrow &\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} - q + c = \kappa q^L - \frac{e}{\Delta} + \frac{\gamma(q^H - q^L)}{\gamma + (1-\delta^*)(1-\gamma)}.
\end{aligned}
$$

Using the implicit-function theorem then leads to

$$
\frac{\partial \delta^*}{\partial c} = \frac{(\gamma + (1-\delta^*)(1-\gamma))^2(\gamma + \delta^*(1-\gamma))^2}{\gamma(1-\gamma)(q^H - q^L)[(\gamma + (1-\delta^*)(1-\gamma))^2 + (\gamma + \delta^*(1-\gamma))^2]} > 0.
$$

This concludes case 2.

We conclude that $\frac{\partial \delta^*}{\partial c} > 0$ if $q^L - (q-c) \geq \kappa q^L - \frac{e}{\Delta}$ and 0 otherwise. □

Proof of Proposition 8

We can proof this proposition using the following two Lemmas

Lemma 8. *Consumers obtain the same signal from a negative rating and no rating.*

Proof of Lemma 8.
This proof holds directly from Restriction 1. Consumers' belief is such that high-quality firms always get a good rating. Hence, on observing no or bad ratings, they would believe this to be obtained by low-quality firms.

$$
E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \\ q^L & \text{if } R_1 = -1 \end{cases}
$$

where δ^* continues to represent the probability that low-quality firms get no rating.

This concludes the proof. □

Lemma 9. *When low-quality firms choose not to receive a good rating, it prefers to receive a bad rating over no rating, if consumers continue to buy at the price level that induces a bad rating.*

Proof of Lemma 9.
First, we discuss the prices set in the second period. Then we look at consumer's beliefs in the first period and the prices set in the first period. We then show that in equilibrium,

there exist some price for which low-quality firms receive no rating, and another price where they receive bad ratings.

When looking at the prices set in the second period, notice first that firms of all types will set the highest possible price in the second period. This is equivalent to consumer's expectation in the second period. Since this is only dependent on ratings, on receiving a good rating in the first period, firms set $\frac{\gamma q^H+(1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)}$; on receiving no rating, they set a price of q^L; on receiving a bad rating, they also set a price of q^L.

Now, consider the possible prices in the first period. In the first period, high-quality firms will set a price $p_1^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_1^H]\}$ and receives a good rating. The proof of this is identical to the proof of high-quality firm's prices in Lemma 3.

Low-quality firm's do one of the following: set a price such that it receives a good rating, no rating or a bad rating.

When a low-quality firm receives a good rating, it sets a unique price and this is $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$. This proof is identical to that of Lemma 3 when low-quality firms receive a good rating.

We turn our attention to the situation when low-quality firms receive a bad rating. When a low-quality firm receives a bad rating, this only occurs if $[\kappa q^L - p_1^L]\Delta' - e \geq 0$. Further, bad ratings imply that consumers are exhibiting negative reciprocity. Therefore, we define $\Delta' = -\Delta$. And consumer's leave a bad rating whenever $p_1^L \geq \kappa q^L + \frac{e}{\Delta}$, gaining a positive rating utility from doing so. From Restriction 2, we know that since this price is larger than $\overline{p_1^L}$, consumers expectations are fixed and from Lemma 3, we know that the firm set the highest possible price. Therefore, when obtaining a bad rating, the low-quality firm sets the price $p_1^L = E[q_1|R_0, p_1^H]$.

Now we turn our attention to the situation when low-quality firms receive no rating. This occurs when $[\kappa q^L - p_1^L]\Delta' - e < 0$. That is the rating utility from leaving a good or bad rating is negative. Recall that the utility from giving no rating is 0.

Suppose that the consumer considers between leaving a good rating and no rating. By reciprocity, the consumer acts with a kindness of their own, implying that $\Delta' = \Delta > 0$. Therefore, no rating only occurs if $[\kappa q^L - p_1^L] < \frac{e}{\Delta}$. In other words, $p_1^L > \kappa q^L - \frac{e}{\Delta}$.

Now suppose that the consumer considers between leaving a bad rating and no rating. By reciprocity, the consumer acts negatively, implying that $\Delta' = -\Delta < 0$. Therefore, no rating only occurs if $p_1^L < \kappa q^L + \frac{e}{\Delta}$ and $p_1^L > \kappa q^L - \frac{e}{\Delta}$ hold together.

Notice that $\kappa q^L - \frac{e}{\Delta} < \kappa q^L + \frac{e}{\Delta}$. This implies that there exist a range of prices, $p_1^L \in (\kappa q^L - \frac{e}{\Delta}, \kappa q^L + \frac{e}{\Delta})$ such that consumers maximising their utility provide no ratings to low-quality firms.

In equilibrium, the total profit of a low-quality firm obtaining a bad rating is $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L$, while the total profit of obtaining no rating is strictly below $\kappa q^L + \frac{e}{\Delta} + q^L$.

We can now show that low-quality firms set some unique price in equilibrium, when they choose not to obtain a good rating. In equilibrium, when consumers observe price of p_1^H, they anticipate a utility of $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$. If this is above $\kappa q^L + \frac{e}{\Delta}$, low-quality firms maximise their profits by extracting the full surplus from consumers, and obtaining a bad rating. Alternatively, if the anticipated utility is below $\kappa q^L + \frac{e}{\Delta}$, it is not profitable for firms to set $\kappa q^L + \frac{e}{\Delta}$ as this would lead to zero demand. Instead, the low-quality firm sets p_1^H and obtains no rating. Therefore, when choosing not to obtain a good rating, low-quality firms always sets the unique price mimicking price of p_1^H.

We conclude that when we allow for negative ratings, there exists a range of prices for which good, bad, and no ratings may occur. However, in equilibrium, a profit maximising low-quality firm mixes between setting a price that allows it to harvest ratings and price mimicking. This allows it to obtain the extreme ratings of no, or negative rating depending on the cost of leaving a rating.

This concludes the proof. □

Together, Lemma 8 and Lemma 9 show that in the equilibrium, negative ratings will replace no ratings when going from a system of 2 ratings options to 3 ratings options, and consumers continue to buy at the price level that induces a negative rating. This concludes the Proof of Proposition 8.

Proof of Lemma 2.
To complete this proof, first consider the possible prices played by the high-quality firm. This is unchanged and follows directly from the Proof of Lemma 9. Second, we now consider the possible prices played by the low-quality firm.

When receiving a good rating, the low-quality firm sets a unique price and this is given by $p_{1,1}^L = \overline{p_1^L} = \kappa q^L - \frac{e_g}{\Delta}$. This proof is identical to that of Lemma 3 when the low-quality firm receives a good rating.

Next, we turn our attention to the low-quality firm receiving a bad rating. This only occurs if $[\kappa q^L - p_1^L](-\Delta) - e_b > 0$. As in Lemma 9 consumers leaving a bad rating exhibit negative reciprocity, this is why the warm glow is negative, and negative ratings occur if $p_1^L > \kappa q^L + \frac{e_b}{\Delta}$.

Notice that applying the same logic as in Lemma A.1, there exists a range of prices such that no rating occurs, and this is $p_1^L \in (\kappa q^L - \frac{e_g}{\Delta}, \kappa q^l + \frac{e_b}{\Delta})$.

This has two implications. First, as e_g is smaller, then from Corollary 1, low-quality firms harvest ratings more.

Second, as e_b is larger, low-quality firms only receive a bad rating if they set a higher price. However, as this runs up against the upper bound of prices that they may set, prices must be less than consumers willingness to pay, firms are unlikely to receive bad ratings.

This concludes the proof. □

Proof of Proposition 9

Proof of Proposition 9.
To begin, we first look at the second period. In period 2, conditional on observing a good rating, consumers form expectations and are willing to pay

$$\overline{p}_2 = \int_0^1 (1 - \delta^*(q))qdq.$$

Here, we define δ^* as the probability that a firm receives no rating.

And conditional on observing no rating, consumers are willing to pay

$$\underline{p}_2 = \int_0^1 \delta^*(q)qdq.$$

Since period 2 is a terminal period there is no more strategic pricing and firms have no incentive to obtain a good rating in period 2. Therefore, in period 2, firms set prices equal to consumers expected quality and consumers choose to buy. Firms are able to extract the full surplus from consumers.

We now turn our attention to period 1. First we show that no firm can set a price higher than the highest quality firm. Second, we show that there exist a group of high quality firms with quality better than $\hat{q}$ that receive a good rating with probability 1. Third, we show that there exist a group of low quality firms with quality lower than $\overline{q}$ that receive a good rating with probability 1.

To show the highest possible price, we first look at the price that the highest quality firm chooses to set. From the modified restriction 1, Restriction 3, the highest quality firm, with

$q = 1$, sets a price weakly below $\kappa - \frac{e}{\Delta}$. At this price, consumers are willing to pay, and the firm receives a good rating with probability 1.

We now show that no firm sets a price above the price that the highest quality firm sets, $p(q = 1)$. Suppose there is some price $p > p(q = 1)$, and all firms setting the price p receive no rating. Firms that receive no rating receive a payoff of $\underline{p}_2$ in period 2. Firms may choose to get a good rating with some probability, and to do so, they set a price $\kappa q - \frac{e}{\Delta}$ and obtain a good rating, allowing them to receive a payoff of $\overline{p}_2$ in period 2. The firm that benefits the most from receiving a good rating is the lowest quality firm of $q = 0$. Supposing that only the highest quality firm receives a good rating, for the firm $q = 0$, they choose to receive no rating if $p + 0.5 \geq -\frac{e}{\Delta} + 1$ and because $p > p(q = 1)$, $p + 0.5 > p(q = 1) + 0.5 \geq -\frac{e}{\Delta} + 1$. Therefore, for $p(q = 1) + \frac{e}{\Delta} \geq 0.5$, the lowest quality firm chooses to set the higher price p and obtain no rating. $p(q = 1) + \frac{e}{\Delta} \geq 0.5$ being the sufficient condition for the lowest quality firm to prefer no rating. Supposing $p(q = 1) + \frac{e}{\Delta} \geq 0.5$ (note that this means $\kappa > 0.5$ is a sufficient condition) and applying the D1 criterion, consumers believe that any firm that sets p is of quality $q = 0$, and choose not to buy the product. Hence, demand at any $p > p(q = 1)$ is zero. Knowing this, no firm sets p. We conclude that the highest price that can be set is the price that the price that the highest quality firm sets.

We now show that there exists a group of high quality firms that sets the price $p(q = 1)$ and obtains a good rating with probability 1. Suppose that the highest quality firm is the only firm that receives a good rating. This means that $p(q = 1) = \kappa - \frac{e}{\Delta}$. However, all other firms can choose to set $\kappa - \frac{e}{\Delta}$ in the first period and obtain $\underline{p}_2 = \int_0^1 \delta^*(q)qdq$ in the second period. Alternatively, they set the price $\kappa q - \frac{e}{\Delta}$ in period 1 and receive $\overline{p}_2 = \int_0^1 (1 - \delta^*(q))qdq$ in period 2. Therefore, for all firms where $\kappa(1 - q) \geq \int_0^1 (1 - 2\delta^*(q))qdq$, these firms choose to set $\kappa - \frac{e}{\Delta}$ with some positive probability. This means that in expectation, consumers choose not to purchase a product at $\kappa - \frac{e}{\Delta}$. Therefore, in order to receive a positive demand and obtain a positive rating, highest quality firm lower the price to some $p(q = 1) < \kappa - \frac{e}{\Delta}$. Recall that no firm can set a price higher than the highest quality firm. Thus, for some firm $\hat{q}$, $p(q = 1) = \kappa\hat{q} - \frac{e}{\Delta}$, and all firms with quality larger than $\hat{q}$, 'naturally' receive a good rating. This means, in equilibrium, they do not manipulate prices in order to obtain a good rating. We call these firms the top firms.

Therefore, we now know that in equilibrium, top firms with quality $q \geq \hat{q}$ play $\delta^*(q) = 0$. And $\overline{p}_2 = \int_{\hat{q}}^1 qdq + \int_0^{\hat{q}} (1 - \delta^*(q))qdq$ and $\underline{p}_2 = \int_0^{\hat{q}} \delta^*(q)qdq$. We now know that $\hat{q}$ is the lowest quality firm that 'naturally' receives a good rating with probability 1. This implies that $\kappa\hat{q} - \frac{e}{\Delta} = q'$, where q' is the highest price for which consumers are willing to buy. This means that $q' = \int_{\hat{q}}^1 qdq + \int_0^{\hat{q}} \delta^*(q)qdq$.

We now look at what non-top firms do.

For all firms with quality $q < \hat{q}$, they choose $\delta^*(q)$ such that they maximise

$$\pi(q) = \delta(q)[q' + \underline{p}_2] + (1 - \delta(q))[\kappa q - \frac{e}{\Delta} + \overline{p}_2]$$

When firms choose $\delta(q)$, and considering that $q' = \int_{\hat{q}}^{1} qdq + \int_0^{\hat{q}} \delta^*(q)qdq = \kappa\hat{q} - \frac{e}{\Delta}$,

$$\begin{aligned}
\frac{\partial \pi(q)}{\partial \delta(q)} &= q' + \underline{p}_2 - \kappa q + \frac{e}{\Delta} - \overline{p}_2 \\
&= \kappa\hat{q} - \frac{e}{\Delta} - \kappa q + \frac{e}{\Delta} + \int_0^{\hat{q}} \delta^*(q)qdq - \int_{\hat{q}}^{1} qdq - \int_0^{\hat{q}} (1 - \delta^*(q))qdq \\
&= \kappa(\hat{q} - q) - \frac{1}{2} + 2\int_0^{\hat{q}} \delta^*(q)qdq \\
&= \kappa(3\hat{q} - q) - \frac{3}{2} - 2\frac{e}{\Delta} + \hat{q}^2
\end{aligned}$$

Setting $\frac{\partial \pi(q)}{\partial \delta^*(q)} = 0$, we obtain the cutoff quality level above which firm's strictly prefer $\delta^*(q) = 0$, and below which $\delta^*(q) = 1$. We label this cutoff as q'', and $q'' = 3\hat{q} + \frac{\hat{q}^2 - \frac{3}{2} - \frac{2e}{\Delta}}{\kappa}$. Therefore, we conclude that for any firm with quality $q \in [q'', \hat{q}]$, firms choose to manipulate prices to obtain a good rating, and for any firm with quality $q \in [0, q'')$, they obtain no rating in equilibrium.

This implies that for any firm of a sufficiently low quality, they choose to play q' and receive no rating. We call these firms the bottom firms. Further, any firm $q \in [q'', \hat{q}]$, they choose to play $\kappa q - \frac{e}{\Delta}$ and obtain a good rating. We call these firms the middle firms.

We can now show that comparative statics of e are qualitatively the same. Reformulating $q' = \kappa\hat{q} - \frac{e}{\Delta}$, we get $\hat{q} = \frac{q' + \frac{e}{\Delta}}{\kappa}$. Using the envelope theorem, since $q' = \int_{\hat{q}}^{1} qdq + \int_0^{\hat{q}} \delta^*(q)qdq$, we do not need to consider the indirect effects. Therefore, we need only consider the direct effect that changes to e has on $\hat{q}$. $\frac{\partial \hat{q}}{\partial e} = \frac{1}{\Delta\kappa}$. Therefore, when it becomes easier to leave a rating, this causes the number of top firms to increase by $\frac{1}{\Delta\kappa}$. This means that the highest quality middle firms now become a top firm.

Next, consider what happens to q'' when e changes. Reformulating $q'' = 3\hat{q} + \frac{\hat{q}^2 - \frac{3}{2} - \frac{2e}{\Delta}}{\kappa}$, we get $q'' = \frac{3q'}{\kappa} - \frac{3}{2\kappa} + \frac{e}{\Delta\kappa} + \frac{(q' + \frac{e}{\Delta})^2}{\kappa^3}$. Again, applying the envelope theorem, we find that $\frac{\partial q''}{\partial e} = \frac{1}{\Delta\kappa} + \frac{2(q' + \frac{e}{\Delta})}{\Delta\kappa^3}$. This means that the highest quality bottom firms now join the group of middle firms.

We have shown that making ratings easier to leave, leads to a higher likelihood of a firm being in the group of middle firms. This is because the proportion of firms that obtain a good rating (sum of top and middle firms) increases by more than the proportion of firms that naturally obtain a good rating (top firms). This difference is $\frac{2(q'+\frac{e}{\Delta})}{\Delta\kappa^3}$. There is an increase in the number of firms that manipulate ratings. This leads to less-informative ratings, and ratings inflation.

This concludes the proof, and we have shown that in a setting with a continuum of firm quality types and 2 ratings, top firms naturally obtain a good rating with probability 1, middle firms manipulate ratings through prices and obtain a good rating with probability 1, and bottom firms choose not to participate in ratings harvesting. We also show that our main effects regarding the cost of leaving ratings pass through in this setting. This is true when the sufficient condition of $\kappa > 0.5$ holds. $\square$

Proof of Proposition 10

To show Proposition 10, we prove a more general proposition, Proposition 12. Further, recall that to save on notation we omit the 0 rising from $t = 0$.

Proposition 12. *All perfect Bayesian equilibria satisfy the following.*

1. *High-quality firms receive a good rating with probability 1 and charge* $p_t^H = E[q_t|R_{t-1}, p_t^H]$, $t \in \{1, 2\}$.
2. *Low-quality firms randomize their strategy in period* $t \in \{1, 2\}$.
   a. *They charge* $\overline{p_t^L} = \kappa q^L - \frac{e}{\Delta}$ *and obtain a good rating with probability* $1 - \delta_t^*$.
   b. *They charge* p_t^H *and obtain no rating with probability* δ_t^*.
3. *Firms set prices equal to expected quality conditional on ratings in the last period.*
4. *Consumer beliefs given equilibrium prices are given by:*
   a. *In period 1,* $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \textit{if } p_1 > \overline{p_1^L} \\ q^L & \textit{if } p_1 \leq \overline{p_1^L} \end{cases}$
   b. *In period 2,*

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \textit{if } p_2 > \overline{p_2^L} \textit{ and } R_1 = 1 \\ q^L & \textit{for any other } p_2,\ R_1 \textit{ combination} \end{cases}$$

c. *In period 3,* $E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}$

Furthermore, we show that $\delta_1^* \in (0,1)$ *and* $\delta_2^* \in (0,1)$ *is an equilibrium if* $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$*, and* $\delta_1^* \in (0,1)$ *and* $\delta_2^* = 1$ *is an equilibrium if* $(1-\kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$*. These equilibria exist if* $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\gamma)(q^H - q^L)}{2-\delta_2^*}$*,* $\gamma > \frac{1}{3}$*, and* $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$.

We proceed as follows. Before proving Proposition 12, we show two Lemmas. First, we use our restrictions to characterize firms' pricing strategies. Second, we pin down equilibrium beliefs. Finally, we use these results to show Proposition 12. Most of the arguments used in proof are similar to the ones from Proposition 1, which is why we briefly sketch them here.

We begin by pinning down the equilibrium prices in period 1 and 2 in Lemma 10.

Lemma 10. *In equilibrium, firms play the following prices in period 1 and 2 with weakly positive probability.*

- *High-quality firm:* $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$
- *Low-quality firm:*

$$p_{1,1}^L = p_{2,11}^L = \kappa q^L - \frac{e}{\Delta}$$

$$p_{1,0}^L = p_{2,10}^L = E[q_t|R_{t-1}, p_t^H]$$

$$p_{2,00}^L = p_{2,01}^L = q^L$$

Proof of Lemma 10.
The proof of Lemma 10 is similar to the proof of Lemma 3.

The key difference lies in the possible rating histories. Given the extension to 3 periods, the possible rating histories are now $R_0 \in \{0\}, R_1 \in \{0,1\}, R_2 \in \{00, 01, 10, 11\}$. As in the proof of Lemma 3, we will consider the pricing strategy of the high-quality firm followed by low-quality firm.

From Restriction 1, a high-quality firm wants to set prices which allows it to get a good rating and therefore in equilibrium her rating's history are $R_0 = \{0\}, R_1 = \{1\}$ and $R_2 = \{11\}$. We show that on the equilibrium path $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$ for $t \in \{1,2\}$.

To show that the high-quality firm sets a unique price in each period $t \in \{1,2\}$ with probability 1, we follow the proof set out in Lemma 3. Suppose towards a contradiction that the high-quality firm sets more than one price with positive probability. Without loss of generality, suppose that the high-quality firm sets a continuous distribution of prices in either period, i.e. it charges prices in some interval $p_t \in [p_t', p_t'']$ such that $p_t'' > p_t'$, and the high-quality firm receives a good rating with probability 1 for all $p_t \in [p_t', p_t'']$. Note that when the high-quality firm gets a rating for all $p_t \in [p_t', p_t'']$, consumers purchase products with probability 1. Notice that for any price $\hat{p}_t > p_t$ such that $\hat{p}_t \in [p_t', p_t'']$, we have $\pi_t^H(\hat{p}_t) > \pi_t^H(p_t)$. The reason is that both prices induce the same demand in period t, and the same rating and therefore the same continuation profits. Thus, the firm can strictly increase profits by shifting all probability mass from $[p_t', p_t'']$ to p_t''. This contradicts the assumption that the high-quality firm sets a uncountably infinite prices with positive probability. Similarly, the firm will not set countably finite or infinite prices that induce the same rating. Therefore, we conclude that the high-quality firm sets a unique price in each period t on the equilibrium path with probability 1. We denote this as p_t^H.

Next, we show that there exist an upper bound on prices, $\overline{p_t^j}$ for $j \in \{L, H\}$ such that j-quality firm receives a positive rating. In order for a firm to induce a positive rating, the rating utility must be positive. Therefore,

$$[\kappa q^j - p_t^j]\Delta - e \geq 0 \iff p_t^j \leq \overline{p_t^j} \equiv \kappa q^j - \frac{e}{\Delta}$$

Finally, consider that prices are bound by consumer's beliefs, $E[q_t|R_{t-1}, p_t^H] < \overline{p_t^H}$. Under such scenarios, by Restriction 1, high-quality firms prefer obtaining a good rating. This can only be achieved if consumers buy. Therefore, p_t^H is bound by $E[q_t|R_{t-1}, p_t^H]$.

We now show that $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$ with probability 1. As with Lemma 3, note that $\overline{p_t^H} > \overline{p_t^L}$ and $q^L > \overline{p_t^L}$. Thus, because for equilibrium expectations we have $E[q_t|R_{t-1}, p_t^H] > q^L$, the high-quality firm sets equilibrium prices strictly larger than $\overline{p_t^L}$. Applying Restriction, 2, consumers have the same beliefs for all prices strictly above $\overline{p_t^L}$ in each period t. Since $p_t^H > \overline{p_t^L}$, these beliefs are given by $E[q_t|R_{t-1}, p_t^H]$, the correct equilibrium beliefs. Further, since consumers have the same beliefs for all prices above $\overline{p_t^L}$ in each period, the high-quality firm optimally sets the highest possible price at which consumers purchase and rate with probability 1. Hence, $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$. We conclude that high-quality firms set this unique price, $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$, with probability 1.

We turn our attention to low-quality firms. First, we show that the price which the low-quality firm sets when it receive a good rating in any period, $t \in \{1, 2\}$, is unique. And this price is $p_{1,1}^L = p_{1,11}^L = \kappa q^L - \frac{e}{\Delta}$. The argument here is the same as the argument used for when high-quality firms receive a good rating. The low-quality firm receives a good rating at any price weakly below $\overline{p_t^L} = \kappa q^L - \frac{e}{\Delta}$, and $\overline{p_t^L} < q^L$. As consumers beliefs are weakly above q^L, they buy at any price weakly below $\overline{p_t^L}$. Since demand and ratings are the same for all prices weakly below $\overline{p_t^L}$, a low-quality firm obtaining a positive rating optimally sets $\overline{p_t^L}$ with probability 1. We can conclude that low-quality firms who obtain a rating in period t set $\overline{p_t^L}$ with probability 1.

Next, we consider the situation where the low-quality firm receives no rating for the first time. The related ratings history are $R_1 = \{0\}, R_2 = \{10\}$. We show that $p_{1,0}^L = E[q_1|R_0, p_1^H]$ and $p_{2,10}^L = E[q_2|R_1, p_2^H]$. We have shown above that all prices above $\overline{p_t^L}$ induces the belief $E[q_t|R_{t-1}, p_t^H]$ - as a result of Restriction 2. Therefore, a low-quality firm receiving no rating optimally sets the highest possible price, $E[q_t|R_{t-1}, p_t^H]$ with probability 1.

Finally, consider the situation where the low-quality firm already has a history of receiving no rating. The related ratings history is $R_2 = \{00, 01\}$. Since by Restriction 1 only low-quality firms receive no rating on the equilibrium path, consumer's belief on observing any history of no rating is that the firm is of a low-quality. Hence $p_{2,00}^L = p_{2,01}^L = E[q_2|R_1 = \{0\}] = q^L$.

This concludes the proof.

$\square$

Lemma 11. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \textit{if } p_1 > \overline{p_1^L} \\ q^L & \textit{if } p_1 \leq \overline{p_1^L} \end{cases},$$

in the second period,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \textit{if } p_2 > \overline{p_2^L} \textit{ and } R_1 = 1 \\ q^L & \textit{for any other } p_2,\ R_1 \textit{ combination} \end{cases},$$

and in the third period,

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \textit{if } R_2 = \{11\} \\ q^L & \textit{for any other } R_2 \end{cases}.$$

Proof of Lemma 11.
As with Lemma 4, we prove this lemma by constructing expected quality using Bayes rule. We start by considering the third period, followed by the second then the first.

We begin with the third period. In the third period, consumers are aware of historical ratings, R_2 and current prices p_3. Given that this is the final period of the game, new ratings are not useful for firms - as there is no future period to signal to. By Restriction 2, this implies that the expected quality in period 3 is independent of prices. Thus, the expected quality in period 3 is independent of prices and only depends on the rating history R_2. As a result, firms set the highest possible price and extract the full consumer surplus.

Note that the possible ratings history are $R_2 \in \{00, 01, 10, 11\}$. When consumers observe a ratings history of $\{00, 01, 10\}$, they expect that the firm is a low-quality firm (Restriction 1).

Using Bayes rule, we pin down consumers' expectations on observing $R_2 = \{11\}$ in period 3. In equilibrium, low-quality firms receive no rating with some probability δ_t^* in period $t \in \{1, 2\}$. Hence, consumers observe a good rating with probability $\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)$ - and know that the probability of a high-quality firm is γ. Hence, $E[q_3|R_2 = 11] = \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$.

We conclude that

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}.$$

Next, we consider the second period. In the second period, consumers observe $R_1 \in \{0, 1\}$ and the price p_2. As with the third period, on observing $R_1 = 0$ Restriction 1 implies that the firm is of a low-quality.

Using Bayes rule, we pin down consumers' expectations in equilibrium in period 2, conditional on $R_1 = 1$. We distinguish between two cases: the low-quality firm choosing between a good or no rating. When the low-quality firm receives no rating, it sets $p_2^L = E[q_2|R_1 = 1, p_2^H]$ and when it receives a good rating, $p_2^L = \overline{p_2^L} = \kappa q^L - \frac{e}{\Delta}$. Note that $p_2^H = min\{\kappa q^H -$

$\frac{e}{\Delta}, E[q_2|R_1, p_2^H]\}$.

We now start with the first case, i.e. the low-quality firm sets $p_2^L = \overline{p_2^L} = \kappa q^L - \frac{e}{\Delta}$ and gets a good rating. Since $\kappa q^H - \frac{e}{\Delta} > \kappa q^L - \frac{e}{\Delta}$ and $E[q_2|R_1 = 1, p_2^H] \geq q^L > \kappa q^L - \frac{e}{\Delta}$, in the equilibrium, consumers who observe $\kappa q^L - \frac{e}{\Delta}$ believe the firm is a low-quality firm, i.e. $E[q_2|R_1 = 1, p_2 = \kappa q^L - \frac{e}{\Delta}] = q^L$.

Next, consider the second case where the low-quality firm sets, by Lemma 10, $p_2^L = E[q_2|R_1 = 1, p_2^H]$ and gets no rating. We distinguish two scenarios. First, suppose $p_2^H = min\{E[q_2|R_1 = 1, p_2^H], \kappa q^H - \frac{e}{\Delta}\} = E[q_2|R_1 = 1, p_2^H]$. By Lemma 10, $p_{2,10}^L = E[q_2|R_1 = 1, p_2^H] = p_2^H$. On observing this price level, Bayes rule implies $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ in equilibrium. Second, consider the scenario where $p_2^H = min\{E[q_2|R_1 = 1, p_2^H], \kappa q^H - \frac{e}{\Delta}\} = \kappa q^H - \frac{e}{\Delta}$. Since $p_2^L = E[q_2|R_1, p_2^H] \neq \kappa q^H - \frac{e}{\Delta} = p_2^H$, consumers who observe p_2^H believe $E[q_2|R_1, p_2^H] = q^H$. Because $p_2^L = E[q_2|R_1, p_2^H] = q_H > \overline{p_2^L}$ and $p_2^H = \kappa q^H - \frac{e}{\Delta} > \overline{p_2^L}$, Restriction 2 implies that both p_2^L and p_2^H induce the same equilibrium beliefs. But then we must have $\delta_2^* = 1$ in equilibrium. Note that beliefs $E[q_2|R_1, p_2^H] = q_H$ are then a special case of $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ for $\delta_2^* = 1$.

This concludes the second case.

We conclude that $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$.

The final stage is to consider period 1. The proof for this is identical to proof in Lemma 11.

This concludes the proof.

$\square$

With Lemma 10 and 11, we can now prove Proposition 12.

Proof of Proposition 12.
We show that all equilibria satisfying our equilibrium restrictions exhibits similar characteristics as those in the main section of the paper. To do so, we show that a perfect Bayesian equilibrium exists, and that all equilibria satisfying our equilibrium restrictions satisfy similar properties as those in the main body. Specifically,

1. High-quality firms receive a good rating with probability 1 and charge $p_t^H = E[q_t|R_{t-1}, p_t^H]$, $t \in \{1, 2\}$.
2. Low-quality firms randomize their strategy in period $t \in \{1, 2\}$.
    a. They charge $\overline{p_t^L} = \kappa q^L - \frac{e}{\Delta}$ and obtain a good rating with probability $1 - \delta_t^*$.

b. They charge p_t^H and obtain no rating with probability δ_t^*.

3. Firms set prices equal to expected quality conditional on ratings in the last period.

4. Consumer beliefs given equilibrium prices are given by:

a. In period 1, $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L} \end{cases}$

b. In period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \overline{p_2^L} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases}$$

c. In period 3, $E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}$

Furthermore, we show that $\delta_1^* \in (0,1)$ and $\delta_2^* \in (0,1)$ is an equilibrium if $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H-q^L)}{\gamma+(1-\delta_1^*)(1-\gamma)}$, and $\delta_1^* \in (0,1)$ and $\delta_2^* = 1$ is an equilibrium if $(1-\kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H-q^L)}{\gamma+(1-\delta_1^*)(1-\gamma)}$. These equilibria exist if $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\gamma)(q^H-q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$, and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$.

From Lemmas 10 and 11, we have shown statement 4 and the low-quality firm's prices in statement 2. What remains, is to show statements 1 and 3, the mixed-strategy in statement 2, and to show the existence of the equilibrium.

We prove statement 3. In period 3, firms are no longer incentivized by future ratings. We have also shown in Lemma 10 that by Restriction 2, in the final period consumers' beliefs are only dependent on past ratings and therefore independent of the price they observe in period 3. Firms set prices in period 3 equal to the expected quality conditional on past ratings. This concludes the proof of statement 3.

Next, we prove statement 1. From Lemma 10, we have shown that $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\}$. To show that $min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\} = E[q_t|R_{t-1}, p_t^H]$, suppose towards a contradiction that $min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\} = \kappa q^H - \frac{e}{\Delta}$. We do this in two parts, for period 2 then period 1.

Note first that in period 2, the low-quality firm with no rating in period 1 always sets the price q^L in periods 2 and 3. Thus, we only consider histories after which the low-quality firm received a good rating in period 1. We know from Lemma 10 that $p_{2,10}^L = E[q_2|R_1, p_2^H]$. Thus, in the candidate equilibrium we have $p_2^H \neq p_{2,10}^L$ and only high-quality firms set $p_2^H = \kappa q^H - \frac{e}{\Delta}$, which is why consumers believe that $E[q_2|R_1 = 1, p_2^H] = q^H$. Given Restriction 2, for any price above $\kappa q^L - \frac{e}{\Delta}$ consumers beliefs are the same. Hence $p_{2,10}^L = E[q_2|R_1 = 1, p_2^H] = q^H$. These beliefs are only correct in equilibrium if $\delta_2^* = 0$. But we cannot have $\delta_2^* = 1$ in equilibrium. To see why, note that in period 2 in the candidate equilibrium the low-quality firms charges a low price and receives a good rating, earning in periods 2 and 3 together $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. Deviating by setting a high price q_H gives no rating, but earns in periods 2 and 3 the profit $q^H + q^L$. Since the firm earns the same profits in either case in period 1, the deviation is profitable if it increases profits in periods 2 and 3. Because $q^L > \kappa q^L - \frac{e}{\Delta}$ and $q^H > \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, this deviation is profitable, contradicting $\delta_2^* = 0$ and that $p_2^H = \kappa q^H - \frac{e}{\Delta}$.

We conclude that $p_2^H = E[q_2|R_1, p_2^H]$.

In period 1, we know from Lemma 10 that $p_{1,0}^L = E[q_1|R_0, p_1^H]$. In the candidate equilibrium, we have $p_1^H \neq p_{1,0}^L$ and only high-quality firms charge $p_1^H = \kappa q^H - \frac{e}{\Delta}$, which is why consumers believe that $E[q_1|R_0, p_1^H] = q^H$. Given Restriction 2, for any price above $\kappa q^L - \frac{e}{\Delta}$ consumers beliefs are the same. Hence $p_{1,0}^L = E[q_1|R_0, p_1^H] = q^H$. These beliefs are only correct in equilibrium if $\delta_1^* = 0$. We now show that we cannot have $\delta_1^* = 0$ in equilibrium. To see why, note that in period 1 in the candidate equilibrium, the low-quality firm sets a low price, receives a good rating and earns total expected profits $\kappa q^L - \frac{e}{\Delta} + (1 - \delta_2^*)(\kappa q^L - \frac{e}{\Delta} + E[q_3|R_2 = 11]) + \delta_2^*(E[q_2|R_1 = 1, p_2^H] + q^L)$. If the firm deviates in period 1 to price q_H and charges q_L in subsequent periods, it will earn $q^H + q^L + q^L$. Because $q^L > \kappa q^L - \frac{e}{\Delta}$, $q^L > (1 - \delta_2^*)(\kappa q^L - \frac{e}{\Delta}) + \delta_2^*(q^L)$, and $q^H > (1 - \delta_2^*)(E[q_3|R_2 = 11]) + \delta_2^*(E[q_2|R_1 = 1, p_2^H])$, this deviation is profitable, contradicting $\delta_1^* = 0$ and that $p_1^H = \kappa q^H - \frac{e}{\Delta}$.

We conclude that $p_1^H = E[q_1|R_0, p_1^H]$.

Overall, this prove statement 1, i.e. that $p_t^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_t|R_{t-1}, p_t^H]\} = E[q_t|R_{t-1}, p_t^H]$.

We have now shown that statements 1,3 and 4 hold in equilibria that satisfy our restrictions. We continue to show statement 2 by characterizing the (mixed) strategies of low-quality firms.

We now characterize the low-quality firms' mixed-strategy in equilibrium in period 2, i.e. δ_2^*. Recall that δ_2^* is the probability of setting a high price of p_2^H that leads to a no rating. Reversely, $1 - \delta_2^*$ is the probability of setting a low price of $\kappa q^L - \frac{e}{\Delta}$ that leads to a good rating. Note that low-quality firms only mix prices after histories where they received a good rating in period 1. This is because consumers' beliefs are such that on observing a period of no rating, they believe the firm to be a low-quality firm.

After a history of a good rating in period 1, when the low-quality firm sets a price of $\kappa q^L - \frac{e}{\Delta}$ in period 2, it obtains a good rating and earns in periods 2 and 3

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}. \tag{16}$$

This is strictly increasing in δ_2^*.

Alternatively, if the low-quality firm sets the high price p_2^H it obtains no rating in period 2 and earns in periods 2 and 3

$$\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L. \tag{17}$$

This is strictly decreasing in δ_2^*.

We now show that $\delta_2^* = 1$ is only possible if no mixed-strategy exists in period 2. First, consider that $\delta_2^* = 1$. Then (16) and (17) become $\kappa q^L - \frac{e}{\Delta} + q^H$ and $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$ respectively. In order for $\delta_2^* = 1$ to be optimal, we must have that at $\delta_2^* = 1$, (17) is weakly larger than (16), i.e.

$$\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L \geq \kappa q^L - \frac{e}{\Delta} + q^H \Leftrightarrow (1-\kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)} \tag{18}$$

Observe that (16) is strictly increasing in δ_2^* and (17) is strictly decreasing in δ_2^*, which is why (18) implies that (17) is larger than (16) for all $\delta_2^* \leq 1$. Therefore, if (18) is met, $\delta_2^* = 1$ is an equilibrium and no mixed-strategy equilibrium exists in period 2.

We now show that $\delta_2^* = 0$ is not a possible equilibrium in period 2. Suppose towards a contradiction that $\delta_2^* = 0$. Then (16) and (17) become $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and $q^H + q^L$ respectively. Since $q^H > \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, the firm is strictly better of by deviating, setting a high price and receiving no rating, which contradicts $\delta_2^* = 0$. We conclude that $\delta_2^* = 0$ cannot be an equilibrium.

We now show that for

$$(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}, \tag{19}$$

there is a unique $\delta_2^* \in (0, 1)$ that characterizes the low-quality firms mixed-strategy in period 2.

Recall that a mixed-strategy equilibrium only exists when (16) and (17) are equal. Observe that for $\delta_2^* = 1$, (17) is strictly below (16) if and only if (19) holds. Next, observe that for $\delta_2^* = 0$, (17) is strictly above (16) because $q^L + q^H > \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$.

Since (16) is strictly increasing and (17) is strictly decreasing in δ_2^*, there is a unique $\delta_2^* \in (0,1)$ such that (16) equals (17) if and only if (19). Otherwise, i.e. if and only if (18), we have $\delta_2^* = 1$.

We now pin down δ_2^*. Note that if $\delta_2^* < 1$, it is determined by (16) equal (17), i.e.

$$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$$

We have two candidates that solve this equation:

$$\begin{aligned}\delta_2^* =& \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} \pm \\ & \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma) + 2\gamma)^2((1-\kappa)q^L + \frac{e}{\Delta})^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})}\end{aligned}$$

Recall that $\delta_2^* \in (0,1)$. In the subtraction case, $1 - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa q^L) + \frac{e}{\Delta})} < 0$. Therefore, we conclude that

$$\begin{aligned}\delta_2^* =& \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} + \\ & \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma) + 2\gamma)^2((1-\kappa)q^L + \frac{e}{\Delta})^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})}\end{aligned}$$

Further, $\delta_2^* > \frac{1}{2} + \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} - \frac{2\gamma(q^H - q^L)}{2(1-\delta_1^*)(1-\gamma)((1-\kappa)q^L + \frac{e}{\Delta})} = \frac{1}{2}$.

Therefore, $\delta_2^* \in (\frac{1}{2}, 1)$ if and only if $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, and $\delta_2^* = 1$ otherwise.

We now turn our attention to period 1 and characterize δ_1^*. We first show that δ_1^* is unique. Recall that in period 1, the low-quality firm charges a low price $\kappa q^L - \frac{e}{\Delta}$ with probability $1 - \delta_1^*$ and obtains a rating, and it charges a high price $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$ with probability δ_1^* and obtains no rating in period 1.

When the low-quality firm charges $\kappa q^L - \frac{e}{\Delta}$, the total continuation profit of the low-quality firm is

$$\pi_1^L(R_1=1) = \kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}] + \delta_2^*[\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L], \tag{20}$$

where the first term are the profits in period 1, the second term are the continuation profits if the firm charges a low price and obtains a good rating in period 2, and the third term are the continuation profits if the firm charges a high price and obtains no rating in period 2.

We show that (20) is strictly increasing in δ_1^* if $\gamma > \frac{1}{3}$.

To show that (20) is strictly increasing in δ_1^*, we first show that $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$.

Taking the derivative of (16) and (17) and solving for $\frac{\partial \delta_2^*}{\partial \delta_1^*}$. We find that

$$\frac{\partial \delta_2^*}{\partial \delta_1^*} = \frac{\delta_2^*(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 - (1-\delta_2^*)(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2}{(1-\delta_1^*)((\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2)}. \tag{21}$$

Notice that the denominator is positive. The numerator simplifies to $(2\delta_2^*-1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2)$. We show that if $\gamma > \frac{1}{3}$, we have $(2\delta_2^*-1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2) > 0$. Since $\delta_2^* \in (\frac{1}{2}, 1]$, $2\delta_2^* - 1 > 0$. We know that $\delta_1^* \in (0,1]$ and that $\delta_2^*(1-\delta_2^*)$ is maximum at $\delta_2^* = 0.5$. Therefore, $\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2 > \gamma^2 - 0.25(1-\gamma)^2$ and $\gamma^2 - 0.25(1-\gamma)^2 > 0$ whenever $\gamma > \frac{1}{3}$. Thus, if $\gamma > \frac{1}{3}$, we have $(2\delta_2^*-1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2) > 0$ and (21) > 0. $\gamma > \frac{1}{3}$ is a sufficient condition.

Next, we show that total derivative of (20) with respect to δ_1^* is greater than 0.

$$\frac{\partial \pi_1^L(R_1=1)}{\partial \delta_1^*} = \frac{\partial \delta_2^*}{\partial \delta_1^*}[q^L(1-\kappa) + \frac{e}{\Delta} - \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}] + \frac{\gamma(1-\gamma)\delta_2^*(q^H - q^L)(\delta_2^* - \frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*))}{(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2} + \frac{\gamma(1-\gamma)(1-\delta_2^*)(q^H - q^L)(\frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*) + (1-\delta_2^*))}{(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2}$$

We show that $\frac{\partial \pi_2^L}{\partial \delta_1^*} > 0$ in three parts.

Consider first that $\frac{\partial \delta_2^*}{\partial \delta_1^*}[q^L(1-\kappa) + \frac{e}{\Delta}]$ and show that this is greater than 0. Since $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$ when $\gamma > \frac{1}{3}$ and $q^L > \kappa q^L - \frac{e}{\Delta}$, we can conclude that $\frac{\partial \delta_2^*}{\partial \delta_1^*}[\kappa q^L - \frac{e}{\Delta} - q^L] > 0$.

Second, we reformulate $\frac{\partial \pi_2^L}{\partial \delta_1^*}$.

To do so, consider $\frac{\gamma(1-\gamma)(1-\delta_2^*)(q^H-q^L)(\frac{\partial\delta_2^*}{\partial\delta_1^*}(1-\delta_1^*)+(1-\delta_2^*))}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2}+\frac{\gamma(1-\gamma)\delta_2^*(q^H-q^L)(\delta_2^*-\frac{\partial\delta_2^*}{\partial\delta_1^*}(1-\delta_1^*))}{(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$. Substituting (21), simplifies to $\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$. This leaves us with the simplified equation of

$$\begin{aligned}\frac{\partial\pi_1^L(R_1=1)}{\partial\delta_1^*}=&\frac{\partial\delta_2^*}{\partial\delta_1^*}[q^L(1-\kappa)+\frac{e}{\Delta}-\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}+\\&\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}]+\\&\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}.\end{aligned}$$

Finally, since $\frac{\partial\delta_2^*}{\partial\delta_1^*}[q^L(1-\kappa)+\frac{e}{\Delta}]>0$, we show that $\frac{\partial\delta_2^*}{\partial\delta_1^*}[\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}-\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}]+\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}>0.$ Substituting (21), we get

$$\begin{aligned}&\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}[1+\\&\frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2-(1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))}]\end{aligned}$$

We check that this is positive. Notice tha $\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}>0$. Therefore, what remains is to show $1+\frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2-(1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))}>0$, and this is true if and only if

$$\begin{aligned}&(1-\delta_1^*)(1-2\delta_2^*)^2[\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2]\\<&\gamma^2+\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*).\end{aligned}$$

We can verify that this is always true. Since $\delta_1^*\in[0,1]$ and $\delta_2^*\in[\frac{1}{2},1]$, $\delta_1^*(1-2\delta_2^*)^2\in[0,1]$. Therefore, $(1-\delta_1^*)(1-2\delta_2^*)^2[\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2]<\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2$. It is easy to see that $\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2<\gamma^2+\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)$. For $-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2<\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)$, the left hand side is negative and the right hand side is positive.

We conclude that $\frac{\partial\pi_1^L(R_1=1)}{\partial\delta_1^*}>0$ when $\gamma>\frac{1}{3}$. Note that $\gamma>\frac{1}{3}$ is a sufficient but not necessary condition.

Next consider the situation when the low-quality firm charges a high price $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ with probability δ_1^* and obtains no rating in period 1. The firm then charges q^L in subsequent periods. Thus, the total continuation profits are given by

$$\pi_1^L(R_1 = 0) = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L \tag{22}$$

This is strictly decreasing in δ_1^*.

We now show that $\delta_1^* = 1$ is only possible if no mixed-strategy equilibrium in period 1 exists. Suppose that $\delta_1^* = 1$. Then (20) and (22) become $\kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta} + q^H] + \delta_2^*[q^H + q^L]$ and $\gamma q^H + (1-\gamma)q^L + q^L + q^L$ respectively. For $\delta_1^* = 1$ to be optimal, we must have (20) lower that (22) for $\delta_1^* = 1$,

$$\kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta}] + \delta_2^*[q^L] + q^H \leq \gamma q^H + (1-\gamma)q^L + q^L + q^L$$
$$\iff (2-\delta_2^*)((1-\kappa)q^L + \frac{e}{\Delta}) \geq (1-\gamma)(q^H - q^L).$$

Further, when $\gamma > \frac{1}{3}$, (20) increases in δ_1^* and (22) decreases in δ_1^*. Hence, when $(2-\delta_2^*)((1-\kappa)q^L + \frac{e}{\Delta}) > (1-\gamma)(q^H - q^L)$, (22) is larger than (20) for all $\delta_1^* \leq 1$. This indicates that $\delta_1^* = 1$ is an equilibrium and no mixed-strategy equilibrium exists in period 1. We conclude that $\delta_1^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists.

We now show that $\delta_1^* = 0$ is not an equilibrium. Suppose towards a contradiction that $\delta_1^* = 0$. Then (20) and (22) become $\kappa q^L - \frac{e}{\Delta} + (1-\delta_2^*)[\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_2^*)(1-\gamma)}] + \delta_2^*[\frac{\gamma q^H + \delta_2^*(1-\gamma)q^L}{\gamma + \delta_2^*(1-\gamma)} + q^L]$ and $q^H + q^L + q^L$ respectively. Since $\kappa q^L - \frac{e}{\Delta} < q^L$, $\frac{\gamma q^H + (1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_2^*)(1-\gamma)} < q^H$, and $\frac{\gamma q^H + \delta_2^*(1-\gamma)q^L}{\gamma + \delta_2^*(1-\gamma)} < q^H$ then (20) $<$ (22). Therefore, if $\delta_1^* = 0$ and the low-quality firm participates in ratings harvesting with probability 1, deviating to the high price and obtaining no ratings is profitable. This contradicts $\delta_1^* = 0$. We conclude that $\delta_1^* = 0$ cannot be an equilibrium.

We now show that for

$$(2-\delta_2^*)((1-\kappa)q^L + \frac{e}{\Delta}) < (1-\gamma)(q^H - q^L), \tag{23}$$

there is a unique $\delta_1^* \in (0,1)$ that characterises the low-quality firms mixed-strategy in period 1. Recall that a mixed-strategy equilibrium only exists when (20) and (22) are equal. First, observe that for $\delta_1^* = 1$, (22) is strictly below (20). Next, observe that $\delta_1^* = 0$, (22) is strictly above (20). Since (20) is strictly increasing in δ_1^* when $\gamma > \frac{1}{3}$ and (22) is strictly decreasing in δ_1^*, there is a unique $\delta_1^* \in (0,1)$ such that (20) and (22) are equal if (23) and $\gamma > \frac{1}{3}$ hold.

We have shown that we cannot have an equilibrium where $\delta_1^* = 0$ or $\delta_2^* = 0$, and that $\delta_2^* = 1$

can be an equilibrium if no mixed-strategy equilibrium exists. Additionally, if $\gamma > \frac{1}{3}$, then $\delta_1^* = 1$ can be an equilibrium if no mixed-strategy equilibrium exists and if a mixed-strategy exists, $\delta_1^* \in (0,1)$ exists such that (20) and (22) are equal, it is unique if $\gamma > \frac{1}{3}$. And, if $\delta_2^* \in (0,1)$ exists such that (16) and (17) are equal, it must be unique. Therefore, we either have a unique pure-strategy in equilibrium or a unique mixed-strategy in equilibrium in each period when $\gamma > \frac{1}{3}$. We have also characterised the mixed-strategy equilibrium for period 2 and shown that $\delta_2^* \in (\frac{1}{2}, 1]$. We conclude that statement 2 holds, and therefore conclude that statements 1-4 hold in equilibrium.

We now show that equilibria satisfying statements 1-4 indeed exist.

We begin by considering the scenario where (19), (23),
$\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$ and $\gamma > \frac{1}{3}$ hold together.

Consumers' beliefs are as follows. In period 1,

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}, \end{cases}$$

and in period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \kappa q^L - \frac{e}{\Delta} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination,} \end{cases}$$

and in period 3,

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2. \end{cases}$$

These beliefs follow Bayes rule on the candidate equilibrium path of play. The candidate equilibrium is consistent with our restrictions. Because high-quality firms obtain a good rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Further, whenever low-quality firms obtain no rating, consumers' beliefs are independent of prices (in both periods 1 and 2), and in period 3, beliefs are the same for all period 3 prices and they only depend on the history of ratings. Therefore, the candidate equilibrium is consistent with Restriction 2.

The candidate equilibrium is such that high-quality firms always play $p_t^H = E[q_t|R_{t-1}, p_t^H]$ for all t and get a good rating. Low-quality firms mix between playing a low price $\kappa q_L - \frac{e}{\Delta}$

and getting a good rating with probability δ_t^*, and setting a high price p_t^H and getting no rating with probability $(1-\delta_t^*)$ in each period $t \in \{1,2\}$.

More precisely, in the candidate equilibrium in period 1, the high-quality firm sets a price $p_{1,1}^H = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains a good rating with probability 1. In period 2, conditional on receiving a good rating in period 1, the high-quality firm sets a price $p_{2,11}^H = \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ and obtains a good rating with probability 1. If the high-quality firm did not receive a good rating in period 1, consumers believe the firm is of a low quality and thus the maximum price that the high-quality firm can set is q^L and receives a good rating. To see that the firm receives a good rating, note that by assumption $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$ and both of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and $\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ are larger than q^L. Therefore, when consumers pay q^L for a high quality product, they receive a sufficient amount of excess surplus and leave a good rating. In the third period of the candidate equilibrium, having received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, the high-quality firm sets a price of $\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. If the firm did not receive a continuation of good ratings, i.e. a rating history of $R_2 \in \{00, 01, 10\}$, then it sets a maximum price of q^L.

In period 1 of the candidate equilibrium, the low-quality firm sets $p_{1,0}^L = \frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains no rating with some probability δ_1^*, and $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$ and obtains a good rating with some probability $1-\delta_1^*$. In period 2, conditional on having obtained a good rating in period 1, the low-quality firm sets a price $p_{2,11}^L = \kappa q^L - \frac{e}{\Delta}$ with some probability $1-\delta_2^*$ and obtains a good rating, and a price $p_{2,10}^L = \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ with some probability δ_2^* and obtains no rating. If the low-quality firm obtained no rating in period 1, then it sets a price q^L in period 2 and obtains no rating. In period 3, if the low-quality firm received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, it sets a price of $\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, and when it receives a rating history of $R_2 = \{00, 10, 01\}$, it sets a price of q^L. This fully characterizes the firms' prices and consumers' beliefs.

We now show that the firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns a total profit of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} + \frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$.

In period 3, deviations to a higher price would reduce demand to zero and earn a total maximum profit of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} + 0$, and deviations to a lower price would reduce profit margins in the third period without increasing demand. Neither deviation increases profits.

In period 2, deviations to a higher price would reduce demand to zero and result in no rating, this leads to a total maximum profit of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}+0+q^L$, and deviations to a lower price would reduce profit margins in the second period without increasing demand. Neither deviation increases profits.

In period 1, deviations to a higher price would reduce demand to zero and result in no rating, this leads to a total maximum profit of $0+q^L+q^L$, and deviations to a lower price would reduce profit margins in the first period without increasing demand. Neither deviation increases profits.

Therefore, there are no profitable deviations for the high-quality firm in any period.

Next, we show that low-quality firms have no profitable deviation.

In the candidate equilibrium, the low-quality firm earns a total profit of $\kappa q^L-\frac{e}{\Delta}+\kappa q^L-\frac{e}{\Delta}+\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. For the equilibrium $\delta_1^*\in(0,1)$ and $\delta_2^*\in(0,1)$, the low-quality firm is indifferent between setting the lower price that obtains good rating and setting a higher price that obtains no rating in all periods. Hence, the firm is indifferent between the total profits of $\kappa q^L-\frac{e}{\Delta}+\kappa q^L-\frac{e}{\Delta}+\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, $\kappa q^L-\frac{e}{\Delta}+\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}+q^L$ and $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}+q^L+q^L$. Recall that (19) implies $\delta_2^*\in(0,1)$ and (23) implies $\delta_1^*\in(0,1)$, and $\gamma>\frac{1}{3}$ implies $\delta_1^*\in(0,1)$ is unique. Showing that there exists no profitable deviation from any of these cases shows that there is no profitable deviation for the low-quality firm.

In period 3, deviations towards a price above $\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ would result in zero demand and a maximum total profit of $\kappa q^L-\frac{e}{\Delta}+\kappa q^L-\frac{e}{\Delta}+0$, and a deviation towards a lower price results in smaller profit margins without increasing demand. Hence, there is no profitable deviation in period 3.

In period 2, the low-quality firm is indifferent between setting the price $\kappa q^L-\frac{e}{\Delta}$ and $\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$, where $\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}>\kappa q^L-\frac{e}{\Delta}$ and her total profits are $\kappa q^L-\frac{e}{\Delta}+\kappa q^L-\frac{e}{\Delta}+\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ and $\kappa q^L-\frac{e}{\Delta}+\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}+q^L$ respectively. Deviations towards a price above $\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ leads to zero demand in period 2 and the maximal profits of $\kappa q^L-\frac{e}{\Delta}+0+q^L$, which is not a profitable deviation. Deviations towards a price $p_2\in\left(\kappa q^L-\frac{e}{\Delta},\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\right)$, leads to the same rating as when the firm sets $\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$; hence, any deviation to p_2 lowers margin without improving demand or future profit, and are therefore not profitable. Deviations towards a price below $\kappa q^L-\frac{e}{\Delta}$ decreases profits in period 2 but does not increase demand or change the rating the firm receives when setting a price of $\kappa q^L-\frac{e}{\Delta}$, which is why this is also not a profitable deviation. Therefore, in period 2, there is no profitable deviation for the firm.

In period 1, the low-quality firm is indifferent between setting the price $\kappa q^L - \frac{e}{\Delta}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$, where its total profits are $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + q^L + q^L$ respectively. If it deviates to a price above $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$, demand falls to zero and it makes a total profit of $0 + q^L + q^L$, which is not a profitable deviation. If it deviates to a price $p_1 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)})$, then it receives the same rating as when it sets the price of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$; hence such a deviation reduces margins in the first period without improving demand or future profit and is not profitable. When deviating to a price below $\kappa q^L - \frac{e}{\Delta}$, the firm receives a good rating; however, the deviation does not increase demand and her margins are lower than when setting the price of $\kappa q^L - \frac{e}{\Delta}$, which is why the deviation is not profitable.

We conclude that there are no profitable deviations for either high- or low-quality firms from the candidate equilibrium.

We conclude that if $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\delta_1^*)(1-\gamma)(q^H-q^L)}{\gamma+(1-\delta_1^*)(1-\gamma)}$, $(1-\kappa)q^L + \frac{e}{\Delta} < \frac{(1-\gamma)(q^H-q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$ and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$, the candidate equilibrium exists.

Consider next the case where (18), (23), $\gamma > \frac{1}{3}$ and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$. In this scenario, $\delta_2^* = 1$. The last three inequalities are identical to the previous case and play the same role in this case.

In the candidate equilibrium, consumers' beliefs are as follows. In period 1,

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} & \text{if } p_1 > \kappa q^L - \frac{e}{\Delta} \\ q^L & \text{if } p_1 \leq \kappa q^L - \frac{e}{\Delta}, \end{cases}$$

and in period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} & \text{if } p_2 > \kappa q^L - \frac{e}{\Delta} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination,} \end{cases}$$

and in period 3,

$$E[q_3|R_2] = \begin{cases} q^H & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2. \end{cases}$$

These beliefs follow Bayes rule on the candidate equilibrium path of play. The candidate equilibrium is consistent with our restrictions. Because high-quality firms obtain a good rating with probability 1, the candidate equilibrium is consistent with Restriction 1. Further, whenever low-quality firms obtain no rating, consumers' beliefs are independent of prices (in both periods 1 and 2), and in period 3, beliefs are the same for all prices. Therefore, the candidate equilibrium is consistent with Restriction 2.

In the candidate equilibrium in period 1, the high-quality firm sets a price $p^H_{1,1} = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$ and obtains a good rating with probability 1. In period 2, conditional on receiving a good rating in period 1, the high-quality firm sets a price $p^H_{2,11} = \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and obtains a good rating with probability 1. If the high-quality firm did not receive a rating in period 1, consumers believe the firm is of a low quality and thus the maximum price that the high-quality firm can set is q^L and the firm receives a good rating. To see that the firm receives a good rating, note that by assumption $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}\}$ and both of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$ and $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ are larger than q^L. In the third period of the candidate equilibrium, having received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, the high-quality firm sets a price of q^H. If the firm did not receive a continuation of good ratings, i.e. a rating history of $R_2 \in \{00, 01, 10\}$, then it sets a maximum price of q^L.

In period 1 of the candidate equilibrium, the low-quality firm sets a price $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$ and obtains no rating with some probability δ_1^* and $\kappa q^L - \frac{e}{\Delta}$ and obtains a good rating with some probability $1 - \delta_1^*$. In period 2, the low-quality firm sets a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ with probability 1 and receives no rating. In period 3, if the low-quality firm received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, it sets a price of q^H, and when it receives a rating history of $R_2 = \{00, 10, 01\}$, it sets a price of q^L.

We now show that the firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns a total profit of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^H$. In period 3, deviations to a higher price reduces demand to zero, and deviations to a lower price reduces profit margins without increasing demand, thus there is no profitable deviation in period 3. In period 2, deviations to a higher price reduces demand to zero, which induces no rating and also reduces continuation profits; deviations to a lower price reduces profit margins without increasing demand. Thus, neither deviation is profitable in period 2. In period 1, deviations to a higher price reduces demand to zero, induce no rating and therefore reduces continuation profits in all future periods to q_L, and deviations to a lower price reduces profit margins without increasing demand. Neither deviation is profitable in period 1. Therefore, there are no profitable deviations for the high-quality firm.

In the candidate equilibrium, the low-quality firm earns a total profit of
$\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$. For the equilibrium $\delta_1^* \in (0, 1)$, the low-quality firm is

indifferent between setting a lower price that obtains a good rating and setting a higher price that obtains no rating in period 1, and for $\delta_2^* = 1$, the low-quality firm always sets a high price and obtains no rating in period 2. The low-quality firm is therefore indifferent between the total profits of $\kappa q^L - \frac{e}{\Delta} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$. Recall that (18) implies $\delta_2^* = 1$, (23) implies $\delta_1^* \in (0,1)$, and $\gamma > \frac{1}{3}$ implies $\delta_1^* \in (0,1)$ is unique. In period 3, deviations towards a higher price, above q^L, reduces demand to zero, and deviations towards a lower price results in a lower profit margin without improving demand, which is why there is no profitable deviation in period 3.

In period 2, conditional on receiving a good rating in period 1, deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ leads to zero demand and results in the same rating (and hence future profit) as setting a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$, which is why it is not a profitable deviations. Further, deviations towards a price of $p_2 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)})$ results in the same rating (and hence future profit) as setting a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$, therefore this reduces margins without providing any additional continuation profit, and is not a profitable deviation. Deviations towards a price $p_2 \leq \kappa q^L - \frac{e}{\Delta}$ leads to a maximal total profit of $\kappa q^L - \frac{e}{\Delta} + \kappa q^L - \frac{e}{\Delta} + q^H$, but since (18) holds, this is not a profitable deviation. Therefore, there are no profitable deviations in period 2 conditional on receiving a good rating in period 1. In period 2, conditional on having received no rating in period 1, deviations towards a price above q^L leads to zero demand and does not change rating and continuation profits, which is why this is not a profitable deviation. Further, since consumers beliefs on observing a single period of no rating is that the firm is of a low-quality, setting a price lower than q^L only reduce the margins in period 2 without increasing demand or future profits, thus this is not a profitable deviation. Therefore, there are no profitable deviations in period 2 conditional on not having received a rating in period 1.

In period 1, the low-quality firm is indifferent between setting the price $\kappa q^L - \frac{e}{\Delta}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$. If it deviates to a price above $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$, demand falls to zero, it gets no rating and it makes a total profit of $0 + q^L + q^L$, which is not a profitable deviation. If it deviates to a price $p_1 \in (\kappa q^L - \frac{e}{\Delta}, \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)})$, then it receives the same rating as when it sets the price of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$; hence such a deviation reduces margins in the first period without improving demand or future profit and is not a profitable deviation. When deviating to a price below $\kappa q^L - \frac{e}{\Delta}$, the firm receives a good rating; however, the deviation does not increase demand and her margins are lower than when setting the price of $\kappa q^L - \frac{e}{\Delta}$. Therefore, we conclude that there is no profitable deviation in period 1.

We conclude that there are no profitable deviations for either the high- or low- quality firm from the candidate equilibrium.

We conclude that if $(1-\kappa)q^L + \frac{e}{\Delta} \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H-q^L)}{\gamma+(1-\delta_1^*)(1-\gamma)}$, $(1-\kappa)q^L + \frac{e}{\Delta} < (1-\gamma)(q^H-q^L)$, $\gamma > \frac{1}{3}$ and $\kappa q^H - \frac{e}{\Delta} \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}\}$, the candidate equilibrium exists.

This concludes the proof. $\square$