# Philips Curve and Okun's Law in Mexico

## Empirical Estimations on the Mathematical Aspect of the Macroeconomic Models

Robin Mao, 1008267475, robin.mao@mail.utoronto.ca

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**Abstract**

This report will focusing on analyzing the Philips Curve and Okun's Law for Mexico and give interpretations on the relations between the inflation rate change, unemployment rate and output growth. To ensure the mathematical accuracy, Python and its query-focused libraries such as Pandas, Geopandas and Numpy will be used, and the codes will be provided in a JupyterNotebook in Appendix.
**Keywords:** Philips Curve, Okun's Law, Inflation Rate, Unemployment Rate, Inflation Rate, Output Growth, Mexico

## 1 Introduction

The major economic concepts in this report are the Philips Curve and Okun's Law. The Philips Curve shows the relation between the inflation rate and the unemployment rate, and is often used by policymakers to understand economic conditions such as nominal wages and the inflationary environment. Okun's Law describes the relation between the change in unemployment rate and the change in real gross domestic product (referred later to as GDP), which is another useful tool for authorities to examine the national economic development.

This report will focus on the mathematical aspects of both concepts and interpret. It will provide empirical estimations on the mathematical equations of the models, visualize and examine using statistics. We will begin by examining the individual economic factors (inflation rate, unemployment rate and real GDP) to enhance our understandings in the Mexican economy, give comparison then plot the relations and calculate the estimators.

## 2 Inflationary Environment in Mexico

The Mexican inflation rate data set provided by the World Bank [(1)] is calculated using the Consumer Price Index (referred latter to as CPI), which is a common inflation measurement that tracks the price of a goods basket and the services that are typically consumed by a household. It estimates the inflation rate by calculating the change in price over the change in time, which is an literal explanation of the following equation,

$$CPI_t = \frac{C_t}{C_0} * 100 \tag{1}$$

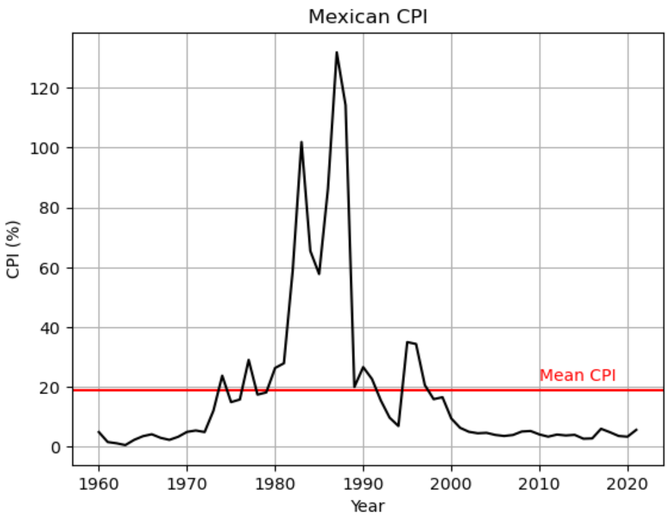
Where $C_t$ represents the basket price at time t, and $C_0$ represents the basket price in the base year. World Bank data set provides the Mexican inflation data from 1960 to 2021. Here below is the table of the most recent 14 years. Note that the inflation is estimated calculating the CPI annually.

Table: The Mexican Inflation Rate from 2008 to 2021

| 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| 5.124983 | 5.297356 | 4.156727 | 3.407378 | 4.111510 | 3.806391 | 4.018616 |
| **2015** | **2016** | **2017** | **2018** | **2019** | **2020** | **2021** |
| 2.720641 | 2.821708 | 6.041457 | 4.899350 | 3.635961 | 3.396834 | 5.689208 |

Data above shows two major inflationary events, one in 2008 and one in 2021. However, to ensure the precision, let us visualize the entire data set.

The following visualization is generated using Python and World Bank data set [1].




Plot: the Mexican inflation rate measured by CPI. Note that the graph is original.

The plot above demonstrates the variation in the Mexican inflation. From the plot, we can see a major inflation node between 1980 and 1990, which was during the Mexican Financial Crisis[2]. It was caused by an earthquake in Mexico City that resulted in more than 10,000 deaths and heavy economic damages. Furthermore, the ignorance from the governmental institutes on this tragedy had lead to another civic action during the time period, followed by another massive hurricane in 1988. There still exist fluctuations after the major events happened during the 80s and the 90s, however, they are relatively weak.

# 3 Changes in the Mexican Unemployment Rate

Similar to its inflationary environment, Mexico's unemployment rate has varied significantly over time reflecting the country's economic and political changes. Historically, Mexico has experienced relatively high levels of unemployment, with peaks occurring during times of economic crisis and recession.

Unemployment is the essential connection between the Philips Curve and Okun's Law, as former shows the relation between inflation and the unemployment and the latter shows the relation between the changes unemployment rate and the change in GDP.

Unemployment rate is calculated by calculating the ratio between the unemployed population and the labour force, in which labour force is defined to be all the social members that are eligible to work and is actively seeking for jobs. Its mathematical calculation is given below,

$$U = \frac{UnemployedCount}{LabourForce} * 100 \tag{2}$$

In economic medium run (and long run), the national unemployment rate should remain at its **natural rate of unemployment**, which represents the unemployment rate of an economy when it is in its healthiest condition.

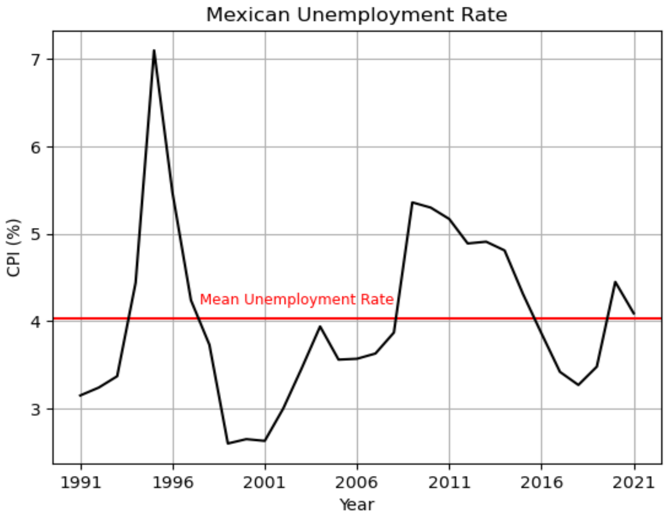
World Bank provides data on the Mexican unemployment rate from 1991 to 2021[3]. The detailed DataFrame is shown in the coding file in the end. The table below contains the most recent 14 years of the Mexican unemployment rate data.

Table: The Mexican Unemployment Rate from 2008 to 2021

| 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| 3.870000 | 5.360000 | 5.300000 | 5.170000 | 4.890000 | 4.910000 | 4.810000 |
| 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
| 4.310000 | 3.860000 | 3.420000 | 3.270000 | 3.480000 | 4.450000 | 4.090000 |

The table above shows a visible node between 2009 to 2011 more than 1 percent higher than the rest. However, let us further visualize the data to show the precise distribution.

Using the World Bank data set[3], we can generate the graph as below.




We can see several unemployment rate fluctuations in the graph. One is the historical peak around 1996 exceeding 7%, which is resulted from the Mexican Peso Crisis when the overall household income decreased over 30% [4]. We can infer that the Mexican economy is in a recession around 1996.

The unemployment rate dropped below the historical mean around 2001 when the economy was in its boom state. Then another recession hits around 2008 with a rising unemployment rate.

The National Institute of Statistics and Geography (5) shows that the Mexican natural rate of unemployment is expected to be between 3% and 5%, in which our unemployment rate falls within the expected interval.

Compare to the CPI graph in section two where there exists a inflation node around 1995, we can see some relation between the two economic factors. However, we will first analyzed each factor individually, then make comparison and correlations.

# 4 The Mexican Real GDP Growth Rate

Gross Domestic Product (GDP), probably the most well-known macroeconomic statistic, is a measurement of the national output. Compare to the nominal GDP, real GDP (referred later to as RGDP) is adjusted to inflation to demonstrate a more accurate representation on the aggregate output using fixed prices from the base year. One way to calculate this is the RGDP growth rate **Chain Weighting Method**:

$$g_1 = \frac{\sum P_{t-1} * Y_t}{\sum P_{t-1} * Y_{t-1}} \tag{3}$$

$$g_2 = \frac{\sum P_t * Y_t}{\sum P_t * Y_{t-1}} \tag{4}$$

$$g_t = (\sqrt{g_1 * g_2} - 1) * 100 \tag{5}$$

Where $g_1$ represents the growth rate using last year as the base year while $g_2$ represents it using this year as the base year. Then equation (5) calculates the geometric average and calculate the change.

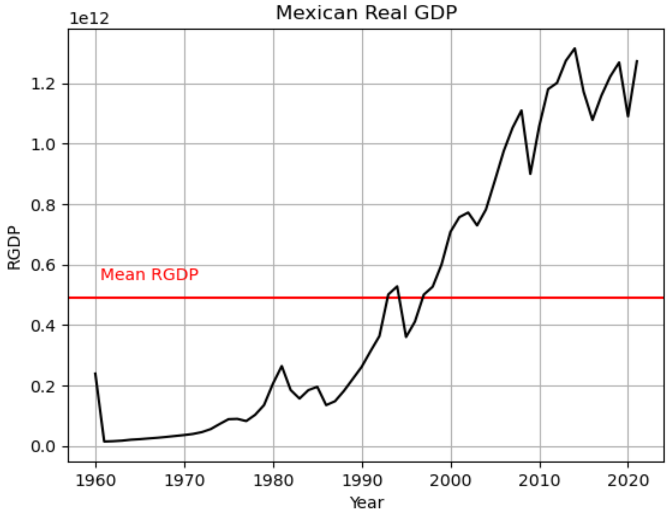
World Bank provides the Mexican Real GDP data from 1960 to 2021, in USD billions. Thus the data is manually queried into the Mexican peso (shown in the Appendix Notebook). Again, let us input the most current 14 years data into the table below.

Table: The Mexican Real GDP from 2008 to 2021 (in trillion Mexican Peso)

| 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| 1.1099 | 0.9000 | 1.0578 | 1.1804 | 1.2010 | 1.2744 | 1.3153 |

| 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
|---|---|---|---|---|---|---|
| 1.1718 | 1.0784 | 1.1589 | 1.2224 | 1.2690 | 1.0905 | 1.2738 |

From table, we can see an increasing trend after the sudden drop in 2009, when the unemployment rate is relatively high (from graph above). Let us visualize the whole DataFrame.




(Note that the all graphs are original.)

The graph above demonstrates the Mexican Real GDP changes in trillion Peso. It shows mainly an increasing trend with frequent fluctuations distributed within. With the current DataFrames, we can now make comparison between the in three variables.

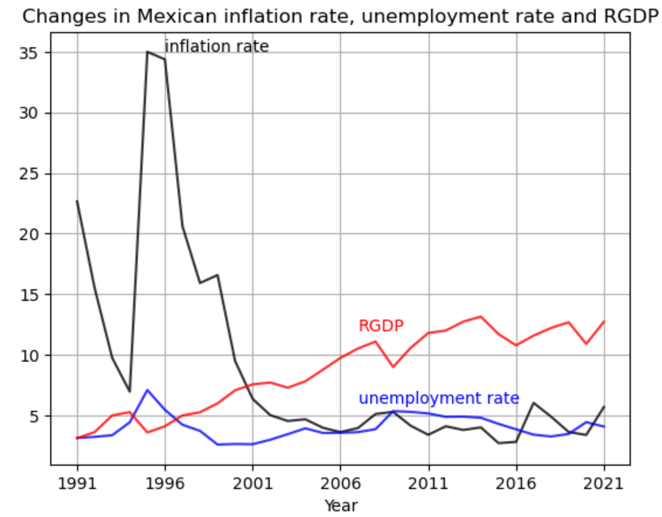
## 5 Comparison among the Factors

To ensure the statistical precision, the data sets are committed outer merges (all observations must have all three factors available to be remained in the DataFrame). After the merges, we have all available data from 1991 to 2021.

Table: The Mexican RGDP, unemployment rate and inflation rate over time

| | **RGDP** | **unemployment rate** | **inflation rate** |
|---|---|---|---|
| 1991 | 3.131428e+11 | 3.15 | 22.662359 |
| 1992 | 3.631576e+11 | 3.24 | 15.507896 |
| 1993 | 5.007361e+11 | 3.37 | 9.751460 |
| 1994 | 5.278132e+11 | 4.44 | 6.965812 |
| 1995 | 3.600739e+11 | 7.10 | 34.999271 |
| 1996 | 4.109756e+11 | 5.47 | 34.378383 |
| 1997 | 5.004135e+11 | 4.24 | 20.625629 |
| 1998 | 5.265021e+11 | 3.73 | 15.928395 |
| 1999 | 6.002329e+11 | 2.60 | 16.585617 |
| 2000 | 7.079067e+11 | 2.65 | 9.491561 |
| 2001 | 7.567063e+11 | 2.63 | 6.367738 |
| 2002 | 7.721064e+11 | 3.00 | 5.030727 |
| 2003 | 7.293363e+11 | 3.46 | 4.546900 |
| 2004 | 7.822406e+11 | 3.94 | 4.688409 |
| 2005 | 8.774762e+11 | 3.56 | 3.988057 |
| 2006 | 9.753871e+11 | 3.57 | 3.629468 |
| 2007 | 1.052696e+12 | 3.63 | 3.966849 |
| 2008 | 1.109989e+12 | 3.87 | 5.124983 |
| 2009 | 9.000454e+11 | 5.36 | 5.297356 |
| 2010 | 1.057801e+12 | 5.30 | 4.156727 |
| 2011 | 1.180490e+12 | 5.17 | 3.407378 |
| 2012 | 1.201090e+12 | 4.89 | 4.111510 |
| 2013 | 1.274443e+12 | 4.91 | 3.806391 |
| 2014 | 1.315351e+12 | 4.81 | 4.018616 |
| 2015 | 1.171868e+12 | 4.31 | 2.720641 |
| 2016 | 1.078491e+12 | 3.86 | 2.821708 |
| 2017 | 1.158913e+12 | 3.42 | 6.041457 |
| 2018 | 1.222408e+12 | 3.27 | 4.899350 |
| 2019 | 1.269012e+12 | 3.48 | 3.635961 |
| 2020 | 1.090515e+12 | 4.45 | 3.396834 |
| 2021 | 1.272839e+12 | 4.09 | 5.689208 |

(Note that all data are provided in the Appendix Notebook.)




Note that to analyze the trend, the RGDP has been simplified to its $10^{-11}$ in Peso.

The graph demonstrates a major inflation around 1996 with an decreased RGDP and increased unemployment rate. Most other minor fluctuations also follows the negative correlation between the unemployment rate and RGDP growth rate (as change in unemployment rate rises the RGDP growth rate decreases) and the positive correlation between the inflation rate and unemployment rate(the unemployment rate increases as the inflation rate rises). These trends are useful towards the estimation on the Philips Curve and Okun's Law in the upcoming section.

# 6 Deriving the Philips Curve

Note that for this assignment, our target Philips Curve is modified into the relation between the inflation change and the real output as the following equation,

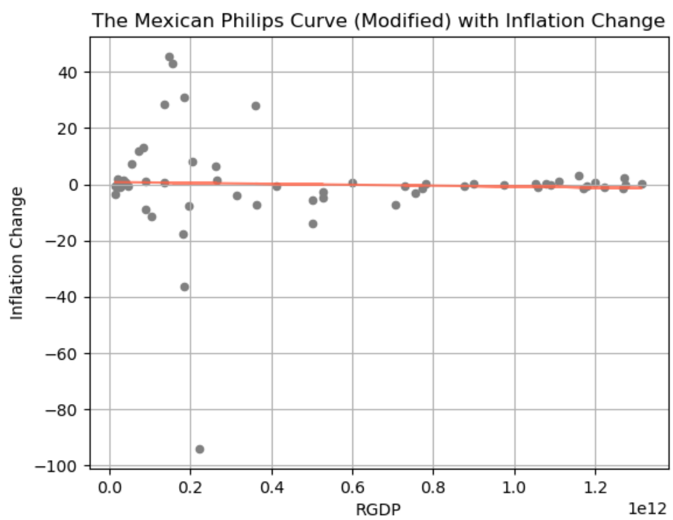
$$\pi_t - \pi_{t-1} = \alpha(Y_t - \bar{Y}) \tag{6}$$

Where $\pi_t$ is the inflation rate at time t, $Y_t$ is the RGDP at time t and $\bar{Y}$ is the output gap, which is the real national output when the economy is at its maximum capacity. The economy goes to a boom if its RGDP exceeds it output gap; vice versa, it goes to a recession. The goal here is to estimate the coefficient $\alpha$ and the output gap $\bar{Y}$.

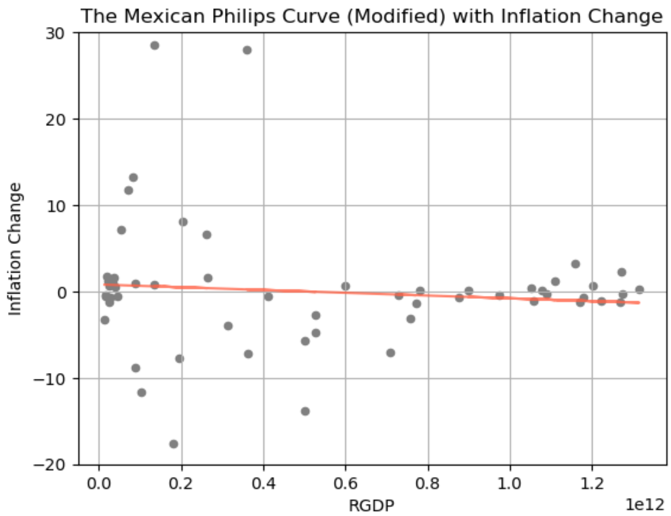
To begin with, the Philips Curve is converted into a more lienar equation as the following,

$$\pi_t - \pi_{t-1} = a + bY_t \tag{7}$$

Visualization is generated as below showing the relation between the changes in Mexican inflation change and the RGDP.



A zoomed in version is provided below as there exists too many outliners.




Plot: zoomed in

The scatter plot above demonstrates the Mexican Philips Curve (with the unemployment rate modified to output) with its linear regression. Note that the graph is original and obtained using DataFrames from above while linear regression is calculated using Numpy.polyfit().

The coefficients from equation (7) are obtained as below (detailed calculations and codes are provided in the Appendix Notebook),

$$a = 0.804313644 \tag{8}$$

$$b = -1.60257340 * 10^{-12} \tag{9}$$

Thus the linear equation is obtained as follow,

$$\pi_t - \pi_{t-1} = 0.804313644 - 1.60257340 * 10^{-12} Y_t \tag{10}$$

With the unit of the output is Mexican Peso (individually) and the unit of the unemployment rate is percent. Different from the classic assumption, the computed $b$ coefficient is negative due to the huge fluctuations in the provided data.

Now let us manipulate the equation (10) into the format of equation (6).

$$\pi_t - \pi_{t-1} = 0.804313644 - 1.60257340 * 10^{-12} Y_t = -1.60257340(Y_t - 5.0189055 * 10^{11}) \tag{11}$$

Where we can conclude that $\alpha$ = -1.60257340 and $\bar{Y}$ = 5.0189055 * $10^{11}$ Peso.

As $\alpha$ is negative and relatively large, it implies that for each $1 * 10^{11}$ Peso (demonstrated as $0.1 * 10^{12}$ in the graph above) increase in $Y_t$, the unemployment change decreases by 1.6025734%.

As the $\alpha$ coefficient is relatively small, the graph demonstrates a significantly elastic relation between the RGDP and the inflation rate change. The size of $\bar{Y}$ implies that the estimated output gap of the Mexican economy is $5.0189055 * 10^{11}$ which is the national output when the economy is in its medium run (or long run) equilibrium. Overly, the sizes are expected due to the significant variation while the negative correlation is unusual.

To examine the accuracy, we calculate the coefficient of determination $R^2$, which represents the proportion of y-variation that can be explained by the model, error sum of squares $SSE$, total sum of squares $SST$ which sums the total y-variation and the residual variance $\sigma^2$.

$$SSE = \sum (e_i)^2 = 18133.32493122715 \tag{12}$$

$$SST = \sum (y_i - \bar{y})^2 = 18165.025555294313 \tag{13}$$

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{18133.32493122715}{18165.025555294313} = 0.001745146130990305 \tag{14}$$

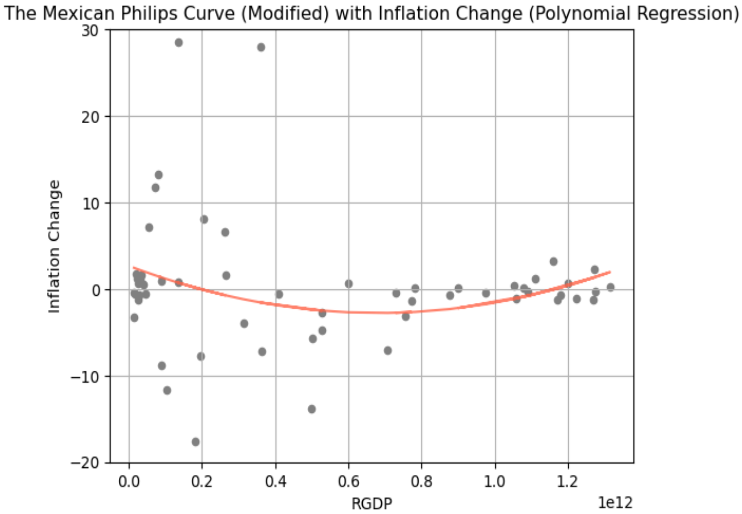
$$\sigma^2 = \frac{SSE}{n-2} = \frac{18133.32493122715}{61-2} = 297.2676218233959 \tag{15}$$

More detailed statistics are provided in the Appendix$^{(2)}$.

Where n is the number of observations. Thus we can conclude that the linear regression explains 0.1745146130990305% of the y-variation. This is due to the high variance in the Mexican inflation change that a simple linear regression model is not capable of accurate estimations. However, utilizing **polynomial regression**, we can conclude a more accurate version as below,

$$\pi_t - \pi_{t-1} = 2.67317472 - 1.59850531 * 10^{-11} Y_t + 1.17305673 * 10^{-23} Y_t^2 \tag{16}$$

This polynomial regression provides a more accurate estimation on the two variables with the $R^2$ of 0.010034378706649805, which is almost 10 times higher than the simple linear regression model from above. The visualization is provided as below,




Note that detailed calculations are included in the Notebook in Appendix.

# 7 Deriving Okun's Law for the Mexcian Economy

Okun's Law is another macroeconomic concept that shows the (mostly negative) relation between the change in unemployment rate and the Real GDP growth. The mathematical relation is provided as below,

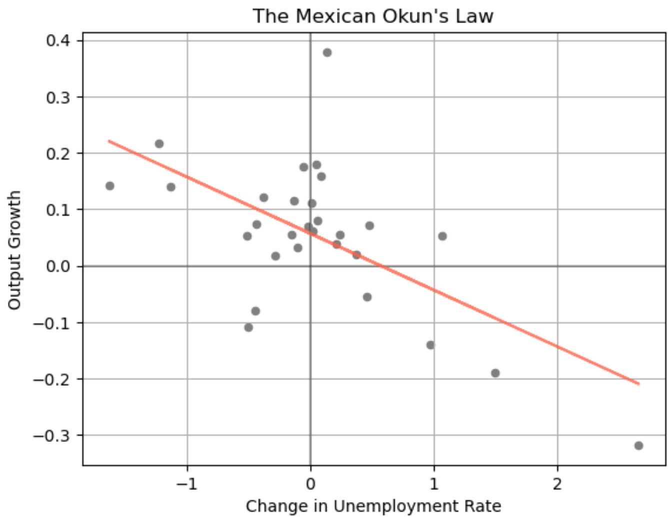
$$u_t - u_{t-1} = -\beta(g_{yt} - g) \tag{17}$$

Where $u_t$ stands for the unemployment rate at time t, $u_{t-1}$ represents the unemployment for the previous year. $g_{yt}$ stands for the output growth at time t and $g$ represents the normal growth rate of the economy. The goal of this section is to provide an accurate estimation on the coefficient $\beta$ and $g$.

Again, let us manipulate the equation (11) into a more linear form as below,

$$u_t - u_{t-1} = a + b g_{yt} \tag{18}$$

With the World Bank DataFrames obtained in the previous sections, a visualization of the Mexican Okun's Law is provided as below,




The graph above demonstrates the negative relation between the Mexican output growth and the unemployment rate which is consistent with the classic assumptions. Several outliners in the output growth rate are recorded.

The coefficients are calculated as below (detailed calculation in Appendix Notebook),

$$a = 0.05724077 \tag{19}$$

$$b = -0.10027533 \tag{20}$$

Thus equation (18) becomes

$$u_t - u_{t-1} = 0.05724077 - 0.10027533 * g_{yt} \tag{21}$$

This equation implies that for each 1% increase in the RGDP growth, the unemployment rate will decrease by 0.10027533% unit with the initial value of 0.05724077%. With the DataFrames provided by World Bank, this is as much as we could estimate.

Now let us manipulate the equation (21) into the Okun's Law format,

$$u_t - u_{t-1} = 0.05724077 - 0.10027533 * g_{yt} = -0.10027533 * (g_{yt} - 0.57083602) \tag{22}$$

Which concludes that $\beta = 0.10027533$ and g = 0.57083602%. Note that both their signs are positive. Compare to other countries, the Mexican $\beta$ coefficient is relatively small, which implies a more elastic relation between the unemployment rate and the RGDP growth rate while g being relatively large. The $g$ value implies that the normal growth rate of the economy is estimated to be 0.57083602%.This is consistent with the classic assumption and the initial expectation because as the RGDP growth rate increases, labour demand rises, resulting in a lower unemployment rate.

To examine the accuracy same as before, the statistics are obtained as below,

$$R^2 = 0.37746473436939865 \tag{23}$$

With the total sum of squares $SST = 0.4971973087547591$ and error sum of squares $SSE = 0.30952285867646406$. The estimated residual variance is given as,

$$\sigma^2 = \frac{SSE}{n-2} = \frac{0.30952285867646406}{29-2} = 0.011463809580609781 \tag{24}$$

More detailed statistics are provided in Appendix$^{(3)}$.Thus we can conclude that the linear regression model can explain 37.746473436939865% of the y-variation, which is significantly higher than the previous Philips Curve model in section 6.

# 8 Conclusion

This report concludes model for the Mexican Philips Curve and the Mexican Okun's Law as below,

**Philips Curve**(with unemployment rate changed to output):

$$\pi_t - \pi_{t-1} = -1.60257340(Y_t - 5.0189055 * 10^{11}) \tag{25}$$

With $R^2$ = 0.001745146130990305, $SSE$ = 18133.32493122715, $SST$ = 18165.025555294313 and $\sigma^2$ = 297.2676218233959. There is also a more precise polynomial estimation (equation (16)).

**Okun's Law:**

$$u_t - u_{t-1} = -0.10027533 * (g_{yt} - 0.57083602) \tag{26}$$

With $R^2$ = 0.37746473436939865, $SSE$ = 0.30952285867646406, $SST$ = 0.4971973087547591, $\sigma^2$ = 0.011463809580609781.

The results indicates that there exists huge fluctuations in all the Mexican's inflation rate, unemployment rate and real gross domestic product with correlations countering some classic assumptions.

Different from the classic assumption, the obtained Philips Curve shows an elastic, negative correlation between the inflation change and the RGDP due to the huge fluctuations in the Mexican economy. As for the Okun's Law, the negative correlation between the unemployment rate change and the output growth is consistent with the textbook expectations.

Overall, the model accuracy could be improved but our estimations are the best we could conclude out of the data sets given by the World Bank. Models show elastic relation between the inflation rate and the RGDP and relatively inelastic relation between relation between the unemployment rate and the output growth.

# References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3. World Bank. (2021). Unemployment, total (% of total labor force) (modeled ILO estimate) — Data. Worldbank.org. https://data.worldbank.org/indicator/sl.uem.totl.zs

4. What Happened in 1996 - Significant Events, Prices, 25 years ago Top Movies, TV and Music. (n.d.). Www.thepeoplehistory.com. https://www.thepeoplehistory.com/1996.html

5. CEICdata.com. (2018). Mexico Unemployment Rate. Ceicdata.com; CEICdata.com. https://www.ceicdata.com/en/indicator/mexico/unemployment-rate

# Appendix

Disclaimer: all graphs are original. All calculations, DataFrames and codes are in the Notebook below.

1. **JupyterNotebook:** https://github.com/robinasaki/Philips-Curve-and-Okun-s-Law-in-Mexico.git

2. Philips Curve Statistics

```
                            OLS Regression Results
==============================================================================
Dep. Variable:         inflation rate   R-squared:                       0.002
Model:                            OLS   Adj. R-squared:                 -0.015
Method:                 Least Squares   F-statistic:                    0.1031
Date:                Wed, 29 Mar 2023   Prob (F-statistic):              0.749
Time:                        16:22:29   Log-Likelihood:                -260.24
No. Observations:                  61   AIC:                             524.5
Df Residuals:                      59   BIC:                             528.7
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.8043      3.334      0.241      0.810      -5.868       7.476
RGDP       -1.603e-12   4.99e-12     -0.321      0.749   -1.16e-11    8.38e-12
==============================================================================
Omnibus:                       54.792   Durbin-Watson:                   1.856
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              543.975
Skew:                          -2.132   Prob(JB):                    7.54e-119
Kurtosis:                      16.994   Cond. No.                     9.93e+11
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 9.93e+11. This might indicate that there are
strong multicollinearity or other numerical problems.
```

3. Okun's Law Statistics

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   RGDP   R-squared:                       0.377
Model:                            OLS   Adj. R-squared:                  0.354
Method:                 Least Squares   F-statistic:                     16.37
Date:                Wed, 29 Mar 2023   Prob (F-statistic):           0.000392
Time:                        16:31:22   Log-Likelihood:                 24.681
No. Observations:                  29   AIC:                            -45.36
Df Residuals:                      27   BIC:                            -42.63
Df Model:                           1
Covariance Type:            nonrobust
=====================================================================================
                        coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
const                 0.0572      0.020      2.875      0.008       0.016       0.098
unemployment rate    -0.1003      0.025     -4.046      0.000      -0.151      -0.049
==============================================================================
Omnibus:                        8.392   Durbin-Watson:                   1.247
Prob(Omnibus):                  0.015   Jarque-Bera (JB):                8.101
Skew:                           0.729   Prob(JB):                       0.0174
Kurtosis:                       5.140   Cond. No.                         1.25
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```