Tutorial 0

Anna Veselovska

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Let us emphasise that the simultaneous streaming and recording of the lecture is an additional service we offer, which does require a certain amount of effort to the lecturer (in terms of concentration and focus). **For this reason, questions from the online audience won't be addressed as the attention of the lecturer will be on the audience in the class (in presence).**

Exercise sessions:

We offer diffent type of exercise tutorials that take place on a weekly basis and are held according to the following schedule:

Group D1: Tuesday, 16:15 -17:45,
Room: 02.08.011, Seminarraum (M2/M3)

Group D2: Wednesday, 08:30 -10:00,
Room: 03.06.011, Seminarraum (M6/M8)

Group D3: Wednesday, 16:15 -17:45,
Room: 03.10.011, Seminarraum

Group T: Session "All you would like to know about Math"
Friday, 08:30 -10:00,
Room: 03.08.011, Seminarraum (M1/M7)
Zoom link: https://tum-conf.zoom.us/j/65100925172?pwd=QVU4cjZJejBRc1NRNytLOHRlblhPdz09

Meeting ID: 651 0092 5172
Passcode: 470648

Group S: Friday, 10:00 -11:30, (!!! except on April 28, at 16:15)
Central Exercise on Zoom
Zoom link: https://tum-conf.zoom.us/j/61465066450?pwd=Tm5ZQk8vL0FLWlR6clNsSkc2N3lTQT09
Meeting ID: 614 6506 6450
Passcode: 385248

Description of Tutorial Types :

Groups D1-D3 are discussion-based exercise sessions, where we expect students' initiative and see these sessions as a perfect place for having interesting discussions, sharing ideas about the problem sets, and deepening your understanding of the concepts discussed in the course. These sessions will be held in person only.

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Group T "All you would like to know about Math" is a special exercise session where you can revisit/learn basic mathematical contents or topics (from Analysis 1, Analysis 2, and Linear Algebra), for which you wish to get some extra teaching. We set up a Moodle forum, on which you can indicate anytime concepts and topics, that need to be revised. Please note that the topic needs to be posted on the Forum at least Monday noon in order to be discussed in the coming Friday session. This session will be held in a hybrid form.

Group S on Friday is the exercise session with which our week of exercises will be concluded. In this session, possible ways of solvi[redacted] a tutor, with some hints and

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Tutorial 0

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Goal: build a foundation for your expertise in Data Analysis.

Ex 0.1 (convex sets)

a) Given: $a \in \mathbb{R}^n$, $a \neq \vec{0}$, $b \in \mathbb{R}$

hyperplane

$$H = \{ x \in \mathbb{R}^n : a^T x = b \}$$

$$a_1 x_1 + \dots + a_n x_n = b$$

$$a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

Show: H is a convex set.

SS: Recall: a set X is convex if

$\forall\, u, v \in X$, $\lambda u + (1-\lambda) v \in X$

for $0 \leq \lambda \leq 1$

Ex 0.1 (convex sets)

a) Given: $a \in$

hyperp

$H =$

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$$\forall\, u, v \in X, \quad \lambda u + (1-\lambda) v \in X$$

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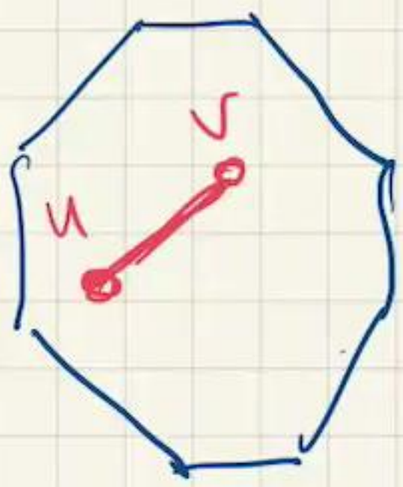
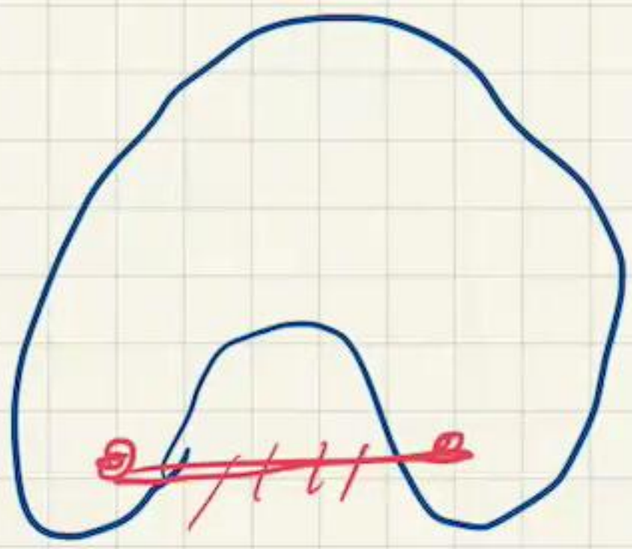
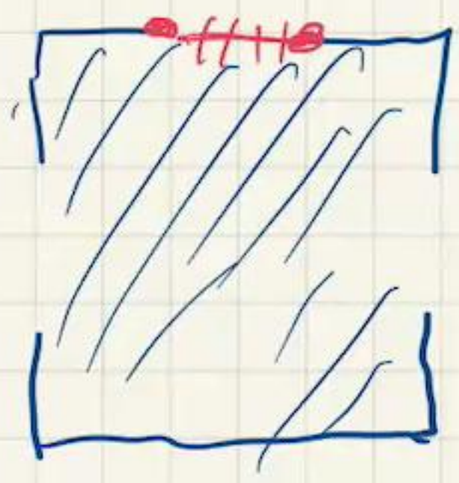
contains every line segment



convex set



not convex



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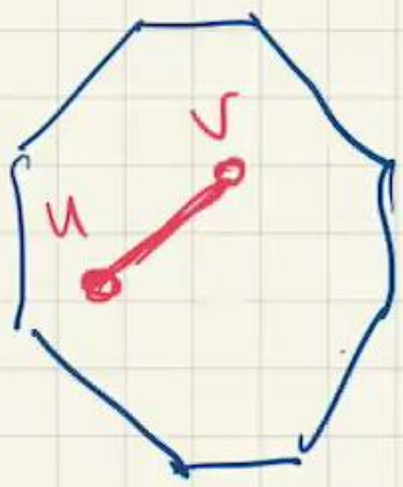

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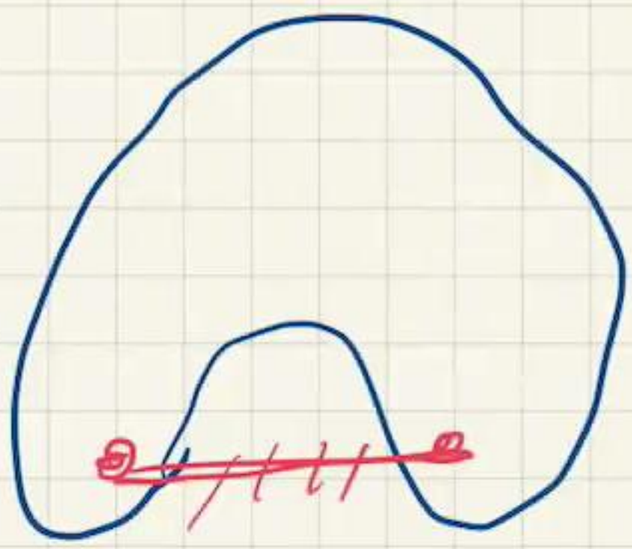

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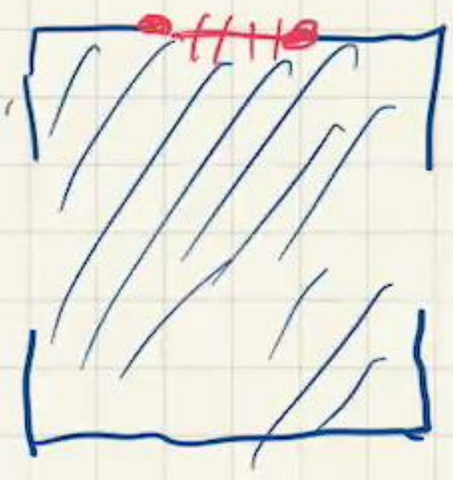

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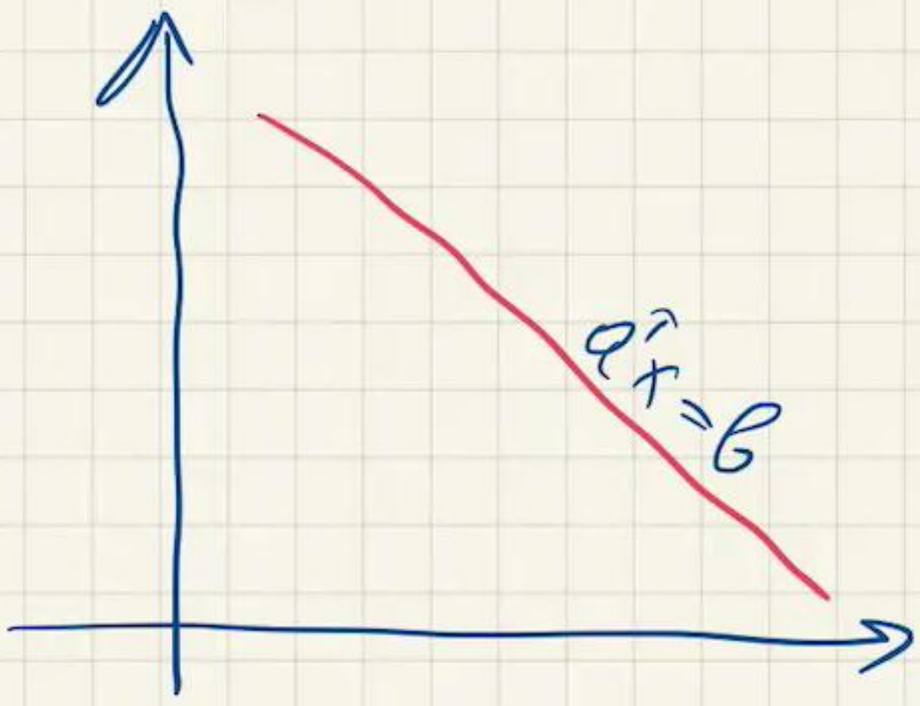

$\forall\ x, y \in H$, $z := \lambda x + (1-\lambda) y \overset{?}{\in} H$

$$z = \lambda x + (1-\lambda) y = \begin{pmatrix} \lambda x_1 + (1-\lambda) y_1 \\ \vdots \\ \lambda x_n + (1-\lambda) y_n \end{pmatrix}$$

Since $x \in H \Rightarrow a^T x = b \Rightarrow \lambda a^T x = \lambda b$

$y \in H \Rightarrow a^T y = b \Rightarrow (1-\lambda) a^T y = (1-\lambda) b$

$\Downarrow$

$a^T(\lambda x) = \lambda b$

$a^T((1-\lambda) y) = (1-\lambda) b$

$a^T z = a^T(\lambda x + (1-\lambda) y)$

$= a^T(\lambda x) + a^T((1-\lambda) y)$

$= \lambda b + (1-\lambda) b = b \Rightarrow z \in H$

Release to
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B

$= (1-\Pi) B$

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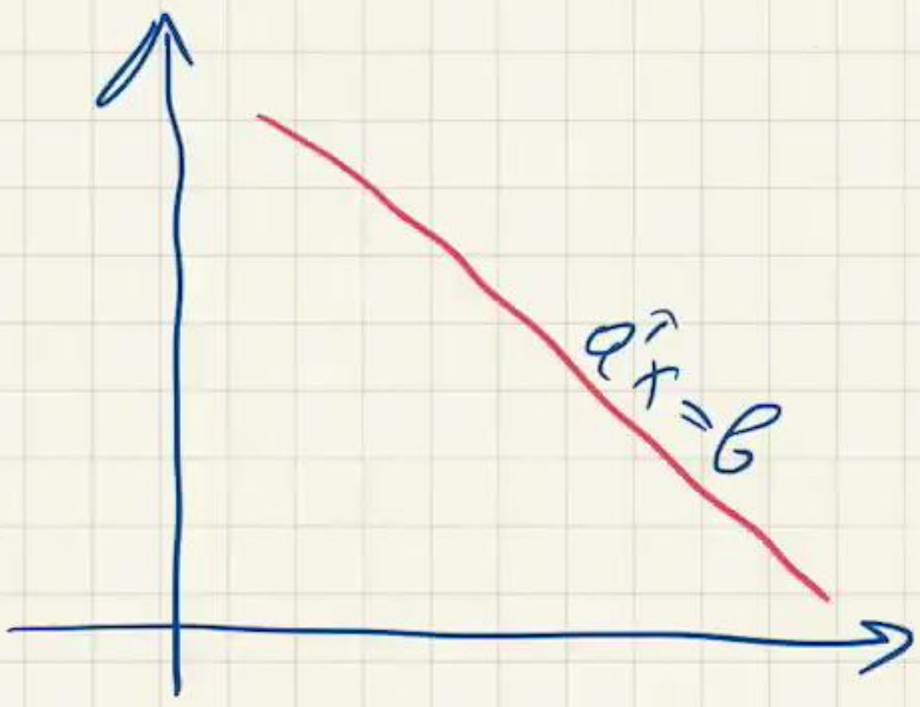

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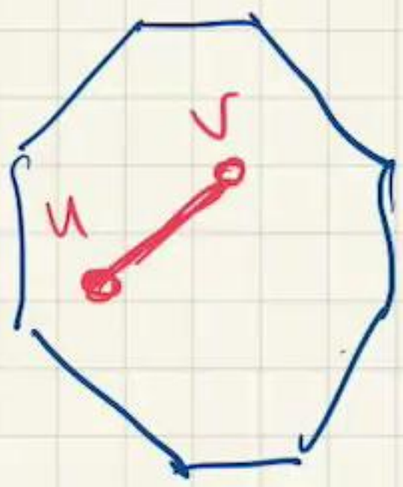
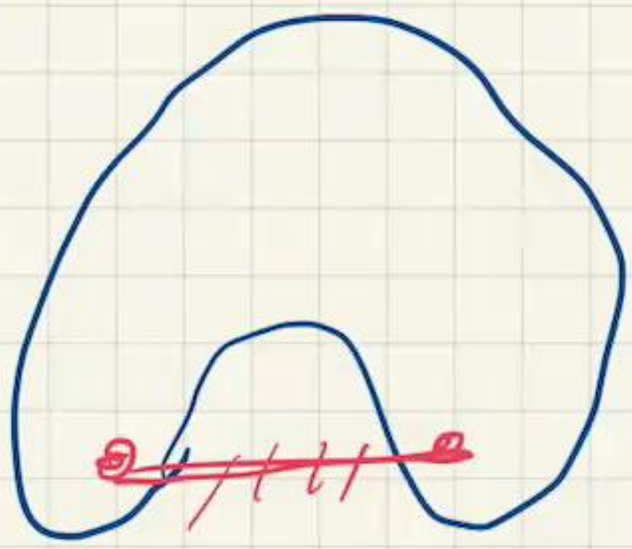
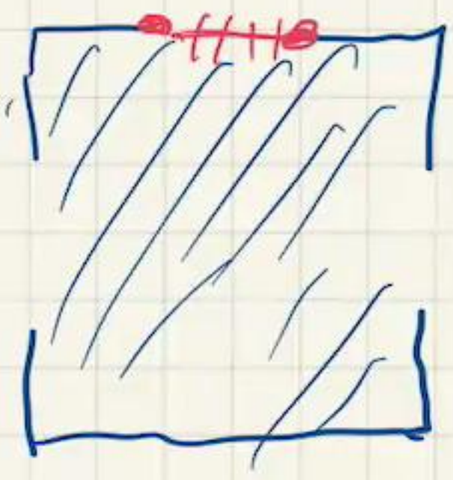


convex set



not convex



Ex 0.1. 8)

Given: $C \subseteq \mathbb{R}^n$ convex set

$x_1, \dots x_k \in C$

$\theta_1, \dots, \theta_k \in \mathbb{R}$, $\theta_j \geq 0$

$\theta_1 + \dots + \theta_k = 1$

Show: $\theta_1 x_1 + \dots + \theta_k x_k \in C$

Note: if $k=2 \Rightarrow$ the definition of convexity

$\theta_1 x_1 + \theta_2 x_2 \in C$

$\theta_1 + \theta_2 = 1$

$\theta_2 = 1 - \theta_1$

SS:

We start first with $k=3$.

Suppose:

$x_1, x_2, x_3 \in C$

$\theta_1 + \theta_2 + \theta_3 = 1$, $\theta_j \geq 0$

We need to show

$\theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 \in C$