Relativistic Vlasov Equation for Heavy-Ion Collisions

Che Ming Ko, Qi Li, and Renchuan Wang[a]
Cyclotron Institute and Center for Theoretical Physics, Texas A&M University, College Station, Texas 77843
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In the local-density and semiclassical approximation, a relativistic Vlasov equation is constructed from the Walecka model. Applying it to heavy-ion collisions at intermediate energies, we obtain results that are similar to those from the more complicated time-dependent Dirac equation. Compared with the results from the nonrelativistic Skyrme interaction, the relativistic effect is evident in the enhanced transverse-momentum distribution during the collision.

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The study of heavy-ion collisions offers the possibility of studying the properties of nuclear matter under extreme conditions. Such knowledge is of interest not only in nuclear physics but also for the study of the evolution of the early Universe and neutron stars. To describe the collision dynamics, the model based on the Vlasov-Uehling-Uhlenbeck (VUU) equation[1,2] has been shown to be very useful as it has both the correct mean-field dynamics at low-energy collisions and collision-dominated dynamics at high-energy collisions. In the VUU model, the mean field is constructed from the Skyrme interaction which describes remarkably well the low-energy dynamics of nuclear matter. However, recent studies of intermediate-energy proton-nucleus reactions indicate that relativistic effects[3] are important in such reactions. A relativistic mean-field theory, introduced originally by Walecka,[4,5] has proved to be very successful in taking into account the relativistic effects. Since the nuclear-matter density encountered in heavy-ion collisions at intermediate energies is much higher than the normal nuclear-matter density, it is expected that relativistic effects will become even more important in such reactions.

In the mean-field limit, a time-dependent Dirac equation based on the Walecka model has been studied recently by Cusson *et al.*[6] These authors have found that the relativistic theory leads to a collision dynamics which is drastically different from that predicted by the normal VUU model based on the Skyrme interaction. In particular, the transverse-momentum transfer predicted by their calculations is much larger than that from the normal VUU model. They attribute such results to the out-of-phase oscillation of the scalar- and vector-meson distributions, an effect which is not included in the conventional VUU model. Since the time-dependent Dirac approach is extremely computationally intensive, they could only study the collision between ions with low masses. Also, for heavy-ion collisions at intermediate energies, the residual nucleon-nucleon collisions are very important, but the complexity of the time-dependent Dirac approach makes it unlikely that the collision term can be realistically included. However, if a relativistic Vlasov equation can be derived from the time-dependent Dirac equation, then it will be possible to include the collision term so that a relativistic VUU equation can be derived. In this Letter, we shall present a method for constructing a relativistic Vlasov equation from the time-dependent Dirac equation and compare the results so obtained with the latter approach.

Following the Walecka model, we consider only the nucleon field ψ, the scalar-meson field ϕ, and the vector-meson field V_μ. The Lorentz-invariant Lagrangean density is then given by

$$L=\bar{\psi}[\gamma^\mu(i\partial_\mu-g_v V_\mu)-(M-g_s\phi)]\psi+\tfrac{1}{2}(\partial_\mu\phi\,\partial^\mu\phi-m_s^2\phi^2)-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_v^2V_\mu V^\mu, \quad (1)$$

where $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$. The masses are M, m_s, and m_v for the nucleon, the scalar meson, and the vector meson, respectively. The parameters of the Lagrangean are given in Refs. 4 and 5, i.e., $g_s^2=91.64$, $g_v^2=136.2$, $m_s=550$ MeV, and $m_v=783$ MeV. In the mean-field approximation, this model describes reasonably well both the bulk nuclear-matter properties and the single-particle properties of nuclei. The compressibility of the nuclear matter given by the model is, however, around 540 MeV which is larger than the value ≈ 200 MeV determined from the energy of the nuclear giant monopole resonance. Introducing the self-interaction of the scalar meson will give different values for the compressibility. In this exploratory work, we shall not attempt such refined studies. The equations of motion for the fields can be obtained as usual from the Lagrangean density.

In the mean-field approximation, the mesons are considered as classical fields with the nucleons acting as their sources. If the nuclear density does not change appreciably in a time and spatial interval of the inverse of the meson masses, then the time and space dependence of the meson fields will follow closely that of the nucleons. In this case, one can then neglect the time and spatial derivatives in the mesonic equations of motion and write $m_v^2 V_0 \approx g_v \langle \psi^\dagger \psi \rangle \equiv g_v \rho_B$, $m_v^2 \mathbf{V} \approx g_v \langle \bar{\psi} \boldsymbol{\gamma} \psi \rangle \equiv g_v \boldsymbol{\rho}_v$, and $m_s^2 \phi \approx g_s \langle \bar{\psi} \psi \rangle \equiv g_s \rho_s$. Here, $\langle \cdots \rangle$ denotes the expectation value in the nuclear many-body state. We have also introduced the definitions for the baryon density ρ_B, the scalar density ρ_s, and the current density $\boldsymbol{\rho}_v$. Substituting these equations into the equation for the nucleon, and expressing explicitly in terms of the large (ψ_L) and small (ψ_S) components of the nucleon field, we obtain the Dirac equations

$$i\partial_t \psi_L = \boldsymbol{\sigma} \cdot \mathbf{P}^* \psi_S + [M^* + (g_v/m_v)^2 \rho_B] \psi_L, \tag{2a}$$

and

$$i\partial_t \psi_S = \boldsymbol{\sigma} \cdot \mathbf{P}^* \psi_L + [-M^* + (g_v/m_v)^2 \rho_B] \psi_S, \tag{2b}$$

where $\mathbf{P}^* = \mathbf{P} - (g_v/m_v)^2 \boldsymbol{\rho}_v$ and $M^* = M - (g_s/m_s)^2 \rho_s$. For heavy-ion collisions at small impact parameters, the dynamics is not expected to be sensitive to the detailed surface properties of the nuclei. In this case, we can introduce the local-density approximation in which the nucleon is taken as moving in locally constant fields and has thus the following approximate relations between its small and large components,

$$\psi_S \approx [\boldsymbol{\sigma} \cdot \mathbf{p}^*/(e^* + M^*)] \psi_L,$$

and

$$\psi_L \approx [\boldsymbol{\sigma} \cdot \mathbf{p}^*/(e^* - M^*)] \psi_S, \tag{3}$$

where $\mathbf{p}^* = \mathbf{p} - (g_v/m_v)^2 \boldsymbol{\rho}_v$ and $e^* = (\mathbf{p}^{*2} + M^{*2})^{1/2}$, with $\mathbf{p}$ the value of the local momentum. These relations eliminate the coupling between the small and large components and reduce Eqs. (2) to

$$i\partial_t \psi = \{E^* + (g_v/m_v)^2 \rho_B\} \psi, \tag{4}$$

where $E^* = (\mathbf{P}^{*2} + M^{*2})^{1/2}$. It shows that the motion of the nucleon is described by an effective one-body Hamiltonian, $h = E^* + (g_v/m_v)^2 \rho_B$.

To derive a Vlasov equation from Eq. (4), we make a classical approximation by considering the one-body phase-space distribution function $f(\mathbf{r},\mathbf{p})$. From the classical Liouville equation and the Ehrenfest theorem, a Vlasov equation can then be derived,

$$\partial_t f + \mathbf{v} \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f = 0, \tag{5}$$

where

$$\mathbf{v} = \langle \partial_t \mathbf{R} \rangle = -i \langle [\mathbf{R}, h] \rangle = \mathbf{p}^*/e^*, \tag{6a}$$

and

$$\nabla_r U = -\langle \partial_t \mathbf{P} \rangle = i \langle [\mathbf{P}, h] \rangle = \nabla_r [e^* + (g_v/m_v)^2 \rho_B]. \tag{6b}$$

Derivation of a similar equation from the quantum Liouville equation is in progress. In terms of the phase-space distribution function, the nuclear density is given by

$$\rho_B(\mathbf{r}) = \int d^3p\, f(\mathbf{r},\mathbf{p}), \tag{7a}$$

while the nuclear scalar and current density are determined in the local-density approximation by

$$\rho_s(\mathbf{r}) = \int d^3p (M^*/E^*) f(\mathbf{r},\mathbf{p}), \tag{7b}$$

and

$$\boldsymbol{\rho}_v(\mathbf{r}) = \int d^3p (\mathbf{p}^*/E^*) f(\mathbf{r},\mathbf{p}), \tag{7c}$$

respectively.

To solve the Vlasov equation, we use the method of pseudoparticles, which was first introduced to heavy collisions by Wong.[7] In this method, each nucleon is replaced by a collection of test particles and the Wigner distribution function is given by the distribution of these test particles in the phase space. To solve the Vlasov equation is then equivalent to the solution of the classical equations of motion for all these test particles, which are given exactly by Eq. (6). The baryon density is simply determined by the density of the nucleons in the coordinate space. For the scalar and current densities, it is very hard to evaluate them exactly because of their nonlinear forms. We shall therefore calculate them approximately by substituting $\mathbf{p}^*$ and $\mathbf{p}^{*2}$ with their mean values at $\mathbf{r}$ and multiplying the resulting expressions by the local baryon density. Then the current density has the same direction as the mean momentum and we introduce therefore $\boldsymbol{\rho}_v = \rho_v \hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is a unit vector in the direction of $\mathbf{p}$. This leads to

$$\rho_s \approx \{M^*/[p^{*2} + M^{*2} + (\delta p)^2]^{1/2}\} \rho_B, \tag{8a}$$

and

$$\rho_v \approx \{p^*/[p^{*2} + M^{*2} + (\delta p)^2]^{1/2}\} \rho_B, \tag{8b}$$

where $p^* = p - (g_v/m_v)^2 \rho_v$ and $(\delta p)^2 = \langle \mathbf{p}^2 \rangle - \langle \mathbf{p} \rangle^2$. For infinite nuclear matter, we have thus $\rho_v = 0$ and $\rho_s \approx 0.92 \rho_B$, which should be compared with the exact values $\rho_v = 0$ and $\rho_s \approx 0.93 \rho_B$ at normal density. At twice the normal density, we have $\rho_v = 0$ and $\rho_s \approx 0.72 \rho_B$ while the exact values are $\rho_v = 0$ and $\rho_s \approx 0.74 \rho_B$. We have calculated the scalar and the current densities for different values of baryon density ρ_B, mean momentum p, and root-mean-square deviation δp. For later applications, values of the scalar and current densities at other values of ρ_B, p, and δp are determined by extrapolation from the tabulated results.

We have carried out a calculation for the collision between two oxygen nuclei at an incident energy of 600 MeV/nucleon which has been previously studied in Ref. 6 with use of the time-dependent Dirac approach. We solve the Vlasov equation using 80 test particles for each nucleon; the cell size in coordinate space is taken to be 1 fm^3 for the evaluation of the density. The classical equations of motion are solved as second-order difference equations with a time step of 0.5 fm/c. The initial distribution of the test particles is assumed to be uniform in coordinate space with the nuclear radius given by $1.14A^{1/3}$; the momentum distribution is then determined

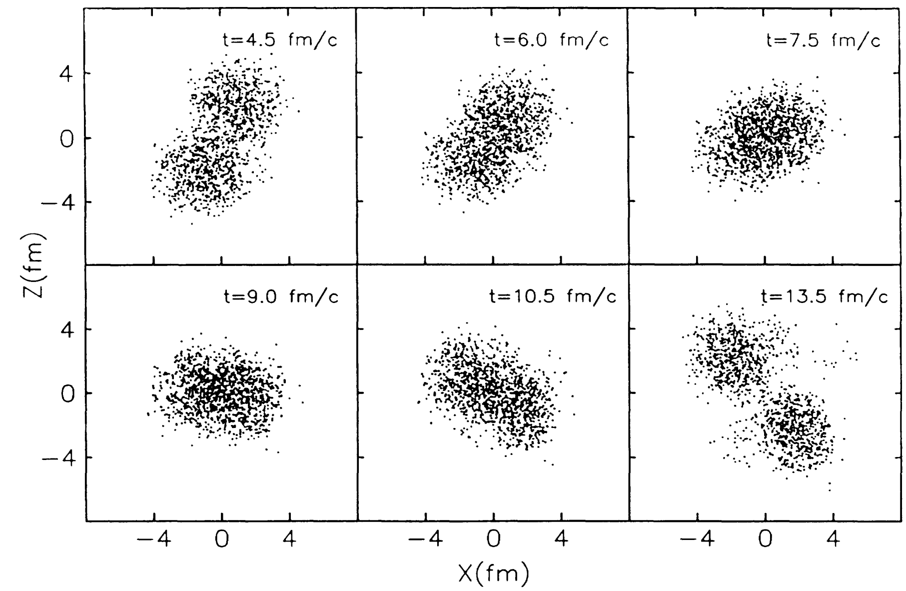

FIG. 1. Evolution of the test particles in the scattering plane for the reaction $^{16}O+^{16}O$ at an incident energy of 600 MeV/nucleon and an impact parameter $b=2$ fm. The beam direction is taken to be the z axis.

by the local Fermi-gas model. These distributions are generated by the standard Monte Carlo algorithm. We consider the collision at an impact parameter of 2 fm and the two nuclei are initially separated by 13 fm from each other. However, we start to measure the time when the two nuclei are only 10 fm apart in order to compare the results directly with those of Ref. 6. By starting with a larger separation distance, we allow the nuclei to reach their self-consistent distribution in the phase space as they are not affected by each other during this stage. In Fig. 1, the evolution of the test particles in the scattering plane is shown. We find that the maximum density during the collision is about 0.3 fm^{-3} and is reached at a time of ≈ 7.5 fm/c, which is similar to that of Ref. 6. In Fig. 2, we show the evolution of the momentum distribution of the test particles. We see an appreciable transfer of the momentum towards the direction perpendicular to the incident direction. This is also demonstrated in Fig.

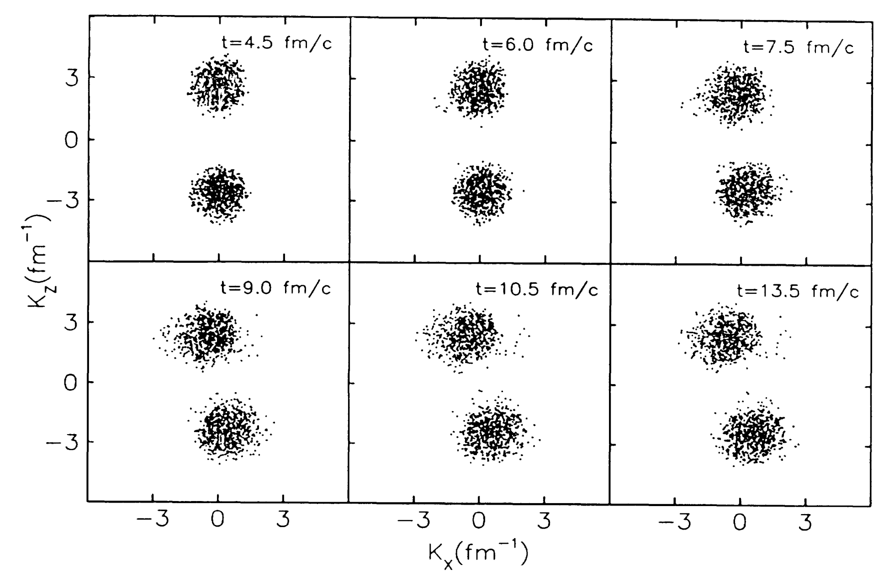

FIG. 2. Evolution of the test particles in the momentum space for the same reaction as in Fig. 1.

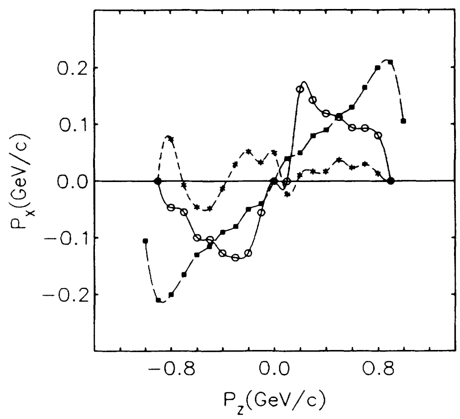

FIG. 3. The transverse momentum projected onto the reaction plane and averaged over all perpendicular momentum P_x, as a function of the longitudinal momentum P_z for the same reaction as in Fig. 1. The open circles are from the relativistic Vlasov model while the filled squares are from the time-dependent Dirac approach. The asterisks are from the Vlasov equation based on the Skyrme interaction.

3 in which the final transverse momentum projected onto the scattering plane and averaged over all perpendicular momenta is shown by the open circles as a function of the longitudinal momentum. Compared with the prediction of Ref. 6 which is also shown in the same figure by the solid squares, the transverse-momentum distribution from our calculation is somewhat different from theirs. However, the difference may not be very significant as there are very few test particles that have transverse momentum $|P_z| \approx 0.2$ GeV/c in our case and the probability is small for $|P_z| \approx 0.8$ GeV/c in their case.

We have also carried out calculations of the Vlasov equation based on the normal Skyrme effective interaction. To make the comparison meaningful, we have readjusted the parameters of the Skyrme interaction such that it has a compressibility of 540 MeV as well. The density evolution from this calculation is similar to that of the relativistic Vlasov model except the final nuclear density is lower. As to the evolution of the momentum distribution, it does not show any significant transfer of the momentum in the transverse direction. This is shown by the asterisks in Fig. 3 for the transverse-momentum distribution. The oscillation in the transverse-momentum distribution is not physically significant as there are only a few percent of the total number of test particles in the momentum regions of $|P_z| \leq 0.2$ GeV/c and $P_z| \geq 0.7$ GeV/c.

In conclusion, we have constructed a relativistic Vlasov equation for heavy-ion collisions based on the Walecka model. Its predictions are similar to the more difficult calculations based on the time-dependent Dirac equation. From comparison with the results from the Vlasov equation based on the Skyrme interaction, the importance of the relativistic effect is clearly demonstrated for heavy-ion collisions at intermediate energies. With our relativistic Vlasov equation, it is possible to include the collision term in the same way as the VUU model. Furthermore, the study of collisions between heavier ions will also be possible as the calculation is much faster than that of the time-dependent Dirac approach.

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[a]Permanent address: Center for Astrophysics, University of Science and Technology of China, Hefei, Anhui, People's Republic of China.

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