Institute of Information Systems

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Quiz

Formal Foundations of Information Systems

Winter Semester 2025, 11.11.2025

(Group C

Name & Last Name: Matriculation Number:

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Signature

Good luck!

1 Set Theory

A.a) Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = x + 1, \qquad g(x) = 2x + 2.$$

(a) Compute the composition $g \circ f$

(b) Compute the composition $f \circ g$

(c) Compare your results from (a) and (b). Is function composition commutative? Explain briefly. (3 Points)

A.b) Relations and Set Operations (3points)

Consider the set $H = \{A, B, C\}$ and the relation

$$R = \{(A, A), (A, B), (B, C), (C, B)\}$$

on H.

1. What is the reflexive closure R_{RC} of R?
2. What is the symmetric closure R_{SC} of R?
3. What is the transitive closure R^* of R?

1.1 A.c)

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{6, 8\}$. Find the following: (4 Points)

(a) $A \cup B$

(b) $A \cap B$

(c) $A \cap B \cap C$

(d) $B - A$

(e) $C - B$

(f) $A \cap C$

2 Summation and Sequences

B.a) Write out the following sums and calculate them (3 points).

1. $\sum_{i=3}^{6} 9$
2. $\sum_{k=1}^{3} (3k)$
3. $\sum_{i=1}^{3} \sum_{j=3}^{6} (i * j)$

B.b) Compute the following limits. Explain your approach in your own words (3 points).

1. $\lim_{n \to \infty} \frac{2}{n} + 5$
2. $\lim_{n \to \infty} \frac{n}{n^2+1}$
3. $\lim_{n \to \infty} \frac{n^3+4n+1}{3n^3+n^2}$

B.c)

Given is the function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = 2x - 3$.

1. Show that f is injective. Explain your approach in your own words.
2. Show that f is surjective. Explain your approach in your own words.
3. Is f bijective? Justify your answer (3 points).

3 Combinatorics

(a) Counting Number Lock Combinations (3 points)

A number lock consists of 5 independent rings, each labeled with the digits 0 through 9.

1. How many possible five-digit codes can be formed if digits may repeat?
2. How many possible codes exist if each digit can be used only once (i.e., all digits must be distinct)?

(b) Arranging Colored Lights (3 points)

A Christmas tree is to be decorated with a light chain consisting of 5 red, 3 blue, and 4 yellow bulbs. How many different arrangements of the bulbs are possible?

(c) Combinations with Repetition (4 points)

An ice cream shop offers 6 different flavors. A customer wants to order a cup containing 4 scoops, where flavors can be repeated. How many different flavor combinations can the customer choose?

C. Linear Algebra (12 points)

C.a) Solve the following system of linear equations using the Gauss–Jordan elimination method. (3 points)

$$\begin{cases} 2x + 3y - z = 5 \\ 4x + y + 2z = 11 \\ -2x + 5y + 3z = -1 \end{cases}$$

Show all steps leading to the reduced form and provide the final values of x, y, and z.

C.b) Compute the determinant of the following 3×3 matrix using the Rule of Sarrus. (3 points)

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\det(A) = ?$$

C.c) Find the inverse of the following matrix using the Gauss–Jordan elimination method. (3 points)

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 2 & 0 & 1 \end{bmatrix}$$

Perform all necessary row operations to transform $[B \mid I]$ into $[I \mid B^{-1}]$, and report B^{-1}.

C.d) Consider a large lower triangular matrix C of size 10×10. (3 points)

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & -2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 3 & 6 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 2 & 7 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 3 & 8 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & -1 & 0 & 2 & 9 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 10 \end{bmatrix}$$

What is the determinant of C? Briefly explain why this property holds for all triangular matrices.

$$\det(C) = ?$$

4 Graph Theory

Create a graph with 8 vertices a to h and the edges a-b, a-c, a-f, a-h, b-d, b-g, c-e, c-f, c-g, c-h, d-g, e-f, e-h, f-g, f-h, g-h, h-h.

(a) Is the graph a finite or infinite graph? Is the graph directed or undirected?

(b) What are the order and the size of the graph?

(c) Define the sets of vertices and edges.

(d) Consider the edge between a and b. How would you describe the edge?

(e) Are the vertices a and g adjacent? What about the vertices a and h? Name all the vertices that are adjacent to the vertex a.

(f) Are there any self-loops in the graph?

(g) Are the edges a–h and f–h adjacent? Which edges would be adjacent to the edge f–h?

(h) Is the graph a simple graph or a general graph/multigraph?

(i) What is the neighbor set of the vertex g? What is the open and closed neighborhood?

(j) What are the degrees of the vertices? Are there any isolated vertices?

(k) What is the sum of the degrees of the vertices? How is that related to the number of edges?

(l) If the vertices b and c are deleted from the graph, is the new graph isomorphic to the initial graph?

(m) Is the graph a regular graph? If yes, what is the degree of each vertex? Is it a cubic graph? Is it a complete graph?

(n) Is the graph a bipartite graph? If yes, is it also a complete bipartite graph?

(o) What is the complement of the graph?

(p) If you consider the graph from (l), is it a subgraph of the initial graph? If yes, is it also a spanning subgraph?

4.1 Solutions:

(a) The graph is a finite graph because it has a finite vertex set and a finite edge set. It is undirected because the edges do not have orientations and are unordered pairs.

(b) The order of the graph is 8 (number of vertices). The size of the graph is 17 (number of edges).

(c) The vertex set is $V = \{a, b, c, d, e, f, g, h\}$. The edge set is $E = \{(a, b), (a, c), (a, f), (a, h), (b, d), (b, g), (c, e), (c, f), (c, g), (c, h), (d, g), (e, f), (e, h), (f, g), (f, h), (g, h), (h$

(d) The edge between a and b is an undirected edge incident to vertices a and b, meaning it connects them and they are adjacent.

(e) Vertices a and g are not adjacent because there is no edge between them. Vertices a and h are adjacent because there is an edge between them. The vertices adjacent to a are b, c, f, and h.

(f) Yes, there is a self-loop in the graph at vertex h.

(g) The edges a–h and f–h are adjacent because they share the common vertex h. The edges adjacent to f–h are those sharing a vertex with it (f or h), excluding itself: (a, f), (a, h), (c, f), (c, h), (e, f), (e, h), (f, g), (g, h), (h, h).

(h) The graph is a general graph or multigraph because it contains a self-loop, which is not allowed in simple graphs. It is not a simple graph.

(i) The neighbor set of vertex g is $\{b, c, d, f, h\}$. The open neighborhood is the neighbor set $N(g) = \{b, c, d, f, h\}$. The closed neighborhood is $N[g] = \{b, c, d, f, g, h\}$.

(j) The degrees of the vertices are: $\deg(a) = 4$, $\deg(b) = 3$, $\deg(c) = 5$, $\deg(d) = 2$, $\deg(e) = 3$, $\deg(f) = 5$, $\deg(g) = 5$, $\deg(h) = 7$ (with the self-loop contributing 2). There are no isolated vertices, as all have degree at least 2.

(k) The sum of the degrees is 34. By the handshaking lemma, this is equal to twice the number of edges ($2 \times 17 = 34$).

(l) After deleting vertices b and c, the new graph has vertices $\{a, d, e, f, g, h\}$ and edges $\{(a, f), (a, h), (d, g), (e, f), (e, h), (f, g), (f, h), (g, h), (h, h)\}$. This graph is not isomorphic to the initial graph because they have different numbers of vertices (6 vs. 8).

(m) The graph is not a regular graph because not all vertices have the same degree. Therefore, it is not a cubic graph (which requires degree 3 for all vertices). It is not a complete graph because not every pair of vertices is connected by an edge.

(n) The graph is not a bipartite graph because it contains a self-loop (a cycle of length 1, which is odd) and cannot be partitioned into two sets with no edges within each set. Thus, it is not a complete bipartite graph.

(o) The complement of the graph, generalized for a graph with a loop (as standard complements are defined for simple graphs without loops or multiple edges), has the same vertex set and includes edges between pairs of distinct vertices not connected in the original graph: $\{(a, d), (a, e), (a, g), (b, c), (b, e), (b, f), (b, h), (c, d), (d, e), (d, f), (d, h), (e, g)\}$, and loops at vertices where the original has none: a, b, c, d, e, f, g.

(p) The graph from (l) is a subgraph of the initial graph because it consists of a subset of vertices and the edges between them from the original. It is specifically an induced subgraph. It is not a spanning subgraph because it does not include all vertices of the original graph.

5 Logic

5.2

Let $A =$ "Aldo is Italian" and $B =$ "Bob is English".
Formalize the following sentences:

1. "Aldo isn't Italian"
2. "Aldo is Italian while Bob is English"
3. "If Aldo is Italian then Bob is not English"
4. "Aldo is Italian or if Aldo isn't Italian then Bob is English"
5. "Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English"

5.3

Let us consider a propositional language where:

- p means "Paola is happy",
- q means "Paola paints a picture",
- r means "Renzo is happy".

Formalize the following sentences:

1. "If Paola is happy and paints a picture then Renzo isn't happy"
2. "If Paola is happy, then she paints a picture"
3. "Paola is happy only if she paints a picture"

5.1 5.4

Compute the truth table of $(F \vee G) \wedge \neg(F \wedge G)$.
The formula models an exclusive or!