Type theory and language
From perception to linguistic communication

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Part I

From perception and action to grammar

Part II

Towards a dialogical view of semantics

Chapter 6

Modality and intensionality without possible worlds

6.1 Possible worlds, modality and intensionality

Montague (1973) uses possible worlds to analyze both modality (represented in his fragment by the adverbs *possibly* and *necessarily*) and a variety of intensional constructions in addition to the temperature and price examples discussed in Chapter 5: intensional transitive verbs such as *seek*, intensional adverbs such as *voluntarily*, verbs of propositional attitudes such as *believe* and *assert* and verbs taking infinitival complements such as *try (to)* and *wish (to)*.

A short introduction to the use of possible worlds in modal logic and philosophical conceptions of possible worlds is given by Menzel (2015). As he points out at the beginning of this article possible worlds are considered to be totalities (or at least a limit) which include the situations which we are aware of around us.

The notion of possible world is intuitively appealing. We talk of living in the best (or worst) of all possible worlds. But equally we talk of the best (or worst) possibility. When we talk in such terms we normally have a small finite number of possibilities in mind which we are contrasting. This has led some authors to use the term "possible world" to refer not to a total universe but to a small set of facts that might obtain in some version of the world. This appears to be standard usage in probability theory (e.g. Halpern, 2003). It is important not to confuse this notion with the notion of possible world as a totality which is used in semantics, inherited from modal logic. This point is made by Cooper *et al.* (2014a) and Lappin (2015).

Problems have been raised for the notion possible world. These have to do with how you individuate and count them and how many possible worlds there must be. Rescher (1999) takes up these problems from a philosophical perspective. He argues that it is impossible to individuate possible worlds and therefore impossible to count them. Lappin (2015) takes up the representation problem for possible worlds. If you cannot represent possible worlds then you cannot individuate them. The central problem for possible worlds as they are talked about in the semantics literature seems to be that the intuitive way to distinguish one possible world from another is to find a proposition that is true in the first world but false in the second. This would be fine except that we now have the corresponding problem for propositions. Unfortunately the intuitive way of distinguishing between one proposition and another (if you are a possible worlds theorist) is to find a possible world in which the first proposition is true and the other is false. This, of course, is circular and will not give us an individuation of either possible worlds or propositions. The standard version of possible worlds semantics as proposed by Montague does not, of course, fall into this obvious trap. Worlds are not represented in terms of sets of propositions which are true in them. Rather we just define an interpretation to include a set of possible worlds and leave aside the question of how they have been individuated. In a sense it is fine from a technical point of view to have an arbitrary set whose membership we cannot represent as a central component of our semantic theory. But it leaves us with the suspicion that we are left with an abstract theory which we do not really know how to connect to any empirical observations of the world. If you take a mathematical view of the semantic enterprise as Montague did, this may be acceptable. But if you are interested in semantics as an aspect of human cognitive ability it can appear problematic. Traditional possible world semantics is a theory based on an assumed set of possible worlds. But it is not a theory of the possible worlds as such, beyond the claim that there is a set of them.

Despite this, there is an intuition about the set of possible worlds which possible world theorists hold onto: that they represent all the logical possibilities. This, at least, gives us a way of considering the required cardinality of the set of possible worlds. The issue of the cardinality of the set of possible worlds and its relationship to a psychological theory of language is something that is already taken up by Partee (1977). Here she refers to Lewis's (1973) argument that there must be at least $\beth_2$ (the cardinality of the power set of the power set of natural numbers) possible worlds. The argument[1] goes like this: suppose we have a family that goes on for ever. That is, there would be $\aleph_0$ members of the family. Now consider that in a logically possible world (though possibly not in biologically possible worlds) any subset of these family members might have blue eyes (none of them, all of them and all the possibilities in between). This gives us a set of possible worlds whose cardinality is the same as the power set of the natural numbers $2^{\aleph_0}$ or $\beth_1$, that is, the cardinality of the set of real numbers. Now consider the logical possibility that each of those possible worlds is biologically plausible. Again, logically speaking, any subset of those worlds could be biologically plausible. This will yield a set of possible worlds of cardinality $2^{2^{\aleph_0}}$ or $\beth_2$. In principle one could create sets of possible worlds of any of the infinitely many infinite cardinalities although as Lewis claims $\beth_2$ is probably sufficient for normal purposes.

Another argument for the uncountability of the set of possible worlds comes from usual assumptions about space and time. We normally assume that the set of moments of time has the same cardinality as the set of points on the real line, that is, that time is continuous. Similarly we

[1] which I first heard from Barbara Partee but for which I cannot find a published reference

also assume that space is continuous. Now for any possible world where an object is at a certain location at a certain time there is another logically possible world where that object is located at a different location or occupies its location in the first world at a different time. For each such world there are uncountably many different logically possible worlds in which the object is located elsewhere.

How do we manage to reason about such large numbers of possibilities? The answer we want to propose here is that we reason in terms of types. A single type has a set of witnesses and there are no constraints on the cardinality of the set of witnesses. Types which have infinitely many witnesses are not more complex than types which have a small finite number of witnesses. Reasoning with a type involves manipulating the structural object which is the type itself not the set of its witnesses. Thus, for example, reasoning with a record type may be more complex than reasoning with a basic type that has no components. But still a record type is always a finite structure and so we are not entering into the complexity of manipulating uncountable sets, even though the record type may be thought of as a "representation" for its set of witnesses which may indeed be an uncountable set. It is here that our approach connects with proof theoretic approaches. In proof theory we manipulate expressions in a language which may represent sets of objects. Our types are not expressions in a language but they are objects in our type theoretic universe which could be thought of as "representing" the set of their witnesses. This approach also makes it possible to have a learning theory where agents can be acquainted with a type without being acquainted with the complete set of its witnesses. Knowing a type whose witnesses are dogs does not mean that you are acquainted with the set of all dogs, but rather that you know a dog when you see one, that is, you have a reliable dog *classifier*. An important aspect of human cognitive processing is that it involves reasoning with the types themselves, treating them as first class citizens which can be arguments to predicates. This is what gives rise to modality and intensionality. Possibly this higher level reasoning is unique, or at least, most fully developed in humans.

We think of types like record types as being types of situations. If we want to keep to the idea of possible worlds as total universes it is straightforward to convert a type of situations, T, to a type of worlds, T^W, as long as we have a way of defining worlds as maximal situations. We could say that a world, w, is of type T^W just in case some part, s of w is of type T. Actually, we do not need to do this because of the way we have set up subtyping. If T is a record type and $s : T$, then if $s < s'$, that is s is a proper part of s' in the sense defined in Chapter 5, then $s' : T$. If we had a way of defining maximal situations, that is, situations s such that there is no s' such that $s < s'$, we could take these to be our worlds. The problem is, though that it is not clear that it is desirable, or even possible, to characterize a notion of maximal situation in this sense. Certainly, there is no notion of maximal record so our choice of modelling situations as records suggests that there is no notion of maximal situation. Our axioms say that given any record it is always possible to add a new field to it.[2]

[2]This fact is parallel to Proposition 2 in Barwise (1989), Ch. 8: *Every situation, s, is a proper part of some other situation, s'.*

6.2 Modality without possible worlds

Montague (1973) introduces *necessarily* and *possibly* as sentence adverbs, that is, they combine with a sentence to produce another sentence. If α is a sentence, then *necessarily* α is true in a possible world, w, just in case α is true in every possible world and *possibly* α is true in a possible world, w, just in case there is some possible world in which α is true.[3]

In Chapter 1, Section 1.6 and Appendix A.10 we introduce modal type systems which are families of type systems, which we call *possibilities*, differing in their assignments of witnesses to basic types and ptypes. The important difference between possible worlds and possibilities is that for possibilities the parameters along which they can vary are fixed by the available types introduced in the type system, a well-defined notion, and one which varies depending on the particular type system. Thus we have a way of characterizing the dimensions along which the possibilities associated with a given type system vary and thus we have a way of representing the possibilities whereas we do not have such a way of characterizing possible worlds. We introduced modal notions relating to such modal type systems: essentially a type is necessary if it has a non-empty set of witnesses in every possibility and a type is possible if there is some possibility in which it has a non-empty set of witnesses. (For precise definitions see Appendix A.10.) Corresponding to the operators in modal logic we can introduce type constructors '$\Box$' and '$\Diamond$' as in (1).

(1) If T is a type, then $\Box T$ and $\Diamond T$ are types

These types should obey the constraints in (2).

(2) a. $\Box T$ is non-empty iff T is necessary (non-empty in all possibilities)

b. $\Diamond T$ is non-empty iff T is possible (non-empty in some possibility)

In order to see how we can meet these constraints we have to first note that in a modal type system we cannot talk of an object a being of a type T *tout court* as we have done so far. a may be of type T in some possibilities but not others. This means that we have to relativize being of a type to possibilities, p which are members of a modal type system, $\mathcal{P}$. Instead of writing $a : T$,

[3]This simple treatment of modality corresponds to the modal logic system S5 where there is no restriction on accessibility between possible worlds (Hughes and Cresswell, 1968, 1996).

we will write $a :_{p,\mathcal{P}} T$ ("a is of type T in possibility p within modal type system $\mathcal{P}$").[4] We also correspondingly relativize our notation for the set of witnesses of a type as in (3).

(3) $[\check{T}]_{p,\mathcal{P}} = \{a \mid a :_{p,\mathcal{P}} T\}$

We introduce two basic types of types, *Nec* and *Poss*, the types of necessary and possible propositions respectively. The witness conditions for these types are given in (4).[5]

(4) a. $T :_{p,\mathcal{P}}$ *Nec* iff for all $p' \in \mathcal{P}$, $[\check{T}]_{p',\mathcal{P}} \neq \emptyset$

b. $T :_{p,\mathcal{P}}$ *Poss* iff for some $p' \in \mathcal{P}$, $[\check{T}]_{p',\mathcal{P}} \neq \emptyset$

Now consider that the inclusion of singleton types in our system (Appendix A.7) allows for the types Nec_T and Poss_T for any type, T. These types have a single witness, T, if T is necessary or possible respectively and otherwise have no witnesses. These types thus meet the constraints on $\Box T$ and $\Diamond T$ given in (2). We propose therefore to make the identifications given in (5).

(5) a. $\Box T = \mathit{Nec}_T$

b. $\Diamond T = \mathit{Poss}_T$

Note that we could also have defined ptypes corresponding to (5) by introducing predicates of 'nec' and 'poss' with arity $\langle \mathit{Type} \rangle$ which obey the constraints in (6).

(6) a. $[\check{\ }\mathrm{nec}(T)] \neq \emptyset$ iff T : *Nec*

b. $[\check{\ }\mathrm{poss}(T)] \neq \emptyset$ iff T : *Poss*

The option of using ptypes will become important below where we wish to add additional arguments to the predicate.

[4]Note that in Appendix A we have throughout the formal development of TTR always relativized the of-type relation to the type system being considered, and in the case of modal type systems in addition to the possibility (identified by the model associated with the possibility).

[5]Note that it is important for these types that we have introduced stratified types (Appendix A.11) since *Nec* and *Poss* can themselves be necessary and possible types. For example, instead of *Nec* $:_{p,\mathcal{P}}$ *Nec* we have $\mathit{Nec}^n :_{p,\mathcal{P}} \mathit{Nec}^{n+1}$ to avoid the danger of running into a version of Russell's paradox. As usual we will suppress discussion of stratification in the text in order to simplify the presentation.

How many possibilities are there in a modal type system? The answer to this question is that there can be as many as you choose for the given type system, ranging from a small finite number of possibilities to a higher order infinity. The definition of a modal type system given in Appendix A.10 only requires that there be a family of possibilities. Thus this definition includes the kind of restricted sets of "possible worlds" differing along a small finite set of parameters which probability theorists talk of and indeed also linguistic semanticists talk of informally when they are in pedagogical explanatory mode (see, for example, Dowty *et al.*, 1981 and a lot of recent literature on inquisitive semantics such as Groenendijk and Roelofsen, 2012).

It is important in a modal type system that the identity criteria for the possibilities are determined by the types provided by the system. Two possibilities are distinct only if they differ in the witnesses associated with some basic type or ptype. It is not possible to make distinctions for which you do not have appropriate types available. Thus the range of possibilities is limited by the types which are available to classify objects.

This is not to say that we have eliminated all potential decidability problems from modal type systems. Of course, if the types that we use to construct the system are not decidable it may not be possible to decide on identity for possibilities. Even if all the types are guaranteed to be decidable, given an inifinite set of possibilities there cannot be any general guarantee that we can decide whether an arbitrary type is necessary or possible or not since we cannot visit every possibility in a finite amount of time. We can only be sure if we have some general argument about the possibilities which does not involve inspecting each possibility individually. But having a way of distinguishing between possibilities which may in the limit be undecidable is better than not having a way of distinguishing between possibilities, other than that they are distinct members of a set.

The work on modality in natural language which has followed after Montague's original work all points to a more restricted kind of modality which involves arguing from some basic assumptions to a conclusion rather than considering all logical possibilities. This view of modality in natural language has been put forward by Kratzer in a body of work beginning with Kratzer (1977). This and other papers by Kratzer on modality are collected in revised and commented form in Kratzer (2012) and there is much other literature which builds on Kratzer's ideas. An excellent introduction to Kratzer's work is given in Chapter 3 of Portner (2009). The essential idea is that modals like *must* (corresponding to necessity) and *can* (corresponding to possibility) must be interpreted relative to a "conversational background" which in Kratzer (1981) (Chapter 2 of Kratzer, 2012) is split into two components, a *modal base* and an *ordering source*. The modal base is a set of propositions[6] which characterize the assumptions from which we are arguing. The ordering source is a set of propositions[7] which determine an ideal which we are trying to get as close to as possible. It is called an ordering source because Kratzer, following Lewis (1981), thinks of it as inducing a partial ordering on possible worlds, in terms of their closeness to the

[6]Actually, a function which determines a set of propositions for each possible world.
[7]Again relativized to possible worlds.

ideal. Kratzer's insight is that necessity and possibility in natural language should be defined relative to a modal base and an ordering source. In simple terms, a proposition, p, is necessary with respect to a modal base, b, and an ordering source (ideal), i, just in case p follows from the conjunction of b and i. A proposition, p, is possible with respect to b and i just in case p is consistent with the conjunction of b and i.

We shall construe Kratzer's propositions as types and we shall take modal bases and ideals to be types as well. To recreate a Kratzerian semantics for necessity and possibility we let the predicates 'nec' and 'poss' have arity ⟨*Type*, *Type*, *Type*⟩ and require that they obey the constraints in (7).

(7) a. $[\check{}\mathrm{nec}(T, B, I)]_{p,\mathcal{P}} \neq \emptyset$ iff for any $p' \in \mathcal{P}$, if both $[\check{}B]_{p',\mathcal{P}} \neq \emptyset$ and $[\check{}I]_{p',\mathcal{P}} \neq \emptyset$ then $[\check{}T]_{p',\mathcal{P}} \neq \emptyset$

b. $[\check{}\mathrm{poss}(T, B, I)] \neq \emptyset$ iff for some $p' \in \mathcal{P}$, $[\check{}B]_{p',\mathcal{P}} \neq \emptyset$, $[\check{}I]_{p',\mathcal{P}} \neq \emptyset$ and $[\check{}T]_{p',\mathcal{P}} \neq \emptyset$

Building on a basic example from Portner, 2009, p. 49, suppose that T is *Mary-eat-her-broccoli*, B is *Mary-has-broccoli-on-her-plate* and I is *Mary-eats-everything-on-her-plate*. Then according to the definitions in (7) nec(T,B,I) is non-empty just in case for any of the possibilities we are considering if both B and I are non-empty then T is non-empty, that is if there's a situation where Mary has brocolli on her plate and there's a situation where Mary eats everything on her plate then there's a situation in which Mary eats her brocolli. Similarly, poss(T,B,I) is non-empty just in case there is some possibility that we are considering where there's a situation in which Mary has brocolli on her plate, a situation in which Mary eats everything on her plate and a situation in which Mary eats her brocolli.

A more restrictive notion of necessity than is given in (7a) would be in terms of subtyping as in (8).

(8) $[\check{}\mathrm{nec}(T, B, I)]_{p,\mathcal{P}} \neq \emptyset$ iff $B \not\perp I$ and $(B \wedge I) \sqsubseteq_{\mathcal{P}} T$

(Here we are using $\not\perp$ for "does not preclude", that is, it is possible for something to be of both types.)

(8) requires that anything of type $B \wedge I$ will also be of type T (no matter what gets assigned to the basic types and ptypes) whereas (7) only requires that if there is something of type B and there is something of type I then there will also be something of type T, though not necessarily the same thing. The subtyping variant is interesting because if you have a way of (at least approximately) computing whether one type is a subtype of another simply by looking at the types, then you

will not have to look at the different possibilities. Similarly, for possibility we may have a way of computing (at least approximately) that a type is instantiable simply by looking at the type and doing a consistency check without having to inspect the possibilities. This points towards a more proof theoretic oriented approach to modality. Part of the important insight of Kratzer's approach to modality is that it involves arguments which can be constructed from the modal base and the ideal.

(8) also seems to fit better with the particular brocolli example we are discussing. nec(T,B,I) will be non-empty just in case Mary having broccoli on her plate does not preclude her eating everything on her plate and in all of the possibilities under consideration any situation in which she has brocolli on her plate and eats everything on her plate is also a situation in which she eats her brocolli.

When Kratzer talks of the conversational background consisting of the base and the ideal she often talks about rules that might be encoded there (bodies of laws or regulations in the case of deontic modality). This idea of rules being involved actually fits better with the brocolli example. It is not so much that we are considering possibilities where Mary eats everything on her plate, but rather that we are considering possibilities where there is a rule that Mary eats whatever is on her plate.

It is important for Kratzer that such rules not be logical laws in the sense that they always hold true. For example, a law that cars not park on double yellow lines does not entail that cars do not park on double yellow lines – this is only something that holds true in deontically ideal worlds. This suggests that there could be a role for what Breitholtz (2014) calls topoi. A topos in her terms is a dependent type, that is, a function which maps an object of some type to a type. Given a situation of the domain type of the topos, the topos will return a new type. The standard licensing condition associated with a topos is similar to the licensing condition we have for, for example, sign combination functions in Chapter 3 (see also Appendix B.1.4.2). This is given in (9).

(9) If $\tau : (T \rightarrow Type)$ is a topos available to agent A, then for any s, $s :_A T$ licenses $:_A \tau(s)$

That is, if an agent, A, judges a situation s to be of the domain type of the topos, then A is licensed to judge that there is something of type $\tau(s)$.

We will use the same trick that we used for the polymorphism of properties in Chapter 5 in characterizing the type *Topos*. That is, we will define *Topos* to be the type in (10).

(10) $\left[\begin{array}{lll} \text{bg} & : & \textit{Type} \\ \text{fg} & : & (\text{bg}\rightarrow\textit{Type}) \end{array}\right]$

This means that we can reformulate the licensing condition in (9) as (11).

(11) If τ is a topos available to agent A, then for any s, $s :_A \tau.\mathrm{bg}$,
licenses $:_A \tau.\mathrm{fg}(s)$

We will say that topoi associated with this condition are *epistemic*. The condition has to do with increasing our knowledge on the basis of a previous judgement. If we judge something, s, to be of the type which is the background of the topos then we can judge that there is something of the type resulting from applying the foreground of the topos to s.

Topoi can also be *deontic*, that is, they are associated with a condition which involves an obligation to carry out a certain act (create something of a given type). This condition is as in (12).

(12) If τ is a topos available to agent A, then for any s, $s :_A \tau.\mathrm{bg}$
obliges $:_A \tau.\mathrm{fg}(s)!$

That is, if an agent, A, judges a situation, s, to be of the background type of the topos, then A is obliged to create (contribute to the creation of) something which is of the type resulting from applying the foreground of the topos to s.

Topoi can be associated with either of these conditions and they can be associated with both which means that they can be used either epistemically or deontically.

We now replace the third "ideal" type argument to the predicates 'nec' and 'poss' with a topos argument, giving them the arity $\langle$*Type*, *Type*, *Topos*$\rangle$. If we need to recreate the option provided by the type rather than the topos we can use a topos whose background type is the type *Rec*. That is, it does not place any constraints on the situations in its domain and thus will return a type for any situation. If such a function is a constant function, that is, it returns the same type for any situation, then this will give us the same effect as we obtained when the argument was a type rather than a topos.

We define the witness conditions in (13) for the new versions of 'nec' and 'poss'.

(13) a. If T and B are types and τ is a topos, then

$$s : \mathrm{nec}(T, B, \tau) \text{ iff}$$
$$s : B,$$
$$B \sqsubseteq \tau.\mathrm{bg} \text{ and}$$
$$\tau.\mathrm{fg}(s) \sqsubseteq T$$

b. If T and B are types and τ is a topos, then

$$s : \mathrm{poss}(T, B, \tau) \text{ iff}$$
$$s : B,$$
$$B \sqsubseteq \tau.\mathrm{bg} \text{ and}$$
$$\tau.\mathrm{fg}(s) \not\perp T$$

In informal terms, (13) says that a situation, s, witnesses that a type, T, is necessary with respect to a background type, B, and a topos, τ, just in case s is of the type B, τ is defined on situations of type B and the type resulting from the application of τ to s is such that any situation of that type will be of type T. It says that T is possible under the same conditions except that the third condition is changed to requiring that the type resulting from the application of τ to s does not preclude T, i.e. that it is possible for a situation to be of both types.

Let us see how this might play out in our basic example (taken from Portner, 2009, p. 49). Consider (14).

(14) Mary should eat her broccoli

Portner points out that this sentence can receive a bouletic (having to do with desires) intepretation if "we are talking about the fact that Mary loves brocolli" while "if we are trying to enforce the idea that children should eat everything on their plates, it naturally receives a deontic interpretation". Suppose that b is the brocolli on Mary's plate. For simplicity we will assume b : *Ind*. Let m be Mary and p her plate. Then the type, B, of the base situation could be (15).

$$(15)\quad \begin{bmatrix} \mathrm{x}{=}b & : & \textit{Ind} \\ \mathrm{c}_1 & : & \mathrm{brocolli(x)} \\ \mathrm{y}{=}m & : & \textit{Ind} \\ \mathrm{c}_2 & : & \mathrm{child(y)} \\ \mathrm{z}{=}p & : & \textit{Ind} \\ \mathrm{c}_3 & : & \mathrm{plate(z)} \\ \mathrm{e}_1 & : & \mathrm{have(y,z)} \\ \mathrm{e}_2 & : & \mathrm{on(x,z)} \end{bmatrix}$$

Let us in addition assume that brocolli is food, that is, (16) holds.

(16) For any a, brocolli(a) $\sqsubseteq$ food(a)

Now let us introduce two topoi, τ_1 and τ_2. We represent their foregrounds in (17a and b) respectively.

(17)

a. $$\lambda r{:}\begin{bmatrix}\text{x:}\mathit{Ind}\\ \text{c}_1\text{:food(x)}\\ \text{y:}\mathit{Ind}\\ \text{c}_2\text{:child(y)}\\ \text{z:}\mathit{Ind}\\ \text{c}_3\text{:plate(z)}\\ \text{e}_1\text{:have(y,z)}\\ \text{e}_2\text{:on(x,z)}\end{bmatrix} . \begin{bmatrix}\text{e} \ : \ \text{eat}(r\text{.y}, r\text{.x})\end{bmatrix}$$

b. $$\lambda r{:}\begin{bmatrix}\text{x:}\mathit{Ind}\\ \text{c}_1\text{:food(x)}\\ \text{y:}\mathit{Ind}\\ \text{c}_2\text{:child(y)}\\ \text{e:love(y,x)}\end{bmatrix} . \begin{bmatrix}\text{e} \ : \ \text{eat}(r\text{.y}, r\text{.x})\end{bmatrix}$$

(17a) associates the type of situation where a child has food on her plate with the type of situation where the child eats that food. This topos is naturally associated with a deontic condition, that is, a child is obliged to create a situation of the type returned by the topos, to eat the food on her plate. (17b) associates the type of situation where there is food which the child loves with the type of situation where the child eats that food. This topos is naturally associated with what we might call a *bouletic* condition, that is, we can use the topos to reason that the child has a desire to create a situation of the type returned by the topos, that is, the child wants to eat the food. This involves a kind of condition which we have not talked about yet which associates types with mental states rather than actions. We will discuss this more in Section 6.3.

The type corresponding to *Mary should eat her brocolli* based on these resources could be either of the types in (18), where T_{broc} is (15) and τ_1 and τ_2 are (17a and b) respectively.

(18) a. nec($\begin{bmatrix}\text{e:eat}(m, b)\end{bmatrix}$, T_{broc}, τ_1)

b. nec($\begin{bmatrix}\text{e:eat}(m, b)\end{bmatrix}$, T_{broc}, τ_2)

We can now check the witness conditions in (13). Any s which is of the type (18a) has to fulfil the conditions in (19).

(19) a. $s : T_{\text{broc}}$

b. $T_{\text{broc}} \sqsubseteq \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{c}_1\text{:food(x)} \\ \text{y}:\textit{Ind} \\ \text{c}_2\text{:child(y)} \\ \text{z}:\textit{Ind} \\ \text{e}_1\text{:have(y,z)} \\ \text{e}_2\text{:on(x,z)} \end{bmatrix}$

c. $\tau_1(s) \sqsubseteq \begin{bmatrix}\text{e:eat}(m, b)\end{bmatrix}$

Assuming that s meets (19a), we can check that (19b) holds by noting that anything of the first type will also be of the second type. (In this case, the two types are identical except for (i) 'brocolli' in the first type corresponds to 'food' in the second, but we know from (16) that brocolli is food (ii) the manifest fields in the first type correspond to non-manifest fields in the second, but we know from the definition of singleton types represented by manifest fields that they are subtypes of the corresponding non-singleton type.) We can see that (19c) will hold given our characterization of τ_1 in (17) since $\tau_1(s)$ will be (20a) and given that $s : T_{\text{broc}}$, s.y will be m and s.x will be b. Thus $\tau_1(s)$ is identical with (20b).

(20) a. $\begin{bmatrix} \text{e} & : & \text{eat}(s.\text{y}, s.\text{x}) \end{bmatrix}$

b. $\begin{bmatrix} \text{e} & : & \text{eat}(m, b) \end{bmatrix}$

Thus (19) is checking that the type $\begin{bmatrix}\text{e:eat}(m, b)\end{bmatrix}$ is a subtype of itself and, of course, any type is a subtype of itself.

We can make a similar argument for (18b).

This is an inferential view of modality in the sense that the topoi, which correspond to patterns of inference, have taken over the work of the accessibility relations between possible worlds which Kratzer uses. Note that while it might appear from our formulation of the witness conditions for 'nec' and 'poss' that we have a definition of modal predicates which does not use the previous notion of modality that we had in terms of possibilities defined in varying the assignments to basic types and ptypes, this is in fact not the case since our definitions of subtyping and preclusion rely on this kind of modality. Thus these definitions have both an inferential flavour (in that they use topoi which are similar to rules of inference) and also a Kripke model flavour in that they use sets of possibilities.

While the use of topoi here gives us something corresponding to accessibility relations in Kratzer's treatment of modality in Kratzer (1977) (Kratzer, 2012, Chapter 1), it does not yet give us anything corresponding to the notion of ordering source introduced in Kratzer (1981) (Kratzer, 2012, Chapter 2) to deal with the different degrees of modality expressed in examples like

(21) a. Mary absolutely must eat her brocolli

b. Mary must eat her brocolli

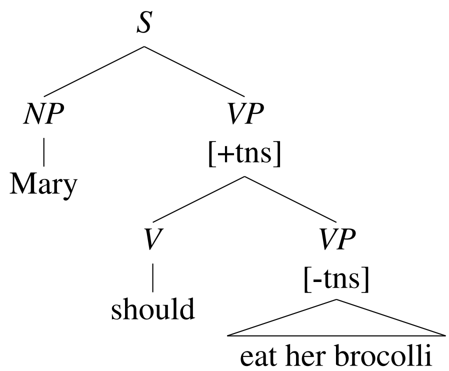
c. Mary ought to eat her brocolli

d. Mary should eat her brocolli

While it is not obvious that there is a fixed order of strength in (21) it is nevertheless the case that speakers of English will perceive differences of strength in the modalities having to do with how necessary it is for Mary to eat her brocolli. For that we need the notion of preference structure as it is discussed in Condoravdi and Cooper (????).

Let us take a look at how these ideas can be exploited in a compositional semantics. In order to do a compositional semantics for modal verbs we need to distinguish between tensed and non-tensed verbs. Our strategy for the structure of sentences with modal verbs is represented by the informal tree in (22).

(22)



That is, we treat the modal *should* as combining with a non-tensed verb phrase to form a tensed verb-phrase.

For simplicity of discussion let us consider the intransitive verb *eat* rather than the complex verb phrase *eat her brocolli*. The version of the operation 'SemIntransVerb' defined in Chapter 5 (given in Appendix B.1.4.1) yields (23) when applied to the predicate 'eat' with arity $\langle Ind \rangle$ and no restrictions introduced on the domain of the content or background conditions on the context. That is, (23) is SemIntransVerb(eat, *Ind*, *Ind*, *Rec*).

(23) $$\left[\begin{array}{lcl} \text{bg} & = & \textit{Rec} \\ \text{fg} & = & \lambda c{:}\textit{Rec}\;.\; \left[\begin{array}{lcl} \text{bg} & = & \textit{Ind} \\ \text{fg} & = & \lambda r{:}\left[\text{x}{:}\textit{Ind}\right]\;.\;\left[\begin{array}{lcl} \text{e} & : & \text{eat}(r.\text{x}) \end{array}\right] \end{array}\right] \end{array}\right]$$

This parametric content for an utterance of *eat* requires that a sentence such as *Mary eats* has a content which is the event type (24) (assuming m is Mary).

(24) $$\left[\begin{array}{lcl} \text{e} & : & \text{eat}(m) \end{array}\right]$$

This type does not require any relationship between the eating event and the utterance event. We therefore conclude that this corresponds best to a tenseless expression. It says nothing about when an event of this type needs to occur. Note that this is something that is natural in a system based on types whereas in a semantics based on the kind of tense operators we find in tense logic it is not so straightforward to represent that content of a non-tensed utterance. How could we modify the type in (24) to represent the relationship of the eating event to some particular speech event, s? In a simple-minded tense system there are basically three possibilities. The eating event is either required to be simultaneous with s, prior to s or after s. We model this by creating event types for events which have two components, the speech event, s and the eating event. The types are given in (25a–c) corresponding to *Mary eats*, *Mary ate* and *Mary will eat*, respectively.

(25)

a. $$\left[\begin{array}{lcl} \text{s-event}{=}s & : & \textit{SEvent} \\ \text{e} & : & \text{eat}(m) \end{array}\right]$$

b. $$\left[\begin{array}{lcl} \text{e} & : & \text{eat}(m) \end{array}\right]^\frown\left[\begin{array}{lcl} \text{s-event}{=}s & : & \textit{SEvent} \end{array}\right]$$

c. $$\left[\begin{array}{lcl} \text{s-event}{=}s & : & \textit{SEvent} \end{array}\right]^\frown\left[\begin{array}{lcl} \text{e} & : & \text{eat}(m) \end{array}\right]$$

Of course, we would expect an actual tense and aspect system for a natural language to involve more complex types than this, for example, allowing partial overlap between the eating event and the speech event. Our aim here is not to develop a realistic account of tense but rather to show how we can distinguish between tensed and tenseless contents in the kind of system we are proposing. The types in (25) can be derived from (24) by tense operators which take a speech event and a type as arguments and return a new type. These operators are defined in (26).

(26) If s : *SEvent* and T is a type, then

1. $\text{pres}(s)(T) = T \mathbin{\dot{\wedge}} \left[\ \text{s-event}{=}s \quad : \quad \textit{SEvent}\ \right]$
2. $\text{past}(s)(T) = T^\frown\left[\ \text{s-event}{=}s \quad : \quad \textit{SEvent}\ \right]$
3. $\text{fut}(s)(T) = \left[\ \text{s-event}{=}s \quad : \quad \textit{SEvent}\ \right]^\frown T$

In a more complete treatment of tense we might want to generalize these operators so that they can relate types to other kinds of events in addition to speech events in order to be able to deal with embedded tenses and phenomena like the historic present (as in *So I was in the pub and this man **comes** up to me . . .*).

In addition to non-tensed contents for verbs, as illustrated by the result of applying 'SemIntransVerb' given in (23), we will also have tensed contents for verbs. Thus in addition to 'SemIntransVerb' we will also have 'SemIntransVerb$_\alpha$' where α is one of 'pres', 'past' and 'fut'. These functions will return a function from speech events to parametric contents as given by the example in (27).

(27) SemIntransVerb$_\alpha$(eat, *Ind*, *Ind*, *Rec*) =

$$\lambda s{:}\textit{SEvent}\,.\left[\begin{array}{lll}\text{bg} & = & \textit{Rec} \\ \text{fg} & = & \lambda c{:}\textit{Rec}\,.\left[\begin{array}{lll}\text{bg} & = & \textit{Ind} \\ \text{fg} & = & \lambda r{:}\left[\text{x:}\textit{Ind}\right]\,.\,\alpha(s)(\left[\ \text{e} \quad : \quad \text{eat}(r.\text{x})\ \right])\end{array}\right]\end{array}\right]$$

This indicates that the contents of tensed expressions depend on speech event in a way that non-tensed expressions do not.

We now turn our attention to how information about tense plays a role in sign types. Recall that in Chapter 3 we defined *Sign* as a recursive type whose witness condition is as in (28). (See also Appendix B.1.)

(28) $\sigma : \textit{Sign}$ iff $\sigma : \left[\begin{array}{lll}\text{s-event} & : & \textit{SEvent} \\ \text{syn} & : & \textit{Syn} \\ \text{cnt} & : & \textit{Cnt}\end{array}\right]$

Here the type *Syn* (for "syntax") was defined as in (29).

(29) $\left[\begin{array}{lll}\text{cat} & : & \textit{Cat} \\ \text{daughters} & : & \textit{Sign}^*\end{array}\right]$

Now we are going to add a further field to this type to indicate whether a sign is tensed or non-tensed. The new definition of *Syn* is given in (30).

(30) $\left[\begin{array}{lll} \text{cat} & : & \textit{Cat} \\ \text{tns} & : & \textit{Bool} \\ \text{daughters} & : & \textit{Sign}^* \end{array}\right]$

The definitions of the category sign types in Chapter 3 (see Appendix B.1) for *S*, *V* and *VP* can remain the same, since these categories are underspecified for tense; they can be either tensed or non-tensed. We will use (31a) to represent the type (31b) and (31c) to represent the type (31d) and we will do similarly for *VP* and *S*.

(31) a. $\underset{[+\text{tns}]}{V}$

b. $\textit{Sign} \mathbin{\dot{\wedge}} \left[\begin{array}{lll} \text{syn} & : & \left[\begin{array}{lll} \text{cat=v} & : & \textit{Cat} \\ \text{tns=1} & : & \textit{Bool} \end{array}\right] \end{array}\right]$

c. $\underset{[-\text{tns}]}{V}$

d. $\textit{Sign} \mathbin{\dot{\wedge}} \left[\begin{array}{lll} \text{syn} & : & \left[\begin{array}{lll} \text{cat=v} & : & \textit{Cat} \\ \text{tns=0} & : & \textit{Bool} \end{array}\right] \end{array}\right]$

We will assume that the categories *NP*, *Det* and *N* are universally untensed[8] and therefore take *NP* to be the type (32) and similarly for *Det* and *N*.

(32) $\textit{Sign} \mathbin{\dot{\wedge}} \left[\begin{array}{lll} \text{syn} & : & \left[\begin{array}{lll} \text{cat=np} & : & \textit{Cat} \\ \text{tns=0} & : & \textit{Bool} \end{array}\right] \end{array}\right]$

We now define tensed versions of the universal resource for lexical sign type construction, $\text{Lex}_{\text{IntransVerb}}$ as defined in Chapter 5 (also in Appendix B.1.4.1). Letting α stand for 'past', 'pres' or 'fut' we characterize $\text{Lex}_{\text{IntransVerb}_\alpha}$ as in (33).

[8]This is something of an open question. See Tonhauser (2007) for discussion.

(33) $\text{Lex}_{\text{IntransVerb}_\alpha}(T_{\text{phon}}, p, T_{\text{arg}}, T_{\text{restr}}, T_{\text{bg}})$, where T_{phon} is a phonological type, p is a predicate with arity $\langle T_{\text{arg}}\rangle$, $T_{\text{restr}} \sqsubseteq T_{\text{arg}}$ and T_{bg} is a record type is defined as

$$\text{Lex}(T_{\text{phon}}, \textit{VP}) \mathbin{\dot{\wedge}} \begin{bmatrix} \text{s-event:}\textit{SEvent} \\ \text{syn:}\begin{bmatrix}\text{tns=1:}\textit{Bool}\end{bmatrix} \\ \text{cnt=SemIntransVerb}_\alpha(p,\ T_{\text{arg}},\ T_{\text{restr}},\ T_{\text{bg}})(\text{s-event}):\textit{PPpty} \end{bmatrix}$$

We will use ‘$\text{Lex}_{\text{IntransVerb}}$’ (without the α) to construct sign types for non-finite verbs characterized by (34).

(34) $\text{Lex}_{\text{IntransVerb}}(T_{\text{phon}}, p, T_{\text{arg}}, T_{\text{restr}}, T_{\text{bg}})$, where T_{phon} is a phonological type, p is a predicate with arity $\langle T_{\text{arg}}\rangle$, $T_{\text{restr}} \sqsubseteq T_{\text{arg}}$ and T_{bg} is a record type is defined as

$$\text{Lex}(T_{\text{phon}}, \textit{VP}) \mathbin{\dot{\wedge}} \begin{bmatrix} \text{syn:}\begin{bmatrix}\text{tns=0:}\textit{Bool}\end{bmatrix} \\ \text{cnt=SemIntransVerb}_\alpha(p,\ T_{\text{arg}},\ T_{\text{restr}},\ T_{\text{bg}}):\textit{PPpty} \end{bmatrix}$$

We now turn our attention to the modal verbs. The parametric content of a modal verb (such as *should*) is a function which requires a background with a modal base (a type) and a topos. Given such a background this function returns a function from properties (such as *eat*) to properties (such as *should eat*). We define ‘$\text{SemModalVerb}_{\text{nec}}$’ and ‘$\text{SemModalVerb}_{\text{poss}}$’ as (35a and b) respectively.

(35) a. $\left[\begin{array}{lll} \text{bg} & = & \left[\begin{array}{lll} \text{base} & : & \textit{Type} \\ \text{topos} & : & \textit{Topos} \end{array}\right] \\ \text{fg} & = & \lambda c{:}\left[\begin{array}{l} \text{base:}\textit{Type} \\ \text{topos:}\textit{Topos} \end{array}\right] . \\ & & \left[\begin{array}{lll} \text{bg} & = & \textit{Ppty} \\ \text{fg} & = & \lambda P{:}\textit{Ppty} . \\ & & \left[\begin{array}{lll} \text{bg} & = & \textit{Ind} \\ \text{fg} & = & \lambda r{:}\left[\text{x:}\textit{Ind}\right] . \\ & & \left[\begin{array}{lll} \text{e} & : & \text{nec}(P.\text{fg}(r), c.\text{base}, c.\text{topos}) \end{array}\right] \end{array}\right] \end{array}\right] \end{array}\right]$

b. $\left[\begin{array}{lll} \text{bg} & = & \left[\begin{array}{lll} \text{base} & : & \textit{Type} \\ \text{topos} & : & \textit{Topos} \end{array}\right] \\ \text{fg} & = & \lambda c{:}\left[\begin{array}{l} \text{base:}\textit{Type} \\ \text{topos:}\textit{Topos} \end{array}\right] . \\ & & \left[\begin{array}{lll} \text{bg} & = & \textit{Ppty} \\ \text{fg} & = & \lambda P{:}\textit{Ppty} . \\ & & \left[\begin{array}{lll} \text{bg} & = & \textit{Ind} \\ \text{fg} & = & \lambda r{:}\left[\text{x:}\textit{Ind}\right] . \\ & & \left[\begin{array}{lll} \text{e} & : & \text{poss}(P.\text{fg}(r), c.\text{base}, c.\text{topos}) \end{array}\right] \end{array}\right] \end{array}\right] \end{array}\right]$

The type, *Modal*, of modal parametric contents, that is, a type of objects like those in (35), is given in (36).

(36) $\left[\begin{array}{lll} \text{bg=}\left[\begin{array}{l} \text{base:}\textit{Type} \\ \text{topos:}\textit{Topos} \end{array}\right] & : & \textit{Type} \\ \text{fg} & : & (\text{bg}\rightarrow(\textit{Ppty}\rightarrow\textit{Ppty})) \end{array}\right]$

We will introduce a syntactic category for modal verbs, ‘vm’. Thus we will now characterize the type, *Cat* as in (37).

(37) s, np, det, n, v, vp, vm : *Cat*

We will use the symbol $\underset{[+M]}{V}$ to represent the type (38).

(38) $\textit{Sign} \mathbin{\dot{\wedge}} \left[\begin{array}{lll} \text{syn} & : & \left[\begin{array}{lll} \text{cat=vm} & : & \textit{Cat} \end{array}\right] \end{array}\right]$

We can now characterize a universal resource, $\text{Lex}_{\text{ModalV}}$, for creating lexical sign types for modal verbs. This is done in (39).

(39) If T_{phon} is a phonological type and p is either 'nec' or 'poss',
then $\text{Lex}_{\text{ModalV}}(T_{\text{phon}}, p)$
is defined as
$\text{Lex}(T_{\text{phon}}, \underset{[+M]}{V}) \mathbin{\dot\wedge}$

$$\left[\begin{array}{l}\text{s-event:}\mathit{SEvent}\\ \text{syn:}\left[\text{tns=1:}\mathit{Bool}\right]\\ \text{cnt=}\left[\begin{array}{l}\text{bg=}\left[\begin{array}{l}\text{base:}\mathit{Type}\\ \text{topos:}\mathit{Topos}\end{array}\right]\\ \text{fg}=\lambda c\text{:}\left[\begin{array}{l}\text{base:}\mathit{Type}\\ \text{topos:}\mathit{Topos}\end{array}\right] .\ \text{pres(s-event)(SemModalVerb}_p\text{.fg}(c))\end{array}\right]\text{:}\mathit{Modal}\end{array}\right]$$

The lexical resources for English can now tell us that *should* is a modal verb of necessity, as in (40).

(40) $\text{Lex}_{\text{ModalV}}$("should", nec)

Finally, English resources will need to include the tense and modal sensitive phrase structure rules in (41) (using the abbreviatory conventions of Appendix B.2.4).

(41) a. $\underset{[+\text{tns}]}{S} \longrightarrow NP\ \underset{[+\text{tns}]}{VP} \mid NP'@VP'$

b. $\underset{[+\text{tns}]}{VP} \longrightarrow \underset{[+M]}{V}\ \underset{[-\text{tns}]}{VP} \mid V'@VP'$

6.3 Intensionality without possible worlds

In Section 6.1 we discussed problems that have to do with individuating and counting possible worlds. Here we discuss well-known problems that arise when you consider propositions to be the sets of possible worlds[9] which make them true. The central problem is that the sets of possible worlds provide a too coarse-grained analysis of propositions. There are intuitively distinct propositions which are true in the same sets of possible worlds. Standard examples of this are mathematical propositions. Mathematical propositions are not contingent, that is, they are either true in every possible world or false in every possible world. The view of propositions as sets of possible worlds has the consequence that there are only two mathematical propositions: the necessarily true proposition and the necessarily false proposition. It seems unintuitive to reduce a rich field of continuing investigation where new "propositions" are still being discovered and proved or disproved to a field where just two propositions are being discussed. Clearly,

[9] Or, if we are concerned with tensed propositions, sets of pairs of possible worlds and times.

mathematics involves a different intuitive notion of proposition that is not modelled by a set of possible worlds. One might be tempted to think that this is a problem about mathematics rather than natural language and that for normal every day dialogue we can ignore this problem. Perhaps we just do not normally talk about necessary propositions or at least what we think of as being necessarily true is in fact relativized in the way that we discussed above in relation to Kratzer's semantics for modality. This is a dangerous route to pursue, not least perhaps because, although many of us do not spend a lot of our time talking about mathematical propositions, we are nevertheless able to express mathematical propositions in natural language and to ignore them would be to rule out something that is part of linguistic activity. There are many of us who are not mathematicians who can nevertheless understand that there is a difference in the content of the two examples in (42).

(42) a. Andrew Wiles proved that two plus two equals four

b. Andrew Wiles proved that Fermat's last theorem is true

If the correct notion of proposition for natural language was that propositions are sets of possible worlds then we should have difficulty in distinguishing the content of these two sentences.

There are non-mathematical candidates for propositions that would be true in all possible worlds. King (2014) points to examples like (43).

(43) a. Bachelors are unmarried

b. Brothers are male siblings

These are examples of what are sometimes called analytic sentences, true in virtue of their meaning. Despite the considerable difficulties with the notion of analyticity (see Rey, 2015, for discussion), it is nevertheless hard to think of a possible world where one of these sentences is true and the other is false. Yet they seem to correspond to different propositions. It does not seem attractive to say that all analytic sentences express the one and only analytic proposition (which in addition is identical with the true mathematical proposition).

There are also examples of sentences, such as those in (44), which we can argue that they express different propositions although they are true in the same possible worlds.

(44) a. Kim sold *Syntactic Structures* to Sam

b. Sam bought *Syntactic Structures* from Kim

An early reference to the equivalence relationship between *buy* and *sell* in the linguistic literature is Fillmore (1970) where it is stated:

> There are no situations that can in themselves be distinguished as buying situations or selling situations; but the choice of one or another of these verbs seems to make it possible to speak of a buying/selling transaction from one of the participant's point of view.

In our terms we would want to say that the ptypes buy(a,b,c) and sell(c,b,a) are distinct types which have the same witnesses. In terms of propositions as sets of possible worlds we would be committed to claim that these sentences express the same proposition.

The problem is not just a matter of what we intuitively consider to be distinct propositions. It has consequences for the truth of sentences with sentential complements after verbs like *believe* and *know*, the verbs of propositional attitude. If we analyze these verbs in terms of relations between individuals and propositions and we treat propositions as sets of possible worlds then for some individual, a, if a believes/knows p and p is logically equivalent to q (that is, is true in the same possible worlds which in turn means that p and q are the same proposition) then a believes/knows q. This has the unfortunate consequence that once you know one logical truth you know them all. So, for example, somebody who knows that the sum of 2 and 2 is 4 also knows any other mathematical truth (since they are all the same proposition), as well as any analytic truth and any logically valid truths. The problem extends beyond propositions that are true in all, or no, possible worlds. For any two propositions that are true in the same possible worlds (that is, are logically equivalent) if you know or believe one of them then you also know or believe the other. It interacts with the idea (originally advanced by Kripke, 1972) that proper names should be rigid designators, that is, that they should have the same denotation in every possible world. One of the puzzles goes back to discussion by Frege (1892). In the ancient world people believed that the morning star and the evening star were distinct heavenly bodies, whereas they are in fact both the planet Venus. The "morning star" had the name *Phosphorus* and the "evening star" had the name *Hesperus*. If both these names refer to the same planet Venus in all possible worlds then *Phosphorus rose in the morning* expresses the same proposition as *Hesperus rose in the morning*, that is, the two sentences are true in the same possible worlds, though they are not true in all possible worlds. Yet it seems reasonable to say that the Ancients believed that Phosphorus rose in the morning but that they did not believe that Hesperus rose in the morning. Frege's original puzzle, which is also problematic for the view that propositions are sets of possible worlds concerned the difference between *The Ancients believed that Hesperus is Hesperus* (true if they believed in the law of self identity which they presumably did) and *The Ancients believed that Hesperus is Phosphorus* (false, since it was an astronomical discovery that both Hesperus and Phosphorus denote the planet Venus). Yet both *Hesperus is Hesperus* and *Hesperus is Phosphorus* represent the same proposition, the one that is true in all possible worlds. As we noted in Chapter 4 this problem is related to Kripke's Paderewski puzzle which we discussed there and we will build on our analysis of proper names in that chapter in our analysis of the attitudes in this chapter.

The example of the equivalence of *buy* and *sell* may initially seem like an argument for the straightforward possible worlds approach when we consider propositional attitudes like *believe* and *know*. It seems impossible that any rational agent who believes or knows one of (44) would not know or believe the other. However, there are other attitude predicates where it does seem feasible to make the distinction. The sentences in (45) do not seem to be contradictory.

(45) a. Chris was happy that Kim bought *Syntactic Structures* from Sam

b. Chris was not happy that Sam sold *Syntactic Structures* to Kim

There are other non-attitude predicates which also make the distinction. For example, in Sweden it is illegal to buy sex but not illegal to sell sex which has important consequences for who gets punished in a situation where sex is bought and sold. Thus the sentences in (46) are consistent when considering Swedish law.

(46) a. It was illegal that Kim bought sex from Sam

b. It was not illegal that Sam sold sex to Kim

These problems have been well known since the early days of formal semantics. There is an excellent overview of the discussion up to the end of 1970's in Dowty *et al.* (1981), 170ff. Partee (1979) provides an important account of relevant issues. For a modern update of Partee's view see Partee (2014). For some modern philosophical views of propositions which go in somewhat similar directions to the proposals here, linking propositions to perception and action, see King *et al.* (2014).

Our basic strategy here is to replace the notion of propositions as sets of possible worlds with the notion of propositions as types, which goes back to work in intuitionistic logic (see discussion by Ranta, 1994, for a relation of this idea to linguistic semantics, and Wadler, 2015, for an overview of the history of the idea from the perspective of logic and computer science). There is a more sophisticated view of propositions in TTR which was advanced by Ginzburg (2012) and used, for example, in Cooper *et al.* (2015). This is that we should regard propositions as pairs of a situation and a type (that is, a record with two fields). This is the notion of Austinian proposition which goes back to Barwise and Perry (1983) who coined the term because of the proposal in Austin (1961) that propositions should incorporate the part of the world which they are true (or false) of. Both of these notions of proposition exploit the intensionality of types, the fact that you can have two distinct types with the same set of witnesses. A type used as a proposition is true just in case there is something of the type. This makes types as propositions parallel to what was called a Russellian proposition in Barwise (1989), Chap. 11. An Austinian proposition is true just in case the situation in the proposition is of the type of the proposition. An Austinian proposition is a way of reifying a judgement, that is, it gives us an object in our type theoretic universe which corresponds to the act of judging a particular situation to be of a type (a record of such a judgement). This means that if a Russellian proposition is true then there is an Austinian proposition containing the same type which is true. If an Austinian proposition is true then the corresponding Russellian proposition is true. If a Russellian proposition is false then any Austinian proposition containing the same type is also false. However, if an Austinian proposition is false, then we cannot conclude from this either the truth or falsity of the corresponding Russellian proposition. We know that the particular situation in the Austinian proposition is not of the type in the Austinian proposition but this tells us nothing about whether there is some other situation of the type.

Neither "proposition" nor "Russellian proposition" are technical terms in TTR. This is because we can judge any type to be non-empty ("true") or empty ("false") and thus any type can be used as a proposition. In practice, however, we will take record types (intuitively, types of situations) to be what corresponds to the intuitive notion of propositions that can be expressed in natural language. The simplest theory of verbs of propositional attitude like *believe* and *know* on this kind of view would be that they correspond to predicates which express relations between individuals and record types, that is, there are predicates 'believe' and 'know' with arity $\langle$*Ind*,*RecType*$\rangle$. This means that we will have a ptype like (47) where a is an individual and T is a record type.

(47) believe(a, T)

What does it mean for this type to be non-empty? We will say that it involves finding a match, in the sense introduced in Chapter 4, for T in a's long term memory. In the terms introduced in Chapter 4 this means that if r is a's total information state, then a's long term memory will be r.ltm, which is a record type, a type representing how the world would be if a's long term memory were true. Thus we are matching the type T, a record type which is the second argument of 'believe', against another record type corresponding to a's long term memory. Note that according to the proposal for matching in Chapter 4 this involves finding a relabelling for the flattened version of T, $\varphi(T)$. The match obtains if there is a relabelling, η, of $\varphi(T)$, such that $\varphi(r.\text{ltm}) \sqsubseteq \eta(\varphi(T))$.

The fact that relabelling is involved in the matching process is important for the analysis of belief because it means that (48) holds.

(48) If believe(a, T) is non-empty, then for any relabelling, η, of T, believe(a, $\eta(T)$) is non-empty

This means that the choice of particular labels in a record type is not relevant when we compute whether an agent stands in the belief (or other attitude) relation to a record type. Note also that, given the way we have defined relabelling in Appendix A.14 via relabellings of the flattened type, that record types which are structured differently, as in (49a,b), will also count as relabellings of each other, in this example in virtue of the relabelling (49c).

(49)

a. $\left[\begin{array}{lll} \ell_1 & : & \left[\begin{array}{lll} \ell_2 & : & T_1 \\ \ell_3 & : & T_2 \end{array}\right] \\ \ell_4 & : & T_3 \end{array}\right]$

b. $\left[\begin{array}{lll} \ell_1 & : & T_1 \\ \ell_2 & : & \left[\begin{array}{lll} \ell_3 & : & T_2 \\ \ell_4 & : & T_3 \end{array}\right] \end{array}\right]$

c. $\ell_1.\ell_2 \rightsquigarrow \ell_1$
$\ell_1.\ell_3 \rightsquigarrow \ell_2.\ell_3$
$\ell_4 \rightsquigarrow \ell_2.\ell_4$

Thus any agent who stands in the belief-relation to (49a) will also stand in the belief-relation to (49b) and *vice versa*. The intuition here is that two agents will have the same beliefs even though they structure the information differently in their separate long term memories.

This can be contrasted with proposals for structured meanings in the possible worlds literature, starting with Lewis (1972), who based his idea on the notion of intensional isomorphism from Carnap (1956), and developed by Cresswell (1985). The idea here is that you alleviate the coarse-grainedness of the possible worlds analysis of propositions by keeping around the functions and arguments that are used to compute the set of possible worlds corresponding to a sentence during its derivation. (A computer scientist could usefully compare this notion of structured meaning to *lazy evaluation*, discussed in relation to computational semantics by van Eijck and Unger, 2010.) The structured meaning is then this semantic derivation structure which is used to calculate synonymy and as the second argument of predicates like *believe*. One problem with this approach is that sentences with radically different structure which nevertheless intuitively express the same proposition may correspond to different structured meanings. One possible example is the active and passive sentences in (50).

(50) a. Kim sold the book to Sam

b. Sam was sold the book by Kim

It is hard to think of a way in which a competent native speaker of English could believe one of these but not the other. Such examples depend very much on the way in which you analyze them and how you set up the relation between syntax and semantics. For example, if you believe that compositional semantics is not defined directly on English syntax but on a logical form derived from English syntax and you are careful to relate both sentences to the same logical form, then, of course, both sentences could be related to the same structured meaning. Another kind of example which is possibly more difficult to handle with such machinery is cases of speakers of different languages with radically different structure who nevertheless intuitively share the same belief.

This kind of theory when viewed from the perspective of the theory presented in this book presents a rather odd view of the phenomena. It first proposes a theory of propositions which is obviously too coarse-grained to model the propositional attitudes. It then tries to fix this by using the derivational structure involved in reading these propositions off the syntax of the natural language. When this turns out to be too fine-grained a wholly new representation, logical form, is introduced to fix this new problem. The status of logical form is in our terms mysterious. It is neither based on the utterance situation nor on the situation types used to construct the content associated with the utterance situation. It is an additional language introduced in order to fix problems involved in interpreting utterances directly, a language which mediates between the utterance and the content. If logical form is more amenable to semantic interpretation than natural language one might raise the question why we do not speak in logical forms rather than the way we do. It is hard to imagine what the realistic status of this intermediate language should be either in terms of the utterance situation, the type of situation associated with the content or neurological events associated with perceiving or conceiving either of these.

A second problem for the structured meaning approach is that it tells us nothing about cases where no syntactic structure is involved, for example, proper names which have the same referent like *Hesperus* and *Phosphorus* or synonymous words like *groundhog* and *woodchuck*. This is pointed out by Dowty *et al.* (1981).

The fact that matching is involved in the logic of belief rules out two important ways (relating to labelling and the internal structure of record types) in which record types could be too fine-grained to give an analysis of intuitive propositions. In general it seems preferable to start from objects that are too fine-grained since we can then set about finding ways of collapsing distinctions rather than starting out with something (like sets of possible worlds) which are not fine-grained enough and try to add things to it to make the finer distinctions.

Another advantage of this strategy is that it offers possibilities for varying the fineness of the grain for different cases. Thus while we can understand that (46) can be consistent, it is much harder to think that both of (51a,b) could be true.

(51) a. Chris believes that Kim bought sex from Sam

b. Chris does not believe that Sam sold sex to Kim

The best we can do to make sense of (51) as a pair of consistent sentences is that Chris is either irrational in her beliefs or does not have sufficient understanding of the language, or that somehow the equivalence between *buy* and *sell* has been suspended. This seems very different from (46) where the two sentences can be seen as equivalent while preserving standard meanings of *buy* and *sell*. In the case of *believe* we have suggested that the type represented by the complement has to be matched against the long-term memory of the believer in order for the sentence to be true. The kind of matching introduced in Chapter 4 involves not only relabelling but also subtyping. Suppose that Chris's long term memory is modelled by the type (52a) and that the content of an utterance of *Sam sold sex to Kim* is the type (52b).

(52)

a. $\left[\begin{array}{lll} \vdots & & \\ \mathrm{id}_i & : & \left[\begin{array}{lll} \mathrm{e} & : & \mathrm{buy(kim, sex, sam)} \end{array}\right] \\ \vdots & & \end{array}\right]$

b. $\left[\begin{array}{lll} \mathrm{e} & : & \mathrm{sell(sam, sex, kim)} \end{array}\right]$

Is there a match for (52b) in (52a)? The answer is "yes". The relevant relabelling is (53a) and the result of applying that relabelling to (52b) is (53b).

(53) a. $\mathrm{e} \rightsquigarrow \mathrm{id}_i.\mathrm{e}$

b. $\left[\begin{array}{lll} \mathrm{id}_i & : & \left[\begin{array}{lll} \mathrm{e} & : & \mathrm{sell(sam, sex, kim)} \end{array}\right] \end{array}\right]$

We can see that (52a) is a subtype of (53b) in virtue of the fact in (54) — any event of buying is also an event of selling.

(54) $\mathrm{buy(kim, sex, sam)} \sqsubseteq \mathrm{sell(sam, sex, kim)}$

In this way we can obtain the correct level of granularity for *believe*. Consider now (55a) where we have the verb *say* instead of *believe* and a situation where the actual utterance that Chris made was (55b).

(55) a. Chris said that Sam sold sex to Kim

b. Kim bought sex from Sam

Is (55a) true in this case? It seems that we answer this question differently depending on how close the match between the reported speech and the original utterance has to be for the purposes at hand. Ginzburg and Cooper (2014) treat direct quotation in terms of a similarity metric on types which is associated with the context. In different contexts we require different similarity metrics. In some contexts (56) might be considered close enough given that what Chris had said originally was (55b).

(56) Chris said, “Sam sold sex to Kim.”

This might be especially be true if Chris’s original utterance was in a language other than English. Here I would like to say that indirect speech cases like (55) also involve a similarity metric given by the context and that similarity metrics associated with indirect speech in general can be looser that those associated with direct speech where we often look not only at the content of the original utterance but also its exact form of words. So according to some similarity metrics (55a) will be true and for others it will be false. It will be true intuitively if the content of its complement is close enough for current purposes to the content of Chris’s original utterance.

We can assimilate our treatment of belief to this general treatment involving similarity metrics by defining a similarity metric that says that the type representing an agent’s long term memory is similar to the type which is the content of the belief complement if the complement content matches the long term memory type in the way we have described. We will argue below that there is an advantage to making this assimilation since the criteria we use for whether an agent has a certain belief seem to vary according to the purposes we have at hand in the current context.

One of the distinctions that it seems to be possible to make in similarity metrics seems to involve different kinds of subtyping. We have defined subtyping so that for two types, T_1 and T_2, T_1 is a subtype of T_2 just in case for any a, if $a : T_1$ then $a : T_2$ and that this holds no matter what assignment is made to basic types and ptypes (Appendix A.2). Now consider the two examples of subtyping in (57).

(57)
a. $\begin{bmatrix} \ell_1 & : & T_1 \\ \ell_2 & : & T_2 \end{bmatrix} \sqsubseteq \begin{bmatrix} \ell_1 & : & T_1 \end{bmatrix}$

b. sell(a, b, c) $\sqsubseteq$ buy(c, b, a)

(57a) holds because of the general characterization of our type theory. It is, if you like, “hard-wired” into the type theoretic system. There is no way that you could construct a type system of the kind TTR characterizes which does not require (57a). (57b), on the other hand, holds only in virtue of a “postulate” that we have added to the general system relating to the particular predicates ‘buy’ and ‘sell’. Just as Montague (1973) introduced what have come to be known as meaning postulates in his system as “restrict[ing] attention to those interpretations of intensional logic in which the following formulas are true”, a postulate concerning the equivalence of selling and buying events in TTR means that we are restricting attention to possibilities (assignments to basic types and ptypes) in which the equivalence holds. According to the general definitions of TTR (not including such postulates) it is possible to construct a system where the equivalence does not hold. We will refer to (57a) as an instance of *structural subtyping* and (57b) as an instance of *postulated subtyping*. It appears that natural languages can distinguish between these different kinds of subtyping in the kind of matching that is required by predicates which take types as arguments. In the case of believe(a, T) we say that this is instantiated (non-empty) just in case a's long term memory is characterized by a type which, modulo relabelling, is a subtype (either structural or postulated) of T. On the other hand, if we think of a set of laws as characterizing, among other things, a set of forbidden types of situations, then illegal(T) would be instantiated just in case T is, modulo relabelling, a structural subtype of one of the forbidden types.

The distinction between structural and postulated subtyping also gives us a clue on how to deal with groundhogs and woodchucks. Structural subtyping is hardwired into the system. Any cognitive system which implements types will also have structural subtyping, assuming TTR is the right type theory for cognitive systems. Any such system will also have the capability to include postulated subtyping. But exactly which postulates the system has is a matter of learning. Different agents will acquire different postulates depending on their experience. While it is hard to imagine a competent speaker of English not knowing the equivalence between buying and selling it is very easy to suppose that a competent speaker does not know the equivalence between woodchucks and groundhogs. Indeed it would be natural for speakers to assume that the words *woodchuck* and *groundhog* are associated with distinct types and an agent would need some kind of evidence to establish an equivalence between the types. It would be possible for an agent who has not acquired the postulates that establish the equivalence it to believe that a woodchuck is in the garden but not to believe that a groundhog is in the garden. However, an agent who has acquired the equivalence would have to believe or disbelieve both. Thus the claim in (58) seems contradictory.

(58) Kim knows that woodchucks are the same as groundhogs and believes that a woodchuck is in the garden but does not believe that a groundhog is in the garden

The only way we can make sense of Kim believing something about a woodchuck but not about a groundhog is that Kim is unaware that woodchucks and groundhogs are the same animal. Thus getting the semantics of these attitude reports right is not simply a matter of having a finegrained enough semantics to distinguish between *woodchuck* and *groundhog* but also in linking this finegrainedness to a lack of knowledge about equivalence on the agent's part.

Suppose that Kim believes a woodchuck is in the garden and does not have the postulated equivalence between woodchuck and groundhog. It would seem from what we have said above that it does not follow that Kim believes that a groundhog is in the garden, and indeed there is a sense in which this is right, if we are taking account of subtyping according to Kim's postulates. Suppose, however, that I do know that woodchucks and groundhogs are the same animal. It seems that I can truthfully report that Kim believes that a groundhog is in the garden, using my knowledge that woodchucks and groundhogs are the same, even though Kim would not herself necessarily assent to a claim: "There's a groundhog in the garden". There is a systematic ambiguity in reports of this kind as to whether the match with Kim's long term memory is computed using the postulates available in Kim's resources or the postulates available in the reporter's resources. Most of the time we do not notice this distinction because it only arises in the case where there is this particular discrepancy between the resources available to the two agents. But it is important to note that in this case there is no one answer to the question *Does Kim believe that a groundhog is in the garden?*. In one sense she does not, and in another sense she does. On the reading where the reporter uses her own postulates it seems that there is a relationship with quotation in translation. Suppose that Kim is a monolingual speaker of German and has a belief which would be reported in German as "Ein Waldmurmeltier ist im Garten". The way in which this belief should be reported in English has to depend entirely on the reporter's resources concerning the correspondences between the contents of *Waldmurmeltier*, *groundhog* and *woodchuck*.

There is a similar systematic ambiguity to that we saw with reporting beliefs about woodchucks and groundhogs in our reporting of ancient beliefs about Hesperus and Phosphorus. Did the ancients believe that Venus rose in the morning? In one sense they did not, since they did not know that the heavenly body which they called Hesperus was in fact Venus. In another sense they did, since the heavenly body which they called Hesperus is in fact (according to the reporter's resources) Venus. The change in long term memory of an ancient who learns that Hesperus and Phosphorus are identical is parallel to that discussed in relation to example (53) and subsequent examples in Chapter 4 except that two proper names are involved rather than one. The type of the ancients' long term memory in their state of ignorance could be a subtype of (59) for some natural numbers i, j, k and l.

(59) $$\begin{bmatrix} \text{id}_i : \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named(x, “Hesperus”)} \end{bmatrix} \\ \text{id}_j : \begin{bmatrix} \text{e:rise\_in\_the\_evening}(\Uparrow\text{id}_i\text{.x}) \end{bmatrix} \\ \text{id}_k : \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named(x, “Phosphorus”)} \end{bmatrix} \\ \text{id}_l : \begin{bmatrix} \text{e:rise\_in\_the\_morning}(\Uparrow\text{id}_k\text{.x}) \end{bmatrix} \end{bmatrix}$$

Upon the ancients' learning that Hesperus and Phosphorus are the same object (59) would be updated to (60a) which is identical with (60b).

(60)

a. $\begin{bmatrix} \text{id}_i : \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named(x, “Hesperus”)} \end{bmatrix} \\ \text{id}_j : \begin{bmatrix} \text{e:rise\_in\_the\_evening}(\Uparrow\text{id}_i.\text{x}) \end{bmatrix} \\ \text{id}_k : \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named(x, “Phosphorus”)} \end{bmatrix} \\ \text{id}_l : \begin{bmatrix} \text{e:rise\_in\_the\_morning}(\Uparrow\text{id}_k.\text{x}) \end{bmatrix} \end{bmatrix} \dot{\wedge} \begin{bmatrix} \text{id}_i : \begin{bmatrix} \text{x}:\textit{Ind} \end{bmatrix} \\ \text{id}_k : \begin{bmatrix} \text{x}{=}\Uparrow\text{id}_i.\text{x}:\textit{Ind} \end{bmatrix} \end{bmatrix}$

b. $\begin{bmatrix} \text{id}_i : \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named(x, “Hesperus”)} \end{bmatrix} \\ \text{id}_j : \begin{bmatrix} \text{e:rise\_in\_the\_evening}(\Uparrow\text{id}_i.\text{x}) \end{bmatrix} \\ \text{id}_k : \begin{bmatrix} \text{x}{=}\Uparrow\text{id}_i.\text{x}:\textit{Ind} \\ \text{e:named(x, “Phosphorus”)} \end{bmatrix} \\ \text{id}_l : \begin{bmatrix} \text{e:rise\_in\_the\_morning}(\Uparrow\text{id}_k.\text{x}) \end{bmatrix} \end{bmatrix}$

Note that (60) could be construed as corresponding to a state of mind where an ancient would still refer to Venus as “Hesperus” in connection with evening rising events and “Phosphorus” in connection with morning rising events even though she was aware that they were the same object. The structure of the memory associates the different name with certain types of events. This seems intuitively correct.

Recall that SemPropName in defined in Chapter 4, example (11), also Appendix B.1.4.1, as (61).

(61) $$\begin{bmatrix} \text{bg} & = & \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named}(\text{x}, T) \end{bmatrix} \\ \text{fg} & = & \lambda r{:}\ \begin{bmatrix} \text{x}:\textit{Ind} \\ \text{e:named}(\text{x}, T) \end{bmatrix} . \\ & & \qquad \lambda P{:}\textit{Ppty} \ . \ P(r) \end{bmatrix}$$

According to the account we gave in Chapter 4 (see example (52)), the type in the ‘bg’-field has to be matched against the gameboard or failing that against the long-term memory or failing that added to the gameboard before the new information can be integrated. The assumption in that discussion was that the relevant long-term memory was that of the agent integrating the utterance. Now we are raising the issue of whose long-term memory is the relevant one to check. There are three long-term memories which can be relevant in a belief report: that of the agent integrating the utterance of the report (that is, the same long-term memory as we were considering in Chapter 4), the long-term memory of the reporter and the long-term memory of the subject of the report (the “believer”). Obviously it is the information state of the agent integrating the report that we are primarily concerned with as it is this integration process which we are trying to explain. This agent does not, of course, have direct access to the long-term memories of either the reporter or the subject of the report. (The integrator’s brain is not wired to either the reporter's or the subject's brain.) However, the integrator can form views of the nature of their long-term memories using, among other things, utterances made by them or utterances made my others about them as evidence. Such information about the long-term memories of the reporter and subject can be incorporated in the integrator's long-term memory. That is, among the beliefs we have encoded in long-term memory we have beliefs concerning what others believe. Consider the type characterizing long-term memory in (62).

(62)
$$\left[\begin{array}{l}
\mathrm{id}_1:\left[\begin{array}{l}\mathrm{x}:\mathit{Ind}\\ \mathrm{e}:\mathrm{named(x, “Venus”)}\end{array}\right]\\
\mathrm{id}_2:\left[\mathrm{e}:\mathrm{rise_in_the_evening}(\Uparrow\mathrm{id}_1.\mathrm{x})\right]\\
\mathrm{id}_3:\left[\mathrm{e}:\mathrm{rise_in_the_morning}(\Uparrow\mathrm{id}_1.\mathrm{x})\right]\\
\mathrm{id}_4:\left[\begin{array}{l}\mathrm{x}:\mathit{Ind}\\ \mathrm{e}:\mathrm{named(x, “Homer”)}\end{array}\right]\\
\mathrm{id}_5{=}\left[\begin{array}{l}
\mathrm{id}_1:\left[\begin{array}{l}\mathrm{x}{=}\Uparrow^2\mathrm{id}_1.\mathrm{x}:\mathit{Ind}\\ \mathrm{e}:\mathrm{named(x, “Hesperus”)}\end{array}\right]\\
\mathrm{id}_2:\left[\mathrm{e}:\mathrm{rise_in_the_evening}(\Uparrow\mathrm{id}_1.\mathrm{x})\right]\\
\mathrm{id}_3:\left[\begin{array}{l}\mathrm{x}{=}\Uparrow^2\mathrm{id}_1.\mathrm{x}:\mathit{Ind}\\ \mathrm{e}:\mathrm{named(x, “Phosphorus”)}\end{array}\right]\\
\mathrm{id}_4:\left[\mathrm{e}:\mathrm{rise_in_the_morning}(\Uparrow\mathrm{id}_3.\mathrm{x})\right]
\end{array}\right]:\mathit{RecType}\\
\mathrm{id}_6:\left[\mathrm{e}:\mathrm{believe}(\Uparrow\mathrm{id}_4.\mathrm{x},\ \Uparrow\mathrm{id}_5)\right]
\end{array}\right]$$

[Liz' references on buy and sell]

[what happens if the embedded ltm has one and the outer one has two?]

[matching a parametric content for a proper name. How to get the case where the name is in the reporter's resources?]

[Jonathan's comments on ch 1 - reply?, believe vs. legal, matching vs relabelling, believe vs know (Austinian propositions)]

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