Modelling the Universe including Neutrinos

ASTR 403

Robin Dhillon and Yuze Zhang

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Introduction

Based on the fact that our universe is homogeneous and isotropic on large scales, the Robertson-Walker metric was developed and used to obtain the Friedmann's equation. From measurements of density parameters for different components of the universe, one can establish the Benchmark model to describe our universe with a high precision. Density parameters can also be changed to speculate different types of universe models such as the Big bounce, Big Crunch, Loitering universe, and more.

The Friedmann Equation

The Robertson-Walker metric based on the characteristics of a homogeneous and isotropic universe takes the forms below:

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2 \qquad (1)$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \qquad (2)$$

One can obtain Riemann Curvature Tensor and Ricci Scalar for the Einstein's Field equations:

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \qquad (3)$$

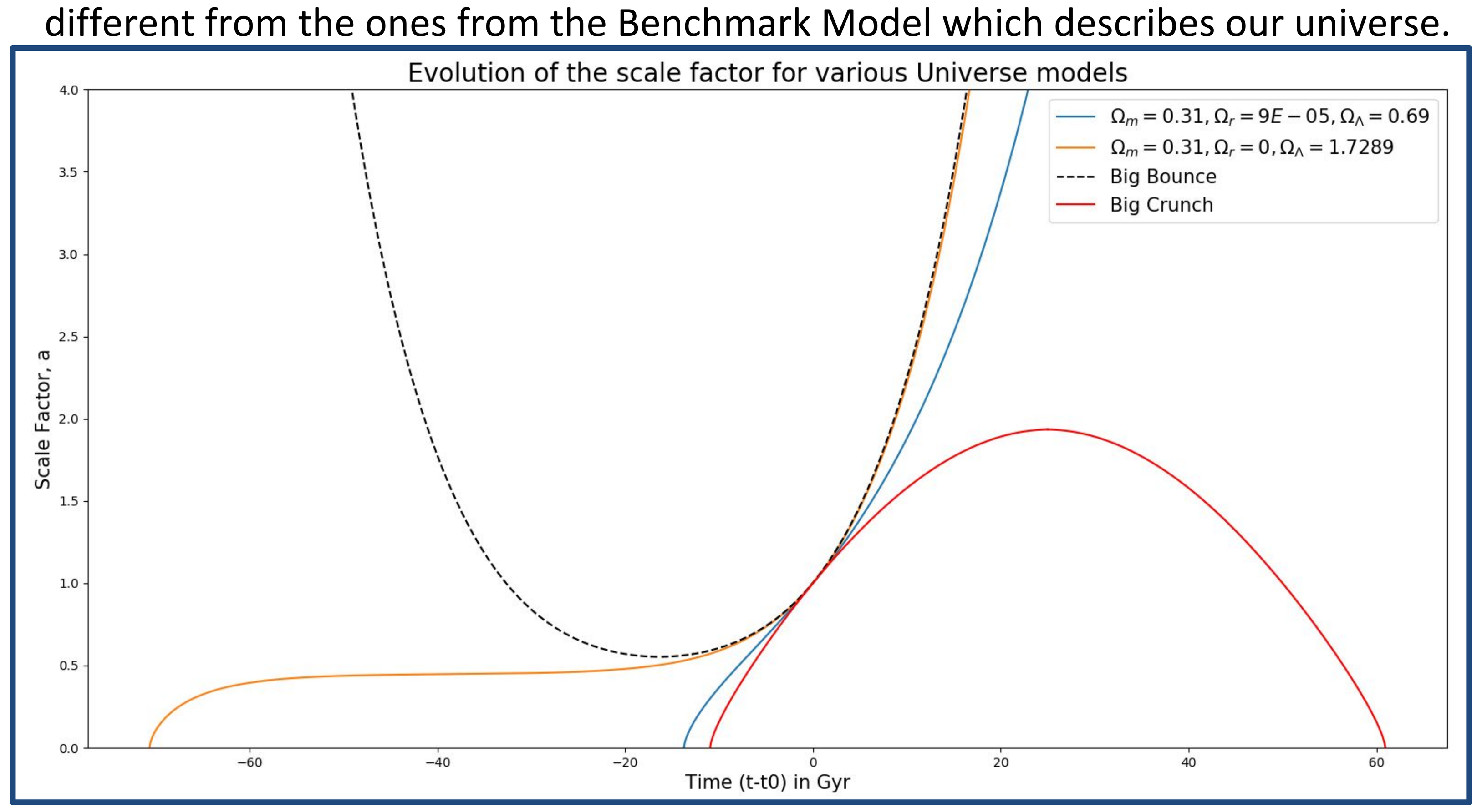
The field equation gives the relation between the density distribution and the curvature of the spacetime. From the set of equations, we can derive the Friedmann's equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \qquad (4)$$

In our universe, the Friedmann's equation can be approximated as the following:

$$H(a)^2 = H_0^2\left(\frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{\gamma,0}}{a^4} + \Omega_{\Lambda,0} + \frac{1-\Omega_{m,0}-\Omega_{\gamma,0}-\Omega_{\Lambda,0}}{a^2}\right) \qquad (5)$$

which includes the density parameters of matter, radiation, cosmological constant, and curvature. These numbers can also be set to numbers that are different from the ones from the Benchmark Model which describes our universe.



Neutrinos in the Universe

In the radiation density parameter of the Benchmark Model, both photon energy density and the neutrino energy density are considered. However, in reality, even though neutrino has negligible mass, it still is not massless. Based on observations of large scale structures of the universe, the range of the average neutrino mass ranges from 0.019eV/c^2 to 0.1eV/c^2. As a result, neutrinos are considered ultra-relativistic in early universe and not completely relativistic today. Also, neutrinos are considered as fermions, and here, one can consider neutrinos in the universe as a neutrino fermi gas. The energy density of a fermi gas (therefore the neutrinos) is:

$$\epsilon_\nu = \frac{g_s c}{h^3}\int \frac{\sqrt{p^2+m_\nu^2c^2}}{e^{\frac{pc}{k_bT_\nu}}+1}d^3p \qquad (6)$$

where the temperature of the neutrinos relates to the scale factor by

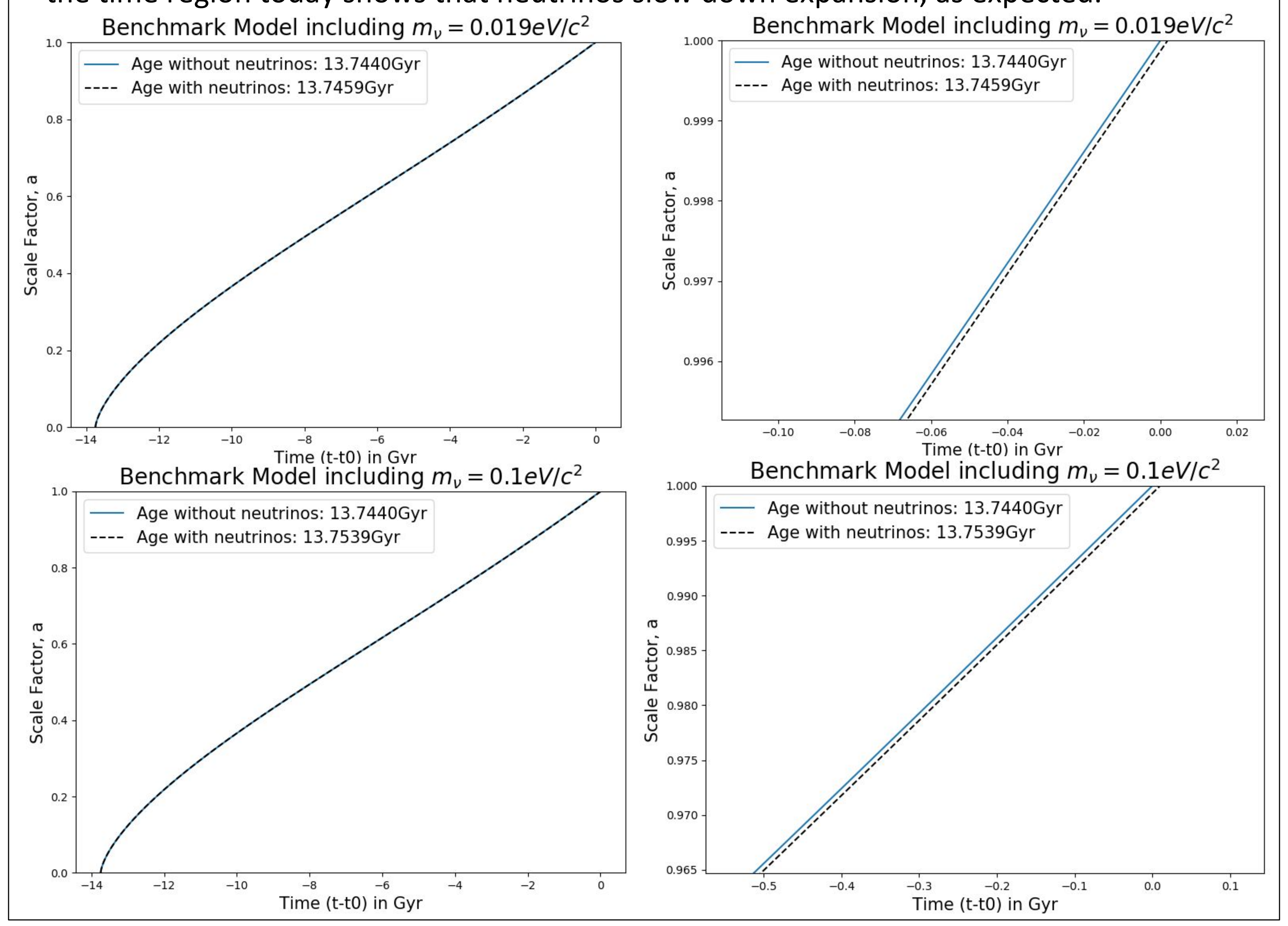
$$T_\nu(a) = \left(\frac{4}{11}\right)^{\frac{1}{3}}\frac{T_{cmb,0}}{a} = \frac{1.95K}{a} \qquad (7)$$

and since d^{3}p = 4πp^{2}dp and gs = 2 for neutrinos, equation (6) becomes

$$\epsilon_\nu = \frac{8\pi c}{h^3}\int \frac{p^2\sqrt{p^2+m_\nu^2c^2}}{e^{\frac{pca}{(1.95)k_b}}+1}dp \qquad (8)$$

Benchmark Model with Neutrino Density

Including neutrinos into the Benchmark Model gives the following plots. Zooming into the time region today shows that neutrinos slow down expansion, as expected.



Revised Friedmann's Equation with Neutrinos

Including the density parameter, which depends on the scale factor a, of the neutrinos in the Freidmann's Equation gives us a more complete model of our universe. To make sure one has the correct density parameter for curvature, the density parameter of neutrinos today is subtracted along with other density parameters from (5) in the last part of the equation. This was used for the previous figures of different neutrino masses.

$$H(a)^2 = H_0^2\left(\frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{\gamma,0}}{a^4} + \Omega_{\Lambda,0} + \Omega_\nu(a) + \frac{1-\Omega_{m,0}-\Omega_{\gamma,0}-\Omega_{\Lambda,0}-\Omega_{\nu,0}}{a^2}\right) \qquad (9)$$

Generate Universe Models

To obtain the universe model, one has to obtain the relation between the scale factor of the universe and the time. From equation (5):

$$H_0t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{m,0}}{a} + \frac{\Omega_{\gamma,0}}{a^2} + \Omega_{\Lambda,0}a^2 + \Omega_\nu(a)a^2 + (1-\Omega_{m,0}-\Omega_{\gamma,0}-\Omega_{\Lambda,0}-\Omega_{\nu,0})}} \qquad (10)$$

In this project, we used numerical integration via python to integrate both the integral of the density parameter of the neutrinos and the model of the universe. When running the code, a necessary substitution was made when integrating the density parameter of the neutrino, since the exponential part of the integrand in (8) encounters error of overflow for large numbers. One could also use natural units to circumvent this overflow, which serves the same purpose as our substitution.

Conclusions

The goal of this project was to investigate the effects that neutrinos have on the universe's expansion. In the Benchmark Model, we included neutrinos in radiation, assuming they were massless. However, they do have a mass and thus one expects that the age of the universe must be larger (slightly, if not a lot). This is because there would be more mass present which would slow down the expansion. This is what we discovered while conducting this experiment, which confirms our expectations. Furthermore, it was confirmed that the larger the mass of the neutrino, longer the age of the universe, and the density parameter corresponding to the largest and smallest masses ranges from 0.002 to 0.0004, respectively, which roughly agrees with the values provided by Ryden.

References

[1] Ryden, B., 2017. Introduction To Cosmology. 2nd ed. New York, NY: Cambridge University Press.
[2] Lattanzi, M. and Gerbino, M., 2017. Status Of Neutrino Properties And Future Prospects - Cosmological And Astrophysical Constraints. [online] arXiv.org. Available at: <https://arxiv.org/abs/1712.07109> [Accessed 2 April 2020].