Compact Closed Categories without Units

Robin Houston · March 18, 2007

In recent work (Houston et al., 2005) we have defined a notion of *semi star-autonomous category*. It is, of course, a routine matter to specialise this definition to the compact closed case. This note gives an elementary axiomatisation of semi compact closure, and shows its equivalence to the 'abstract' notion. The present definition is completely algebraic, and is perhaps easier to understand and use. The definition itself is not really new: Hines (1999, §3.5) has a similar-looking definition, which seems to be strictly weaker than the present one, and Došen and Petrić (2005) give a more general version,[1] though it appears to include redundant axioms.

Definition 1. A *category with tensor* $\mathbb{C}$ is a category equipped with a tensor product

$$\otimes : \mathbb{C} \times \mathbb{C} \to \mathbb{C},$$

together with natural isomorphisms having components

$$\begin{aligned} &\alpha_{A,B,C} : A \otimes (B \otimes C) \to (A \otimes B) \otimes C, \\ &\sigma_{A,B} : A \otimes B \to B \otimes A \end{aligned}$$

such that $\sigma^{-1}_{B,A} = \sigma_{A,B}$, and subject to the pentagon and hexagon conditions found in the usual definition of symmetric monoidal category.

Although the development is stated in terms of a symmetric tensor, it is perfectly possible – with only a little more work – to carry it through when the tensor is merely braided. The string diagrams, in particular, should make it clear which direction of braiding is required in any particular definition. Note that an additional axiom is needed in the braided case, specifically the braid dual of the second cancellation condition, and braid-dual versions of the lemmas need to be proved. We also introduce an abbreviation that will be useful in the next definition: let θ denote the unique canonical natural isomorphism with components

$$\theta_{A,B,C} : A \otimes (B \otimes C) \to (C \otimes A) \otimes B.$$

(This may be defined as either $\alpha_{C,A,B}.\sigma_{A\otimes B,C}.\alpha_{A,B,C}$ or $(\sigma_{A,C} \otimes B)\alpha_{A,C,B}(A \otimes \sigma_{B,C})$; the hexagon condition says precisely that these must be equal.)

Definition 2. A *semi compact closed category* is a category $\mathbb{C}$ with tensor, equipped with: for every object $A \in \mathbb{C}$, a *dual object* A^*, and natural transformations η^A and ε^A with components

$$\begin{aligned} \eta^A_X &: X \to X \otimes (A^* \otimes A) \\ \varepsilon^A_X &: (A \otimes A^*) \otimes X \to A \end{aligned}$$

These natural transformations are called the *unit* and *counit* of A, and are required to satisfy the four axioms shown in Fig. 1.

The plan for the rest of this paper is as follows. In §1 we develop the theory of semi compact closed categories directly from the axioms, since it is instructive to see how readily this may be done, and how similar it is to the ordinary theory of compact closure. (But see later for an alternative, indirect, approach.) §2 then shows that every semi compact closed category is (degenerately) semi star-autonomous in the sense of Houston et al. (2005).

§3 is independent of the previous sections, and shows how an arbitrary semi compact closed category may be fully and faithfully embedded in an ordinary compact closed category (which has one additional object playing the role of the unit). This embedding preserves the tensor and duality on the nose, which makes it possible to transfer most of our knowledge about compact closed categories to the unitless situation, and in particular to deduce the main results of §1.

[1] The Došen-Petrić axioms are intended to define a semi star-autonomous category, therefore assume two tensors $\otimes$ and $\invamp$, related by a linear distributivity. If one takes the two tensors to be equal, and the linear distributivity to be the ordinary associativity, then one recovers the present definition with some redundancy.

$$
\begin{tikzcd}
X \otimes Y \arrow[r, "X \otimes \eta^A_Y"] \arrow[dr, "\eta^A_{X\otimes Y}"'] & X \otimes (Y \otimes (A^* \otimes A)) \arrow[d, "\alpha_{X,Y,A^*\otimes A}"] \\
& (X \otimes Y) \otimes (A^* \otimes A)
\end{tikzcd}
\qquad
\begin{tikzcd}
(A \otimes A^*) \otimes (X \otimes Y) \arrow[d, "\alpha_{A^*\otimes A,X,Y}"'] \arrow[dr, "\varepsilon^A_{X\otimes Y}"] & \\
((A^* \otimes A) \otimes X) \otimes Y \arrow[r, "\varepsilon^A_X \otimes Y"'] & X \otimes Y
\end{tikzcd}
$$

$$
\begin{tikzcd}
A \arrow[r, "1"] \arrow[d, "\eta^A_A"'] & A \\
A \otimes (A^* \otimes A) \arrow[r, "\alpha_{A,A^*,A}"'] & (A \otimes A^*) \otimes A \arrow[u, "\varepsilon^A_A"']
\end{tikzcd}
\qquad
\begin{tikzcd}
A^* \arrow[r, "1"] \arrow[d, "\eta^A_{A^*}"'] & A^* \\
A^* \otimes (A^* \otimes A) \arrow[r, "\theta_{A^*,A^*,A}"'] & (A \otimes A^*) \otimes A^* \arrow[u, "\varepsilon^A_{A^*}"']
\end{tikzcd}
$$

Figure 1: Coherence conditions for a semi compact closed category

Figure 2: Diagrammatic notation for η and ε

1 Direct development

We shall use string diagrams (Joyal and Street, 1991), to make the calculations easier to follow. Our diagrams are to be read from left to right, and we notate η and ε as in Fig. 2. Diagrammatic forms of the axioms are shown in Figs. 3–5. The first task is to show how the duality operation can be extended to a contravariant functor, in such a way that η and ε are both dinatural in A. Given an arrow $f : A \to B$, we define $f^* : B^* \to A^*$ as shown in Fig. 6. Note that, directly from the second cancellation axiom, we have $1^*_A = 1_{A^*}$ for all $A \in \mathbb{C}$, thus our putative functor preserves identities (which is a good start). It is surprisingly complicated to prove directly that it also preserves composition, but it will be easy once we have the right lemmas.

Lemma 3. *For all X, A and $Y \in \mathbb{C}$, the following diagrams commute.*

$$
\begin{tikzcd}
(X \otimes (A \otimes A^*)) \otimes Y \arrow[r, "\alpha^{-1}_{X,A\otimes A^*,Y}"] \arrow[d, "\sigma_{A\otimes A^*,X} \otimes Y"'] & X \otimes ((A \otimes A^*) \otimes Y) \arrow[d, "X \otimes \varepsilon^A_Y"] \\
((A \otimes A^*) \otimes X) \otimes Y \arrow[r, "\varepsilon^A_X \otimes Y"'] & X \otimes Y
\end{tikzcd}
$$

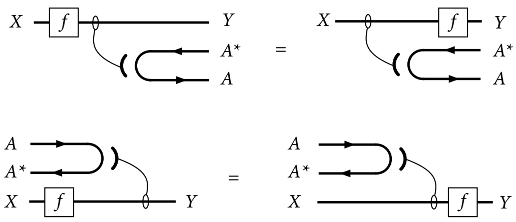


Figure 3: Diagrammatic form of the naturality conditions

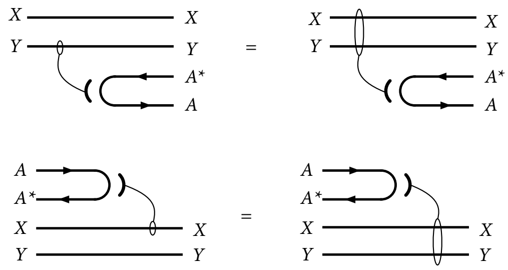


Figure 4: Diagrammatic form of the associativity conditions

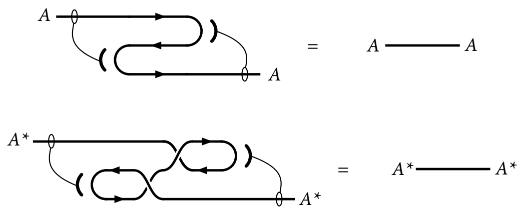


Figure 5: Diagrammatic form of the cancellation conditions

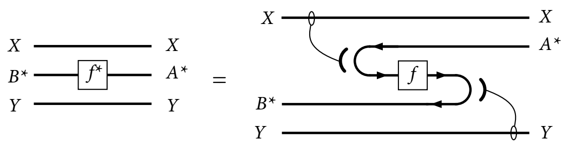
$$\begin{tikzcd}
X \otimes Y \arrow[r, "X \otimes \eta^A_Y"] \arrow[d, "\eta^A_X \otimes Y"'] & X \otimes (Y \otimes (A^* \otimes A)) \arrow[d, "X \otimes \sigma_{A^* \otimes A, Y}"] \\
(X \otimes (A^* \otimes A)) \otimes Y \arrow[r, "\alpha^{-1}_{X, A^* \otimes A, Y}"'] & X \otimes ((A^* \otimes A) \otimes Y)
\end{tikzcd}$$

Proof. The proof of the first diagram, by string diagram manipulation, is shown in Fig. 7. (Perhaps the least obvious step is the penultimate one, which uses the naturality of σ.) The second is proved by a symmetrical argument: Fig. 8 shows the diagrammatic form of its statement. □

Lemma 4. *For any objects X, A, B, Y, and arrow $f : A \to B$, the following diagram commutes. (The associativities have been suppressed to make it more comprehensible.)*

$$\begin{tikzcd}
X \otimes B^* \otimes Y \arrow[r, "X \otimes f^* \otimes Y"] \arrow[d, "\eta^A_X \otimes B^* \otimes Y"'] & X \otimes A^* \otimes Y \\
X \otimes A^* \otimes A \otimes B^* \otimes Y \arrow[r, "X \otimes A^* \otimes f \otimes B^* \otimes Y"'] & X \otimes A^* \otimes B \otimes B^* \otimes Y \arrow[u, "X \otimes A^* \otimes \varepsilon^B_Y"']
\end{tikzcd}$$

In string diagram terms, this says



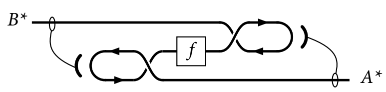


Figure 6: Given $f : A \to B$, we define $f^* : B^* \to A^*$ using this diagram

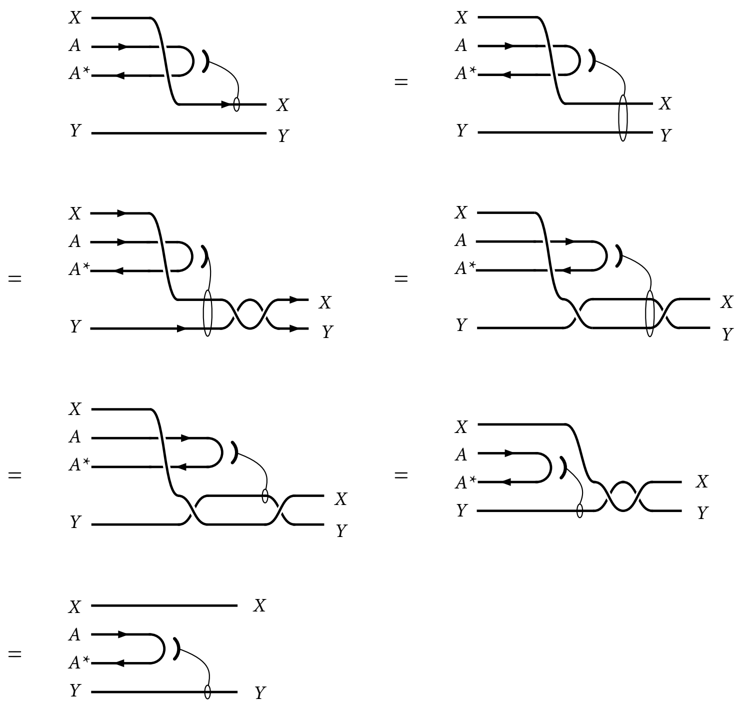


Figure 7: A diagrammatic proof of Lemma 3

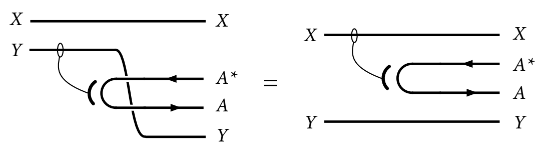


Figure 8: The second part of Lemma 3

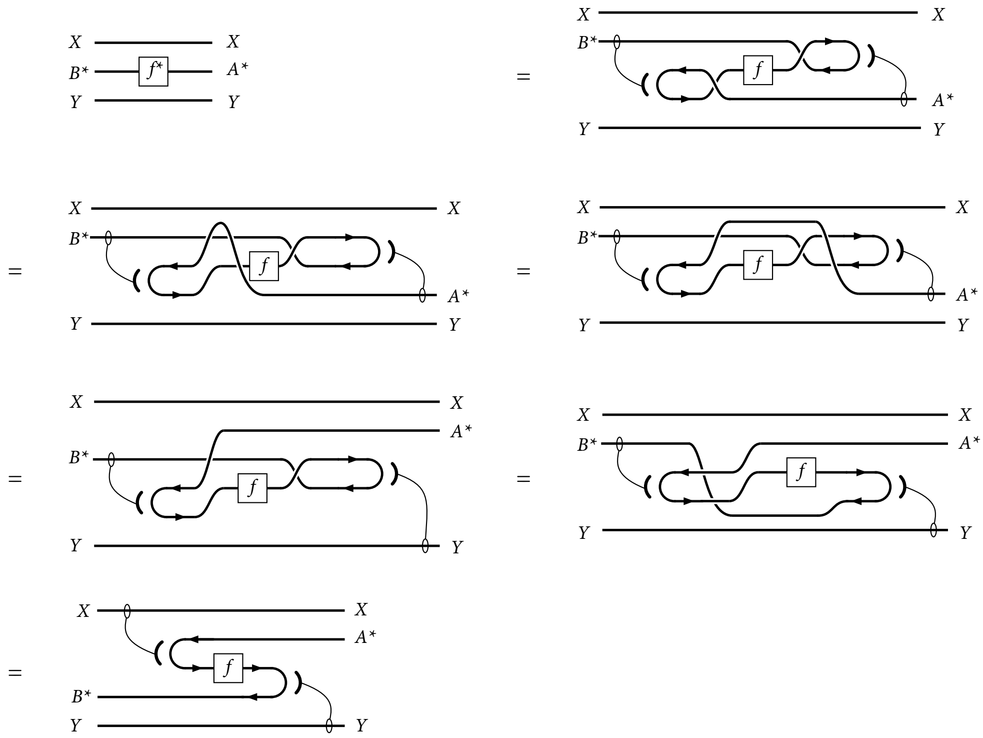


Figure 9: A diagrammatic proof of Lemma 4

Proof. The proof is again by string diagram manipulation, shown in Fig. 9. Both parts of Lemma 3 are used. □

The final lemma is not needed immediately, but this seems the right place to prove it.

Lemma 5. *For all X, A, $Y \in \mathbb{C}$, the following diagram commutes.*

$$\begin{tikzcd}
X \otimes A \otimes Y \arrow[r, "1"] \arrow[d, "\eta_X^A \otimes A \otimes Y"'] & X \otimes A \otimes Y \\
X \otimes A^* \otimes A \otimes A \otimes Y \arrow[r, "X \otimes \sigma_{A^*, A\otimes A, Y}"'] & X \otimes A \otimes A \otimes A^* \otimes Y \arrow[u, "X \otimes A \otimes \varepsilon_Y^A"']
\end{tikzcd}$$

(The associativities have again been suppressed.)

Proof. See Fig. 10. The first step uses both parts of Lemma 3.[2] □

All the hard work was in the lemmas: everything else is comparatively straightforward.

Proposition 6. *The duality preserves composition, i.e. given $f : A \to B$ and $g : B \to C$, we have $(gf)^* = f^*g^*$.*

Proof. See Fig. 11. □

Proposition 7. *The natural transformations η and ε are also dinatural in the superscript variable.*

Proof. See Fig. 12 for a proof that ε is dinatural. The proof for η may be obtained by turning the string diagrams upside down. □

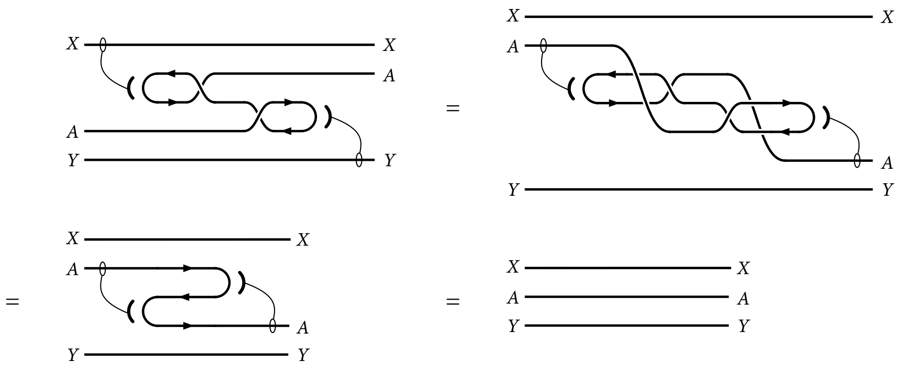


Figure 10: Proof of Lemma 5

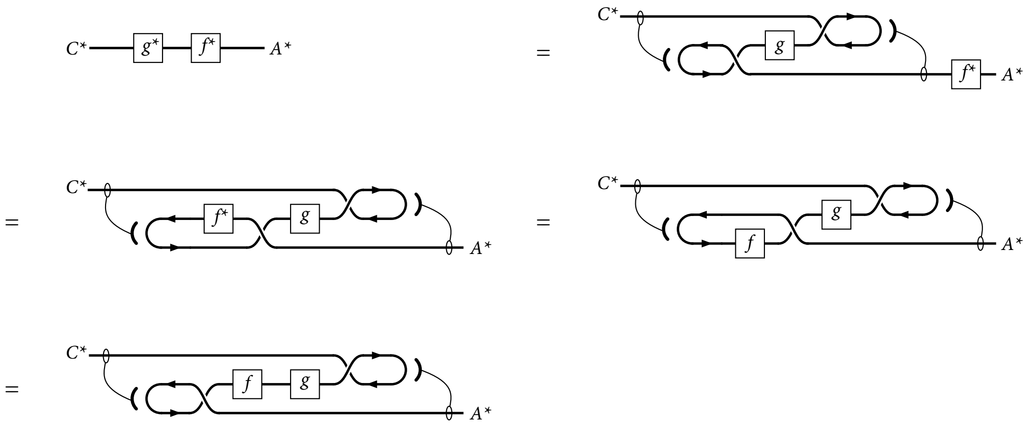


Figure 11: Proof of Prop. 6

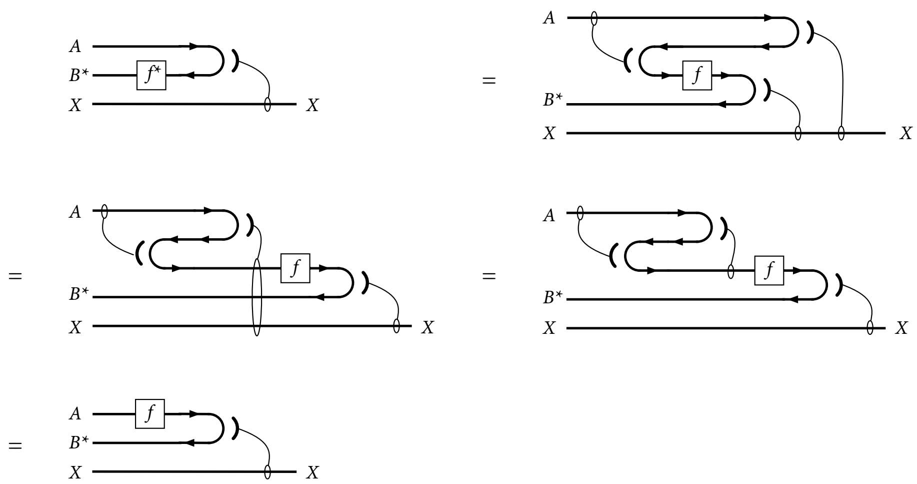


Figure 12: Proof that ε is dinatural (Prop. 7)

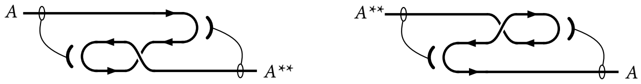


Figure 13: How to construct a natural isomorphism $A \cong A^{**}$

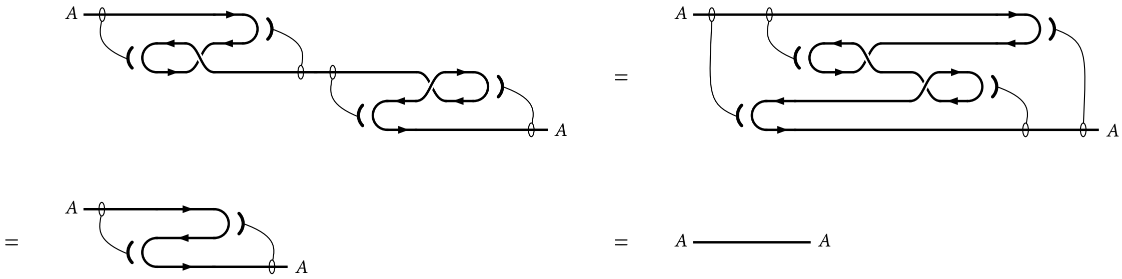


Figure 14: The maps from Fig. 13 compose to give the identity on A

Proposition 8. *There is a natural isomorphism $A \cong A^{**}$.*

Proof. Fig. 13 shows how to construct a natural isomorphism $A \cong A^{**}$. Naturality is immediate from the naturality and dinaturality of η and ε, and the naturality of σ. Figs. 14 and 15 show that these maps are indeed mutually inverse, hence determine an isomorphism. Notice that the second step in Fig. 14 uses Lemma 5.[2] □

Proposition 9. *For each object A, there is an adjunction $A \otimes - \dashv A^* \otimes -$, which determines a natural isomorphism*

$$\mathbb{C}(B \otimes A, C) \cong \mathbb{C}(A, B^* \otimes C)$$

Proof. There are obvious natural transformations

$$\begin{aligned} \gamma_{A,B,C} &: \mathbb{C}(A \otimes B^*, C) \to \mathbb{C}(A, C \otimes B), \\ \delta_{A,B,C} &: \mathbb{C}(A, B^* \otimes C) \to \mathbb{C}(B \otimes A, C) \end{aligned}$$

illustrated in Figs. 16–17. We need to show that one of these natural transformations is invertible, which we shall do by showing that they are in some sense mutually inverse. We begin by showing that the composite

[2] In the braided case, it uses the braid-dual analogue of that lemma.

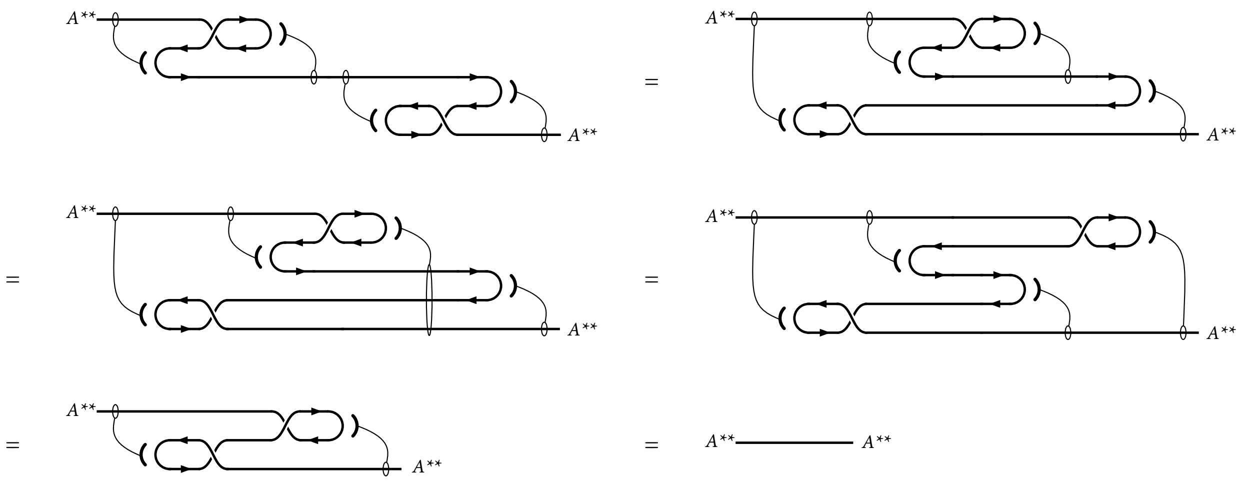


Figure 15: The maps from Fig. 13 compose to give the identity on A^{**}

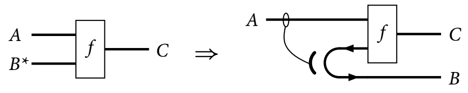


Figure 16: The natural transformation γ

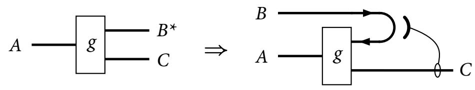


Figure 17: The natural transformation δ

$$\mathbb{C}(A \otimes B^*, C) \xrightarrow{\gamma} \mathbb{C}(A, C \otimes B) \xrightarrow{\cong} \mathbb{C}(A, C \otimes B^{**}) \xrightarrow{\cong} \mathbb{C}(A, B^{**} \otimes C) \xrightarrow{\delta} \mathbb{C}(B^* \otimes A, C) \xrightarrow{\cong} \mathbb{C}(A \otimes B^*, C)$$

is the identity (where the unlabelled isomorphisms are symmetry or involution maps). Consider some $f : A \otimes B^* \to C$: the result of applying this composite to f is shown in Fig. 18. Fig. 19 shows that this is equal to f. (The first step combines several uses of naturality and associativity conditions.) Therefore γ has a post-inverse and δ a pre-inverse.

Similarly one may take a map $g : A \to B^* \otimes C$, and apply to it the composite

$$\mathbb{C}(A, B^* \otimes C) \xrightarrow{\delta} \mathbb{C}(B \otimes A, C) \xrightarrow{\cong} \mathbb{C}(B^{**} \otimes A, C) \xrightarrow{\cong} \mathbb{C}(A \otimes B^{**}, C) \xrightarrow{\gamma} \mathbb{C}(A, C \otimes B^*) \xrightarrow{\cong} \mathbb{C}(A, B^* \otimes C)$$

as shown in Fig. 20. This is equal to g – the proof is obtained by turning all the diagrams in Fig. 19 upside down – hence γ also has a pre-inverse and δ a post-inverse. Therefore both are invertible, as claimed. □

2 The promonoidal structure

This section shows that a semi compact closed category is semi star-autonomous in the sense of Houston et al. (2005). The proof relies on the characterisation of semi monoidal categories stated in §2.2 of *op. cit.* and proved by Houston (2006), which we briefly recall here.[3]

Definition 10. Let $\mathbb{C}$ be a category with tensor. A *linear element* a of the object $A \in \mathbb{C}$ is a natural transformation with components

$$a_X : X \to X \otimes A$$

such that

$$\begin{tikzcd} X \otimes Y \arrow[r, "X \otimes a_Y"] \arrow[dr, "a_{X \otimes Y}"'] & X \otimes (Y \otimes A) \arrow[d, "\alpha_{X,Y,A}"] \\ & (X \otimes Y) \otimes A \end{tikzcd}$$

[3]The perceptive reader will note that the definitions below are not precisely identical to those of Houston et al. (2005), but they are clearly equivalent.

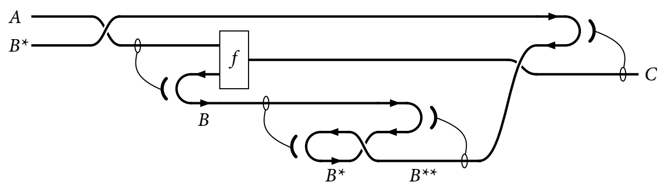


Figure 18: The result of applying γ and then δ to some $f : A \otimes B^* \to C$

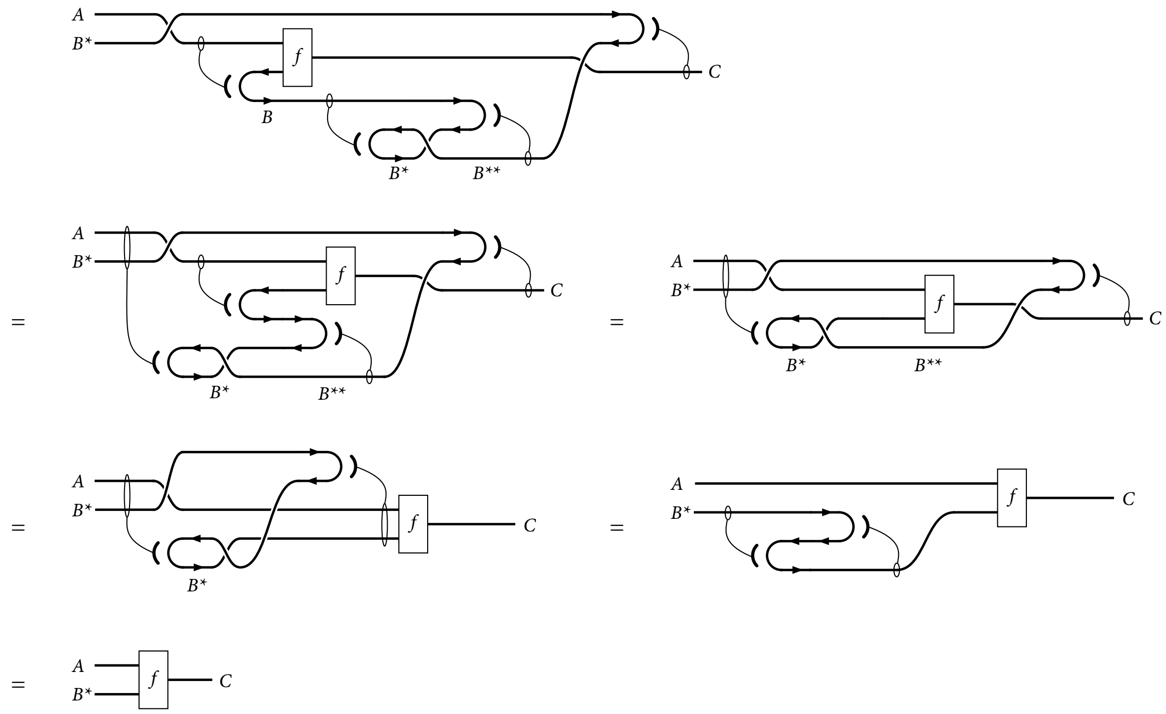


Figure 19: The map shown in Fig. 18 is equal to f

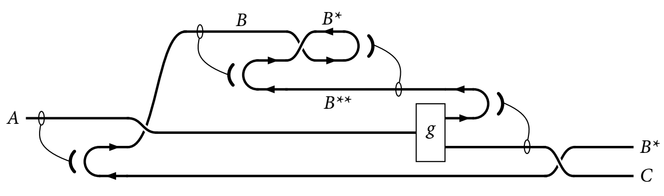


Figure 20: The result of applying δ and then γ to some $g : A \to B^* \otimes C$,

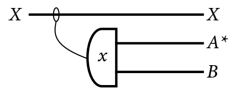


Figure 21: The diagrammatic representation of a linear element $x \in \mathrm{Lin}(A^* \otimes B)$

commutes for all $X, Y \in \mathbb{C}$.

Definition 11. Given a category $\mathbb{C}$ with tensor, define a functor

$$\mathrm{Lin} : \mathbb{C} \to \mathrm{Set}$$

as follows. For $A \in \mathbb{C}$, let $\mathrm{Lin}(A)$ be the set of linear elements of A. For $f : A \to B$ and $a \in \mathrm{Lin}(A)$, let $\mathrm{Lin}(f)(a)$ be the linear element of B with components

$$X \xrightarrow{a_X} X \otimes A \xrightarrow{X \otimes f} X \otimes B.$$

Definition 12. Suppose we have a category $\mathbb{C}$ with tensor, equipped with a functor $\multimap : \mathbb{C}^{\mathrm{op}} \times \mathbb{C} \to \mathbb{C}$ and a natural isomorphism

$$\mathrm{curry}_{A,B,C} : \mathbb{C}(B \otimes A, C) \to \mathbb{C}(A, B \multimap C)$$

with counit $e^A_B : A \otimes (A \multimap B) \to B$.

Define the natural transformation $l_{A,B} : \mathrm{Lin}(A \multimap B) \to \mathbb{C}(A, B)$ as follows: for each $x \in \mathrm{Lin}(A \multimap B)$, let $l_{A,B}(x)$ be the composite

$$A \xrightarrow{x_A} A \otimes (A \multimap B) \xrightarrow{e^A_B} B.$$

Proposition 13 (Houston et al., 2005, Prop. 2.6). *To give a symmetric semi-monoidal closed category is to give:*

- *A category $\mathbb{C}$ with symmetric tensor,*
- *A functor $\multimap : \mathbb{C}^{\mathrm{op}} \times \mathbb{C} \to \mathbb{C}$ with a natural isomorphism*

$$\mathrm{curry}_{A,B,C} : \mathbb{C}(B \otimes A, C) \to \mathbb{C}(A, B \multimap C)$$

such that the natural transformation l of Def. 12 is invertible.

With this machinery available it is easy to prove the main result of this section:

Proposition 14. *A semi compact closed category is semi star-autonomous.*

Proof. Let $\mathbb{C}$ be a semi compact closed category. By assumption it is equipped with a symmetric tensor, and if we define

$$A \multimap B := A^* \otimes B$$

then Prop.9 shows that we have a natural isomorphism

$$\delta_{A,B,C} : \mathbb{C}(B \otimes A, C) \xleftarrow{\cong} \mathbb{C}(A, B \multimap C).$$

It remains only to construct an inverse to the function $l_{A,B} : \mathrm{Lin}(A^* \otimes B) \to \mathbb{C}(A, B)$.

If we represent a linear element $x \in \mathrm{Lin}(A^* \otimes B)$ as shown in Fig. 21, note that, by the definition of δ, the arrow $l_{A,B}(x)$ is as shown in Fig. 22.

Given a map $f : A \to B$, define $l^{-1}_{A,B}(f)$ to be the natural transformation whose component at X is

$$X \xrightarrow{\eta^A_X} X \otimes (A^* \otimes A) \xrightarrow{X \otimes (A^* \otimes f)} X \otimes (A^* \otimes B).$$

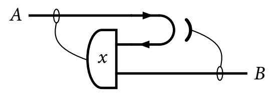


Figure 22: The diagrammatic representation of a $l_{A,B}(x)$

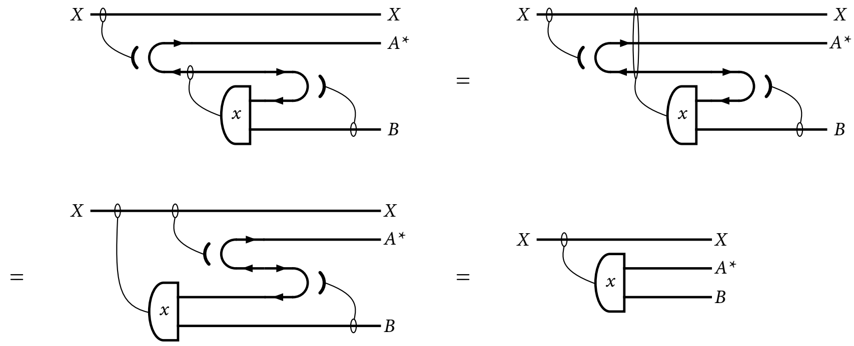


Figure 23: Proof that $l^{-1}_{A,B}(l_{A,B}(x)) = x$. (See Prop. 14)

For any $f : A \to B$, the arrow $l_{A,B}(l^{-1}_{A,B}(f))$ is the composite

$$A \xrightarrow{\eta^A_A} A \otimes (A^* \otimes A) \xrightarrow{A\otimes(A^*\otimes f)} A \otimes (A^* \otimes B) \xrightarrow{\alpha_{A,A^*,B}} (A \otimes A^*) \otimes B \xrightarrow{\varepsilon^A_B} B$$

By the naturality of α and ϵ, and the first cancellation condition, this is indeed equal to f. Conversely suppose we have a linear element $x \in \mathrm{Lin}(A^* \otimes B)$. Since it will be convenient to use a string diagram calculation here, we introduce a diagrammatic notation for this linear element, shown in Fig. 21. Now Fig. 23 shows a proof that $l^{-1}_{A,B}(l_{A,B}(x))$ is equal to x. □

3 Embedding theorem

If we wanted to add a unit object I to a semi star-autonomous category $\mathbb{C}$, we would also have to add an infinite family of other objects such as I^*, $I^* \otimes A$ for $A \in \mathbb{C}$, and so on. In the compact closed case, there is no such obstacle, since I^* is always isomorphic to I, and we may take $I^* = I$ without essential loss of generality. This raises the hope that it may always be possible to fully embed any semi compact closed category $\mathbb{C}$ into a compact closed category $\mathbb{C}'$, in such a way the objects of $\mathbb{C}'$ are essentially just the objects of $\mathbb{C}$ plus a unit object. It turns out that such an embedding is possible, as this section shows.

Recall the '**e**' construction of Joyal and Street (1993): given a monoidal category $\mathbb{C}$, the category $\mathbf{e}(\mathbb{C})$ is defined as follows. An object of $\mathbf{e}(\mathbb{C})$ is a pair (F, γ^F) of a functor $F : \mathbb{C} \to \mathbb{C}$ and a natural isomorphism with components $\gamma^F_{A,B} : F(A \otimes B) \to F(A) \otimes B$. A morphism $\delta : (F, \gamma^F) \to (G, \gamma^G)$ is a natural transformation $F \Rightarrow G$ such that the diagram

$$\begin{tikzcd}
F(A \otimes B) \arrow[r, "\gamma^F_{A,B}"] \arrow[d, "\delta_{A\otimes B}"'] & F(A) \otimes B \arrow[d, "\delta_A \otimes B"] \\
G(A \otimes B) \arrow[r, "\gamma^G_{A,B}"] & G(A) \otimes B
\end{tikzcd}$$

commutes for all A and $B \in \mathbb{C}$.

The tensor product $(F, \gamma^F) \otimes (G, \gamma^G)$ is defined to be (FG, γ^{FG}), where $\gamma^{FG}_{A,B}$ is the composite

$$FG(A \otimes B) \xrightarrow{F\gamma^G_{A,B}} F(GA \otimes B) \xrightarrow{\gamma^F_{GA,B}} FGA \otimes B.$$

The tensor product of two arrows is their horizontal composite as natural transformations. The tensor unit I is simply the identity functor, with the identity natural transformation.

There is an functor $L : \mathbb{C} \to \mathbf{e}(\mathbb{C})$, where $L(A) := (A \otimes -, \alpha_{A,-,-})$ for objects $A \in \mathbb{C}$, and $L(f) := f \otimes -$ for arrows f.

Proposition 15. *L is full and faithful.*

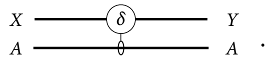
Proof. Joyal and Street's proof of this claim uses the tensor unit in an essential way, so we need to find a new proof that uses semi compact closure instead. The functor L induces, for every X and $Y \in \mathbb{C}$, a function $\mathbb{C}(X, Y) \to \mathbf{e}(\mathbb{C})(LX, LY)$. We'll describe an inverse to this function, showing that it is invertible and hence that L is full and faithful.

Let δ be a natural transformation $LX \Rightarrow LY$. Thus δ consists of components $\delta_A : X \otimes A \to Y \otimes A$, natural in A and such that the diagram

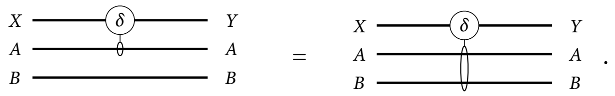
$$\begin{tikzcd}
X \otimes (A \otimes B) \arrow[r, "\alpha_{X,A,B}"] \arrow[d, "\delta_{A\otimes B}"'] & (X \otimes A) \otimes B \arrow[d, "\delta_A \otimes B"] \\
Y \otimes (A \otimes B) \arrow[r, "\alpha_{Y,A,B}"] & (Y \otimes A) \otimes B
\end{tikzcd}$$

commutes for all $A, B \in \mathbb{C}$.

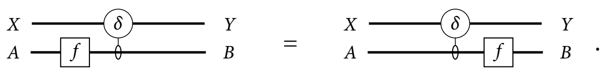
It will be convenient to use string diagrams in the proof: we'll picture δ_A as



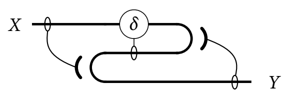
In string diagram terms, the commutative square above is a rewiring condition of the sort we have seen above:



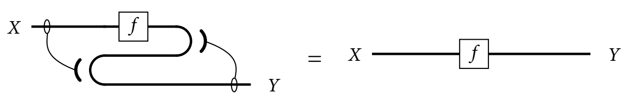
The naturality of δ means that functions can pass through the loop:



Now we can define our inverse to the action of L, to take δ to the following arrow $f : X \to Y$:



We must show that $f \otimes A = \delta_A$, for any $A \in \mathbb{C}$. The proof is a routine string diagram manipulation, shown in Fig. 24. This shows that the passage $\mathbf{e}(\mathbb{C})(LX, LY) \to \mathbb{C}(X, Y) \to \mathbf{e}(\mathbb{C})(LX, LY)$ is the identity. For the other direction, we need to show that



which is immediate. □

Now define $\mathbb{E}$ to be the full subcategory of $\mathbf{e}(\mathbb{C})$ determined by the objects that have adjoints. This is clearly a compact closed category.

Proposition 16. *The image of L is contained in $\mathbb{E}$. Specifically, for every $X \in \mathbb{C}$, the object $L(X)$ is adjoint to $L(X^*)$ in $\mathbf{e}(\mathbb{C})$.*

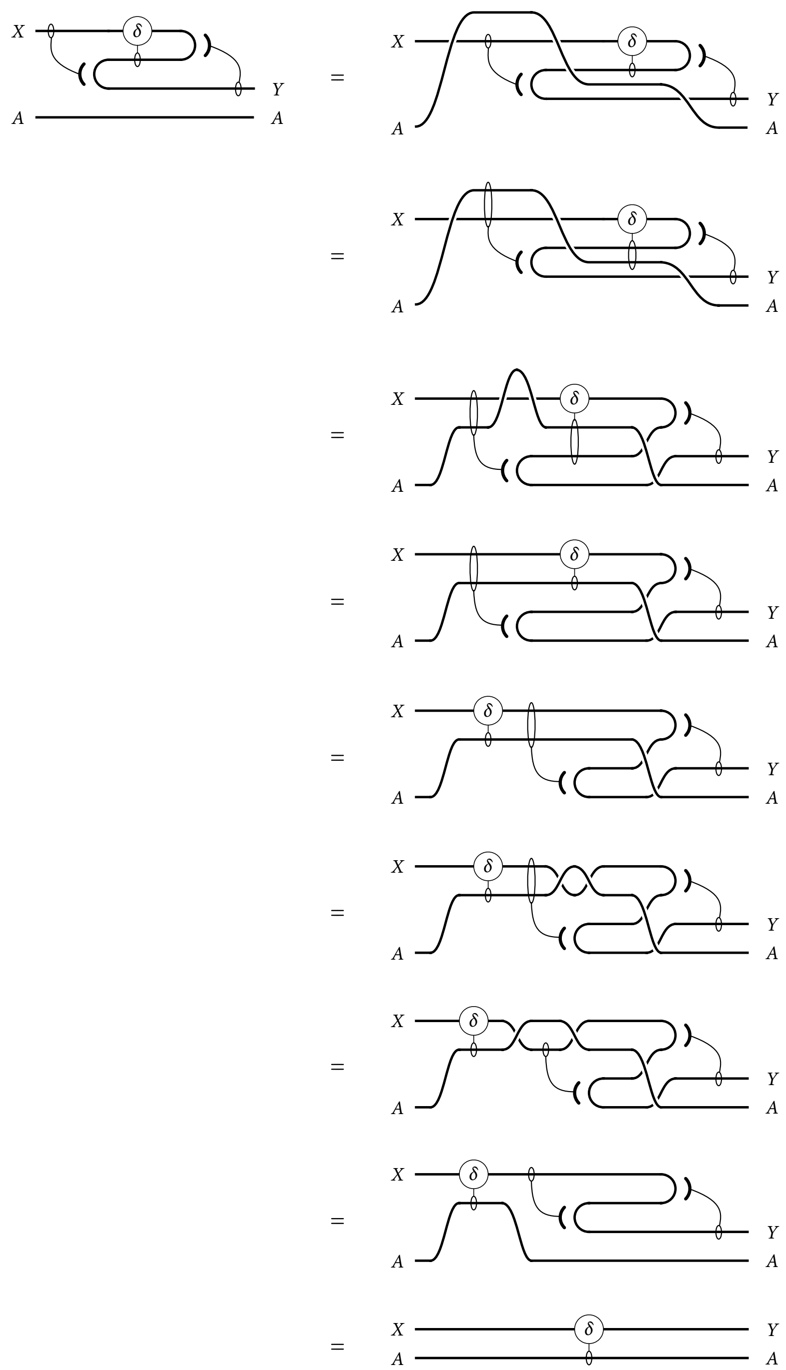
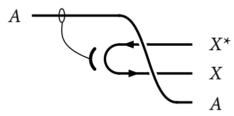
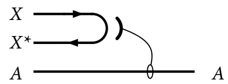


Figure 24: Proof that $f \otimes A = \delta_A$, used in Prop. 15.

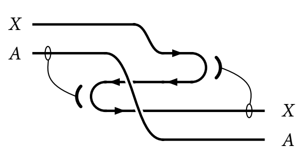
Proof. Define $\eta^{LX} : I \to L(X^*)L(X)$ to have components



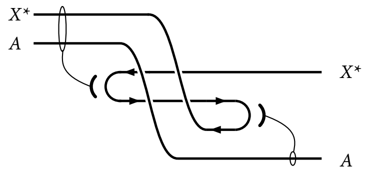
and $\varepsilon^{LX} : L(X)L(X^*) \to I$ to have components



These are clearly natural in A, and it's easy to verify that they satisfy the condition making them maps of $\mathbf{e}(\mathbb{C})$. To show that they really do define an adjunction between $L(X)$ and $L(X^*)$, we need to show that



and



are both equal to the identity. This is an easy exercise in manipulations of the sort that are by now routine. □

Finally, let $\mathbb{C}'$ be the full subcategory of $\mathbb{E}$ determined by those objects that are either isomorphic to $L(X)$ for some X, or isomorphic to I. This subcategory is closed under the tensor and adjoint operations, so it's compact closed. The image of the (full and faithful) functor L is contained in $\mathbb{C}'$ by definition. Thus $\mathbb{C}$ is embedded, in a structure-preserving fashion, in a compact closed category that has essentially only one extra object, the unit object. (If in fact $\mathbb{C}$ had a unit object all along, this functor will be an equivalence.)

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