# A neural network for a binary classification

## import library

In [2]:

```
!pip3 install tqdm
```

```
Collecting tqdm
  Downloading tqdm-4.64.1-py2.py3-none-any.whl (78 kB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 78.5/78.5 kB 2.4 MB/s
 eta 0:00:00
Installing collected packages: tqdm
Successfully installed tqdm-4.64.1
```

In [3]:

```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from matplotlib import ticker, cm
import os
from tqdm import tqdm
```

## load data

In [4]:

```
directory_data  = './'
filename_data   = 'assignment_03_data.npz'
path_data       = os.path.join(directory_data, filename_data)
data            = np.load(path_data)

x_train = data['x_train']
y_train = data['y_train']

x_test  = data['x_test']
y_test  = data['y_test']
```

In [5]:

```
print('*************************************************')
print('size of x_train :', x_train.shape)
print('size of y_train :', y_train.shape)
print('*************************************************')
print('size of x_test :', x_test.shape)
print('size of y_test :', y_test.shape)
print('*************************************************')
print('number of training image :', x_train.shape[0])
print('height of training image :', x_train.shape[1])
print('width of training image :', x_train.shape[2])
print('*************************************************')
print('number of testing image :', x_test.shape[0])
print('height of testing image :', x_test.shape[1])
print('width of testing image :', x_test.shape[2])
print('*************************************************')
```

```
*************************************************
size of x_train : (10000, 28, 28)
size of y_train : (10000,)
*************************************************
size of x_test : (1800, 28, 28)
size of y_test : (1800,)
*************************************************
number of training image : 10000
height of training image : 28
width of training image : 28
*************************************************
number of testing image : 1800
height of testing image : 28
width of testing image : 28
*************************************************
```

## convert data into numpy array

In [6]:

```
x_train = np.asarray(x_train)
y_train = np.asarray(y_train)

x_test  = np.asarray(x_test)
y_test  = np.asarray(y_test)
```

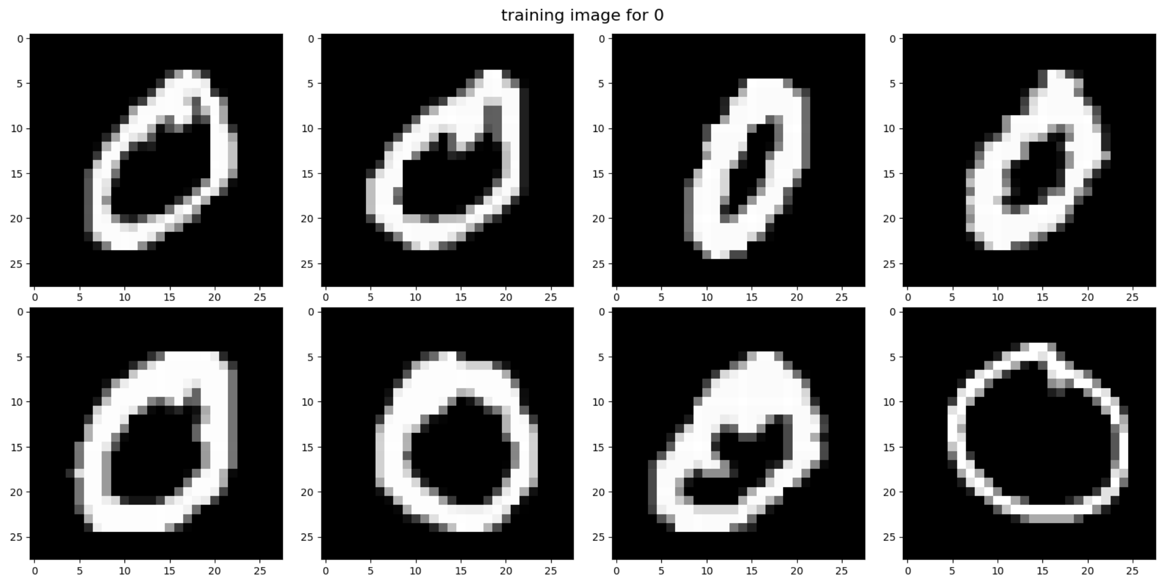
## plot grey image

In [7]:

```
def plot_image(title, data):

    nRow = 2
    nCol = 4
    size = 4

    fig, axes = plt.subplots(nRow, nCol, figsize=(size * nCol, size * nRow))
    fig.suptitle(title, fontsize=16)

    for i in range(nRow):
        for j in range(nCol):

            k = i * nCol + j
            axes[i, j].imshow(data[k], cmap='gray', vmin=0, vmax=1)

    plt.tight_layout()
    plt.show()
```

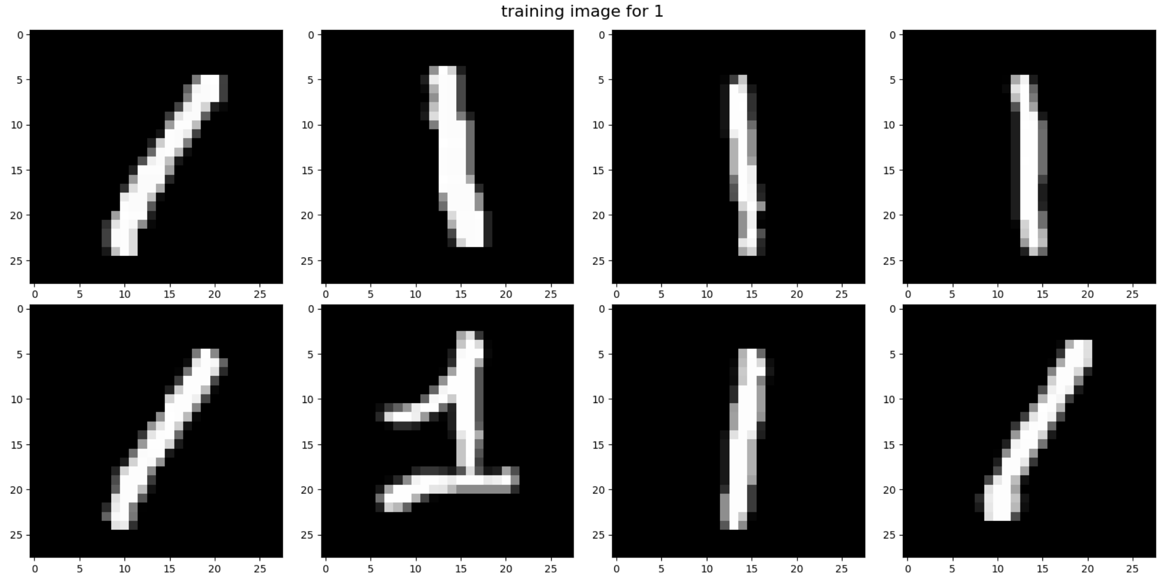
In [8]:

```
plot_image('training image for 0', x_train)
```



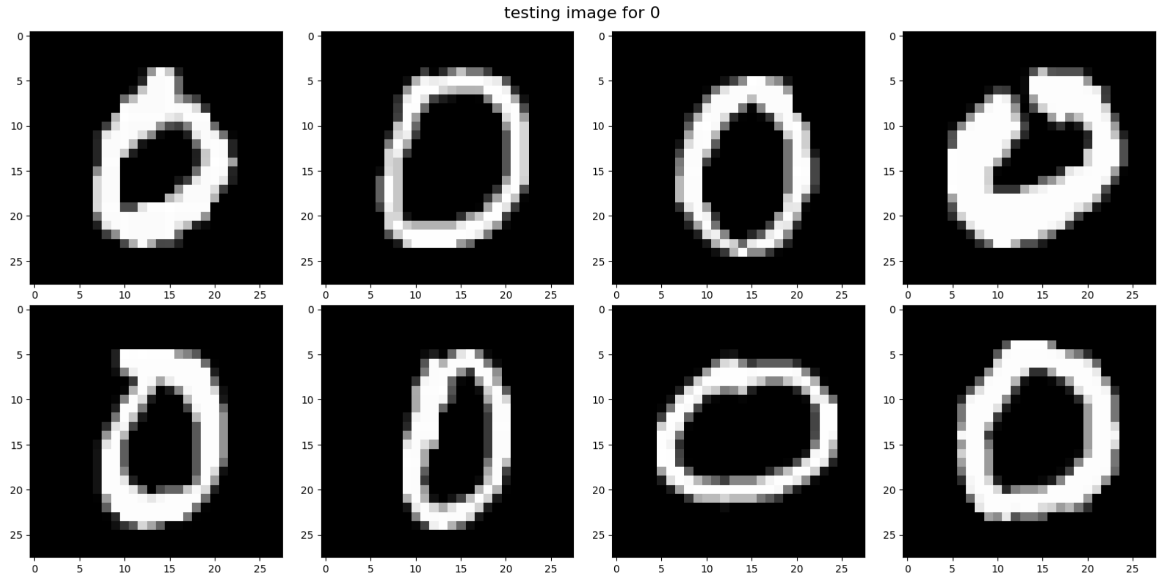
In [9]:

```
plot_image('training image for 1', x_train[5000:,:,:])
```



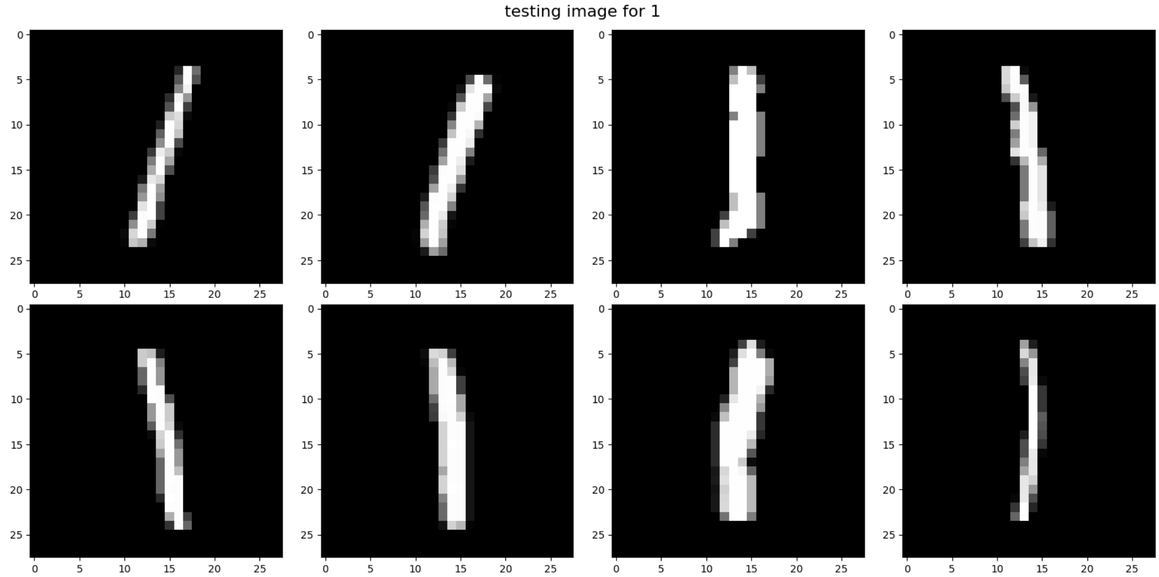
In [10]:

```
plot_image('testing image for 0', x_test)
```



In [11]:

```
plot_image('testing image for 1', x_test[900:])
```




## reshape input data

- vectorize input image matrices into vectors (row-major order)

In [12]:

```
vec_x_train = x_train.reshape(x_train.shape[0], x_train.shape[1] * x_train.shape
[2])
vec_x_test  = x_test.reshape(x_test.shape[0], x_test.shape[1] * x_test.shape[2])
```

In [13]:

```
print('size of training input :', vec_x_train.shape)
print('size of training output :', y_train.shape)

print('size of testing input :', vec_x_test.shape)
print('size of testing output :', y_test.shape)
```

```
size of training input : (10000, 784)
size of training output : (10000,)
size of testing input : (1800, 784)
size of testing output : (1800,)
```

In [14]:

```
x_train = vec_x_train.transpose()
x_test  = vec_x_test.transpose()

length_data         = x_train.shape[0]
number_data_train   = x_train.shape[1]
number_data_test    = x_test.shape[1]

y_train.reshape(1, number_data_train)
y_test.reshape(1, number_data_test)
```

Out[14]:

```
array([[0., 0., 0., ..., 1., 1., 1.]])
```

In [15]:

```
print('size of training input :', x_train.shape)
print('size of training output :', y_train.shape)

print('size of testing input :', x_test.shape)
print('size of testing output :', y_test.shape)

print('number of training data :', number_data_train)
print('number of testing data :', number_data_test)
```

```
size of training input : (784, 10000)
size of training output : (10000,)
size of testing input : (784, 1800)
size of testing output : (1800,)
number of training data : 10000
number of testing data : 1800
```

## reshape the ground truth

In [16]:

```
index_train_0 = np.where(y_train == 0)[0]
index_train_1 = np.where(y_train == 1)[0]

index_test_0 = np.where(y_test == 0)[0]
index_test_1 = np.where(y_test == 1)[0]
```

In [17]:

```
print('size of ground truth for 0 in the training :', index_train_0.shape)
print('size of ground truth for 1 in the training :', index_train_1.shape)

print('size of ground truth for 0 in the testing :', index_test_0.shape)
print('size of ground truth for 1 in the testing :', index_test_1.shape)
```

```
size of ground truth for 0 in the training : (5000,)
size of ground truth for 1 in the training : (5000,)
size of ground truth for 0 in the testing : (900,)
size of ground truth for 1 in the testing : (900,)
```

# initialize the neural network

- neural network consists of fullly connected linear layer followed by sigmoid activation function
- the size of the fully connected layer is input 784 and output 1

# initialize the weights for the fully connected layer

- create one matrix for the weights

In [34]:

```
size_input  = length_data
size_output = 1

weight = np.ones((size_output, size_input))
```

In [35]:

```
print('size of the weight :', weight.shape)
```

```
size of the weight : (1, 784)
```

# define neural network

## define sigmoid function

In [20]:

```
def activation_sigmoid(input):

# ==================================================
# fill up the blank
#
    output = 1. / (1. + np.exp(-1. * input))
#
# ==================================================
    return output
```

In [21]:

```
def derivative_sigmoid(input):

# ==================================================
# fill up the blank
#
    output = activation_sigmoid(input) * (1. - activation_sigmoid(input))
#
# ==================================================
    return output
```

### define the layer

- $n$ : the number of data
- input : $784 \times n$
- output : $1 \times n$

In [22]:

```
def layer_fully_connected(input, weight):

# ==================================================
# fill up the blank
#
    output = np.matmul(weight, input)
#
# ==================================================
    return output
```

## define forward propagation

In [23]:

```
def compute_prediction(input, weight):

# ==================================================
# fill up the blank
#
    prediction  = activation_sigmoid(layer_fully_connected(input, weight))
#
# ==================================================
    return prediction
```

## define the loss function

- cross entropy between the ground truth and the prediction
- cross entropy : $-y \log(h) - (1 - y) \log(1 - h)$
  - $y$ : grount truth
  - $h$ : prediction
- output dimension : $1 \times 1$

In [24]:

```
def compute_loss(prediction, label):

# ===================================================
# fill up the blank
#
    loss = -1. * label * np.log(prediction) - (1. - label) * np.log(1. - predict
ion)
#
# ===================================================
    return loss
```

## compute the accuracy

- note that iterations over the input data are not allowed inside the function

In [25]:

```
def compute_accuracy(prediction, label):

# ===================================================
# fill up the blank
#
    accuracy = np.sum(np.where(prediction >= 0.5, 1, 0) == label) / len(label)
#
# ===================================================
    return accuracy
```

## compute the gradient with respect to the weights

- note that iterations over the input data are not allowed inside the function
- size of input : $m \times n$
- size of label : $1 \times n$
- size of prediction : $1 \times n$
- $n$ denotes the number of data
- $m$ denotes the length of each data

In [27]:

```
def compute_gradient_weight(input, label, prediction):

# ===================================================
# fill up the blank
#
    gradient = np.matmul((prediction - label), input.T) / input.shape[1]
#
# ===================================================
    return gradient
```

# gradient descent algorithm

- hyper-parameters

In [36]:

```
number_iteration    = 500
learning_rate       = 0.01
weight              = weight * 0.01
```

# variables for storing intermediate results

In [37]:

```
accuracy_train   = np.zeros(number_iteration)
accuracy_test    = np.zeros(number_iteration)

loss_train_mean = np.zeros(number_iteration)
loss_train_std   = np.zeros(number_iteration)
loss_test_mean   = np.zeros(number_iteration)
loss_test_std    = np.zeros(number_iteration)

prediction_train_0_mean = np.zeros(number_iteration)
prediction_train_0_std  = np.zeros(number_iteration)
prediction_train_1_mean = np.zeros(number_iteration)
prediction_train_1_std  = np.zeros(number_iteration)
prediction_test_0_mean  = np.zeros(number_iteration)
prediction_test_0_std   = np.zeros(number_iteration)
prediction_test_1_mean  = np.zeros(number_iteration)
prediction_test_1_std   = np.zeros(number_iteration)
```

# run the gradient descent algorithm

In [38]:

```
for i in tqdm(range(number_iteration)):

# ==================================================
# fill up the blank
#
    prediction_train = compute_prediction(x_train, weight)
    loss_train = compute_loss(prediction_train, y_train)
    gradient_train = compute_gradient_weight(x_train, y_train, prediction_train)

    weight = weight - learning_rate * gradient_train

    prediction_test = compute_prediction(x_test, weight)
    loss_test = compute_loss(prediction_test, y_test)
#
# ==================================================
    accuracy_train[i]   = compute_accuracy(prediction_train, y_train)
    accuracy_test[i]    = compute_accuracy(prediction_test, y_test)

    loss_train_mean[i]  = np.mean(loss_train)
    loss_train_std[i]   = np.std(loss_train)
    loss_test_mean[i]   = np.mean(loss_test)
    loss_test_std[i]    = np.std(loss_test)

    prediction_train_0_mean[i]  = np.mean(prediction_train[:,index_train_0])
    prediction_train_0_std[i]   = np.std(prediction_train[:,index_train_0])
    prediction_train_1_mean[i]  = np.mean(prediction_train[:,index_train_1])
    prediction_train_1_std[i]   = np.std(prediction_train[:,index_train_1])
    prediction_test_0_mean[i]   = np.mean(prediction_test[:,index_test_0])
    prediction_test_0_std[i]    = np.std(prediction_test[:,index_test_0])
    prediction_test_1_mean[i]   = np.mean(prediction_test[:,index_test_1])
    prediction_test_1_std[i]    = np.std(prediction_test[:,index_test_1])
```

```
100%|██████████████████████████████| 500/500 [00:15<00:0
0, 31.49it/s]
```

## functions for presenting the results

In [39]:

```
def function_result_01():

    title           = 'loss (training)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'loss'
    color_mean      = 'red'
    color_std       = 'blue'
    alpha           = 0.3

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(loss_train_mean)), loss_train_mean, '-', color = color_me
an)
    plt.fill_between(range(len(loss_train_mean)), loss_train_mean - loss_train_s
td, loss_train_mean + loss_train_std, facecolor = color_std, alpha = alpha)

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [40]:

```
def function_result_02():

    title           = 'loss (testing)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'loss'
    color_mean      = 'red'
    color_std       = 'blue'
    alpha           = 0.3

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(loss_test_mean)), loss_test_mean, '-', color = color_mean
)
    plt.fill_between(range(len(loss_test_mean)), loss_test_mean - loss_test_std,
loss_test_mean + loss_test_std, facecolor = color_std, alpha = alpha)

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [41]:

```
def function_result_03():

    title           = 'prediction for 0 (training)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'prediction'
    color_mean      = 'red'
    color_std       = 'blue'
    alpha           = 0.3

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(prediction_train_0_mean)), prediction_train_0_mean, '-',
color = color_mean)
    plt.fill_between(range(len(prediction_train_0_mean)), prediction_train_0_mea
n - prediction_train_0_std, prediction_train_0_mean + prediction_train_0_std, fa
cecolor = color_std, alpha = alpha)

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [42]:

```
def function_result_04():

    title           = 'prediction for 1 (training)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'prediction'
    color_mean      = 'red'
    color_std       = 'blue'
    alpha           = 0.3

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(prediction_train_1_mean)), prediction_train_1_mean, '-',
color = color_mean)
    plt.fill_between(range(len(prediction_train_1_mean)), prediction_train_1_mea
n - prediction_train_1_std, prediction_train_1_mean + prediction_train_1_std, fa
cecolor = color_std, alpha = alpha)

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [43]:

```
def function_result_05():

    title           = 'prediction for 0 (testing)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'prediction'
    color_mean      = 'red'
    color_std       = 'blue'
    alpha           = 0.3

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(prediction_test_0_mean)), prediction_test_0_mean, '-', co
lor = color_mean)
    plt.fill_between(range(len(prediction_test_0_mean)), prediction_test_0_mean
- prediction_test_0_std, prediction_test_0_mean + prediction_test_0_std, facecol
or = color_std, alpha = alpha)

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [44]:

```
def function_result_06():

    title           = 'prediction for 1 (testing)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'prediction'
    color_mean      = 'red'
    color_std       = 'blue'
    alpha           = 0.3

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(prediction_test_1_mean)), prediction_test_1_mean, '-', co
lor = color_mean)
    plt.fill_between(range(len(prediction_test_1_mean)), prediction_test_1_mean
- prediction_test_1_std, prediction_test_1_mean + prediction_test_1_std, facecol
or = color_std, alpha = alpha)

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [45]:

```
def function_result_07():

    title           = 'accuracy (training)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'accuracy'

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(accuracy_train)), accuracy_train, '-', color = 'red')

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [46]:

```
def function_result_08():

    title           = 'accuracy (testing)'
    label_axis_x    = 'iteration'
    label_axis_y    = 'accuracy'

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(accuracy_test)), accuracy_test, '-', color = 'red')

    plt.xlabel(label_axis_x)
    plt.ylabel(label_axis_y)

    plt.tight_layout()
    plt.show()
```

In [47]:

```
def function_result_09():

    print('final training loss = %6.5f' % (loss_train_mean[-1]))
```

In [48]:

```
def function_result_10():

    print('final testing loss = %6.5f' % (loss_test_mean[-1]))
```

In [49]:

```
def function_result_11():

    print('final training accuracy = %6.5f' % (accuracy_train[-1]))
```

In [50]:

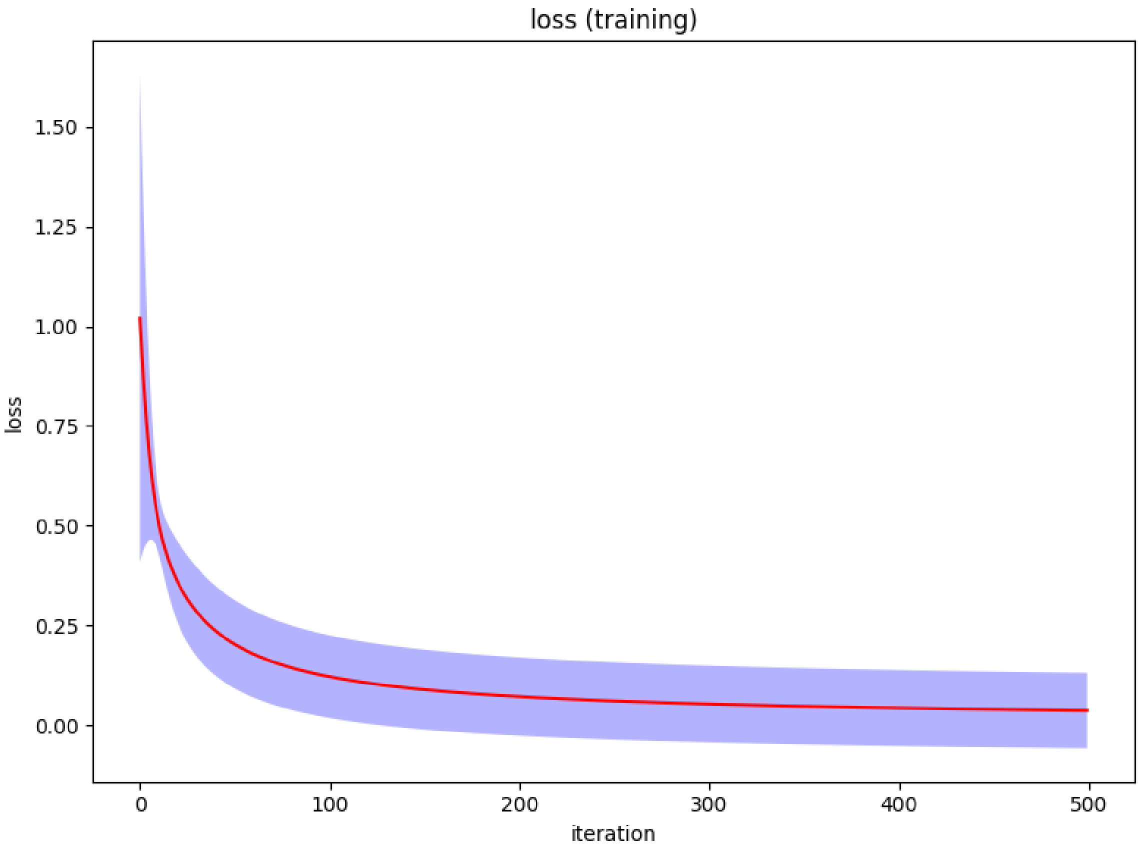
```
def function_result_12():

    print('final testing accuracy = %6.5f' % (accuracy_test[-1]))
```

# results

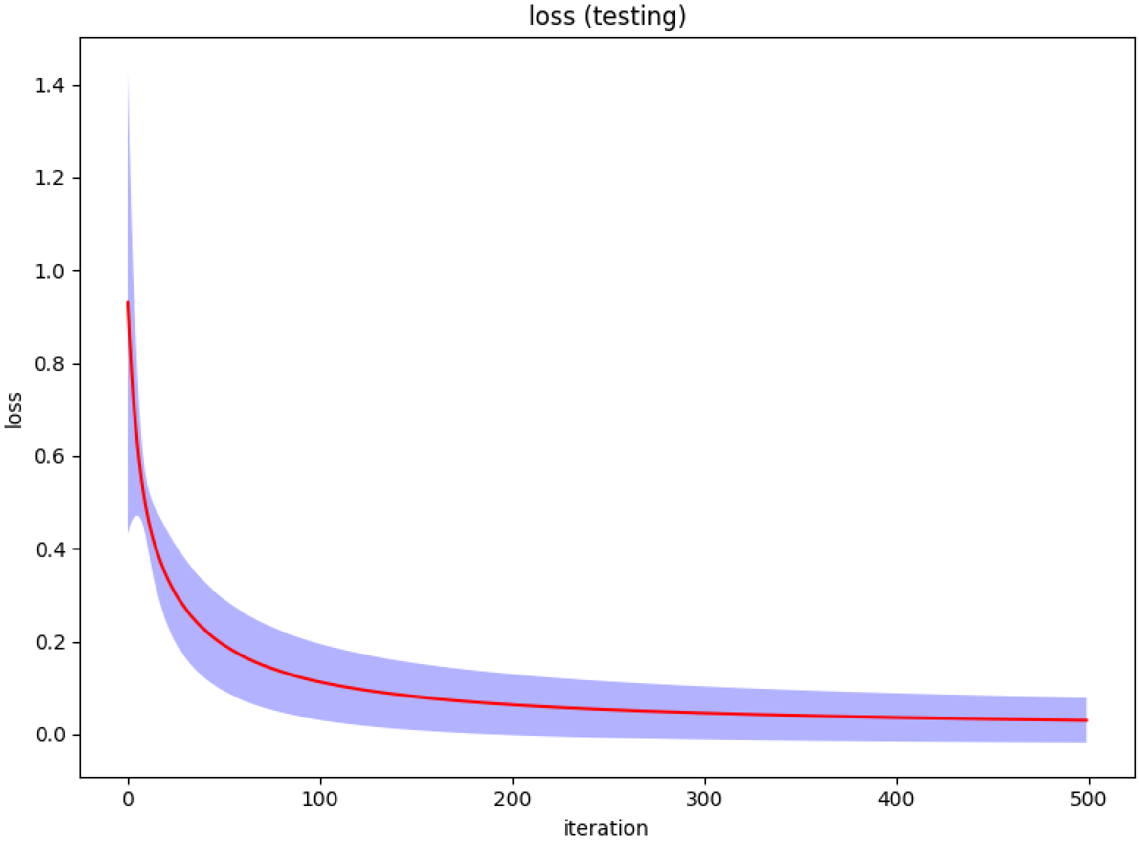
In [51]:

```
number_result = 12

for i in range(number_result):

    title           = '# RESULT # {:02d}'.format(i+1)
    name_function   = 'function_result_{:02d}()'.format(i+1)

    print('')
    print('#####################################################################
###########')
    print('#')
    print(title)
    print('#')
    print('#####################################################################
###########')
    print('')

    eval(name_function)
```

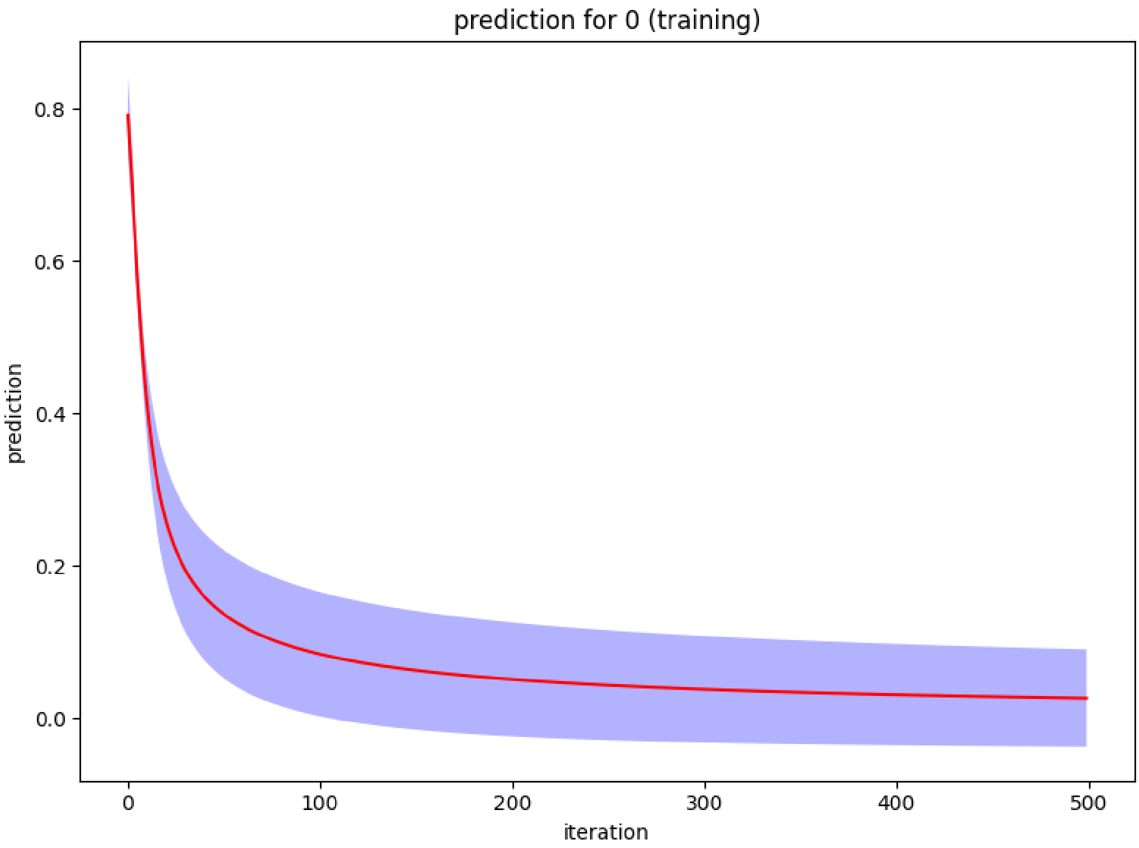
```
################################################################################
#
# RESULT # 01
#
################################################################################
```



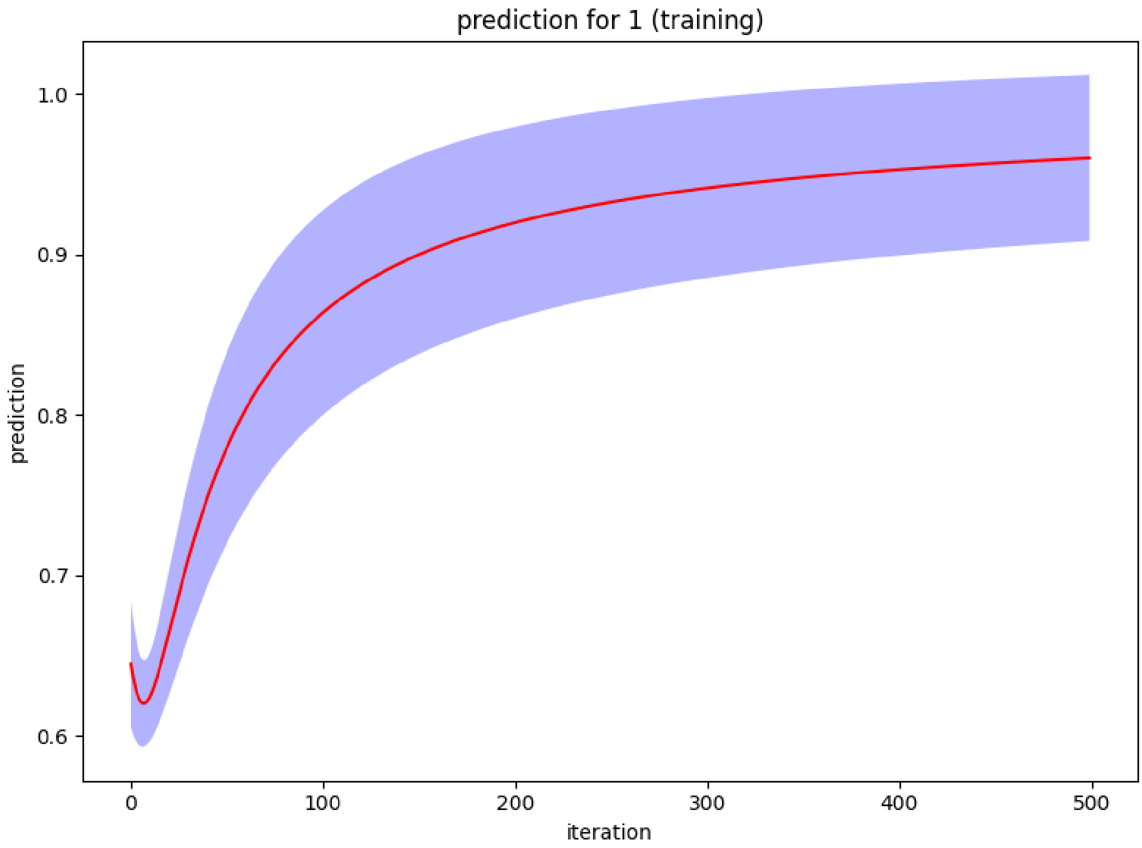

```
################################################################################
#
# RESULT # 02
#
################################################################################
```


loss (testing)
loss
iteration
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500

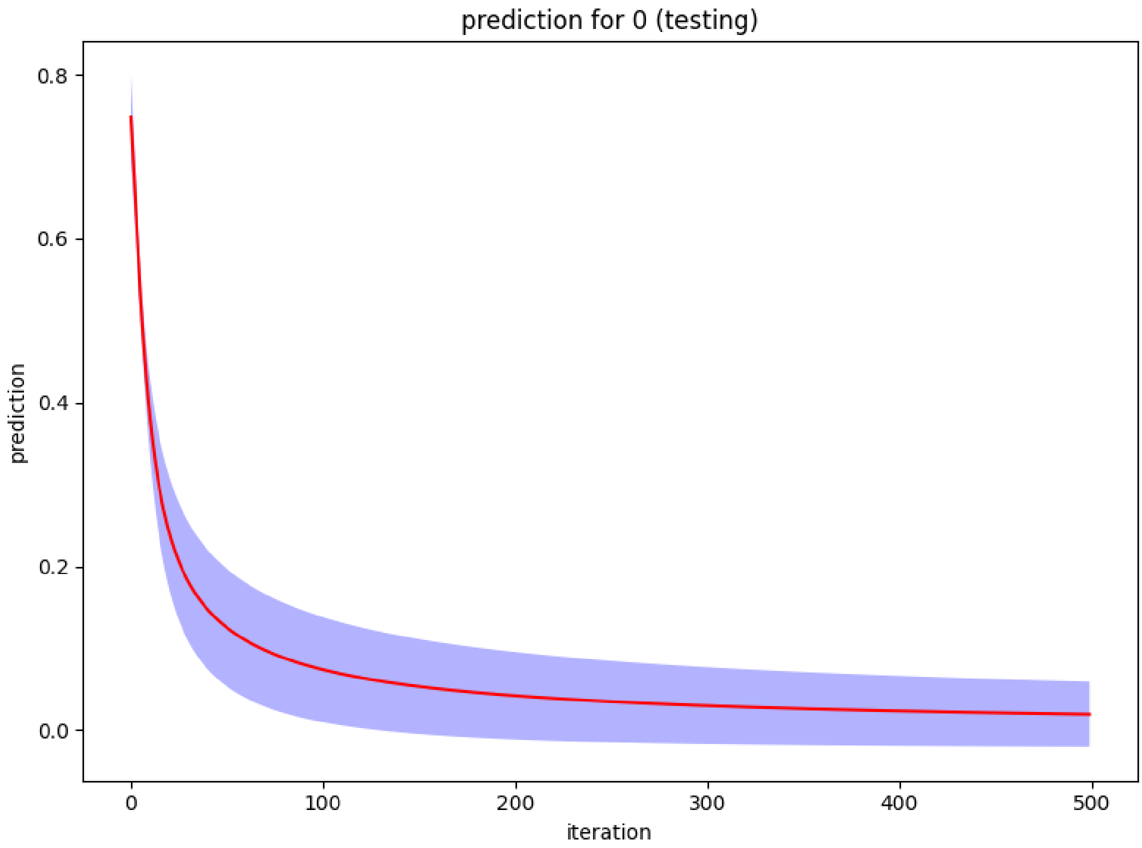
```
######################################################################
############
#
# RESULT # 03
#
######################################################################
############
```


prediction for 0 (training)
prediction
iteration
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500

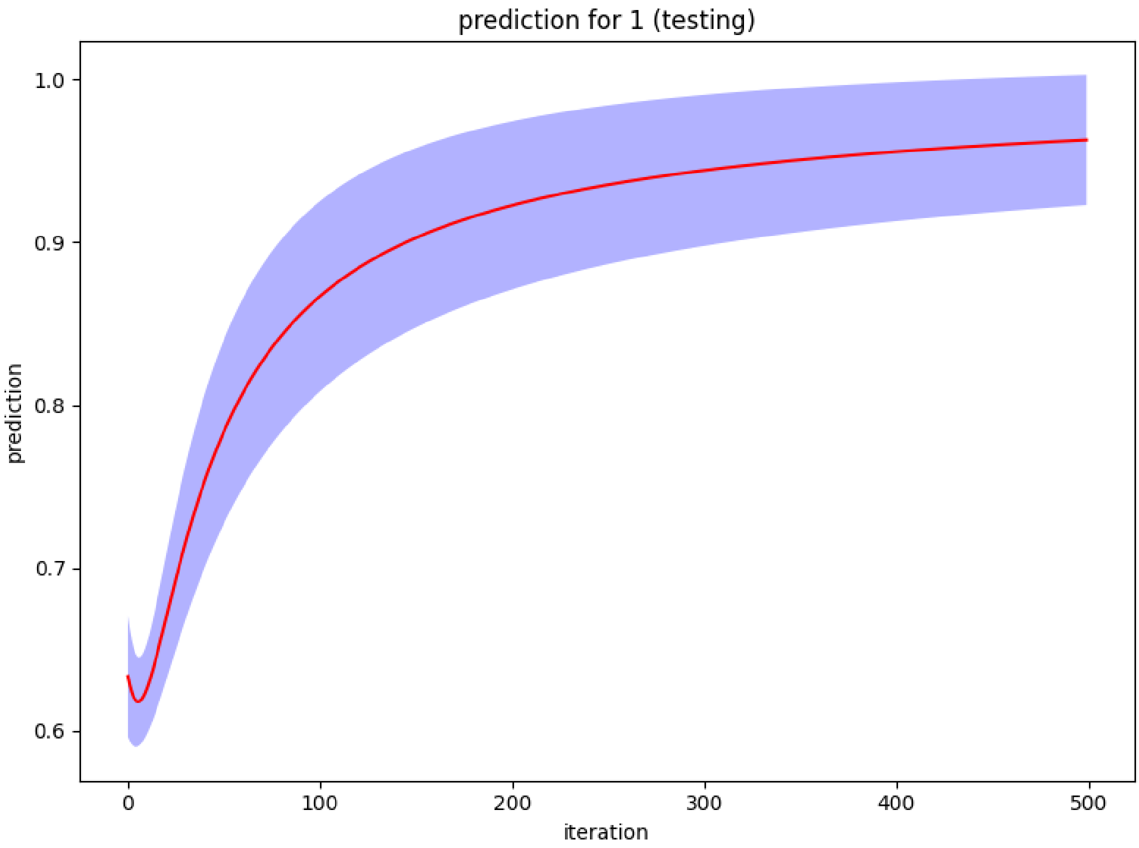
```
########################################################################
###########
#
# RESULT # 04
#
########################################################################
###########
```


prediction for 1 (training)
1.0
0.9
0.8
0.7
0.6
prediction
0
100
200
300
400
500
iteration

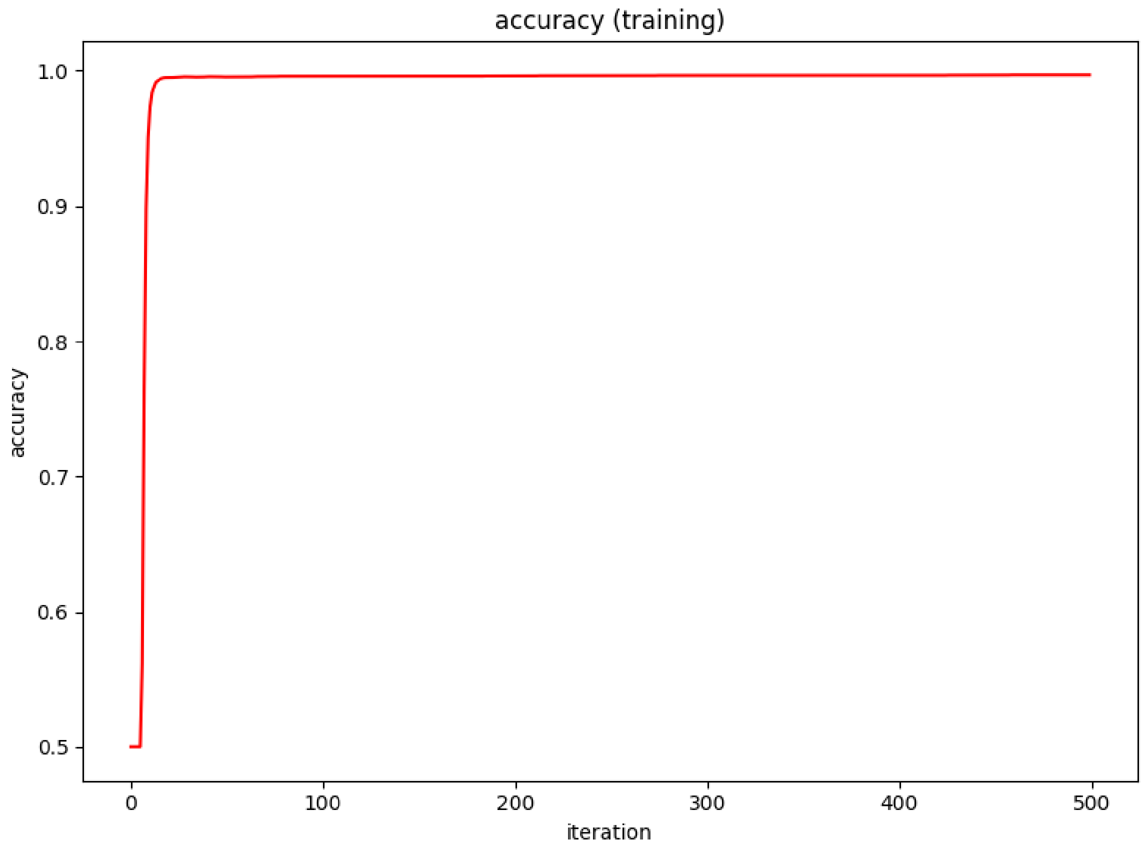
```
############################################################################
############
#
# RESULT # 05
#
############################################################################
############
```


prediction for 0 (testing)
prediction
iteration
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500

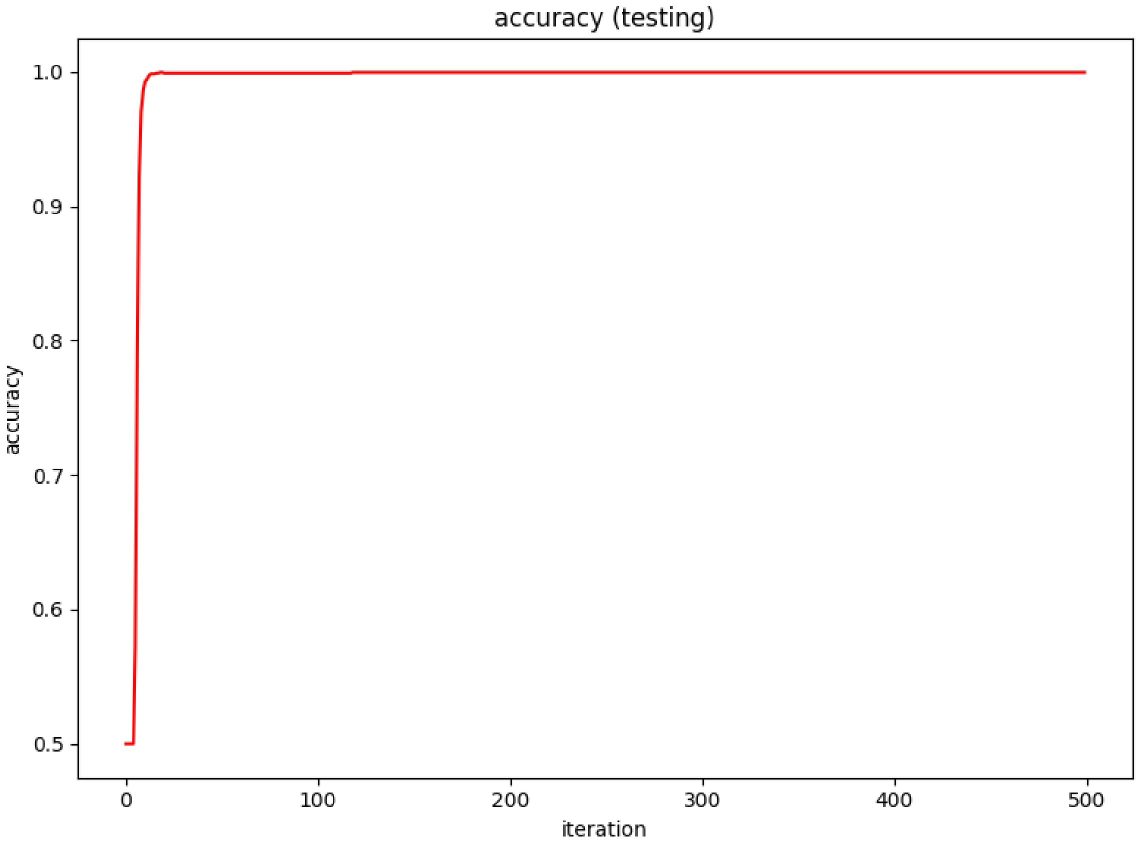
```
####################################################################
############
#
# RESULT # 06
#
####################################################################
############
```


prediction for 1 (testing)
1.0
0.9
0.8
0.7
0.6
prediction
0
100
200
300
400
500
iteration

```
########################################################################
############
#
# RESULT # 07
#
########################################################################
############
```


accuracy (training)
accuracy
iteration
1.0
0.9
0.8
0.7
0.6
0.5
0
100
200
300
400
500

```
################################################################################
#
# RESULT # 08
#
################################################################################
```


accuracy (testing)
accuracy
1.0
0.9
0.8
0.7
0.6
0.5
0
100
200
300
400
500
iteration

```
################################################################################
#
# RESULT # 09
#
################################################################################

final training loss = 0.03675

################################################################################
#
# RESULT # 10
#
################################################################################

final testing loss = 0.02983

################################################################################
#
# RESULT # 11
#
################################################################################

final training accuracy = 0.99700

################################################################################
#
# RESULT # 12
#
################################################################################

final testing accuracy = 0.99944
```

In [ ]: