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Problem 1

1.Function Definition

The Beta function is a special function in Mathematics, which is defined by the following formula
$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$
The domain of the function is for – Re x > 0, Re y > 0. (Positive Real numbers x and y)
The co-domain of the function is also the same Re x > 0, Re y > 0. (Positive Real numbers x and y)
It is also known as the Euler integral of the first kind. The Beta function is important in Mathematics , calculus and analysis due to its relationship with the gamma function and the factorial function. Interestingly a lot of complex integrals can be derived and reduced to simpler expressions involving the Beta function

2. Characteristics of the Beta Function

- $B(m, n) = B(n, m)$
- $B(m, n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\Theta \cos^{2n-1}\Theta d\Theta$
- $B(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}dx$
- $B(m, n) = \int_0^{1} \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$

It is also related to the Gamma Function with the following relationship
$B(p, q) = \frac{\Gamma p \Gamma q}{\Gamma(p+q)}$
Also it possesses the Recurrence Relationship property
$B(x+1, y) = B(x, y)\frac{x}{x+y}$
Relationship with factorial method
$B(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$