Homework #1
Ling495

Robin Clark
Fall 2017

Due: October 3, 2017

Problem 1

In the notes on group selection, we considered Sober and Wilson's approach to the evolution of altruism at the population level. In this exercise, you will convert their population level treatment to an individual level game.

Look up the equations for W_A and W_S. They give the fitness for altruists and selfish agents. Take the equations and convert them into a bimatrix (normal form) game of imperfect information played between Adam and Eve. Suppose that Adam and Eve must choose between playing **Altruist** and **Selfish**. In the first bimatrix, use the variables X (for baseline fitness), c for cost, and b for benefit to show the payoffs. In the second bimatrix, fill in X, c and b with the numeric values assumed by Sober and Wilson. Solve the game by finding the players' best responses.

Is the game a Prisoner's Dilemma? Explain your answer.

Problem 2

In class, we began a probability analysis of rolls of two dice. Complete the analysis to give the probabilities of rolling the number 2 through 12 (the sum of the dots on the upward face of each die). The result should allow you to work out the probabilities of rolling the various numbers.

Suppose I offer to play the following game with you: I will pay you one dollar whenever you roll a 4, 5, 6 or 7. Otherwise, you must pay me a dollar. What is your expected utility for playing the game. Should you play it? (Assume that you should play if your expected utility is greater than 0.)

As a variant, suppose I offer to pay you one dollar if you roll a 2, 4, 5, 6 or 7; otherwise, you must pay me a dollar. What is your expected utility in this case? Would the change make playing the game worthwhile?

Problem 3

Three people—call them Alice, Bob, and Carol—plan to make a garden. Each player can decide whether or not to contribute to the group effort. If all three contribute, the garden is quite pleasant and each player who contributes gets a payoff of 3. If only two players contribute, all three players get a nice garden but the one who contributes is a free-rider who gets to enjoy the nice garden without the effort of contributing: the ones who contribute get a payoff of 3 and the one who doesn't contribute gets a payoff of 4. If only one person contributes, the garden isn't very nice: The player who contributes has the expense and a lousy garden, for a payoff of 1; the two who didn't contribute get to free-ride off a crummy garden, so they get a payoff of 2. Finally, if no one contributes the garden is crummy but no has the expense, so each player gets payoff of 2.

Make a sequential game-tree for the above game. Assume that Alice moves first, Bob move second and Carol moves third. Use backward induction to solve the game.

(Note: I adapted this from an old introduction to game theory.)