Homework #3
Ling495

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Fall 2017

Due: November 14, 2017

Let's return to the Snowdrift game and translate it to an evolutionary setting:

		Eve	
		Dove	Hawk
Adam	Dove	25, 25	10, 40
	Hawk	40, 10	0, 0

Figure 1: The Snowdrift Game

Suppose that we set up a repeated Snowdrift game. Recall that if the probability of repeated play is p, then the expected number of repetitions is $\frac{1}{1-p}$.

For the first version, suppose that players are paired at random from the population and that they play pure strategies: A player either always plays **Dove** or always plays **Hawk**. Show a game matrix for this repeated version of the game.

Next, calculate the expected payoffs for $p = 0.98$. What are the equilibria in this case? Do $< \mathbf{Dove}, \mathbf{Hawk} >$ and $< \mathbf{Hawk}, \mathbf{Dove} >$ have an advantage over $< \mathbf{Dove}, \mathbf{Dove} >$? Suppose the players took turns playing $< \mathbf{Dove}, \mathbf{Hawk} >$ and $< \mathbf{Hawk}, \mathbf{Dove} >$ so the equally split the payoffs. What would their utilities be in that case?

Could a Hawk invade a population of Doves? Could a Dove invade a population of Hawks? Explain and support your answer with utility calculations.

Suppose, now, you modified the Dove strategy to a Vengeful Dove. This is the Snowdrift version of tit-for-tat. The player begins by playing Dove and then echoes the opponents move from the previous turn. Modify your game matrix to show the utilities of Vengeful Dove and Hawk under repeated play with $p = 0.98$.

Suppose now that the players switch from repeated to one-shot play. An estimated 75% of players are cooperators (tend to play **Dove**, while 25% of players are free-riders (tend to play **Hawk**. A cooperator will play **Dove** 90% of the time; a free-rider plays **Hawk** 95% of the time. One player observes another playing **Dove** and getting a payoff of 25. The player then gets to play against the player she observed. Work out a Bayesian Update Table to give the posterior probabilities for:

1. $P(\text{Dove}|\text{cooperator})$
2. $P(\text{Dove}|\text{free-rider})$