Hide and Seek, and the Emergence of Randomness in a Population of Deterministic Agents

Robin Clark

rclark@sas.upenn.edu

Steven O. Kimbrough

kimbrough@wharton.upenn.edu

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Abstract

The game of Hide and Seek is interesting because it affords the study both of the emergence of coordination and anti-coordination. Socially these are important for many reasons, including modeling of the evolution of identity markers and in-group–out-group tagging. We endow our agents with sensible, simple deterministic search methods, in the context of an overall evolutionary dynamic. We find that using these methods, the agents can achieve both coordination and anti-coordination, depending upon the payoffs involved. Further, we find that adding a modicum of randomization to individual search disrupts the population's ability to coordinate. It is hardly surprising that apparently random behavior should arise from individual randomized behavior. We see this in these experiments, but we also see that it is hardly necessary. The deterministic search produced effective anti-coordination. In addition, the deterministic search produced effective coordination, in distinction to what randomized behavior achieved. Throughout, we emphasize that these are emergent population phenomena. We note as well that there is randomization occurring in this model, at the population level, if not at the individual level. These results, then, suggest that an evolutionary dynamic, acting on a population, is sufficient for apparently randomized individual behavior and that adding actual randomized individual behavior may not be necessary or even helpful.

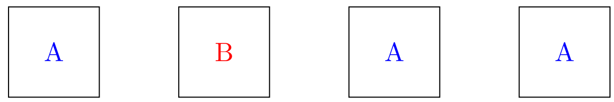
1 Introduction

Coordination games and the evolution of social norms has been the object of intense research (Bicchieri et al., 1997; Skyrms, 2014). In this paper, we propose to extend this research by considering the game of "Hide and Seek" (von Neumann, 1953; Crawford and Iriberri, 2007). While norms allow individuals to coordinate their behavior, in a standard setting "Hide and Seek" involves the avoidance of coordination. We study the game in two *modes*. In Evade mode, the hider seeks to avoid having the seeker find the hidden object, while in Find mode, the hider attempts to place the hidden object so that the seeker *will* find it.

Hide and seek behavior is of more than passing interest: Social groups will play Evade mode hide and seek with outsiders and Find mode with insiders when it comes to vocabulary, fashion and even research topics. In particular, social groups have a vested interest in marking in-groups and signaling social identity in a way that outsiders have trouble counterfeiting (Gambetta, 2009; Gumperz, 1992; Richerson and Boyd, 2005; Henrich, 2016). One efficient method of establishing a signal of in-group membership would be to devise a "secret handshake" known only to members of the group; altruistic behavior could, then, be directed to someone who knows the handshake. Notice, though, that the behavior involved in playing "Hide and Seek" may involve a sophisticated understanding of norms and, in particular, the distribution of behaviors at a population level. Thus, understanding the dynamics of "Hide and Seek" is useful in understanding some fundamental aspects of spontaneous social organization.

In "Hide and Seek" one player has the role of the hider, who privately stows away a prize, and the other player has the role of the seeker, who attempts to find the prize. In Evade mode, the hider attempts to stow the prize so that it will not be found, while in Find mode, the hider stows the prize in hopes that it will be found.

An array of four boxes is available to the players; the boxes are labeled "A" and "B" as below:



The box labeled "B" (shown here in red, to emphasize the point) is a primary focal point (Schelling, 1960; Sugden, 1995; Bacharach and Bernasconi, 1997; Bacharach, 2006); the end boxes, both labeled "A," are secondary focal points (Bacharach and Bernasconi, 1997; Crawford and Iriberri, 2007); while the second to last box, also labeled "A" and referred to as "central A," is non-focal (Crawford and Iriberri, 2007). Notice that the structure of the game is virtually identical to a coordination with focal points, a fact that we will explore in more detail below.

As noted above, the game itself has two roles: A hider who is told that she must stow a prize in one of the four boxes and a seeker who is told his opponent has stowed a prize in one of the four boxes and his sole opportunity of finding it is by opening one and only one box. Reflection shows that the optimal strategy for the hider in Evade mode is to ignore the labeling of the boxes and select a box at random under a uniform distribution; that is, each box has a 0.25 probability of being selected. The seeker's best reply, then, would also be to ignore the labels on the boxes and randomly select a box to open, again under a uniform distribution. It is notoriously difficult for people to randomize their choices (Camerer, 2003, page 127 and *passim*) so their observed behavior is, in fact, quite different from the predicted equilibrium strategy, as witnessed by the observed frequencies in a one-shot game (Crawford and Iriberri, 2007; Rubinstein et al., 1996). See Table 1.

Player	A	B	A	A
Hider (53;$p = 0.0026$)	9%	36%	40%	15%
Seeker (62;$p = 0.0003$)	13%	31%	45%	11%

Table 1: Initial distributions reported in (Crawford and Iriberri, 2007).

It is clear from the table that hiders and seekers are not uniformly distributed over the boxes but, rather, that both hiders and seekers are attracted to the primary focal point and central A, with both groups having a preference for the latter.

Crawford and Iriberri (2007) suggest a Level-k model of the behavior (Camerer et al., 2004). We will explore a rather different approach, one with affinities to the *El Farol* Problem (Arthur, 2015). We will suggest, in particular, that agents use a modified "win-stay, lose-shift" policy defined over a set of moves. Our main interest will be in considering how the behavior of the population evolves over time. We will be interested, in particular, not so much in the initial mix of behaviors exhibited by the agents, but in how the agents modify their strategies over time and how the result is mixed over the population. Such rule-based behavior may well resist equilibria as agents must often change their strategy in response to changes in the population. For example, a preference to hide the prize in central A, as witnessed above in the frequencies of the one-shot game, will soon be discovered and exploited. We will initially be interested in patterns in the behavior of the population in repeated one-shot game play (between normally different players); while we are also interested in iterated game play between stable dyads of players, we will put this case aside for the moment.

2 Technical Description

We constructed an Agent-Based Model in Python 3.[1] Agents come in two types, corresponding to two different Python classes: Hiders and Seekers. The strategies used for hiding may be different from the strategies used for seeking. For instance, a "Hide" agent might use a "purloined letter" strategy of hiding the prize in the primary salience box, while a "Seek" agent might look for the prize in the central A box.

Each agent is equipped with the following computational objects:

1. Salience, providing a salience order of the boxes;
2. History, the entire history of the agent's plays;
3. DecisionSwitch, used to decide whether to change the agent's focal box;
4. SwitchingStrategy, used to change the agent's focal box, given that the agent has decided to change.

Each of these objects, except History, may be instantiated in multiple ways, each its own strategy. The salience order is simply a ranking of the boxes in terms of salience, represented as a list from most salient to least salient; two or more equally salient items would be represented as a sub-list. In the example given above, enumerating the boxes from left to right, we would get the following representation:

$$[1, 0, 3, 2]$$

Box 1, labeled "B" in the figure above, bears primary salience, since it is overtly marked; the first and last boxes have indistinguishable secondary salience; central A , the second to last box, is least salient. In our default setup, all agents have a common salience sequence, $[1, 0, 3, 2]$, representing the suggestion from the data (Crawford and Iriberri, 2007) that between 0 and 3, while close in salience, 0 is slightly more salient.

Each agent has a history of its own past play. This information can be restricted in a variety of ways. First, the agent may have information about

[1]The code is available from the authors.

the success or failure of its own choice, but is not given information about its opponent's choice; in a second variant, the hider is given information about its opponents choice, but the seeker only receives information about the hider's choice when it successfully locates the prize; third, both players are fully informed about their opponent's choice at the end of the round. However, in our default configuration, and for all of this paper, the history is, after initialization, simply a sequence of 1s and 0s indicating success or failure from the agent's perspective.

Decision switches regulate the agent's decision to change strategies. Individual agents may have different tolerances for failure. An example might be a rule that checks a hider's history and switches if, in ten plays, the hiding place was discovered more than twice. Equally, a seeker might change strategies if it fails to find the prize after five rounds. Clearly, there are a large number of possible decision switches. In the default configuration all agents follow a "win-stay, lose-shift" switching strategy, in which they switch after n consecutive loses or failures. During initialization the value of n is randomized (in the default case) from the range $[2, 4]$.

Switching strategies govern the choice of a new box once the agent has decided to shift strategies. Once again there are a large number of possible rules that agents could use. One strategy would be to move from less salient boxes to more salient boxes (call this Forward); another might be to move from more salient boxes to less salient ones (call this Reverse). Another might place the boxes in an order that is not directly related to salience and move up or down in that alternative ordering. In our default setup we employ two distinct switching strategies, Forward and Reverse, which we assign randomly to agents, whether they are Hiders or Seekers.

Besides creating the agents, initialization sets the mode of play, which is either Find or Evade. In Find mode, the interests of the Hiders and Seekers are aligned. They both achieve success if the Seeker finds (opens) the box in which the Hider has hidden the precious object. In Evade mode, Hiders and Seekers are in a game of pure conflict. Hiders are successful if and only if Seekers fail to discover the box with the token. Seekers are successful if and only if the box they choose contains the token.

After initialization of the two populations, of Hiders and Seekers, the game mechanism randomly picks one Hider and one Seeker, and polls them for their play (choice of box). Each agent has a record of its current box choice in the form of an index into its salience sequence (`idxNow`). The agent uses this index to obtain a specific box, from the sequence. In case of ties (represented by a list of boxes within the salience sequence), the agent chooses randomly. Upon receiving the agent's plays, the game mechanism (in our default case) informs the two agents regarding whether the Seeker agent discovered the token. The agents then individually update their histories, and call their DecisionSwitch objects to determine whether or not to switch their focal boxes. If an agent decides to switch, it calls its SwitchingStrategy object to effect the change, which results in a new value of its `idxNow`.

The research project, then, is to explore different policies for how individuals change in response to experience and observe whether, at the population level, the system evolves into a state nearing the equilibrium even if individuals never arrive at an equilibrium strategy.

3 Description of the System

Before turning to the experiments, we need to cover a handful of technical details. Each agent has a level of "stickiness," which corresponds to the number of losses in a row it is willing to endure; the stickiness level varies between 2 and 4 losses in a row. Once the stickiness level of the player is exceeded, the player changes strategy in one of two different ways: We can imagine the salience levels of the boxes as though they are arranged on a clock, with the most salient box (box 1, which is marked by "B") at 12; the next most salient box at 3 (box 0, the first box in the sequence in our model); the next most salient box (box 3, the last box in the sequence) at 6; and the least salient box (box 2) at 9. The players are initially arrayed at these points according to the probability distribution reported by Crawford and Iriberri (2007). The population of agents are divided into two groups: Those that change strategy by moving clockwise to a less salient box, and those that change strategy by moving counterclockwise to a more salient box. Once the policy is set, each agent searches the space in a completely deterministic fashion. Chance simply divides the population into which policy they are assigned, which point on the salience ordering they begin, and which partners they are assigned.

As in a typical model from evolutionary game theory, the individuals play according to a single strategy. First, given the underlying determinacy of the system, will the population find a single box as the norm in "Find" mode? Second, will the population discover the optimal mixed strategy in "Evade" mode? Finally, assuming the first two questions are answered positively, can we precisely characterize the factors that play a role in the evolution of the system. We turn to experiments in the next section.

4 Experiments

4.1 Default Scenario

4.1.1 Evade

In the first experiment, all agents share the same salience hierarchy: [1,0,3,2]; the "stickiness" of agents (how many loses in a row they will tolerate) varies randomly between 2 turns and 4 turns, depending on the agent. Each agent has a default policy for selecting a new strategy. There are two such policies, assigned randomly to agents at initialization: forward and reverse. In addition each agent is initialized with a history: [1,1] (two successes). The initial distributions of chosen boxes are those given in Table 1. The numerical counts obtained in a typical run (replications are not noticeably different) are shown in the following table.

Disposition	Box 0	Box 1	Box 2	Box 3
Starting Hider	13	76	79	32
Starting Seeker	30	63	86	21
Ending Hider	52	48	49	51
Ending Seeker	47	53	61	39

Figure 1 shows the results of this particular run, which is quite representative.

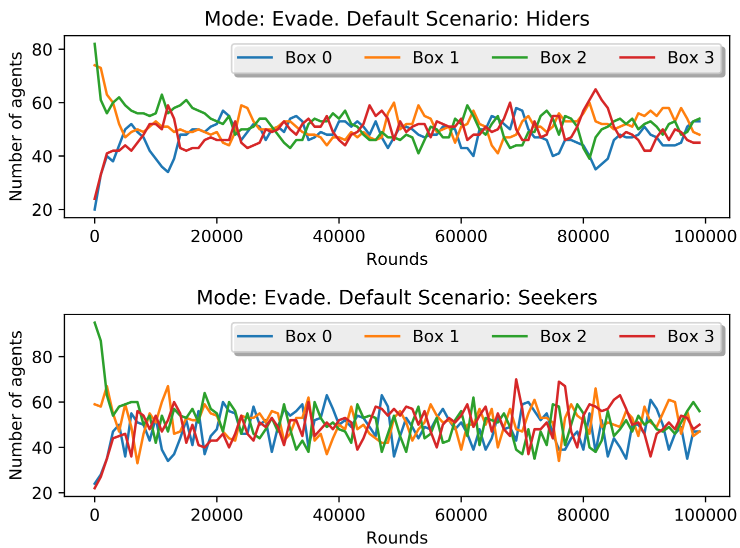


Figure 1: Example run of 100,000 rounds, default distributions: Evade.

The remarkable thing to notice here is that the ending box distributions are not discernibly different from chance, i.e., from what one would expect if the counts for each box were independently drawn from a binomial(50,0.25) distribution. This is despite the fact that the agents follow a deterministic rule in searching their decision space. The agents behave deterministically yet the population appears to behave randomly. As a simple test of this (space limitations prevent a full exploration), we can draw, say 100,000 4-tuples from a binomial(50,0.25) distribution and ask how often the maximum value of an ending box is greater than or equal to our observed 61. To illustrate, we got this result:

```
def binomialTest(threshold):
    aSample = np.random.binomial(200,0.25,(100000,4))
```

```
    test = aSample.max(axis=1)
    count = test > threshold
    return sum(count/100000)

binomialTest(61)
Out[100]: 0.12233999999998527
```

Had the result been less than 0.05 we would reject the null hypothesis of random assortment among boxes at the 5% level. Instead, what this tells us is that more than 12% of the time, under the null hypothesis, the maximum value would be 61 or more. So we fail to reject the null hypothesis. We do note for future reference (below) that at 62 the probability is 0.087 and at 64 the probability is 0.04. That said, the run here is unusually extreme, in our experience.

4.1.2 Find

Here we have the same setup and algorithm as in the previous subsection, but we are in Find mode, so the interests of the agents are aligned. Here are the starting and ending box distributions in a typical run:

Disposition	Box 0	Box 1	Box 2	Box 3
Starting Hider	26	68	76	30
Starting Seeker	22	75	78	25
Ending Hider	0	0	200	0
Ending Seeker	0	0	200	0

Figure 2 plots the round by round progress.

What is most notable here is the perfect coordination achieved, not with randomized search but with deterministic search at the individual level! We note that the coordination point that the population arrives at, box 2, is the most popular box at initialization. This is basically the definition of a focal point (Schelling, 1960, Chapter 2).

4.2 Shuffled Search

In Shuffled Search, each agent is assigned a shuffled search order ($4! = 24$ possible search orders). Although agents vary as to their search order, each agent follows its particular order deterministically. The initial and ending distributions of agents over the boxes are shown below, along with the associated example run:

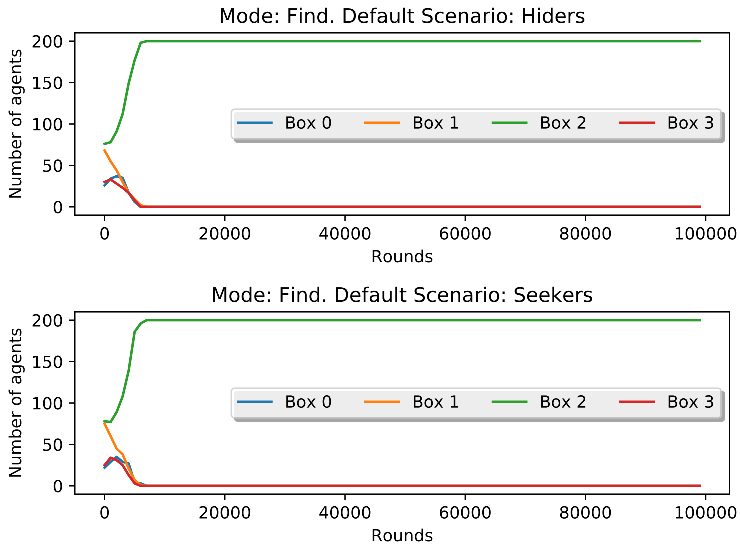


Figure 2: Example run of 100,000 rounds, default distributions: Find.

Disposition	Box 0	Box 1	Box 2	Box 3
Starting Hider	18	66	85	31
Starting Seeker	22	58	98	22
Ending Hider	0	0	200	0
Ending Seeker	0	0	200	0

Although there are a variety of different search protocols, because they are exhaustive and followed deterministically by the agents, the population soon evolves a norm for the coordination game. Again, because the focal box, 2, has the largest initial grouping of agents, it emerges as the point around which the population coordinates.

For the sake of completeness, Figure 4 presents the Evade case, in which as expected apparent randomization emerges.

4.3 Focal Point Focus

We explored shifting the focal points at initialization. If, for example, one box has, across both types of agents, clearly the highest focal point score, then even with shuffling the saliences, the population under Find coordinates on that focal point. If there is just a slight advantage for one box, then the population still tends to coordinate on that box, but with a great deal of noise. Finally, if the agents are distributed equally over the boxes initially, the population settles on a box at random, after a somewhat longer period of disagreement. In the case of

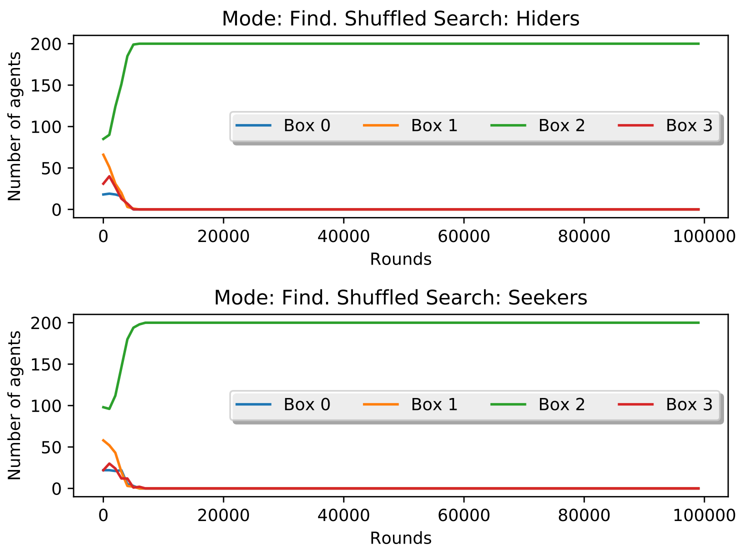


Figure 3: Example run of 100,000 rounds, default distributions but with Shuffled Search: Find.

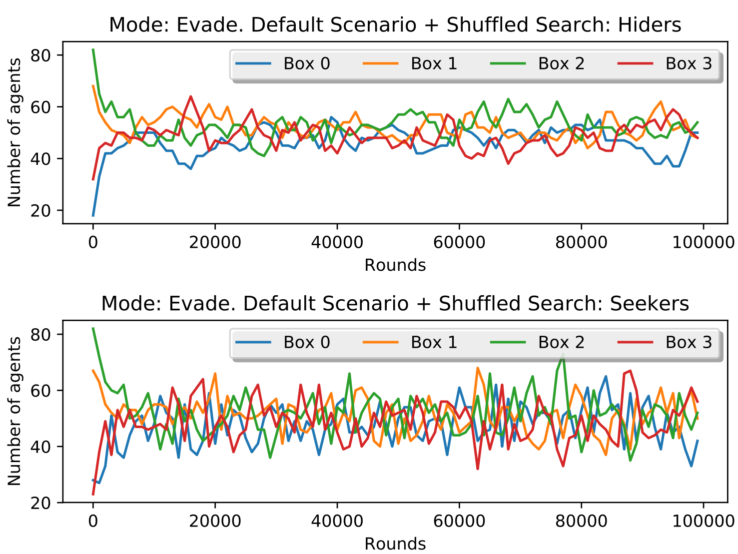


Figure 4: Example run of 100,000 rounds, default distributions but with Shuffled Search: Evade.

Evade, the initial focal point configuration makes no difference in the outcomes, which continue to appear to be random.

4.4 Uniform Searches

In the previous experiments, we explored the dynamics of deterministic searches and discovered that apparent randomness emerges at the population for Hide and Seek, but the same basic algorithm could converge on a population norm for coordination. In the following experiments, we look at random search. In this case, the agents select a new focal box randomly, according to a uniform distribution. Figure 5 shows that the system fails to arrive at a population norm for coordination when the search is truly random; Figure 6 shows that the Evade mode Hide and Seek game is unaffected. This demonstrates that deterministic search works for both Find and Evade, but random search, while it works for evade, is positively harmful for Find. Perhaps it is not surprising that evolution has not produced good randomizers in social animals.

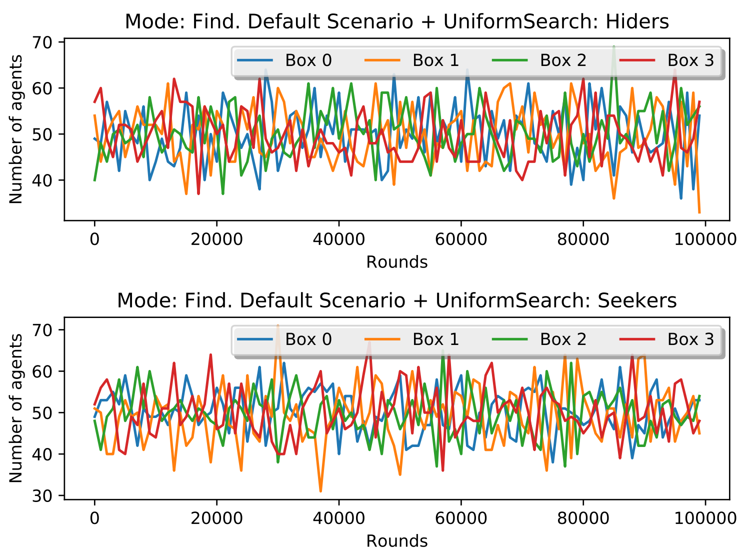


Figure 5: Example run of 100,000 rounds, default distributions but with UniformSearch: Find.

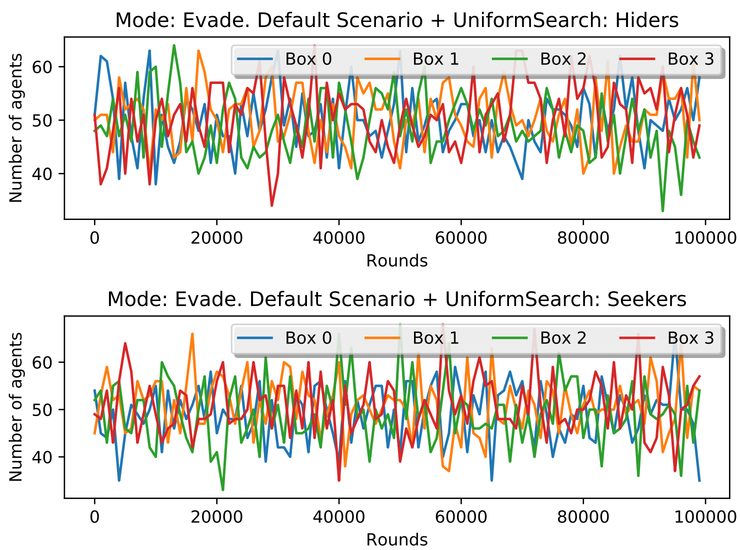


Figure 6: Example run of 100,000 rounds, default distributions but with UniformSearch: Evade.

4.5 Stickiness

Recall that in the default setup, agents are assigned a tolerance for loss of between 2 and 4 turns, a property we will call "stickiness." In the following experiments we look at the role that stickiness plays in the system, by varying the stickiness level of the agents. In the first experiment, the agents are uniformly assigned a stickiness of 1, which means that they change focal after a single loss. As can be seen in Figure 7, this completely disrupts the ability of the population to arrive at a norm for coordination. (Evade mode is not shown, since it looks as above.)

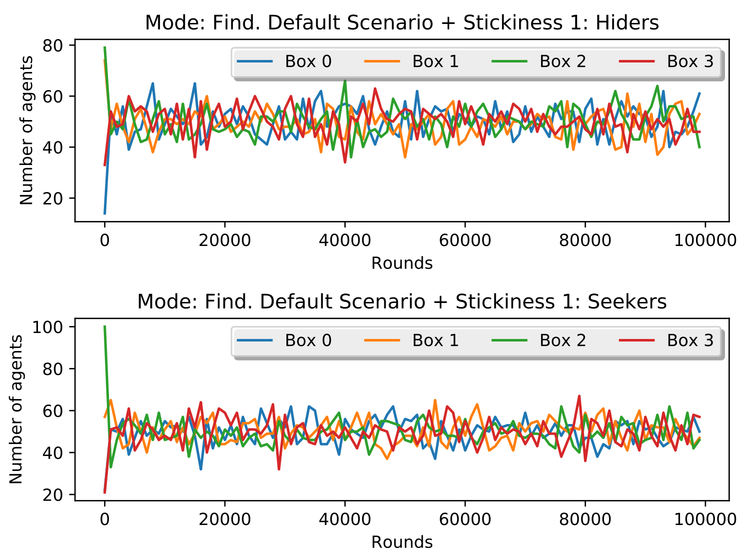


Figure 7: Example run of 100,000 rounds, default distributions, Stickiness 1: Find.

The obvious question here is what could be the lowest level of stickiness that preserves the population's ability to discover a coordination point. In Figure 8 we show that at stickiness level 2 (tolerance for two losses in a row) the population still does not converge to a coordination point in a typical run.

As Figure 9 shows, however, when stickiness reaches 3 the population achieves coordination easily.

Finally, if we randomly assign agents to have stickiness of between 1 to 4, inclusive, achieving coordination is possible but takes a much longer time. See Figure 10.

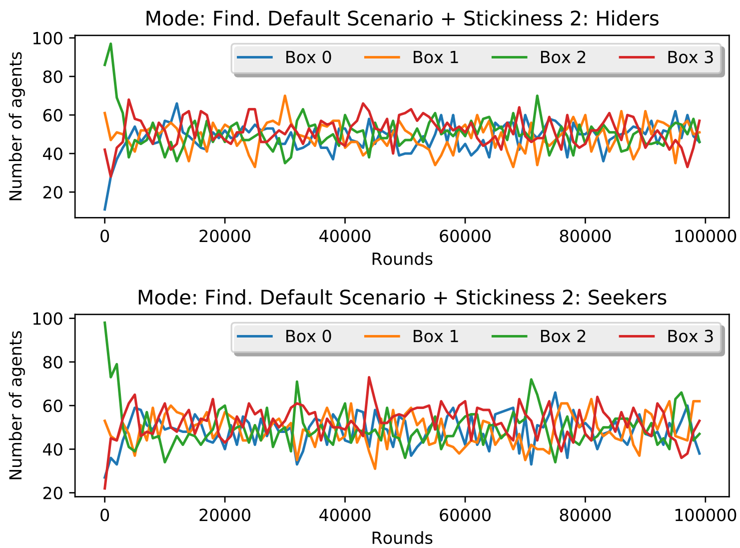


Figure 8: Example run of 100,000 rounds, default distributions, Stickiness 2: Find.

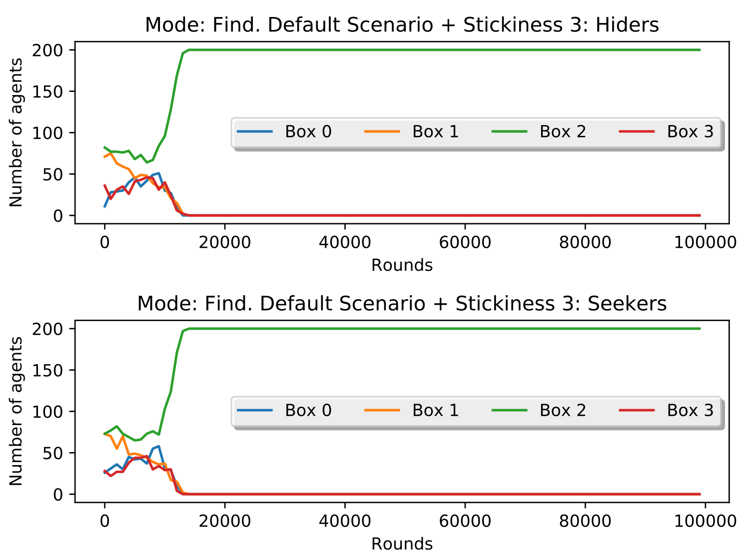


Figure 9: Example run of 100,000 rounds, default distributions, Stickiness 3: Find.

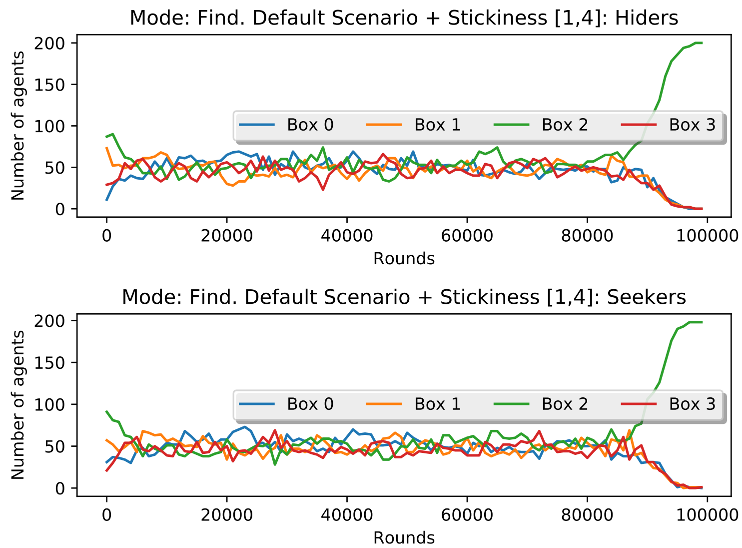


Figure 10: Example run of 100,000 rounds, default distributions, Stickiness in [1,4]: Find.

5 Discussion

The game of Hide and Seek is interesting because it affords the study both of the emergence of coordination (in Find mode) and anti-coordination (in Evade mode). Socially these are important for many reasons, including modeling of the evolution of identity markers and in-group–out-group tagging. We endow our agents with sensible, simple deterministic search methods, in the context of an overall evolutionary dynamic. We find that using these methods, the agents can achieve both coordination and anti-coordination, depending upon the payoffs involved (delimiting the Find and Evade modes). Further, we find that adding a modicum of randomization to individual search disrupts the population's ability to coordinate. It is hardly surprising that apparently random behavior should arise from individual randomized behavior. We see this in these experiments, but we also see that it is hardly necessary. The deterministic search produced effective anti-coordination. In addition, the deterministic search produced effective coordination, in distinction to what randomized behavior achieved. Throughout, we emphasize that these are emergent population phenomena. We note as well that there is randomization occurring in this model, at the population level, if not at the individual level. These results, then, suggest that an evolutionary dynamic, acting on a population, is sufficient for apparently randomized individual behavior and that adding actual randomized individual behavior may not be necessary or even helpful.

In the near term, we see three additional directions in which we will take this research. First, we plan to perform a more detailed statistical analysis of the behavior of the system at both the individual and population levels. In Evade mode, we should be able to show statistically that the individuals show marked non-random behavior while the population passes tests for random behavior, thus settling into the equilibrium for Hide and Seek.

Next, we are designing and running human experiments; we hypothesize that people will not search randomly when playing Hide and Seek (whether in Find or Evade mode). We suspect that individuals will tend to search according to patterns, although there will no doubt be diversity of patterns between individuals. Our plan is to run a repeated Hide and Seek game and analyze the response both of the population and the individuals that make up the population.

Finally, we plan to explore more complex and realistic scenarios. For example, we will try a greater diversity of search methods in the population. In particular, though, we will explore the dynamics of in-group and out-group tagging. The idea is to have a large population of agents who want to coordinate with each other; within that population, we will have a sub-group that seeks to coordinate within the sub-group, but evade with the out-group. Can the integrity of the in-group be maintained and what effect does this sub-group have on the behavior of the larger group, under a variety of different incentives? We hypothesize that mutualism will reign within groups but predation will characterize the relationship between the supergroup and the sub-group, so that the sub-group will need to change their conventions periodically to maintain the effectiveness of their norms.

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