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Games: Unifying Logic, Language, and Philosophy

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- $(\mathbf{M}, \mathbf{L}) \models \varphi$ *for each* MV-*chain* $\mathbf{L}$ *and each* safe $\mathbf{L}$-*structure* $\mathbf{M}$.
- $(\mathbf{M}, \mathbf{L}) \models \varphi$ *for each* MV-*chain* $\mathbf{L}$ *and each* witnessed $\mathbf{L}$-*structure* $\mathbf{M}$.

As in the propositional case we could formulate the strong completeness theorem. However, unlike in propositional case there is no standard completeness theorem. In fact, the set of predicate tautologies of the standard MV-algebra is a Π_2-complete set.

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Chapter 7

GAMES, QUANTIFICATION AND DISCOURSE STRUCTURE

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Abstract Quantifiers in natural language contribute both to the truth conditions of a sentence and to the discourse in which the sentence occurs. While a great deal of attention has been paid to truth conditions, the contributions of quantifiers to the discourse have been little studied. This paper seeks to rectify this by developing a set of game rules that account both for the truth conditional and the discourse contributions of quantified expressions.

7.1 Overview

In this paper, I would like to make some proposals on the treatment of quantifiers and their consequences for discourse in Game Theoretic Semantics.[1] For present purposes, I will mean by *quantifier* any noun phrase which contains both a determiner and a head noun along any modifiers like adjectives and relative clauses. The examples in (1) illustrate the sort of phrases I will be interested in treating[2]:

(1) a. *Aristoteleans*: every dean, all deans, some faculty member, not all students, no provost.
 b. *Cardinals and bounding determiners*: at least five administrators, at most four department chairs, between three and ten trustees.

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[1]For general treatments of GTS, see Hintikka and Sandu (1997), Hintikka and Kulas (1985), and Hintikka (1996), and the references cited there. For a treatment of some generalized quantifiers within GTS see Pietarinen (2001, 2007).

[2]I will put aside definite descriptions like *the dean* and "polyadic quantifiers" like *each...a different...* as in "each dean read a different comic book." The former case requires a more detailed discussion of the discourse model (see Clark (2005) for a game-theoretic treatment) and the latter case is too complex to treat here; but see van Benthem (1989), Keenan (1992) and Pietarinen (2007).

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c. *Majorities*: most monkeys, more than half of the department chairs, more deans than faculty members, fewer provosts than trustees.

The expressions in (1) have long been studied by both linguists and philosophers and a sophisticated and very useful theory of them already exists.[3]

Why bother to recast the project in terms of games? One good answer is that it is always interesting and useful to rework a well-understood theory in a different way. I think, however, that there is a more interesting and telling reason to investigate the game-theoretic properties of quantifiers in natural language. To motivate things, let us consider a small text like the one in (2):

(2) Exactly three students took an exam. One passed it effortlessly. She had clearly studied for it. The others struggled with it and barely passed. They felt relief mixed with shame.

The first sentence in the text in (2) contains two quantifiers: *exactly three students* and *an exam*. Clearly, the sentence is true in some models and false in others. But the contribution of the quantifiers extends beyond the first sentence, as the second sentence illustrates. The indefinite *one* in the second sentence clearly depends on the discourse effects of the quantifier *exactly three students* from the first sentence; equally, the pronoun *it* in the second sentence depends upon the quantifier *an exam* in the first. Continuing to the third sentence, *she* must refer (in the absence of other compelling discourse information) to the student picked out by *one* in the second sentence, and so is ultimately contingent on the first sentence. Consider, next, the contribution of the phrase *the others* in the second to last sentence. This phrase is understood as referring to the remaining two students. Its interpretation is contingent upon the interaction between the quantifier *exactly three students* and the discourse anaphor *one*. Having interpreted *the others* correctly, these two students can become the antecedent of the pronoun *they* in the last sentence.

Clearly, quantifiers make contributions to our understanding of and inferences from texts, contributions that extend far beyond the level of the sentence. Generalized quantifier theory, with its emphasis on truth conditions and the functions that interpret quantifiers, has not been able to clearly address the connections between quantifiers and their effects on the discourse.[4] I will argue, though, that Game Theoretic Semantics provides a natural framework for investigating these relationships. I will be particularly concerned with how quantifier games can establish new discourse entities. I will not, however,

[3]Generalized quantifiers, first proposed in Mostowski (1957), have played a central role in the development of semantic theory. See, among many others, Barwise and Cooper (1981), van Benthem (1986), Keenan and Stavi (1986), and Keenan and Westerståhl (1997).

[4]Indeed, Dynamic Semantics (Groenendijk and Stokhof, 1991; van den Berg, 1996; Beaver, 2001) and Discourse Representation Theory (Kamp and Reyle, 1993) have attempted to address these connections.

discuss reference tracking, the problem of following discourse entities through a discourse. I take this to be a problem of the management of resources in the discourse model, a problem that is amenable to a game treatment. For games played on this level, the reader may consult Clark (2004, 2005).

Throughout this paper, I will make the standard assumptions about Game Theoretic Semantics as applied to natural language. We will suppose that there are two players, Eloïse and Abelard, the initial verifier and the initial falsifier, respectively. They are playing a zero-sum game on a sentence S relative to a model M. Eloïse wins if the sentence is verified relative to M while Abelard wins if the sentence is counter-exemplified. See Hintikka and Sandu (1996) for discussion and references.

7.2 Aristoteleans

The Aristotelean quantifiers are familiar from first-order logic. These include any quantifier synonymous with those in the familiar Aristotelean forms. These quantifiers have been well-studied in GTS and I will restrict my attention to their effects on discourse anaphora. For present purposes, I will restrict my attention to *every/all* and *some/a*, the other Aristoteleans being simple to define on this basis.

I will follow the practice in GTS of allowing players to add entities and sets of entities to a special set, the choice set I_S. This set works as a discourse model, a database of entities and sets of entities that have been invoked by the discourse to date. When a pronoun, definite description or other anaphoric element is encountered in the course of play, one player or the other must select an entity or set from I_S to serve as the target referent of the anaphoric element. For example, in a sequence of games like:

(3) A boy was following a man. The man did not notice.

The verifier will choose elements from the model to act as witnesses for the noun phrases in the first sentence. These elements will be placed in the choice set at the end of a play of the first game. The second game begins with a definite description, *the man*. The verifier is forced to make her choice of referent from I_S; hence, *the man* in the second sentence is understood to refer to an entity already invoked by the discourse. Compare (3) with:

(4) a. Every student thinks he's smart. #He has enormous self-confidence.
b. Every student thinks he's smart. They have enormous egos.

The sequence in (4)a is decidedly peculiar, while the sequence in (4)b is more acceptable.[5] To account for the differences between (3), (4)a and (4)b, we must, first, partition the choice set into two parts. One member of the partition,

[5]See Clark (2004, 2005) for a game-theoretic treatment of such texts.

$I_{current}$, will contain the entities invoked by the current sentence. Certain types of anaphora, an example might be reflexives like *himself* and *herself* constrain the player to choose within $I_{current}$. The other, $I_{discourse}$, will contain entities and sets that have been invoked by previous games in the discourse. We can think of this partition as the discourse model proper. Second, we will impose the following constraint on the passing of discourse entities from $I_{current}$ to $I_{discourse}$:

(5) *Choice Preservation*
An entity is passed from $I_{current}$ to $I_{discourse}$ just in case it was selected by Eloïse. Otherwise, the set X from which the entity was selected is placed in $I_{discourse}$.

To demonstrate the system, let us turn to examples. Denoting a game played on a sentence S relative to a model, M, as $G(S\,;M)$, then we have the following rule for *some*:

(R.some)
If the game $G(S\,;M)$ has reached an expression of the form:

$$Z - \text{some } X \text{ who } Y - W.$$

Then the verifier may choose an individual from the appropriate domain, say b. The game is then continued as $G(Z - b - W, b \text{ is an } X \text{ and } bY; M)$. The individual b is added to the set $I_{current}$.

Given the constraint in (5), if Eloïse is playing verifier when she chooses the witness b, then b will survive in the discourse model and be added to the set $I_{discourse}$. Thus, the following text is acceptable:

(6) Mary saw some student. He jumped out from behind the door.

In (6), Eloïse, playing the initial verifier, selects a witness for *some student* from the model in accord with the rule (R.some). After the first game, the players move to the second sentence in (6). In order to interpret the pronoun, *he*, Eloïse must pick an entity from $I_{discourse}$. Since her choice of student survives in $I_{discourse}$, she can select it as the target referent of *he*.

If Abelard, the initial falsifier, is playing verifier, as would be the case under negation, then his choice is forgotten, as (7) shows[6]:

(7) Mary didn't see some student. {They were/#He was} hiding in the corridor.

In (7), negation forces the verifier and the falsifier to exchange roles. Therefore, at the point where (R.some) must be played on *some student*, Abelard will be playing the verifier. His choice of student is dropped at the end of the game, although the set of students is added to the discourse model.

[6] I will prefix pragmatically odd choices with a '#.'

The rule (R.some) and Choice Preservation allow us to account for the establishment of discourse entities in this simple case. Let us contrast (R.some) with another Aristotelean, *every*:

(R.every)
If the game $G(S\,;M)$ has reached an expression of the form:

$$Z - \text{every } X \text{ who } Y - W.$$

Then the falsifier may choose an individual from the appropriate domain, say b. The game is then continued as $G(Z - b - W, b \text{ is an } X \text{ and } bY; M)$. The individual b is added to the choice set $I_{current}$.

Since the falsifier is playing, there is no question of the particular choice of witness selected under (R.every) surviving into the next game, although the set that contains witness does survive, according to Choice Preservation. We expect to see the following pattern:

(8) a. Every student thinks he's treated unfairly.
b. Every student passed the exam. #She studied very hard.
c. Every student passed the exam. They studied very hard.
d. Every student wrote an essay. One spelled most of the words correctly. He must have had a dictionary.

In (8)a, the falsifier selects an entity as a counterexample to the sentence. This entity is placed into $I_{current}$ and can act as an antecedent for any anaphor that occurs within the game. Thus, the pronoun *he* in the embedded clause can denote the falsifier's choice. At the end of the game, however, the falsifier's particular choice is deleted and the set that falsifier chose from is placed in $I_{discourse}$. Thus, there is no singular referent for the pronoun *she* in (8)b and the text is peculiar, all else being equal.

We can compare the peculiarity of (8)b with the unremarkable acceptability of (8)c. Although the falsifier's choice of witness is dropped, the set from which he chose is placed in $I_{discourse}$ and can serve as the target for a plural pronoun. Consider, finally, the slightly longer text in (8)d. In the first sentence, the falsifier selects a witness and the set he chose from is evoked; by Choice Preservation, *students* is added to $I_{discourse}$. The rule for interpreting *one* is approximately:

(R.one)
When a semantical game has reached a sentence of the form:

$$X - \text{one} - Y$$

an individual, say b, is selected by the verifier from a set in $I_{discourse}$. The game is continued with respect to:

$$X - b - Y.$$

The entity b is then added to $I_{current}$.

In accordance with (R.one), the verifier may find a set in $I_{discourse}$ and pick an element from it to serve as the witness for *one*. Having done so, she establishes a particular discourse entity—one of the students—who survives in $I_{discourse}$ and can then serve as the antecedent for the pronoun in the third sentence.

7.3 Cardinals and bounding determiners

We turn now to the interesting cases of cardinals and bounding determiners, as exemplified in:

(9) a. At least three students passed the exam.
b. At least one (= some) dean drank eau de vie.
c. At most ten graduate students wrote papers.
d. Between three and seven trustees take viagra.
e. Exactly five deans read at the sixth grade level.

These quantifiers involve explicit numeric quantities. In (9)a and (9)b the quantifiers set a lower bound on the number of individuals with the property named in the predicate. In (9)c an upper bound is placed on the number of individuals and in (9)d and (9)e upper and lower limits are placed on the number of individuals. Thus, one might take *exactly five* to mean "more than four but less than three." Notice that we take *at least one* to be equivalent semantically to *some*, although their pragmatic effects may differ. I will, again, restrict my attention to some simple cases, the others being easy to define on their basis.

The game rules that follow differ in form from the rules for Aristoteleans presented in Section 7.2. The games for the Aristoteleans all involve the choice of a witness by one player or the other. I will propose that these quantifiers, as well as the quantifiers that follow, involve two moves. In one move, a player chooses a set of entities and, in the next move, his or her opponent selects a witness from that set.

Consider a simple cardinal determiner like *at least n*, where *n* is a positive integer. We can simulate this kind of a quantifier by allowing the verifier to select a set of entities from the model, each of which could potentially witness the relevant property. The falsifier is then allowed to select an individual from this set as a counterexample. If he is unable to do so, then the sentence must hold in the model and the verifier wins:

(R.at least *n*)
If the game $G(S\,;M)$ has reached an expression of the form:

$$Z - \text{at least } n\ X \text{ who } Y - W$$

then the verifier may choose a set of entities from the domain M, call it ver(M), such that $|\text{ver}(M)| \geq n$. The falsifier then selects an entity $d \in \text{ver}(M)$. Play continues on $Z - d - W, d$ is an X and $d - Y$. Both d and the contents of ver(M) are placed in $I_{current}$.

Notice that both the verifier's choice of the set, ver(M), and the falsifier's choice from ver(M) are placed in $I_{current}$, although only ver(M) will survive in the discourse model. This means that the falsifier's choice of counterexample and the verifier's choice of the set should be available as targets for anaphora within the current game. To motivate this consider the following:

(10) a. #At least three students think he's smart.
(where *he* is one of the students.)
b. At least three students think they're smart.

At first view, it would seem that a singular anaphor is ruled out in (10)a. That is, we cannot utter (10)a intending to mean that each of the set of at least three students believes of himself or herself "I am smart." I submit, however, that this is a fact about the morphosyntax of coreference; the problem is that the pronoun does not agree in number with the antecedent noun phrase, so the two cannot share reference at any level. Compare this with (10)b, which is at least three ways ambiguous. On one reading, the students have a belief about some set of individuals, namely that whose individuals are smart. We need not concern ourselves with this reading. The other two readings involve whether the students believe of the whole set of three or more students (that is, the witness set) that they all are smart or whether each member of the set believes "I am smart." In the former case, the set ver(M) is the target of the anaphor and in the latter case the falsifier's choice of individual, d, is the target of the anaphor.

When the contents of $I_{current}$ are placed in $I_{discourse}$, the falsifier's choice of witness is, of course, deleted as required by Choice Preservation. We can immediately account for the following range of texts:

(11) a. At least five deans smoked crack. They passed out.
b. At least five deans drank Mad Dog. #He passed out.
c. At least five deans dropped acid. One jumped out the window.

Example (11)a is acceptable because the plural pronoun *they* can denote the verifier's witness set, the set of deans that smoked crack. Example (11)b is odd because, all else being equal, there is no entity in $I_{discourse}$ for the singular definite pronoun to denote. Finally, (11)c is acceptable because the verifier can pick a single element out of the witness set, now transfered to $I_{discourse}$. The explanations for each of the small texts in (11) is the same as those given for texts above involving the Aristoteleans.

Let us turn, now, to examples involving *at most n* as in:

(12) At most three politicians smoke crack.

Following Keenan and Stavi (1986), we might try to exploit the boolean structure of the algebra in which natural language determiners take their denotations and treat *at most* $n(P, Q)$ as the boolean complement of *at least* $n + 1(P, \overline{Q})$.

That is, the falsifier and the verifier would exchange roles and play on *at least* $n+1$ when they encounter a sentence containing *at most n*. But consider the discourse effects of a sentence containing *at most n*:

(13) At most five faculty members considered resorting to vegetarianism. They changed their minds when they realized how much work it would be.

The first sentence is true if three faculty members considered resorting to vegetarianism. Suppose that is the case. The pronoun in the next sentence refers to just those three faculty members who considered resorting to vegetarianism and to no others. Our method is to write rules that require the players to select sets that will eventually serve as potential discourse entities; the problem with exchanging roles and playing on *at least* $n+1$ is that it fails to create the needed discourse entities. We must, therefore, reject this approach.

The following rule, however, will do the trick:

(R.at most n)
If the game $G(S; M)$ has reached an expression of the form:

$$Z - \text{at most } n\ X \text{ who } Y - W$$

The verifier chooses a set of entities from the domain M, call it ver(M), such that the cardinality of ver(M) is less than or equal to n. The falsifier chooses a disjoint set of entities from M, call it fal(M), such that $|\text{ver}(M) \cup \text{fal}(M)| > n$. The game then continues on:

Z–every ver(M) $-W$, Z–no fal(M)$-W$, every ver(M) is an X who Y, every fal(M) is an X who Y.

The set ver(M) is placed in I_{current}.

The rule (R.at most n) is based on the idea that the verifier must choose a maximal set of cardinality bounded by n. If she has a winning strategy, then she must pick out every object so described and the falsifier should be unable to select an object matching that description. Since Eloïse in her role as the verifier never selects a single entity—play is carried by selecting sets and then playing on *every*, where the falsifier chooses, and *no*, where Eloïse cannot be playing as initial verifier—we do not expect definite singular discourse anaphora to be licensed by *at most n*, although ver(M) will be in the choice set and available for plural definites and indefinite anaphora. Thus, *at most n* should behave like *at least m*, which it does:

(14) a. At most five trustees know how to play *Candyland*. #He studied it at Harvard Business School.
b. At most five trustees drank Old Crow. They were trying to save money.
c. At most five trustees performed on the kazoo. One did a passable interpretation of *Die Walkyrie*.

In (14)b, the pronoun *they* refers to the five or fewer trustees who drank Old Crow; that is, the pronoun refers to the set selected by the verifier in the previous game. The game rule (R.at most n), when contrasted with the treatment of *at most n* as the complement of *at least* $n+1$, brings out the strategic nature of interpretation.

I will put aside bounding quantifiers as below (but see Clark, 2004); for the present I will merely note some appropriate texts and leave the definition of the game rules as a puzzle for the reader:

(15) a. Between three and seven department chairs exchanged flowers. #He sneezed because he was allergic.
b. Between three and seven department chairs exchanged flowers. They decorated their hats with them.
c. Between three and seven department chairs exchanged flowers. One led a ceremonial procession down the Alps.

7.4 Majority determiners

By *majority determiners* I mean determiners like *most*, *more/less than half of the* and n-ary determiners like *more ... than ...*, as illustrated in (15):

(16) a. Most faculty eat grubs in the winter.
b. More than half of the trustees dine on Andalucian dogs.
c. More deans than faculty resort to prostitution.

These determiners, being higher-order, are of greater complexity than those we have considered up to now. *most P's are Q's* is true when the cardinality of the set of things that are both P and Q is greater than that of the set of things that are P but not Q. It may not be immediately obvious how to construct a simple game rule, based on choice, that will yield the correct result; that is, where verifier has a winning strategy just in case a majority of P's have the property Q.

In addition, the cardinalities involved might be infinite:

(17) Most integers are not divisible by five.

Although Abelard should win on (17), it is far from obvious how to encode the meaning of (17) in a finite game, if such a thing is even possible. For the moment, I will restrict my attention to games involving majorities over finite sets.

We might try the following game rule for *most*:

(R.most)
If the game $G(S : M)$ has reached an expression of the form:

$$Z - \text{most } CN \text{ who } P_1 - W$$

where CN is a common noun and P_1 is a predicate, then the verifier picks a set of objects, call it ver(M), of cardinality at least:

$$\frac{|CN|}{2} + 1$$

The falsifier may choose an individual $d \in$ ver(M) and the game continues as:

$$G(Z - d - W, d \text{ is a } CN \text{ and } d\ P_1; M)$$

The set ver(M) is then added to the choice set I_S.

The game rule (R.most) requires that the verifier select a set whose cardinality is greater than half that of the set denoted by CN. The falsifier may then select an element of that set to test the sentence on. If the falsifier cannot select a counterexample from the set, then it must be that a majority of the elements denoted by CN have the requisite property and the verifier wins. Notice that the difference between (R.most) and the game rules for the cardinal determiners resides in the requirement that ver(M) be of a particular size.

Finally, the rule requires that the set ver(M) be placed in the choice set. The discourse effect of (R.most) should be similar to those of the cardinal determiners. That is, singular pronouns will not match, but plurals and indefinites will:

(18) a. Most deans practice fortune-telling. He is a reader of tarot cards.
b. Most deans are druids. They march about waving mistletoe.
c. Most deans hunt small game. One caught a pigeon.

Finally, let us turn now to examples of quantifiers with multiple heads like "More doctors than lawyers eat pez." Here is a candidate game rule:

(R.more-than)
If the game $G(S; M)$ has reached an expression of the form:

$$Z - \text{more } X \text{ who } Q \text{ than } Y \text{ who } R - W$$

Then the verifier picks a set of entities ver(M) of cardinality n and the falsifier likewise picks a set of entities fal(M) also of cardinality n. The falsifier picks $d \in$ ver(M) and the verifier picks $c \in$ fal(M). The game continues on: $Z - d - W$ and not $Z - c - W$ and d is an X and c is an Y and d Q and c R.
Both ver(M) and fal(M) are added to the choice set I_S.

The effect of (R.more-than) on discourse anaphora is far less clear. As we would expect, use of a singular definite pronoun is not allowed:

(19) More deans than faculty eat three square meals a day. #He is getting fat that way.

The proper interpretation of plural definite pronouns is less clear:

(20) a. More deans than faculty eat three square meals a day. They need to keep up their blood sugar.
(*They* being the deans)

b. More deans than faculty eat three squares a day. They want to keep their weight down.
(*They* being the faculty)

It seems to me that (20)a is somewhat more comfortable than (20)b, but that the latter is still possible. I have, therefore, added both ver(M) and fal(M) to the choice set.

Equally, it seems to me that either argument of *more-than* can provide a basis for indefinite singular anaphora:

(21) a. More deans than faculty eat three squares a day. One danced a merry jig to taunt the assembled faculty.
(*One* being one of the deans.)
b. More deans than faculty eat three squares a day. One whined piteously outside the deans meeting.
(*One* being one of the faculty.)

The game rules given in this section work for finite sets. Can they be adapted for infinite sets? The treatment of generalized quantifiers given in Keenan and Stavi (1986) uses infinitary means, in the guise of arbitrary meets and joins, to derive higher-order quantifiers. For example, *most* is defined as the arbitrary meet of an infinite family of generalized quantifier denotations, built from the basic cardinal determiners.[7]

One might think that infinite would correspond to an infinite round of games over finite samples. Thus, we might let the verifier pick a sample from the model on which the game is played. These sub-games could be repeated infinitely. But, of course, this will not work; the verifier could cheat infinitely. For example, she will always be able to pick a biased set for:

(22) Most numbers are even.

and win each time. Thus, using an infinite round of finite games will not work. Instead, Eloïse and Abelard must be locked in an infinite game, whatever that means. To my mind, a better option would be for Eloïse to offer Abelard a convincing proof (or vice versa) of the truth (or falsity) of the proposition.

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[7] The semantic automata framework of van Benthem (1986) which simulates higher-order quantifiers with push-down automata does not even treat the problem of infinite sets, since the models are encoded as finite strings.

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Part III

DIALOGUES