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Generalized Quantifiers and Number Sense

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Abstract
Generalized quantifiers are functions from pairs of properties to truth-values; these functions can be used to interpret natural language quantifiers. The space of such functions is vast and a great deal of research has sought to find natural constraints on the functions that interpret determiners and create quantifiers. These constraints have demonstrated that quantifiers rest on number and number sense. In the first part of the paper, we turn to developing this argument. In the remainder, we report on work in neurobiology that test the empirical claims of the theory. In particular, we look at fMRI experiments and behavioral experiments with various patient populations that support the intimate connection between natural language quantification and number sense.

Phrases like the ones shown in (1) – quantifiers – are of enduring interest to philosophers, psychologists and linguists:

(1) some student, every book, at least three surgeons, most lawyers, some but not all dentists, no pediatrician

The analysis of quantifiers has stimulated a fruitful collaboration between linguists and philosophers, on the one hand, and psychologists and neuroscientists, on the other.

The puzzle quantifiers present can be stated in terms of three interrelated questions:

(2) a. What do quantifiers denote?
b. What psychological mechanisms underlie our ability to grasp their denotations?
c. How are these mechanisms realized in the brain?

In trying to answer these questions, we will see that our ability to learn and use quantified expressions like those in (1) is intimately tied to our ability to understand precise number; while most animals can readily perceive quantities corresponding to small numbers, up to three or four objects, we are alone in our ability to distinguish accurately between, say, 50 and 51 objects. While the precise nature of this ability –and its relation to language – is still an open question, preliminary results are available and the investigation is one that must be carried out in a collaboration between neuroscientists, psychologists, linguists and philosophers.

1. *Generalized Quantifiers*

Consider a simple sentence consisting of a generalized quantifier, *every dog*, and a predicate, *barks*:

(3) Every dog barks.

The translation into first-order predicate logic is straightforward:

(4) $\forall x[\mathrm{DOG}(x) \rightarrow \mathrm{BARK}(x)]$

which is true if and only if every dog is such that it does, indeed, bark; in terms of a standard truth definition, given a model, *M*, and a function from variables to entities in the model, *g*:

(5) $[[\forall x\varphi]]^{M,g}$ is true if and only if for every $d \in M$, $[[\varphi]]^{M,g[d/x]}$ is true.

Notice, however, the divergence between the simple subject-predicate form of (3) and the logical form in (4). In addition, the rule of interpretation in (5) does not capture the interpretation of *every* in English.

Some insight into this is provided if we use λ-operators to abstract away from the predicates; the quantifier in subject position, *every dog*, would then be:

(6) $\lambda Q \forall x[\mathrm{DOG}(x) \rightarrow Q(x)]$

which is a function from predicates (sets) to truth-values. Abstracting out the head noun, *dog*, gives us a function from sets to functions from sets to truth-values:

(7) $\lambda P \lambda Q \forall x[P(x) \rightarrow Q(x)]$

The interpretation of the English determiner *every* is captured by the interaction of the universal quantifier and the connective.

The analysis in (7) immediately suggests a more direct interpretation of the determiner as function from pairs of sets to truth-values:

(8) $\mathrm{every}(P, Q) = \begin{cases} 1 & \text{iff } P \subseteq Q \\ 0 & \text{otherwise} \end{cases}$

That is, every(*P*, *Q*) holds true if and only if the set *P* is a subset of the set *Q*; for example, every(DOG, BARK) holds just in case the set of dogs is a subset of the set of barking things.

The definition of *every* in (8) is both simpler than the definition in (7) and closer to the normal subject/predicate structure of the English sentence. It localizes the meaning of *every* as a function on two sets, *P* and *Q* instead of distributing the meaning of *every* between the universal quantifier and material implication.

The type of representation illustrated in (8), a *generalized quantifier*, is easily adapted to the definition of other natural language determiners; we illustrate two cases here, a 'cardinal' determiner in (9) and a majority determiner in (10):[1]

(9) At least three students smoked.

$\text{at-least-3(STUDENT, SMOKER)} = \begin{cases} 1 & \text{iff } |\mathrm{STUDENT} \cap \mathrm{SMOKER}| \geq 3 \\ 0 & \text{otherwise} \end{cases}$

(10) Most doctors golf.

$\text{most(DOCTOR, GOLFER)} = \begin{cases} 1 & \text{iff } |\mathrm{DOCTOR} \cap \mathrm{GOLFER}| > |\mathrm{DOCTOR} - \mathrm{GOLFER}| \\ 0 & \text{otherwise} \end{cases}$

The sentence in (9) is true if and only if the number of students who are smokers exceeds two and the sentence in (10) is true if and only if there are more doctors who are golfers than doctors who are not golfers.

The generalize quantifier notation can easily be used to show scope ambiguities between quantifiers:

(11) Every student read some book.
$\text{every}(\{x|\text{STUDENT}(x)\}, \{y|\text{some}(\{z|\text{BOOK}(z)\}, \{w|\text{READ}(y, w)\})\})$
$\text{some}(\{z|\text{BOOK}(z)\}, \{w|\text{every}(\{x|\text{STUDENT}(x)\}, \{y|\text{READ}(y, w)\})\})$

Although the set notation in (11) is a bit cumbersome visually, the reader can verify – assuming that some(P, Q) is true when P and Q have a non-empty intersection – that the two analyses in (11) express the scope ambiguity of the sentence.

Since generalized quantifiers are functional expressions, we can develop a systematic treatment of quantifiers as functions, a treatment not afforded by that of first-order logic. Notice, however, that the space of possible denotations is vast; if a natural language determiner like *every* is a function from pairs of sets to truth-values, then there are $2^{4^{|M|}}$ possible denotations, where $|M|$ is the number of entities in the model. Thus, even in relatively small models there is an abundance of possible determiner denotations; in a model with only two individuals, there are 2^{16} functions that could be possible determiner denotations.

This embarrassment of quantifiers should puzzle us: Languages do not select from this set willy-nilly, rather we find similar functions chosen across languages; children must learn these functions and it is far from clear how they would do so if there were no constraints on the set of possible denotations; finally, speakers and hearers must grasp the conditions under which quantifiers are used appropriately. As a result, researchers began to investigate principled constraints on quantifier denotations.[2]

A number of plausible constraints were soon imposed on the set of determiner denotations; a good summary of the constraints can be found in van Benthem (1986). The cumulative effect of these constraints was that the truth of a determiner in the following way:

(12) The interpretation of the function f_{DET} that interprets a natural language determiner DET, as in $f_{\text{DET}}(P, Q)$ is contingent on $|P \cap Q|$ and $|P - Q|$; that is, the cardinality of the intersection of P and Q and the cardinality of P complement Q.

Thus, the truth of (3) is contingent only on the number of dogs that bark and the number of dogs that don't bark; if the number of dogs that fail to bark is greater than 0, then (3) must be false.

The generalization in (12) certainly holds true for Aristoteleans like *every/all*, *some*, *no* and *not all*. It holds for cardinal determiners like *at least n at most m*, *between m and n* and so on; majority determiners like *most* and *more than half* and so on; as well as parity quantifiers like *an even number of*. Exception determiners like *all but n* also fall under the generalization:

(13) All but three dogs barked.

although examples like *all…but John* require more information than just the cardinality of a set:

(14) All the students but/except John studied for the exam.

More complex determiners like:

(15) More lawyers than doctors drink martinis.

that is, three-place determiners of the form 'more-than(P, Q, R)' can be formed from an obvious generalization; that is, we require only the cardinalities of $|P \cap R|$ and $|Q \cap R|$ in this case.

Even 'vague' determiners like *many* or *few* which are contingent on expectations that must be derived from context seem to rely on the numbers in the sets P and Q; thus:

(16) Few students studied for the exam.

is interpreted relative to my expectations about the number of students that ought to have studied for the exam versus the number that actually studied for it.

Consider, finally, that an apparent exception like *only*(P,Q) as in:

(17) Only frogs croak.

requires that the cardinality of $Q - P$ be 0. Even the exceptional can be understood in terms of number.

Finally, van Benthem (1986) demonstrates that there is an interesting relationship between quantifiers and computation theory; in this case, we imagine that the underlying model has been replaced by strings of symbols generated from the sets $P \cap Q$ and $P - Q$:

1. Aristotelean and cardinal quantifiers can be simulated by finite state machines without loops; that is, an extremely simple memoryless machine consisting of a set of internal states and a function which, when given an item and a state returns a new state;
2. Parity quantifiers (like *an even/odd number of* can be simulated by finite state machines with a loop;
3. Majority quantifiers like *most* require a simulating machine that has at least a simple memory, like a push-down stack.

Although these results only hold of finite strings, their immediate relevance is that different types of determiners use different computational resources; the importance will become clear when we consider the neurobiology of quantifiers. These results have been greatly extended by Szymanik (2009) who brings more general complexity results to bear on the class of functions that can interpret determiners.

2. *Number Sense*

The analysis of generalized quantifiers, then, implies that the interpretation of quantifiers is directly related to the interpretation of number. It is a remarkable fact that we are alone, among all species, in having expressive language – one that contains things like quantifiers – and in having a sense of precise number.

Let us turn to this sense of precise number in a bit more detail. Virtually all animals have a sense of approximate number. In particular, both humans and animals seem to directly perceive small numerosities; quantities of up to three elements are accurately perceived so that a set of two elements is easily distinguished from a set of three elements; it is often claimed, in fact, that the perception of small numbers is 'subitized', meaning that there is a dedicated perceptual mechanism for the perception of small quantities, up to three. Animals have only an approximate sense of larger numbers beyond the subitized range. For example, a pigeon can be taught to peck at a bar 16 times for a reward; if the behavior of the pigeon is charted over numerous trials, it becomes apparent that the pigeon pecks the bar about 16 times, the actual number of pecks being normally distributed around 16.[3]

This approximate sense of number is modulated by the Weber–Fechner law which states that the relationship between a stimulus and its perception is logarithmic; if a stimulus increases as a geometric progression – if it is multiplied by a fixed factor – its perception will be altered in an arithmetic progression. This means that while it is easy to distinguish accurately, say, four things from five things, it should be much harder to distinguish accurately between 50 things and 51 things.[4]

Recent work (see, e.g. Pietroski et al. 2009) has tested the relationship between approximate number sense and the ability to interpret sentences containing the determiner *most*. Recall from our earlier discussion that *most* is a determiner that is not first-order definable; it can only be simulated by a machine with a memory. In these experiments, subjects were briefly presented with an image and then asked to make a truth-value judgment about that image. For example, they might be presented with an image of some number of red and blue balls, the presentation being so brief that the subjects would not have time to count the number of balls. They were then asked to evaluate whether most of the balls were blue. Their performance was shown to be exactly subject to the Weber–Fechner law. This work raises interesting questions about the relationship between the meaning of an expression like *most* and the methods that speakers use in working out the verification of a use of the expression.

While adult humans, like other animals, have an approximate sense of number, we also have a precise sense of number. Thus, if I were to have you tap your finger 20 times without counting, you will behave like a pigeon; your performance will be centered in a normal distribution around 20. Furthermore, your approximate sense of number obeys the Weber–Fechner law. However, if I allow you to count, you can exactly tap your finger 20 times; furthermore, you can distinguish arbitrarily large numerosities, if you are allowed to count the items.

The fact that we have precise number sense and quantifiers immediately suggests that the two capacities are, somehow, linked by an underlying system. As a first approximation, one might suppose that the number words themselves provide the basis for our sense of precise number. Notice, however, that precise number sense must rely on more than just names for numbers; the semantics of precise number requires not just a set of number (or number names) but an ordering relation on them. It is not enough to know that 22 is distinct from 21; I also need to know that 22 is greater than 21. An alternative view, one proposed in Gelman and Gallistel (1978) and developed in much later work, proposes a distinct mechanism, 'numerons', which would underlie our precise sense of number.

Nevertheless, we might suppose that our linguistic abilities and our numeric abilities are connected in some way. We might take as a hypothesis the idea that our language provides the underlying mechanism for building number systems. This is a variant of the Sapir–Whorf hypothesis; that is, that language can influence our cognitive abilities. In some sense, language can 'cause' our ability to perceive precise number; let's call this the *Whorfian hypothesis*. Recent work in the Amazon has singled out two groups – the Pirahã (see, in particular, Gordon (2004) and the Mundurucu (Pica et al. 2004)) – both of whom lack number words; the question is whether these people are capable of numeric cognition, whether precise or approximate.

Other hypotheses are possible. For example, it could be that our ability to perceive and use precise number is a logical prerequisite for our ability to use natural language; that is, the various cognitive operations required for planning and producing utterances are quantitative in character, the same operations that are required for manipulating number.

Alternatively, we might suppose that both language and precise number have some more basic set of capacities at their base. In other words, some set of cognitive operations – perhaps the ability to compute and store partial functions – underlie both language and precise number.

The Whorfian hypothesis has received the most attention in recent years.[5] In part, this is because babies and small children exhibit only approximate number sense, as noted above. Children appear to acquire their sense of precise number with their number language. Equally, languages which lack precise number words (and, possibly, quantifiers) have received some attention recently. It has been shown that speakers of such languages have well-developed approximate number sense.

Notice, though, that a great deal of care must be taken in assessing the Whorfian hypothesis. The fact that someone lacks a vocabulary for precise number makes it extremely difficult to ask them to perform tasks that require precise number judgments. Thus, children might have the capacity to perform a numeric task, but it would be difficult to ask them to do so. Equally, an adult speaker of a language without number words might never have explicitly performed a numeric task and it might be difficult to ask them to do so; it does not follow that they lack the capacity to perform the task simply because their language does not allow easy access to the capacity.

These considerations suggest that we need a different method of investigating these claims, a method that will bring to bear more than just observational and behavioral tests. Instead, we should investigate the neurobiological underpinnings of both number and language. If we understand the neuroanatomy underlying both number sense and quantifier understanding, we can begin to unravel the role that language plays in number sense.

3. *The Neurobiology of Quantifiers and Numbers*

One way to probe the relationship between language and precise number is to consider information both from brain imaging studies and to look at specific populations that show impairment either for number sense or for language.

The model of number knowledge proposed by Dehaene (1997), for example, distinguishes between approximate and precise number knowledge; it suggests that the distinction between close numbers (e.g. 6 and 7) requiring precise number knowledge depends on a verbally mediated representation, and this involves recruitment of left peri-Sylvian language regions (Dehaene 1997); Dehaene et al. (1999) showed activation of inferior parietal cortex bilaterally and left orbital frontal cortex during an assessment of precise addition compared to approximate addition (see also Dehaene et al. 2003; Feigenson et al. 2004). These investigators argue that left orbital frontal activation mediates the verbal representation necessary for precise calculations, although orbital frontal cortex is not peri-Sylvian and is anterior to Brocas area that is traditionally implicated in language processing.

Certainly, neuroimaging studies have associated number knowledge with parietal activation (see, e.g. Burbaud et al. 1995; Dehaene 1997; Dehaene et al. 1999; among many others) and patient studies have demonstrated loss of number knowledge following parietal lobe disease (see Halpern et al. 2003; McMillan et al. 2005, 2006; and the references cited in these articles). A combined use of patient studies and neuroimaging should serve to tease apart the relationship between language and number sense (Clark and Grossman 2007).

The experimental method is relatively simple. Subjects were presented with the pair of an image, which constitutes a model, and a sentence that they were asked to verify. Subjects were presented with 120 grammatically simple propositions each containing a quantifier that probed a color feature of a familiar object in a visual stimulus array (e.g. at least three of the balls are blue). Six different quantifiers were each presented in 20 trials: half were first-order quantifiers (*at least three*, *all*, and *some*) and half were non-first-order quantifiers (*less than half*, *odd*, *even*). Note that we avoided negatives like *no* and *not all*. Nevertheless, these quantifiers allowed us to explore some interesting features that might potentially contribute to quantifier comprehension, such as the presence of an explicit number, and the need for precise quantification rather than approximation. Notice that the quantifiers allow us to compare the different complexity measures discussed above; Aristotelean and cardinal quantifiers being simplest, parity quantifiers being the next most complex and majority quantifiers being the most complex.

Each quantifier problem involved two consecutive 10 s events. In the first event, only the proposition was presented; in the second event, the same proposition in addition to a stimulus array containing eight randomly distributed familiar objects (balls, flowers, cars, etc.). This array acted as a model; the array was presented for 2500 ms followed by a blank screen for 7500 ms. Subjects were asked to decide if the proposition accurately described the stimulus array. They responded by pressing the right button of a fiber optic response pad if true; the left button was pressed if false.

Half of each type of item was true and half false. Also, half of the stimulus arrays contained a quantity of target items near the criterion for validating or falsifying the proposition, therefore requiring a precise judgment (e.g. four targets in at least three). These stimuli would force the subject to use precise number sense in verifying the truth of the proposition. Half of the stimuli were distant from the criterion. Subjects could presumably use approximate number sense to make a judgment; for example, eight target items in the array satisfied the criterion and the proposition to be verified involved the determiner *at least 3*). Debriefing following the experiment revealed that none of the participants were aware that the stimulus array always consisted of eight objects. We monitored behavioral accuracy. All stimuli were counterbalanced and randomly distributed throughout the experiment, divided into four equal runs. The stimuli were presented to the subject using an LCD projector back-projected on to a screen placed at the bore of the magnet.

The subjects were 12 young, right-handed adults, eight males and four females. The results were quite striking. There was no activation of left peri-Sylvian regions typically associated with language during comprehension of quantifiers. In addition, there was no contrast of activation during the comprehension of precise quantifiers as opposed to approximate quantifiers; that is, the results failed to demonstrate evidence for verbally mediated precise number knowledge. Instead, there is right inferior parietal cortex recruitment during quantifier processing, suggesting that quantifier judgments recruited the number processing areas in the parietal lobe.

Comparing the activation patterns associated with non-first-order quantifiers with first-order quantifiers reveals that special physiological mechanisms support the processing of non-first-order quantifiers; in particular, we see additional activation in the frontal cortex. The formal analysis of quantifiers, discussed above, suggested that executive resources required for non-first-order quantifiers, but not first-order quantifiers, are, first, holding one number property in mind while identifying another and, second, comparing properties in order to evaluate the truth-value of the proposition. This analysis is congruent with our hypothesis that a more complex computational mechanism was required for majority and parity determiners. The additional activation in the frontal cortex suggests that executive resources like working memory are being marshaled for these tasks. If so then the lack of differential activation across precise and approximate judgments is unsurprising since precise judgments do not imply more resources. In other words, regardless of the manipulation of the stimulus array, the same executive resources are required for non-first-order quantifier comprehension.

We have two broad conclusions:

1. All generalized quantifiers have in common number knowledge and activation of inferior parietal cortex.
2. Working memory and prefrontal activation occur during non-first-order quantifier comprehension.

In order to further understand these results, we studied several populations of patients with some form of focal neural degenerative disease. One such group were patients with corticobasal degeneration (CBD). CBD patients typically present clinical features like apraxia, cortical sensory loss, and naming difficulty; neuroimaging studies have shown disease in the parietal cortex and, indeed, these patients show number processing deficits (see McMillan et al. 2006). We expect, given our first finding, that CBD patients should show a gross impairments in the understanding of quantifiers in general.

A second population of patients were diagnosed with frontotemporal dementia (FTD). These patients have disease that often includes the dorsolateral and inferior portions of frontal cortex, areas associated with working memory limitations in FTD. In general, FTD patients have difficulty understanding sentences that make demands on working memory. Although FTD patients show no impairment in number knowledge *per se*, we predict that they should have trouble with non-first-order quantifiers, since these make demands on executive resources like working memory.

Finally, as controls, we tested normal healthy seniors and patients with Alzheimer's disease (AD). The former group should show no particular impairment, except the normal limitations of age, while the latter group show limitations in multiple cognitive domains including number calculations and working memory. We expect, then, that healthy seniors should have little difficulty with the verification task, while Alzheimer's patients should show across-the-board impairments.

The materials and methods for the experiment are exactly as above; subjects were presented with a model verification task involving a mix of simple first-order and non-first-order quantifiers. Unlike the first experiment, patients were not put in the scanner; instead they were sorted into the four groups and tested behaviorally.

All patient groups performed worse than healthy seniors on overall quantifier comprehension. Moreover, CBD patients (72% correct) were significantly worse than FTD patients (83% correct) and AD patients (82% correct). This brings out the distinct comprehension deficit for quantifiers in CBD. There was also a significant main effect for Quantifier Class. All groups are more impaired on non-first-order quantifier comprehension than first-order quantifier comprehension.

There was a significant interaction effect of Group by Quantifier Class; this was due to the relative deficit of CBD patients for first-order quantifiers, where CBD patients were significantly more impaired than AD patients. For non-first-order quantifiers, CBD patients were more impaired than both AD patients and FTD patients. However, when the results were correlated with a test of working memory – reverse digit span, where patients are asked to report a sequence of digits in a sequence reversing the order of presentation and the longest sequence that is correctly reversed is recorded – there was a strong correlation between accuracy on reverse digit span and non-first-order quantifier comprehension for all patient groups, but there was no correlation with first-order quantifier comprehension; see McMillan et al. (2005, 2006) for discussion and presentation of the statistical tests.

To make the discussion more concrete, consider the image in Fig. 1. The image on the left shows right hemisphere cortical atrophy in corticobasal syndrome. The aqua line shows where a coronal slice was taken for the image on the right. The right image is a coronal slice showing cortical atrophy, and the aqua blobs on the coronal view shows areas of a significant regression relating performance accuracy in CBD to cortical atrophy on a task where patients matched a brief sentence containing a cardinal quantifier (e.g. 'At least three cows are in the barn'). The figure shows that the areas implicated in quantifier processing are in the right hemisphere, not the left, and overlap significantly with parietal regions that support number sense.

Summarizing the results, then, we see that while number processing areas are used to support quantifier comprehension in natural language, we do not see language areas being used to support precise number judgments. In other words, we have an interesting dissociation between precise number and language. Put more simply, it does not seem as though number language alone is behind our sense of precise number! While it would be hard to see precise number knowledge without precise number language, we nevertheless must conclude that there is no causation flowing from number language to number sense.

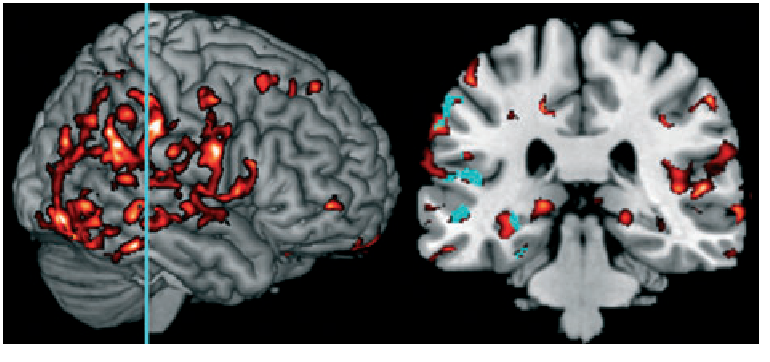

Fig. 1. Corticobasal Degeneration (CBD) and quantifier processing.

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Short Biography

Robin Clark is an associate professor in the Department of Linguistics at the University of Pennsylvania. His research interests, beyond language and number, include game theoretic pragmatics, and the neurobiological underpinnings of linguistic decision-making. He is the author of Meaningful Games (2012, the MIT Press).

Notes

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[1] In order to make the examples read more naturally, I've replaced the verb 'smoke' in (9) with the nominal 'SMOKER' and the verb 'golf' in (10) with the nominal 'GOLFER'. Nothing of substance to the current discussion hinges on this.

[2] The literature on generalized quantifiers is extensive, so I can only cite some of the highlights. The idea was first developed by Mostowski (1957). Arguably, the treatment accorded to quantifiers in Montague (1974) is quite compatible with Mostowski's work, but it is not until Barwise and Cooper (1981) that the connection is really made. The work of Keenan and Stavi (1986) and Keenan and Faltz (1985) developed the framework. van Benthem (1986) also provided a systematic computational treatment of generalized quantifiers. More recently, Keenan and Westerståhl (1997) and Peters and Westerståhl (2006) have summarized and further extended the theory. Extending the work of van Benthem (1986), Szymanik (2009) has brought considerations of computational complexity to bear on the problem.

[3] Dehaene (1997) and Butterworth (1999) provide good overviews of research on number sense in neuroscience. See also Whalen et al. (1999), Dehaene et al. (1999), and Feigenson et al. (2004).

[4] Gallistel and Gelman (1992) is a good source on preverbal counting. See also Whalen et al. (1999), Feigenson et al. (2004), and Dehaene et al. (1999). Xu and Spelke (2000) is a useful source for approximate number sense in infants. An alternative view on primate number sense can be found in Tomasello and Call (1997).

[5] See Whorf (1956) for a core text. More recent discussion includes Gumperz and Levinson (1996), particularly the paper by Slobin (1996). See also Lucy (1996).

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