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Number sense and quantifier interpretation

Robin Clark · Murray Grossman

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Abstract We consider connections between number sense—the ability to judge number—and the interpretation of natural language quantifiers. In particular, we present empirical evidence concerning the neuroanatomical underpinnings of number sense and quantifier interpretation. We show, further, that impairment of number sense in patients can result in the impairment of the ability to interpret sentences containing quantifiers. This result demonstrates that number sense supports some aspects of the language faculty.

Keywords Cognitive neuroscience · Natural language · Quantifiers · Psycholinguistics · Semantics · Numerosity · Number sense

1 Introduction

Humans are, to our knowledge, the sole species both capable of fully expressive speech and endowed with a precise sense of number. By *fully expressive speech*, we mean natural language, of course, however vague that term may be. Humans are the only species that spontaneously learn and use a sign system with a complex recursive structure (example (1)a), capable of reference displaced in time (example (1)b) and space (example (1)c) as well as self-reference (example (1)d where "this" refers to the whole sentence that contains it), endowed with a rich conceptual and semantic structure that includes reference not only to individuals (example (1)e) but also properties of individuals (example (1)f), properties of properties of individuals (example (1)g), and so on:

(1) a. [S Friends of John know that [S his merit lies in his pugnacity]]
b. In the 19th century, the practice of dentistry did not require a license.
c. The grass is always greener on the other side of the fence.
d. Copy this message on five one dollar bills and God
will bless you.
e. <u>Harry</u> left.
f. <u>Being tall</u> brings some financial advantages
g. Being pleasant is <u>pleasant</u>.

Other species, particularly higher primates, can learn to produce abstract signs in relatively complex patterns, but the learning is slow and effortful, requiring extensive tuition, and the results are far less complex in expression and structure than that routinely found in everyday speech. The examples above are well within the grasp of any ordinary speaker without any special tuition.

By precise sense of number, we mean the ability to distinguish sets of cardinality greater than four with great accuracy. Many animals, including humans, are capable of direct and accurate perception of numbers up to about four—we will refer to this as *subitization*—a fact which indicates that perception of these small numbers is supported by a dedicated neural substrate. For most animals, however, perception of

R. Clark (✉)
Department of Linguistics, University of Pennsylvania, Philadelphia, PA 19104, USA
e-mail: rclark@babel.ling.upenn.edu

M. Grossman
Department of Neurology, University of Pennsylvania Medical School, Philadelphia, PA 19104, USA

numbers greater than four is approximate. This means that the perception of numerosities is subject to Weber's law (see Dehaene 1997). In particular, the exact number is not perceived but, rather, a close approximation is made. For example, a rat taught to press a bar 15 times will press about 15 times (usually slightly more if a reward is involved). A graph of the number of presses made on repeated trials would show 15 as the average, with a normal distribution around it. The rat has no precise sense of the number 15; whatever neural system supports number sense is only capable of an approximation of the quantity.

We will argue that there are intimate connections between number sense and language. In particular, our understanding of quantifiers, like the underlined phrases shown in (2), is related to our ability to understand precise number:

(2) a. Some boys eat vegetables.
b. John watered at least three plants.
c. More than half of the students turned in the assignment.

Nevertheless, it is possible to dissociate language and precise number sense in ways that suggest that precise number sense does not rely on language. Furthermore, our evidence is compatible with the hypothesis that precise number sense is required in order to understand quantifiers. Thus, our evidence suggests the possibility that precise number sense is a prerequisite for quantifier understanding. This result is incompatible with the hypothesis that language is responsible for our sense of precise number.

2 Language and basic number sense

Both humans and animals have a sense of number. This number sense appears to be divided into two subsystems. First, there is a subsystem devoted to approximate number. The sense of approximate number is relatively noisy and errorful, as we shall discuss below. Second, there is a subsystem devoted to precise and accurate perception of small numbers, the subitizing system alluded to above. This system allows for direct perception of quantities up to about three or four.[1] The neuroanatomy of the approximate system is well understood (Dehaene 1997; Dehaene et al. 1999, 2003). Approximate number judgments activate the intraparietal sulcus in the parietal lobe (see Fig. 1[2]), near the fingers of the homuncular strip. This location has been used, by the way, to argue that human number sense is intimately tied with counting on the fingers (Butterworth 1999).

In this subsection, we will discuss evidence for both these systems in humans and animals. In addition, we will briefly consider how children develop from animal-like counters—who have only the approximate and subitizing systems—to adults who have a precise sense of number. We will then turn to our main area of interest, the interaction between number sense and language as well as the nature and origin of precise number sense.

2.1 Approximate number sense and subitizing

Let's turn first to the approximate sense of number. This sense of number is modality independent: it detects number from perceptual input whether that input is visual, auditory, or tactile. This perception of number is, however, noisy and errorful, the probability of the error growing as the number to be perceived grows; that is, this perception of number is ratio dependent. As an example, rats trained to press a lever n times produce presses that are normally distributed around n, and the standard deviation of the distribution grows as a linear function of the target number (Platt and Johnson 1971; Whalen et al. 1999; Cordes et al. 2001). Intuitively, if we are prevented from actually counting, it is easier to distinguish between 5 objects and 6 than it is between 35 objects and 36; this result is shown in Whalen et al. (1999) and Cordes et al. (2001). Although both pairs differ by one, in the second case the approximate number sense has difficulty detecting the difference since the probability of error has grown.

A similar effect can be observed in human infants. Xu and Spelke (2000) studied the ability of infants to distinguish between the numerosities of random arrays of dots. They report that 6-month-old infants can discriminate reliably between arrays containing 8 dots and 16 dots, and between arrays containing 16 dots and 32 dots. They cannot, however, distinguish reliably between arrays containing 8 dots and 12 dots or between arrays containing 16 dots and 24 dots. That is, they can discriminate reliably between two arrays when one array is twice as large as the other, but not when one array is only one and a half times as large as the other. Notice, in particular, that it is the ratio and not the absolute number that matters here. The numeric difference between 8 and 16 is equal to that between 16 and 24. Nevertheless, 6-month-olds can distinguish between the former but not the latter, thus

[1] The existence of the subitizing system is still controversial. See Gallistel and Gelman (1992) for a different view. Nothing we present here hinges on this point, although the data we present in Halpern et al. (2003) and Halpern et al. (2004) is relevant here.

[2] This diagram is from Dehaene et al. (2003).

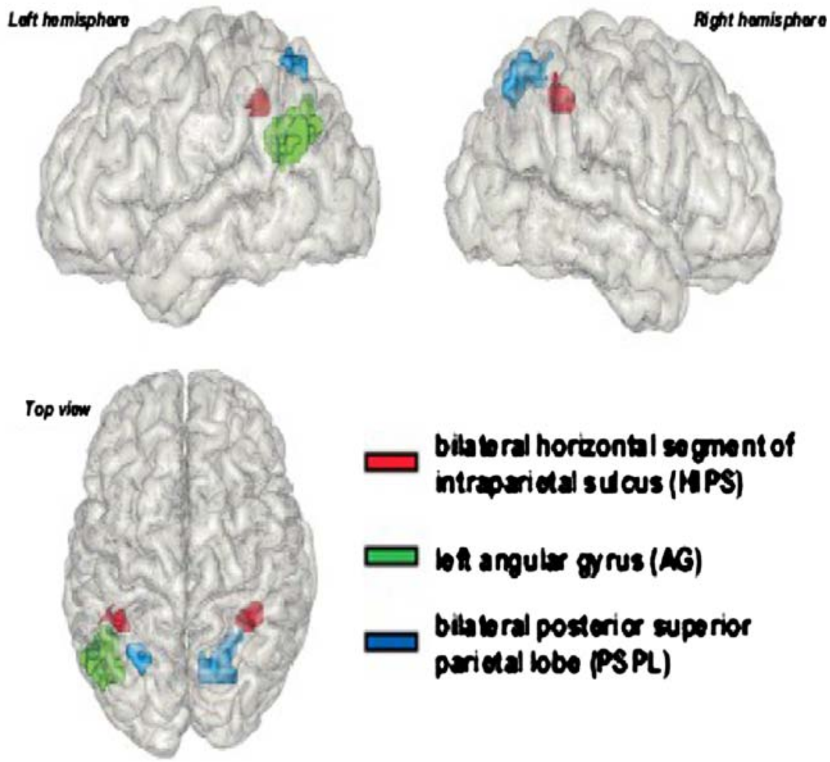

Fig. 1 Number-processing areas of the brain

demonstrating that it is the ratios that matter when it comes to the perception of approximate number.

Adults are similarly ratio dependent when exposed to sequences of sounds or random arrays of dots under conditions that prevent them from counting and are, therefore, forced to rely on the approximate number system (Feigenson et al. 2004). Adults and older children can make finer distinctions than 6-month-olds, of course, but their behavior is similarly ratio-dependent and cross modal. Interestingly, Feigenson et al. (2004) further report that the ratio-dependence is replicated on Arabic numerals. That is, when asked to judge which of two Arabic numerals is larger, they are faster to respond when the two numerals are small or when they are distant from each other.

For our purposes, it is important to note, first, that all species have an approximate sense of number and that humans rely on this system for quick estimates of number. This approximate system is, however, noisy in the sense that it provides an estimate of the neighborhood in which the quantity lies but not a precise number that corresponds to that quantity. Furthermore, since the system is noisy and ratio-dependent it can quickly and accurately distinguish between two quantities only when there is a sufficiently large difference in their numerosity. The approximate system has greater difficulty when the difference between the two quantities is small. For example, the difference between 6 and 12 is easily judged, but the difference between 6 and 8 is harder.

Finally, the approximate system seems to be strictly a representation of discrete number as opposed to some continuous variable like area or volume. For example, infants compute discrete number in tasks involving large numbers of elements. When total surface area, contour length, item size, item density and so on are controlled for, infants respond to changes in 8 versus 16 dots (but not 8 vs. 12 dots, as noted above). This suggests that the approximate system is a pure representation of number (Feigenson et al. 2004). This is not the case with the next subsystem, the subitizing system, which seems to represent both number and continuous variables like area.

The subitizing system provides direct perception of small quantities, numbers up to 3 or 4. Feigenson et al. (2002) performed experiments where 10- and 12-month old infants watched an experimenter hide one cracker in a bucket on one side and two crackers in a bucket on the other, where the crackers were controlled for size. The infants reliably chose the bucket with the greater number of crackers. The result holds for 2 versus 3 equal sized crackers. However, for 3 versus 4 crackers, 2 versus 4 crackers, 3 versus 6 crackers and even (incredibly, perhaps) 1 versus 4 crackers, the infants chose randomly.

The result vanishes, however, when size is not controlled for. Infants will choose one large cracker over two small ones, thus showing that they are (rationally) maximizing food intake over discrete number.

Similar experiments have been run on rhesus monkeys using apple slices again controlled for size. (Feigenson et al. 2002). The monkeys reliably chose the larger quantity in the 1 versus 2 slice condition, the 2 versus 3 slice condition and the 3 versus 4 slice condition. However, the monkeys chose at random in the 3 versus 8 condition and the 4 versus 8 condition. Once the numbers involved exceeded the range of small numbers, the monkeys, like the infants, had trouble distinguishing the quantities.

Adults, of course, are able to marshal precise number in a way that distinguishes them from infants and other species. Nevertheless, the subitizing system is detectable in adult behavior. Adults can detect the number of random dot arrays with near flawless accuracy for numbers up to 4, after which both error rate and response time climb with increasing cardinalities (Mandler and Shebo 1982; Trick and Pylyshyn 1994; Feigenson et al. 2004).

In this section we have considered two number systems that are shared by humans–whether adults or infants—and other species. The two systems interact in a way that makes a number of very clear behavioral predictions that we will make use of these predictions in the studies below:

1. Given two arrays of objects whose cardinality differs by a large amount, it should be relatively easy for the approximate system to distinguish them; response times should be on average low and accuracy should be high.
2. Given two arrays of objects whose cardinality differs by only a small amount, the approximate system should have difficulty distinguishing them; response times should be on average high and accuracy low.
3. In the case of two numerically similar arrays, people should turn to counting—that is, using precise number—if they can.

3 Language and precise number sense

In light of the fact that we appear unique in having both precise number sense and expressive language, it seems sensible to suppose that both are somehow linked causally. The first, and most commonly supposed, possibility is that language is the crucial prerequisite for precise number sense. Number words and their ordering provide a structure on which we can hang our sense of the semantics of number. Unlike other animals, we know what 15 means because 15 fits into a larger structure: It is the successor of 14 and the predecessor of 16. If called upon, we can count objects by establishing a bijection—a one-to-one correspondence—between the objects and this sequence of number words. My ability to count and have a sense of precise number is, then, fundamentally linguistic; language provides a representational system on which I can hang precise number sense. The essential idea is that, while all species have a sense of approximate number based, perhaps, on a continuous number line, language makes precise number sense possible by ''discretizing'' this continuous line through providing number words as a representational system.

The idea that the representational system for numbers and mathematics could be crucial for number sense is quite sensible. Consider, for example, the difference between Roman and Arabic numerals. Both systems can represent the natural numbers. Performing simple arithmetic computations on Roman numerals can be extremely cumbersome. The Arabic system, particularly with positional zeroes, allows for simple and extremely natural algorithms for doing the basic arithmetic computations.

Notice, however, that this hypothesis—namely, that language and number words are at the origin of our precise sense of number—is not the only hypothesis logically available. We could suppose that the human sense of precise number provides the preconditions for expressive language. Still a third possibility is that a more abstract neurological change in the human species paved the way for both precise number sense and expressive language. We should observe that the idea that language is at the source of number sense is an instance of linguistic relativity, the hypothesis that language can be the motive force of thought. We might, for example, hypothesize that causation in the other direction: Our capacity for precise number and mathematics is at the source of our linguistic ability. Alternatively, we might suppose that some deeper capacity underlies both number sense and language; for example, the ability to perform algebraic computations like union, intersection and complementation might provide the basis for both mathematics and language.

Finally, we could suppose that, despite appearances, number and language bear no real relationship; distinct cognitive systems make both possible. For example, our sense of precise number might rely on an abstract structure that maps to counting, on the one hand, and number language, on the other.[3] Notice that this

[3] This is similar to the numeron model of Gelman and Gallistel (1978). We will return to this model below.

hypothesis requires at least two sets of changes from our closest non-human relatives. One set would make expressive language possible while another set would make precise number possible. While this may actually be correct, it seems to us to be unattractive as an initial hypothesis since it requires a diachronic sequence that is both more complex and less constrained than the others.

Natural language quantifiers provide an interesting laboratory for studying the connections between language and number sense. We will provide evidence, below, that our ability to understand quantifiers is related to our ability to understand precise number. Nevertheless, it is possible to dissociate language and precise number sense in ways that suggest that precise number sense does not rely on language. Furthermore, our evidence is compatible with the hypothesis that precise number sense is required in order to understand quantifiers. Thus, our evidence suggests the possibility that precise number sense is a prerequisite for quantifier understanding. This result is incompatible with the hypothesis that language is responsible for our sense of precise number.

For present purposes, we will restrict our attention to a strict subclass of noun phrases that could count as quantifiers. We will exclude names, pronouns, titles and honorifics (like *the right honorable inspector of widgets)*. We will also exclude bare plurals and generics, as in (3)a, pragmatically vague expressions, as in (3)b, which rely on prior expectations about cardinalities, definite descriptions which usually require that the discourse context have special properties, as in (3)c, and possessives, as in (3)d:

(3) a. <u>Dogs</u> chew <u>bones</u>.
  b. <u>Few undergrads</u> drink herbal tea. <u>Many trustees</u> eat toast.
  c. <u>The inventor of the zip</u> was English.
  d. <u>John's favorite goat</u> drinks beer.

Despite these restrictions, we are still left with a fair range of noun phrases to examine, as illustrated by the underlined expressions in (4):

(4) a. <u>Every dean</u> drank <u>between three and five glasses of tea</u>.
  b. <u>Some provost</u> danced at the party.
  c. <u>More than half (of) the faculty</u> dined on pheasants.

We can sort the quantified expressions we are left with into the following groups, sorted according to determiners:

(5) a. Aristoteleans: all, every, some, no, not all
  b. Cardinals: at least three, at most five, between eight and ten,...
  c. Majority: most, more than half, less than a third,...
  d. Parity: an even number of, an odd number of

We do not have space, here, to review the functions that can be used to interpret natural language determiners; the literature is well-known and easily found.[4] We will show, below, that the formal differences in the complexity of the functions which interpret determiners are reflected in brain physiology.

As is standard, we will take a determiner to denote a function from a pair of properties to a truth value; given that P and Q are properties, DET a determiner, and 2 the set of truth values, then:

$$\text{DET}: P \times Q \to 2$$

Crucially for our purposes, the truth value for any instance of the class of determiners we have chosen to study can be established entirely on the basis of the cardinalities of the sets $|P \cap Q|$ and $|P–Q|$.[5] That is, the interpretations of the determiner we are interested in rely completely on the cardinalities of the sets it takes as its arguments. If this formal analysis is correct, then we have good reason to suspect that our understanding of sentences involving quantified expressions should rest, at least in part, on our sense of number.

Indeed, it is quite possible to construct simple neural models of the quantifiers. Consider, first, the Aristoteleans:

all(P, Q)
some(P, Q)
no(P, Q)
not all(P, Q)

The inputs to the system would be provided by the numerosity component, while the output would be a linguistic judgment on the appropriateness of the quantifier. We might suppose that the subitizing system can accurately decide if there is at least one element in the perceptual field that has the property of being both P and Q and if there is at least one element that has the property of being P but not Q. Alternatively, we might suppose that the approximate number system can accurately count the presence of at least one object in

[4] We rely here on work on generalized quantifiers familiar from the work of Barwise and Cooper (1981), Keenan and Stavi (1986), and van Benthem (1986), among others. Generalized quantifiers were first proposed in Mostowski (1957). For a good general summary see Keenan and Westerstahl (1997).

[5] See, in particular, van Benthem (1986) for an extensive discussion of this point. We will rely heavily on his development of semantic automata for simulating natural language quantifiers throughout.

$P \cap Q$ (a P that is Q) and the presence of at least one object in $P–Q$ (a P that is not Q).

To simulate *all* or *every*, we need only detect that $P–Q$ is empty. Let ALL be a node with two inputs A and B. A fires if $P \cap Q$ contains at least one element. B fires if $P – Q$ contains at least one element. Notice that A and B involve number judgments and, so, are supported by the number faculty. Let the connection strength between A and ALL be 1 and the connection strength between B and ALL be –1. Suppose ALL sums its inputs and fires if the result is greater than or equal to 1. Clearly, ALL fires if and only if all the P in the perceptual array have the property Q.

Equally, consider the determiner *some.* Here, we require only that at least one P have the property Q. Let A and B be as above and let SOME be connected to their outputs. Let the connection between A and SOME be 1 and let SOME fire if the sum of the inputs is greater than or equal to 1. SOME fires if and only if at least one P has the property Q.

Next, suppose we model *not all* in a way analogous to *some.* Here we require that $P–Q$ be nonempty, so let A and B be as above. Let the connection between B and NOT-ALL be 1 and let NOT-ALL fire if the sum of the inputs is greater than or equal to 1. NOT-ALL fires if and only if at least one P lacks the property Q, as desired.

Finally, we can model *no* in a way that mirrors our model of *all*. NO is the output node and A and B are networks as above. The connection between A and NO is –1 while the connection between B and NO is 1. NO fires when the sum of its inputs is greater than or equal to 1. Here, NO fires if and only if no P has the property Q.

Notice that the above model immediately predicts that the Aristoteleans should be available either as long as the subitizing system is available or, at the very least, as long as judgments about small numbers are preserved. In particular, as long as patients have preserved access to small numbers—that is, so long as they can observe that at least one object either has or lacks some property—understanding of the Aristoteleans should be spared.

Modeling the cardinals and majority quantifiers is equally straightforward. Let the input to the system be the system of precise number and the output node correspond to the determiner in question. The system tracks the cardinalities of $P \cap Q$ and $P–Q$ using a numeron system (see Gelman and Gallistel 1978). A quantifier like *at least* $n(P, Q)$ would involve a network with input for the numeron corresponding to n. The numeron system would count the elements in $P \cap Q$. The output node AT-LEAST-N would fire if activated by the numeron n. The quantifier *exactly* $n(P, Q)$ would involve inhibition from the numerons corresponding to n–1 and n+1. Other simple cardinal quantifiers can be simulated analogously.

Finally, we should note that there are two ways we could support cardinal determiners. First, we can, of course, use the precise number sense to create networks that fire under the exact truth conditions of the determiner. Suppose, however, that the precise number networks were unavailable. One could marshal the approximate number function φ to simulate the precise system. The result would be "noisy" determiner denotations.

We have some evidence that both systems are available. When making comparative judgments, where the user must decide for an array of m objects whether n of them have a property, the noisy, approximate system might be good enough for "distant" judgments where the difference between n and the observed number exceeds the distribution defined by φ. In cases of near judgments, however, the approximate system won't be good enough and the precise system must be used.

This model immediately predicts that people with precise number deficits should have access to the noisy system and, so, approximate cardinals, albeit with a higher degree of error than normals. The system will break down on near judgments, however, since the approximate system will be unable to distinguish the cardinalities.

Notice, finally, that there is a difference in complexity between the Aristotelean and cardinal determiners, on the one hand, and the majority and parity expressions like *an even/odd number of*, on the other. Majority quantifiers are inherently more complex than either cardinals or Aristoteleans. In order to correctly process a majority quantifier, either the cardinalities $P \cap Q$ and $P–Q$ must be counted and held in memory for comparison, or some structure must be imposed on the scene as will be discussed below. In either event, parity quantifiers place greater computational demands on the system.

To make the contrast clearer, consider a sentence with a simple Aristotelean determiner:

(6) Every dean smokes cigars.

Suppose we wanted to verify (6); we could do so by using a very simple device that tests, for each member of the set of deans, whether or not that member smokes cigars. Thus, we could line the deans up in front of the device and test each one in turn. If a dean tests positive as a cigar smoker, we move on to the next dean. If we run out of deans, we accept (6) as true. If a dean tests negative as a cigar smoker, we stop all testing and reject (6) as false. Notice that no memory of the past is required by our verification procedure and the deans can be tested in any order with the same results.

Compare the case of (6) with the following:

(7) More than half the deans smoke cigars.

Verification of (7) is a more complicated matter than verification of (6), if only because we need some method of tracking the overall number of deans before we can decide whether more than half are cigar smokers. One idea is to use opponent processes to verify the statement. We might, for example, equip all the deans with guns, line up the cigar smoking deans on one side of a field and the deans who don't smoke cigars on the other. Let each dean shoot the dean opposite him through the heart. If any deans are left standing in the cigar smoking line, then more than half the deans smoke cigars. Otherwise, if no deans are left standing or if only non-cigar-smoking deans are left standing, then (7) is false.[6]

In summary, our model of quantifier comprehension uses input from the number processing portions of the brain to support judgments of the truth or falsity of linguistic expressions involving quantifiers. The different types of quantifiers involve different types of numeric input; Aristoteleans require only the ability to count to one, while cardinals involve precise number. Majority quantifiers are the most complex since they require some ability to compare precise cardinalities. Notice though that our model explicitly requires that number sense supports some aspects of linguistic behavior.

4 Experimental evidence

We turn now to the description of a series of experiments designed to probe the relationship between number sense and quantifier understanding.[7] We will consider three groups:

1. Normal subjects; that is healthy adults who have not been diagnosed with deficits in number or language; these subjects are studied with functional magnetic resonance imaging (fMRI), a technique for identifying brain regions involved in performing a cognitive task;
2. Corticobasal degeneration (CBD) patients; these are patients in whom word meaning seems well preserved but who have deficits in number processing;
3. Frontotemporal dementia (FTD) patients; due to temporal lobe disease some of these patients show semantic dementia, a fluent form of progressive aphasia with profound naming difficulty and an impairment of word meaning; we should note that other FTD patients with temporal lobe disease show deficits in working memory, a fact that will be crucial below.

We used a simple experimental paradigm. Subjects were visually presented with a sentence involving familiar objects and properties that could be pictorially represented. Each sentence involved a single quantified expression:

(8) a. Some of the cars are red.
b. At least five of the balls are blue.
c. Less than half of the flowers are yellow.

We divided quantifiers into two groups: First order quantifiers which included the Aristoteleans as in (8)a and cardinals as in (8)b, and higher order determiners as in (8)c. After the initial presentation of the sentence, the subjects were presented with the sentence accompanied by a stimulus array. The subjects were then asked to judge the truth of the sentence with respect to the visual array; they pressed the right button of a response pad if they judged the sentence to be true and the left button if they judged it to be false. Note that we tried to make the supporting models for the sentences as natural as

[6] This is essentially a variant of the Boyer-Moore vote counting algorithm. See Boyer and Moore (1991) for discussion. This algorithm has relatively low complexity but it still requires more in the way of computational resources than is required to test our Aristoteleans and cardinals. Alternatively, we could equip our testing device with a memory that tracks the number of cigar-smoking versus non-cigar-smoking deans. In fact, we could use a push-down automaton (pda) to simulate the quantifier in (7); once again, see van Benthem (1986) for a discussion of this. The behavior of a pda is fully determined by its current input, the state it is in and the content of a memory store. The exact character of this memory need not detain us. The important point, for our purposes, is that natural language quantifiers can be partitioned into two classes that differ as to their complexity. These differences can be seen in expressive power—majority quantifiers can make more finely grained distinctions between models than the cardinals and Aristoteleans. But greater expressive power comes at the price of making greater demands on computational resources than the cardinals and Aristoteleans. Furthermore, these distinctions in expressive power and computational demands will be manifest in any device that implements or simulates these quantifiers. Our firing line simulation of (7) requires us to impose more organization on the model, even if it does not require much in the way of memory.

[7] This section reports work by McMillan et al. (2005) and McMillan et al. (2006).

Table 1

Condition	Activation locus (Brodmann area)	Coordinates			*Z*-value	*P*-value
		X	*Y*	*Z*		
First-order	Right inferior parietal (40)	44	48	36	4.90	0.000
	Bilateral anterior cingulated	8	16	24	4.52	0.000
Higher-Order	Right inferior parietal, lateral occipital (40,19)	44	–48	36	4.91	0.000
	Right dorsolateral prefrontal, inferior frontal (46,45)	32	32	24	4.54	0.000
	Bilateral anterior cingulated (24)	12	0	52	5.14	0.000
	Bilateral thalamus	–28	–20	–12	4.70	0.000
Higher-Order minus First-Order	Left anterior cingulated (24)	–16	–4	52	3.64	0.000
	Left dorsolateral prefrontal (46)	–40	28	20	2.95	0.002
	Right inferior frontal (47)	24	20	–8	3.38	0.000
	Right lateral occipital, inferior parietal (19,40)	36	–88	4	3.22	0.001
Precise minus Approximate	Left lateral occipital (18)	–4	–76	4	3.14	0.001
	Right inferior parietal (40)	32	–28	40	3.83	0.000

possible. For example, if we presented subjects with the sentence ''all the balls are green'' then there was at least one green ball in the stimulus array. Furthermore, the arrays were of relatively low cardinality so that subjects could perceive their contents relatively quickly.

The subjects were 12 young, healthy, right-handed, native speakers of English. They were presented with 120 simple sentences, each containing a single quantifier. Half of the quantifiers were first order *(all, some* and *at least three)* while the other half were not (these included a majority determiner *less than half* and the parity expressions *an even number of* and *an odd number of*). The subjects were presented with arrays of 8 familiar objects (balls, flowers, cars and so on) arrayed at random on the screen; debriefing after the experiment revealed that none of the subjects noticed that the arrays always consisted of 8 objects. As noted above, the subjects' task was to judge the truth of the proposition given the array of objects. Half the stimuli contained numbers of objects that were near the criterion for the truth value judgment—for example, the subject might be presented with 4 target objects in an ''at least three'' judgment—thus requiring the subject to use their precise number sense. The other half of the judgments involved numbers that were far from the criterion—for example 7 objects in an ''at least three'' judgment. This case would require only approximate number sense.

We collected both behavioral data on accuracy and imaging data. The behavioral data indicated that judgments on majority quantifiers and parity expressions were significantly more difficult than judgments on Aristoteleans and cardinals. The mean accuracy on the former was 84.5% with a standard deviation of 8.6%. The mean accuracy on the latter was 92.3% with a standard deviation of 4.5%. The difference between the two groups was significant.[8]

The imaging data were extremely interesting. These data emerged using a ''subtraction'' technique. For example, we subtracted the ''sentence plus array'' from the presentation of the ''array alone'' to factor those areas of the brain that are used in making truth value judgments on sentences containing quantifiers. Furthermore, we could use the subtraction method to compare judgments on parity expressions and majority (''higher order'') quantifiers with Aristotelean and cardinal (''first order'') quantifiers to see how higher order quantifiers differ in their anatomical effects from first order quantifiers. We could use the same method to compare ''near'' judgments from ''far'' judgments, as discussed above. Table 1 summarizes these results. First order quantifiers recruit the right inferior parietal cortex and bilateral anterior cingulate regions (not illustrated) (see Fig. 2, panel A); the parietal lobe is associated with number sense while the anterior cingulate is important for attentional control. Higher order quantifiers also recruited these regions as well as right dorsolateral prefrontal and bilateral inferior frontal regions (see Fig. 2, panel B) that are implicated in quantifier comprehension. We also observed activation in right lateral occipital cortices and the thalamus as well (not illustrated), presumably related to the perceptual demands of these arrays. If we subtract the areas of activation associated with first order quantifiers from those associated with higher order quantifiers, we see activation in large areas of dorsolateral, prefrontal cortices bilaterally; these are the areas involved

[8] The accuracy data were analyzed using a paired-sample t-test. The difference was significant to $t(11) = 3.43$, $P < 0.01$. For a more thorough discussion of these results see McMillan et al. (2005) and McMillan et al. (2006).

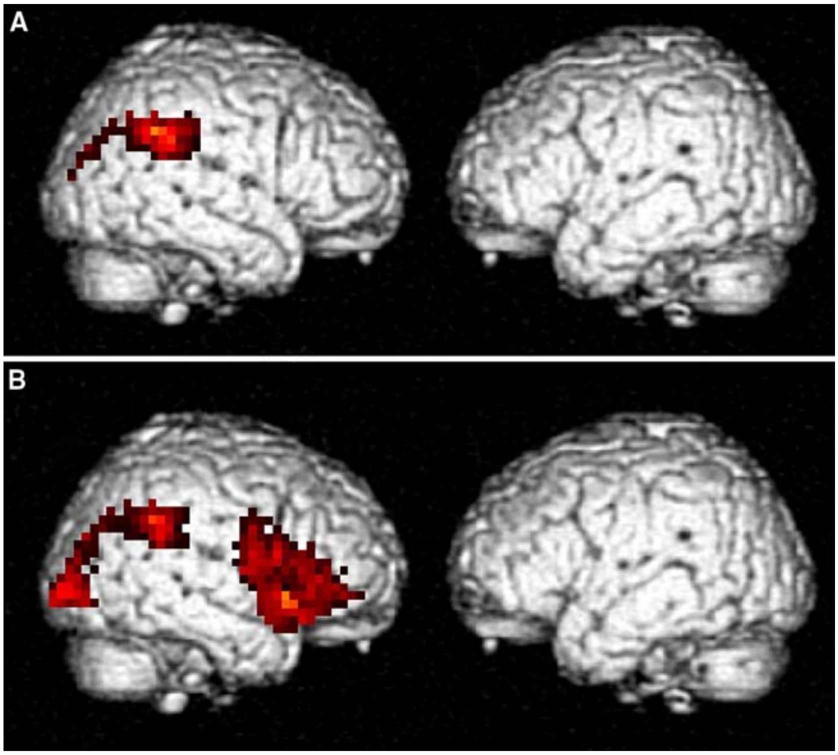

Fig. 2 Regions associated with quantifier interpretation

in ''higher order'' quantifiers. There are also right inferior parietal, right lateral occipital and left anterior cingulate activation.

All well and good, but what do these findings mean? First, they emphasize that comprehension of the types of quantifiers we investigated involves the inferior parietal cortex, an area associated with number sense. This strongly suggests that comprehension of quantifiers draws on number knowledge. The role of number knowledge in quantifier comprehension is underlined by the fact that the parietal region is in the right hemisphere, which is not typically associated with language. Furthermore, the activation of inferior frontal and dorsolateral prefrontal cortices only for ''higher order'' quantifiers suggests that listeners were using working memory in processing majority and parity quantifiers. This is consistent with the model of quantifier processing outlined in section 3.

Let us turn, now, to near versus far judgments. Recall that the former type of judgment demands precise number sense while the latter type of judgment can be approximated. Dehaene et al. (1999) showed activation of inferior parietal cortex bilaterally and left orbital frontal cortex during an assessment of precise addition compared to approximate addition. Dehaene and his colleagues used this to argue that left orbital frontal activation mediates a verbal representation for precise calculation; this is the hypothesis that precise number sense is grounded in language. We should note, however, that the left orbital frontal cortex is not peri-Sylvian and is anterior to Broca's area, the area normally associated with language processing. Our study of precise judgments of quantifiers did not reveal any activation in language regions of the left peri-Sylvian cortex. Indeed, when we subtract approximate from precise judgments of quantification, we observe recruitment of the right inferior parietal cortex. This is a number-related area in the non-language hemisphere. These observations are, then, not consistent with verbal mediation of precise number in quantifier judgments, since language areas are not recruited. A radical suggestion would be that precise number sense underlies language, although we suspect that both language and precise number are made possible by some deeper faculty, perhaps the ability to compute partial functions, hold them in memory and combine them as part of a larger computation.[9]

The imaging data shown in Fig. 2 suggests that the processing of different types of quantifiers have different anatomical correlates. Furthermore, we have shown that, to a degree, the processing of this part of language

[9] A more complete discussion of these fMRI results can be found in McMillan et al. (2006).

is supported by areas of the brain that are associated with number processing. These findings suggest that we should be able to find populations of patients with brain injuries who show deficits in quantifier understanding but who are not aphasic. Furthermore, it should be possible to find patients who have impaired understanding of parity and majority quantifiers, although their comprehension of Aristoteleans and cardinals is intact, since the former recruit specific areas of the brain not recruited by the latter.

In order to explore these possibilities, we examined patients with corticobasal degeneration (CBD). CBD is a progressive neurodegenerative condition that typically begins in the mid-fifties. It affects males and females equally. Clinical features include both a motor disorder and a cognitive disorder. Motor problems may include unilateral limb rigidity, abnormal involuntary movements such as myoclonus (lightning-like jerks), dystonia (periodic posturing of a limb), alien hand phenomena (poor control over limb movement), and gait difficulty. Cognitive problems may involve acalculia and number difficulty, apraxia (a disorder of skilled voluntary movements), naming difficulty, visual-spatial difficulty. These difficulties correspond to the distribution of cortical atrophy, centered in the parietal lobe, early in the disease (see Fig. 3). As the condition progresses, the frontal parts of the brain may become involved as well, resulting in some limitations in planning and mental organization. Crucially for our purposes, CBD patients exhibit acalculia and substantial number deficits, although comprehension of objects, nouns, actions, verbs and the like appears to be preserved. We expect, then, that CBD patients should exhibit selective difficulty with processing quantified expressions since, as we have seen, these expressions draw on number sense, which depends on the integrity of the parietal cortex.

Halpern et al. (2004) demonstrated that CBD patients' knowledge deficit involves numbers but not objects. To study the nature and extent of semantic memory difficulty in CBD patients, we asked them to perform tasks requiring single word meaning of objects and knowledge of numbers. We compared the CBD patients with another group of patients known to have difficulty understanding the meaning of nouns and objects—patients with semantic dementia. There were four tasks tapping number knowledge. The first required patients' judgments of the relative magnitude of a pair of single-digit Arabic numerals. The pairs systematically differed by a small amount or a larger amount. Patients judged which of the pair is largest. The second task required patients to judge the relative magnitude of two sets of 1 cm, blackfilled "dots" that are the same numerosity as the Arabic numerals. Again, patients judged which array is larger. The third task involved patients' addition of the pair of Arabic numerals, and the fourth task required patients to total the two sets of dots. The task probing object knowledge asked patients to name 60 line drawings of familiar objects. Performance was equated across the tasks by generating z-scores based on the performance of the entire cohort of participants. We then examined relative performance across the number tasks and the object task. We found a significant difference between groups: A significantly larger proportion of CBD patients had more difficulty on the number tasks than the object task; however, a significantly larger number of semantic dementia patients had more difficulty on the object task than the number tasks.

We compared the CBD patients with other groups of patients who have a disease affecting other brain regions. Patients with frontotemporal dementia (FTD) show degeneration in the dorsolateral and inferior portions of the frontal cortex, areas that are associated with working memory. Some of these patients, then, have difficulty understanding sentences involving long distance syntactic dependencies, since these dependencies can tax working memory. In addition, some FTD patients show difficult with *n*-back measures of working memory; these tests involve recalling an item in a list that is *n* places back, where *n* is a small integer. We should note that, aside from general problems with

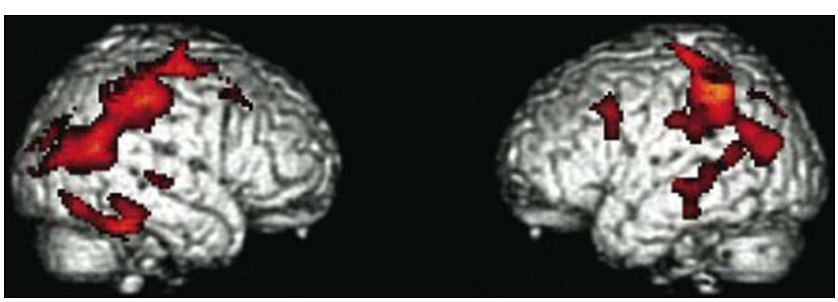

Fig. 3 Cortical atrophy in Corticobasal Degeneration (CBD)

working memory, FTD is not reported to affect number knowledge. Of course, resource demanding tasks like division or handling large, multi-digit numbers are compromised in FTD. All else being equal, we predict that FTD patients should respond well with Aristotelean and cardinal quantifiers but should show impairment with majority and parity quantifiers.

Finally, we examined Alzheimer's disease (AD) patients. These patients show some difficulty in a number of cognitive domains, including both number calculation and working memory. AD patients appear to have a disease in both parietal and prefrontal regions of the brain. We predict, then, that patients with AD should show difficulty across the board in processing quantifiers.

The experiment assessing quantifier comprehension in CBD, FTD and AD was virtually identical to the imaging experiment with normals described above, with one exception. The subjects were presented with the pair consisting of the sentence and the stimulus array for as long as it took for them to make a decision about whether the sentence accurately described the stimulus array. In addition, we collected data on a dot comparison task to assess general number knowledge in the same patients. Patients were presented with 36 pairs of boxes containing arrays of 1 cm dots. They were asked to judge which of each pair contained more dots. Finally, we included a general test of sentence comprehension. Patients were presented with twelve sentences containing a transitive verb and a center-embedded clause. The sentences were presented twice and then a comprehension probe was presented.

The data on quantifier comprehension is summarized in Fig. 4. We first note that CBD patients did significantly worse on quantifier comprehension than any of the other groups. This includes Aristoteleans like ''some cars are red.'' Moreover, CBD patients were significantly impaired on Aristoteleans and cardinals relative to patients with AD. This emphasizes the role of number knowledge in making quantifier judgments since CBD patients have a specific impairment of number sense while AD patients do not. Finally, it is worth emphasizing that these difficulties were related to their impairment judging dot array magnitude, and this quantifier comprehension deficit occurred despite reasonably normal language comprehension.

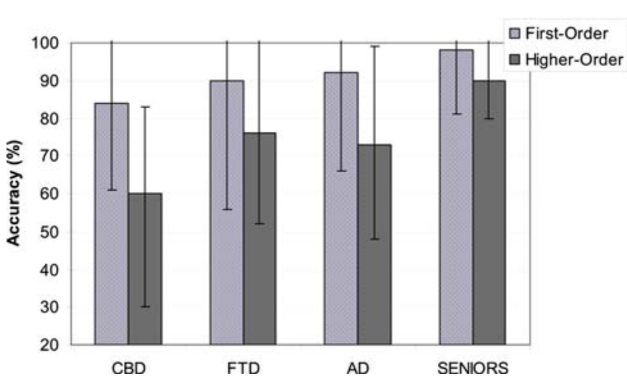

Fig. 4 Mean (standard deviation) accuracy for first-order and higher-order quantifier comprehension in corticobasal degeneration (CBD), frontotemporal dementia (FTD), Alzheimer's disease (AD), and healthy seniors

We also investigated the problem of approximate versus precise number by examining near and far judgments. Recall that these involve whether the cardinality of the target set is near criterion, which requires precise number sense, or far from criterion, in which case approximate number sense can be used. We found no difference in CBD patients' performance as a function of near or far judgments. Their degraded sense of number interfered with quantifier comprehension across the board.

Our model, as well as imaging data from healthy subjects, suggests that majority and parity quantifiers make particular demands on working memory. This is in contrast with Aristoteleans and cardinals which, as we saw above, can be computed by memoryless functions. We predict, then, that FTD patients and AD patients should have particular difficulty with majority and parity quantifiers. This could be either because majority quantifiers, for example, require numbers to be held in memory for raw comparison or, as suggested by the Boyer–Moore treatment of such quantifiers alluded to above, because the objects must be organized in memory. While the data in Fig. 4 show that FTD and AD patients are mildly impaired with Aristoteleans and cardinals, they have much more difficulty with higher order quantifiers, a result that is clearly consistent with our prediction.

Notice that CBD patients also had more difficulty with higher order quantifiers. This could be because of associated compromised frontal lobe functioning that may also be present in CBD or because processing these quantifiers involves resource demands that simply overwhelmed these patients because of their poor number knowledge. Finally, we should note that healthy seniors showed greater difficulty with majority and parity quantifiers, although their difficulty was not as great as in patients with FTD or AD. The source of this difficulty is likely to be working memory limitations often seen in healthy aging.

In summary, we have seen that the processing of at least some quantifiers is supported by the same neuroanatomy that supports number processing. Moreover, different formal classes of quantifiers are associated with different regions of the brain. We have, further, demonstrated that these different classes of quantifiers can be selectively impaired. Finally, we have shown that the neuroanatomy associated with precise number processing supports an aspect of language processing—namely the understanding of quantifiers. This strongly suggests that language itself is not wholly responsible for our sense of precise number. While language may contribute to some aspects of number processing, our observations also show that number knowledge supports other aspects of language processing such as quantifier comprehension. We cannot rule out the seemingly radical hypothesis that special neuroanatomical structures are required for the computation of partial recursive functions and that it is these structures that led to the uniquely human endowments of expressive language and precise number sense.

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