ECE 4200 Assignment #9

Problem #1: 1. Movie-comparison:

Movie assignment: { LotR - 1; HP... 7 - 2; Snatch - 3; 12 Mnkys - 4; The Cent - 5; The Hobbit - 6 }

In Euclidean: $s_e = \frac{1}{|\vec{r}_1 - \vec{r}_2|}$

$$s_p = \frac{(\vec{r}_1 - \hat{r}_1) \cdot (\vec{r}_2 - \hat{r}_2)}{|\vec{r}_1 - \hat{r}_1||\vec{r}_2 - \hat{r}_2|}$$

1-2: $\vec{r}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ $\hat{r}_1 = \begin{bmatrix} 3.666 \\ 3.666 \end{bmatrix}$

$\vec{r}_2 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\hat{r}_2 = \begin{bmatrix} 2.33 \\ \vdots \end{bmatrix}$

$s_e = \frac{1}{r_2} = \boxed{0.707}$

$s_p = \boxed{0.8437}$

1-3: $\vec{r}_1 = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$ $\hat{r}_1 = \begin{bmatrix} 3.667 \\ \vdots \end{bmatrix}$ $s_e = \boxed{0.1717}$

$s_p = \boxed{-0.833}$

$\vec{r}_3 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ $\hat{r}_3 = \begin{bmatrix} 3.1 \\ \vdots \end{bmatrix}$

1-4:

$\bar{r}_1 = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$ $\hat{r}_1 = \begin{bmatrix} 3.667 \\ \vdots \end{bmatrix}$

$\bar{r}_4 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\hat{r}_4 = \begin{bmatrix} 3.25 \\ \vdots \end{bmatrix}$

$s_e = 0.2357$

$s_{pf} = 1$

1-5:

$\bar{r}_1 = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$ $\hat{r}_1 = \begin{bmatrix} 3.667 \\ \vdots \end{bmatrix}$

$\bar{r}_5 = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$ $\hat{r}_5 = \begin{bmatrix} 3.5 \\ \vdots \end{bmatrix}$

$s_e = 0.2132$

$s_p = 0.7385$

1-6:

$\bar{r}_1 = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$ $\hat{r}_1 = \begin{bmatrix} 3.667 \\ \vdots \end{bmatrix}$

$\bar{r}_6 = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$ $\hat{r}_6 = \begin{bmatrix} 3.25 \\ \vdots \end{bmatrix}$

$s_e = 0.4474$

$s_p = 0.6378$

2-3: $\bar{r}_1 = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$ $\hat{r}_2 = \begin{bmatrix} 2.75 \\ \vdots \end{bmatrix}$

$\bar{r}_3 = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 5 \end{bmatrix}$ $\hat{r}_3 = \begin{bmatrix} 3 \\ \vdots \end{bmatrix}$

$s_e = 0.2724$

$s_p = -0.3693$

2-4: $\bar{r}_1 = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$ $\hat{r}_2 = \begin{bmatrix} 2.75 \\ \vdots \end{bmatrix}$

$\bar{r}_4 = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$ $\hat{r}_4 = \begin{bmatrix} 3.25 \\ \vdots \end{bmatrix}$

$s_e = 0.2673$

$s_p = -0.7478$

2-5: $\bar{r}_1 = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$ $\hat{r}_2 = \begin{bmatrix} 2.75 \\ \vdots \end{bmatrix}$

$\bar{r}_5 = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$ $\hat{r}_5 = \begin{bmatrix} 3.5 \\ \vdots \end{bmatrix}$

$s_e = 0.2887$ $s_p = -0.5817$

$s_c = 0.4082$

$s_p = 0.4283$

3 & 6: $\bar{r}_3 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\hat{r}_2 = \begin{bmatrix} 2.75 \\ \vdots \end{bmatrix}$

$\bar{r}_6 = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$ $\hat{r}_6 = \begin{bmatrix} 2.75 \\ \vdots \end{bmatrix}$

3-4:

$\bar{r}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ $\hat{r}_3 = \begin{bmatrix} 3 \\ 3 \\ \vdots \end{bmatrix}$

$\bar{r}_4 = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ $\hat{r}_4 = \begin{bmatrix} 3.25 \\ \vdots \end{bmatrix}$

$s_e = 0.4082$

$s_p = 0.6736$

3-5:

$\bar{r}_3 = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix}$ $\hat{r}_3 = \begin{bmatrix} 3 \\ \vdots \end{bmatrix}$

$s_e = 0.5$

$s_p = 0.8485$

$\bar{r}_5 = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 5 \end{bmatrix}$ $\hat{r}_5 = \begin{bmatrix} 3.5 \\ \vdots \end{bmatrix}$

3-6:

$\bar{r}_3 = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix}$ $\hat{r}_3 = \begin{bmatrix} 3 \\ 3 \\ \vdots \end{bmatrix}$

$s_e = 0.1091$

$s_p = 0.8554$

$\bar{r}_6 = \begin{bmatrix} 4 \\ 5 \\ 3 \\ 1 \end{bmatrix}$ $\hat{r}_6 = \begin{bmatrix} 3.75 \\ \vdots \end{bmatrix}$

4-5:

$\bar{r}_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ $\hat{r}_4 = \begin{bmatrix} 3.25 \\ \vdots \end{bmatrix}$

$s_e = 0.7071$

$s_p = 0.8862$

$\bar{r}_5 = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ $\hat{r}_5 = \begin{bmatrix} 3.5 \\ \vdots \end{bmatrix}$

4-6:

$\bar{r}_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $\hat{r}_4 = \begin{bmatrix} 3.41 \\ \vdots \end{bmatrix}$

$\bar{r}_6 = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$ $\hat{r}_6 = \begin{bmatrix} 3.85 \\ \vdots \end{bmatrix}$

$s_c = 4.2132$

$s_p = 0.8505$

5-6:

$\bar{r}_5 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ $\hat{r}_5 = \begin{bmatrix} 3.5 \\ \vdots \end{bmatrix}$

$\bar{r}_6 = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$ $\hat{r}_6 = \begin{bmatrix} 3.85 \\ \vdots \end{bmatrix}$

$s_c = 0.4925$

$s_p = 0.9027$

Ust-Uzer:

A-B: $\bar{r}_A = [5\ 1\ 2\ 3\ 4]$ $\hat{r}_A = [3\ 3\ \dots\ 3]$

$\bar{r}_B = [5\ 1\ 2\ 2\ 5]$ $\hat{r}_B = [3.33\ \dots\ 3.33]$

$s_c = 0.5774$

$s_p = 0.8165$

A-C: $\bar{r}_A = [5\ 1\ 3\ 4]$ $\hat{r}_A = [3\ 3\ 3\ 3]$

$\bar{r}_C = [1\ 4\ 4\ 3]$ $\hat{r}_C = [2.5\ 2.5\ \dots]$

$s_c = 0.1915$

$s_p = -0.9799$

A-D: $\bar{r}_A = [1\ 2]$ $\hat{r}_A = [3\ 3]$

$\bar{r}_D = [3\ 5]$ $\hat{r}_D = [3.3333\ 3.3333]$

$s_c = 0.7724$

$s_p = -0.2631$

A-E: $\bar{r}_A = [1\ 2\ 3\ 4]$ $\hat{r}_A = [3\ \dots\ 3]$

$\bar{r}_E = [5\ 4\ 5\ 1]$ $\hat{r}_B = [3.6\ \dots\ 3.6]$

$s_c = 0.1741$

$s_p = -0.7192$

B- C:

$\bar{r_B} = [5\ 4\ 2\ 2\ 5]$ $\quad \hat{r_B} = [3.3333\ldots]$

$\bar{r_C} = [1\ 2\ 4\ 4\ 3]$ $\quad \hat{r_C} = [2.8\ldots]$

$s_c = 0.1768$

$s_p = 0.7540$

B- D: $\bar{r_B} = [4\ 2\ 2]$ $\quad \hat{r_B} = [3.333\ldots\ 2.333]$

$\bar{r_D} = [2\ 3\ 1]$ $\quad \hat{r_D} = [3.333\ldots\ ]$

$s_c = 0.2673$

$s_p = 0.6174$

B- E: $\bar{r_B} = [4\ 2\ 2\ 5]$ $\quad \hat{r_B} = [3.333\ldots]$

$\bar{r_E} = [3\ 5\ 4\ 5\ 1]$ $\quad \hat{r_E} = [3.6\ldots]$

$s_c = 0.6601$

$s_p = -0.9194$

C- D: $\bar{r_C} = [2\ 4]$ $\quad \hat{r_C} = [2.8\ \ 2.8]$

$\bar{r_D} = [2\ 3]$ $\quad \hat{r_D} = [3.33\ldots]$

$s_c = 1$

$s_p = 0.3363$

C - E:

$\bar{r}_C = [2\ 4]$ $\quad r_C^4 = [2.8\ldots]$

$\bar{r}_E = [3\ 5]$ $\quad r_E^4 = [3.6\ldots]$

$s_c = 0.378$

$s_p = 0.1246$

D - E: $\bar{r}_D = [2\ 3\ 5]$ $\quad r_D^4 = [3.33\ldots]$

$\bar{r}_E = [3\ 5\ 4]$ $\quad r_F^4 = [3.6\ldots]$

$s_c = 0.4002$

$s_p = 0.2434$

2.

LOTR by D:

(k=1)

A-D : −0.2631
B-D : +0.6172 → largest (+)
C-D : 0.3313
D-E : 0.2938

$\hat{r}_{D,\,LOTR} = 3.33 + \dfrac{-0.6172(5-3.333)}{0.6172}$

$= 1.6663$

Avg Ratings:

$\bar{r}_A = 3$
$\bar{r}_B = 3.33$
$\bar{r}_C = 2.8$
$\bar{r}_D = 5.33$
$\bar{r}_E = 3.6$

LOTR by E:

(k=1)

A-E : $s_p = -0.7192$
B-E : $s_p = -0.9194$ → largest
C-E : $s_p = 0.5298$

$\hat{r}_{E,\,LOTR} = 3.6 + \dfrac{-0.9194(5-3.333)}{0.9194}$

$= 1.933$

HPATP07 rating by A:

k=1:

B-A: 0.8625
C-A: -0.779
D-A: -0.2631
E-A: -0.7196

largest |Sp|.

$$r_{A,HPATP07} = 3 + \frac{0.8625[4-3.333]}{0.8625}$$

$$= 3.666$$

LSATSB by C:

k=1:

A-C: -0.779
B-C: -0.7940
D-C: 0.3363
E-C: 0.5296.

largest |Sp|

$$r_{C,LSATSB} = 2.8 + \frac{-0.7940[2-3.333]}{0.7940}$$

$$= 2.8 + 3.333 - 2 = 4.1333$$

The Cuteness by D:

A-D : -0.2681
B-D : -0.6172 → highest (sp)
C-D : 0.3863
E-D : 0.2937

$$r_{D,\text{cuteness}} = 3.33 + \frac{-0.6172\,[2 - 3.33]}{0.6172}$$

$$= 3.33 + 3.33 - 2 \quad (1.33)$$

$$= \boxed{4.66}$$

The Hobbies by D:

A-D
B-D → highest
C-D
E-D

$$r_{D,\text{Hobbies}} = 3.33 + \frac{-0.6172\,[5 - 3.333]}{0.6172}$$

$$= 3.333 + 3.333 - 5$$

$$= 6.666 - 5 = \boxed{1.666}$$

$k=3$:

LOTR by D:

top 3: A-D: -0.2631
B-D: -0.6172
C-D: 0.3313

$$\hat{r}_{D,LOTR} = \bar{r}_D + \frac{\sum_{j \in \{A,B,C\}} s(D,j)(r_{j,LOTR} - \bar{r}_j)}{\sum_{j \in \{A,B,C\}} |s(D,j)|}$$

$$= 3.33 + \frac{-0.2631[5-3] - 0.6172[5-3.33] + 0.3313[1-2.8]}{0.6172 + 0.3313 + 0.2631}$$

$$= 1.5547$$

LOTR by E:

$k=3$: top 3: A-E: -0.7142
B-E: -0.9194
C-E: 0.5296

$$\hat{r}_{E,LOTR} = 3.6 + \frac{-0.7142[5-3] - 0.9194[5-3.33] + 0.5296[1-2.8]}{0.7142 + 0.9194 + 0.5296}$$

$$= 1.7888$$

+IDAPPO7 by A:

B-A : 0.8625 ←
C-A : -0.799 ←
D-A : -0.2631
E-A : -0.7192 ←

(k=3:)

$$\hat{r}_{A,\,+IDAPPO7} = 3 + \frac{0.8625[4-3.333] + (-0.779)(2-2.8) - 0.7192(5-3.6)}{0.8625 + 0.779 + 0.7192}$$

$$= \boxed{3.6905}$$

LSATSB by E:

(k=3:) top 3: { A-C : -0.779 ; B-C : -0.7940 ; E-C : 0.5496 }

$$\hat{r}_{C,\,LSATSB} = 2.8 + \frac{-0.779(2-3) - 0.7940(2-3.33) + 0.5496(4-3.6)}{0.779 + 0.7940 + 0.5496}$$

$$= \boxed{3.2734}$$

The Castleman by D:

K=3:

top 3: { B-D: -0.6172, C-D: 0.3363, E-D: 0.2939 }

$$\hat{r}_{D,\text{castleman}} = 3.33 + \frac{-0.6172(2-3.33) + 0.3363(4-2.8) + 0.2939(5-3.6)}{0.6172 + 0.3363 + 0.2939}$$

$$= 4.6414$$

The Hobbit by D:

K=3:

top 3: { B-D: -0.6172, C-D: 0.3363, E-D: 0.2939 }

$$\hat{r}_{D,\text{Hobbit}} = 3.33 + \frac{-0.6172(5-3.33) + 0.3363(3-2.8) + 0.2939(1-3.6)}{0.6172 + 0.3363 + 0.2939}$$

$$= 1.945$$

3.

	LOTR	HPATP07	Sratch	LSATSB	The Cat	Hobbit
A	5	3.6905	1	2	3	4
B	5	4	2	2	2	5
C	1	2	4	3.7734	4	3
D	1.5527	2	3	5	4.6414	$1.94\bar{5}$
E	1.7888	3	5	4	5	1

k-means Clustering of Movies using $k=2$:

Criterion: Euclidean distance for $k=2$

Initial Clusters: LOTR and HPATP07

Centers:

Repeat:
- Assign each cluster pt $\bar{x}_i$ to the cluster $\bar{c}_j$ to get clusters.
- Change $\{\bar{c}_1, \bar{c}_2\}$ to new cluster means

Done: until no new cluster assignment or less than eps

Done using K-means Algorithm ~~Clustering~~ for Assignment 8

Iteration 0:

Clus: {0, 1} for k=2

Label:
- LOTR - 0
- HPATPO7 - 1
- Snatch - 1
- LSATSB - 1
- The Cat - 1
- Hobbit - 0

Centroids: $\begin{bmatrix} 4.5 \\ 5 \\ 2 \\ 1.2485 \\ 1.3944 \end{bmatrix}$ $\begin{bmatrix} 2.4226 \\ 2.5 \\ 3.44 \\ 3.66 \\ 4.25 \end{bmatrix}$

Iteration 1:

Labels:
- LOTR - 0
- HPATPO7 - 0
- Snatch - 1
- LSATSB - 1
- The Cat - 1
- Hobbit - 0

Centroids: $\begin{bmatrix} 4.23 \\ 4.66 \\ 2 \\ 1.8325 \\ 1.9298 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 2 \\ 3.924 \\ 4.218 \\ 4.666 \end{bmatrix}$

∴ Final Clusters:

Iteration 2:

Labels: $\begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ LOTR, HPATPO7, Snatch, ⋮

Centroids: $\begin{bmatrix} 4.23616 \\ 4.666 \\ 2 \\ 1.8305 \\ 1.9196 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 2 \\ 3.924 \\ 4.218 \\ 4.166 \end{bmatrix}$

LOTR
HPATPO7
Hobbit

Snatch
The Cat
LSATSB

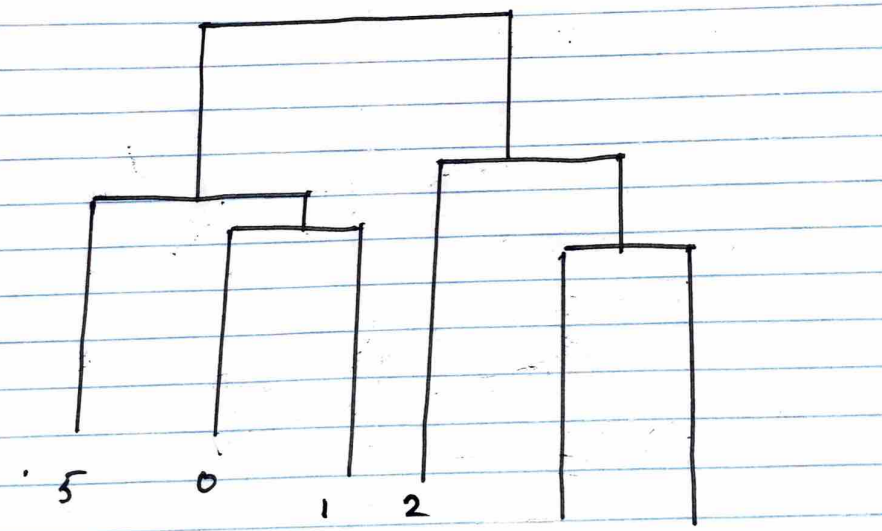
4. Inverse Euclidean Similarity Distance:

Inverse Euclidean similarity:

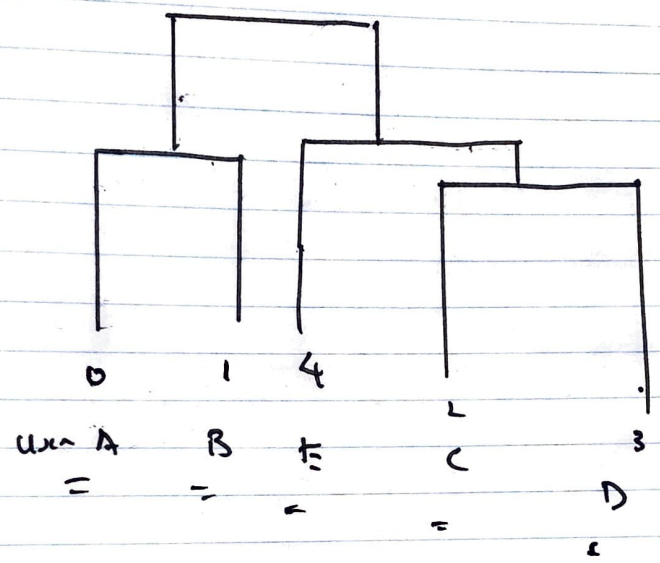
$$sc(u,v) = \frac{1}{\|\vec{r_u} - \vec{r_v}\|}$$

Using a Python script:

Hierarchical Clustering for Movies:



Hierarchical Clustering for Users:



Problem 2:

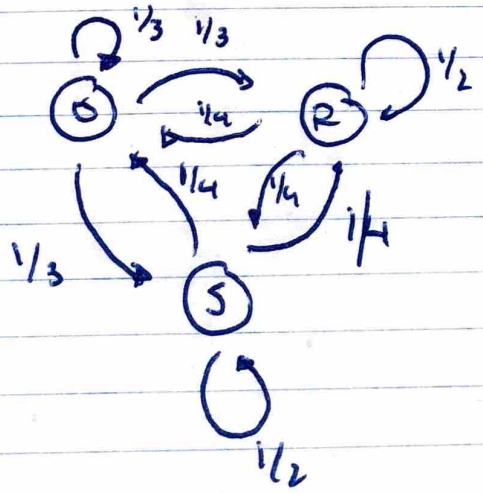
{ O - Overcast
R - Rain
S - Sunny }

April 29 is Rain

Transition Matrix:

	O	S	R
O	1/3	1/3	1/3
S	1/4	1/2	1/4
R	1/4	1/4	1/2

1. State Transition Diagram:



2. Probability it will be sunny on April 30th, 2020

$P(x_1 = S \mid x_0 = R) = \boxed{1/4}$

transition from R to S

$\therefore$ 25% probability.

3.

April 29	April 30	May 1	May 2
R	?	?	R

can be S, R, or O.

↓

Account for every permutation

04/30	05/01
O	O
O	S
O	R
⋮	
R	R

9 permutations

$P(05/02 \text{ is } R) = P_{RO}P_{OO}P_{OR} + P_{RO}P_{OS}P_{SR} + P_{RO}P_{OR}P_{RR} +$

$P_{RS}P_{SO}P_{OR} + P_{RS}P_{SS}P_{SR} + P_{RS}P_{SR}R_{RR} +$

$P_{RR}P_{RO}P_{OR} + P_{RR}P_{RS}P_{SR} + P_{RR}P_{RR}P_{RR}$

$= 0.371527$

$\therefore$ 37.1527% probability.

4. Rain every day until May 5, 2020 (including it):

04/24	04/30	05/01	...	05/05	[illegible]
R	R	R	... in R	R	

6 transitions

given (R) ↻ 1/2 = 50% ∴ P(rain every day until May 5) = $(1/2)^6$

$= 0.015625$

$\therefore$ 1.5625% probability.

$\therefore$ 1.5625%

5. Rain on May 6, 2020:

04/24	04/30	05/01	...	05/05	05/06
R					R

O/S/R → 3^6 permutations (transition outcomes leading to R on 05/06) ⟹

Using Python to Calculate (6 nested for loops):

$$P(\text{Rain on May 6, 2020}) = \sum_i \sum_j \sum_k \sum_a \sum_b \sum_c P_{Rc} P_{cb} P_{ba} P_{ak} P_{kj} P_{ji} P_{iR}$$

$\therefore$ 36.56%

$i, j, k, a, b, c \in \{S, O, R\}$

$\therefore$ 36.56% it will rain on May 6, 2020