

Markov models, Hidden Markov Models

ECE 4200

Until now ...

We did not (?) have a notion of time ... yet.

However, in many problems, things happen one after another.

- Reinforcement learning: How to play chess/go/etc.
- Predicting the next word to be typed,
- And so on ...

Markov models

Probabilistic models that capture dependent events

Outlook: Sunny, Overcast, Rain

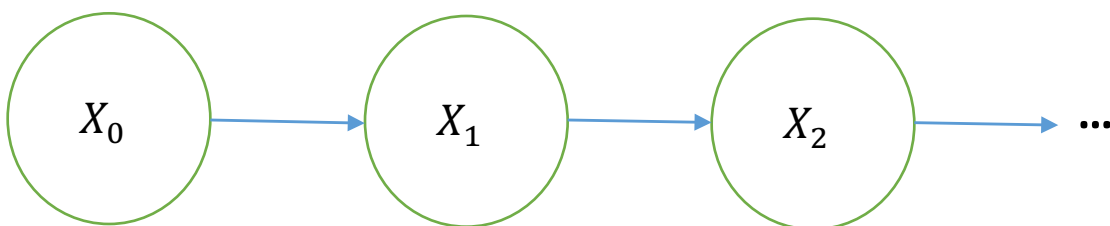
If we know it rained today, what is the probability that it will rain tomorrow?

Markov Models

Used to model a process X_0, X_1, \dots , as follows:

$$\Pr(X_{t+1} | X_t, X_{t-1}, \dots, X_0) = \Pr(X_{t+1} | X_t).$$

Given the present, future is independent of the past.



Markov Models

Used to model a process X_1, X_2, \dots , as follows:

$$\Pr(X_{t+1} | X_t, X_{t-1}, \dots, X_1) = \Pr(X_{t+1} | X_t).$$

Given the present, future is independent of the past.

Example: Gambler

X_t : amount of money won by a gambler

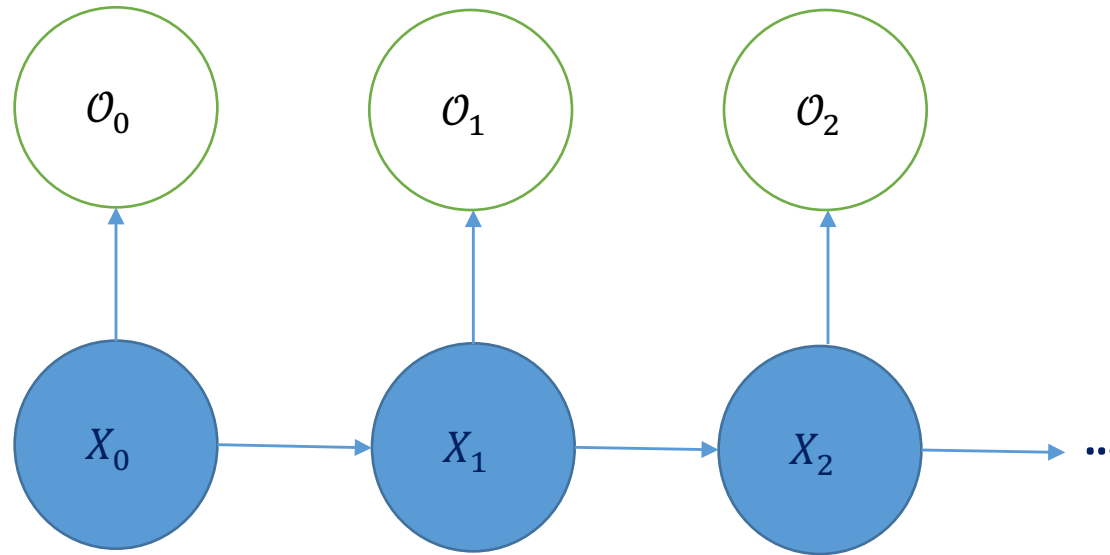
In each round, wins or loses 1₹ with equal probability

$$X_0 = 0$$
$$\Pr(X_{t+1} | X_t, X_{t-1}, \dots, X_1) = \Pr(X_{t+1} | X_t)$$

$$\Pr(X_{t+1} = x + 1 | X_t = x) = \Pr(X_{t+1} = x - 1 | X_t = x) = \frac{1}{2}$$

Representation – draw

Hidden Markov Models



X_0, X_1, \dots : Unobserved hidden states

O_0, O_1, \dots : Observed states.

Examples: ...

Applications

- Computational finance^{[20][21]}
- Single-molecule kinetic analysis^[22]
- Cryptanalysis
- Speech recognition, including Siri^[23]
- Speech synthesis
- Part-of-speech tagging
- Document separation in scanning solutions
- Machine translation
- Partial discharge
- Gene prediction
- Handwriting recognition
- Alignment of bio-sequences
- Time series analysis
- Activity recognition
- Protein folding^[24]
- Sequence classification^[25]
- **Metamorphic virus detection**^[26]
- DNA motif discovery^[27]
- Chromatin state discovery^[28]
- Transportation forecasting^[29]
- Solar irradiance variability ^{[30][31][32]}

Resources

Rabiner:

http://ai.stanford.edu/~pabbeel/depth_qual/Rabiner_Juang_hmms.pdf