

Technology

Problem 1

True or false? If $V(y)$ is a convex set, then the associated production set Y must be convex.

Answer:

False.

If Y is a convex set, then for x and x' that makes $(y, -x)$ and $(y, -x')$ in Y , there must be $(ty + (1-t)y, -tx - (1-t)x')$ in Y . i.e. $(y, -tx - (1-t)x')$ in Y .

If x and x' in $V(y)$, then $tx + (1-t)x'$ also in $V(y)$, then $V(y)$ is convex.

Counterexample that shows convex input set doesn't mean convex production set.

Consider $f(x) = x^2$ production function. Production set $Y = \{y, -x) : y \leq x^2\}$ is not convex. But input set $v(y) = \{x : x \geq \sqrt{y}\}$ is convex

Problem 2

What is the elasticity of substitution for the general CES technology $y = (a_1x_1^\rho + a_2x_2^\rho)^{1/\rho}$ when $a_1 \neq a_2$?

Answer:

To calculate elasticity of substitution, we need to calculate substitution of technology.

$$TRS = -\frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = -\frac{a_1x_1^{\rho-1}}{a_2x_2^{\rho-1}}$$

Take log

$$\ln\{TRS\} = \ln\frac{a_1}{a_2} + (1 - \rho)\ln\frac{x_2}{x_1}$$

$$\sigma = \frac{d\ln(x_2/x_1)}{d\ln TRS} = \frac{1}{1 - \rho}$$