Risk Analytics - Practical 1

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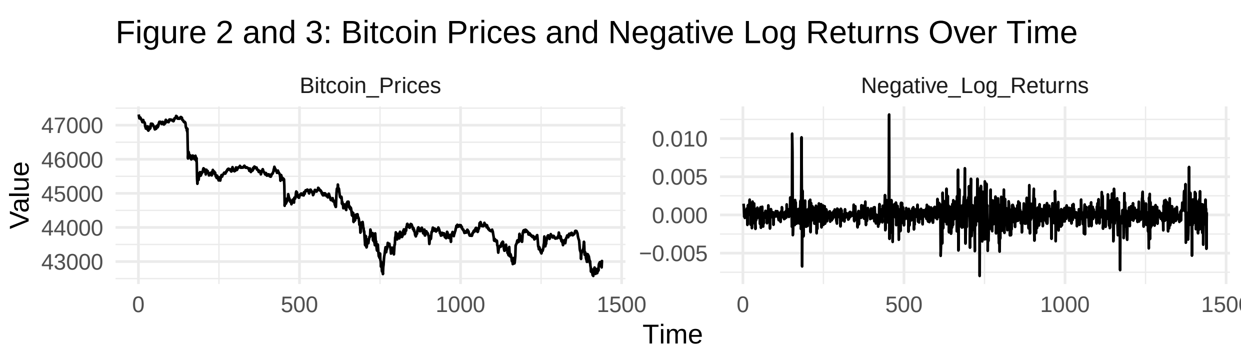
Part 1: Financial Returns and Normality

a) and b) Assessing Stationarity of Bitcoin Prices and Negative Log Returns

In this section, we assess the stationarity of the raw Bitcoin prices and their negative log returns (see Figure 1), as stationarity is crucial for time series modeling.

We first plotted the Bitcoin prices (see Figure 2) to visually inspect for trends or patterns. The plot showed a clear trend, suggesting non-stationarity. To confirm this, we applied the Augmented Dickey-Fuller (ADF) test (see Table 1), which resulted in a p-value of 0.3885, indicating that the raw Bitcoin prices are non-stationary.

To address this, we computed the negative log returns, a transformation commonly used in financial time series analysis to obtain a stationary series. Visual inspection of the negative log returns (see Figure 3) suggested stationarity. This was further confirmed by the ADF test (see Table 2), which gave a p-value of 0.01, leading us to reject the null hypothesis of non-stationarity and confirming that the negative log returns are stationary.



c) Assessing the Normality of Negative Log Returns

To evaluate whether the negative log returns of Bitcoin follow a normal distribution, we first examined the data visually using a histogram (see Figure 4) and a QQ-plot (see Figure 5). The histogram of the negative log returns suggests that the data may be approximately normally distributed, though some deviations from normality could exist. Additionally, the QQ-plot shows that the returns are normally distributed for non-extreme values, but there are noticeable deviations in the tails, indicating that the negative log returns may not follow a perfect normal distribution.

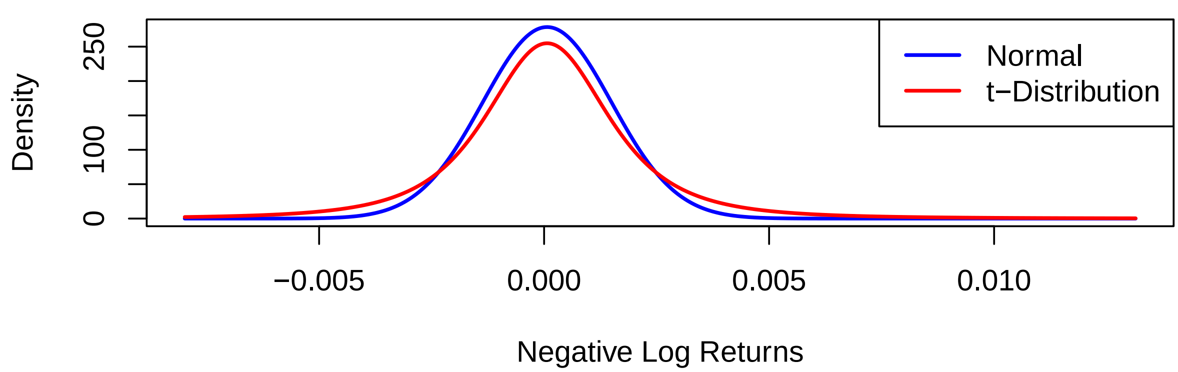
To formally test for normality, we applied the Anderson-Darling test (see Table 3), which gave a p-value less than 0.05. As a result, we reject the null hypothesis (H0) that the data is normally distributed. This confirms that, despite appearing somewhat normal in the central part of the distribution, the negative log returns are not normally distributed, especially due to the extreme values.

d) and e) Fitting a t-Distribution and Comparing Tails

Since the negative log returns deviate from normality, particularly in the extremes, we fit a t-distribution to the scaled data to check if it better captures these extreme values. A QQ-plot was generated to compare the negative log returns with the theoretical t-distribution (see Figure 6), which showed that the data fits the t-distribution quite well, including in the tails. For comparison, we also generated a QQ-plot for the normal distribution (see Figure 5), which demonstrated a poorer fit, particularly for extreme values. This suggests that the t-distribution, with its ability to model heavy tails, is a more appropriate fit for the data (see Figure 8).

Next, we compared the density plots of the normal and t-distributions. As expected, the t-distribution exhibited heavier tails than the normal distribution, meaning we should expect more extreme, unexpected events in a t-distribution (see Figure 9).

Figure 8: Density Comparison: Normal vs t−Distribution



Since the Bitcoin data follows the t-distribution more closely, and the t-distribution has fatter tails than the normal distribution, this indicates that extreme values (large deviations from the mean) are more likely in the Bitcoin data than if it were normally distributed.

Part 2: Financial time series, heteroscedasticity and the random walk hypothesis

Appendices

Practical 1

Figures

Figure 1: Bitcoin Prices and Negative Log Returns Over Time on Common Scale

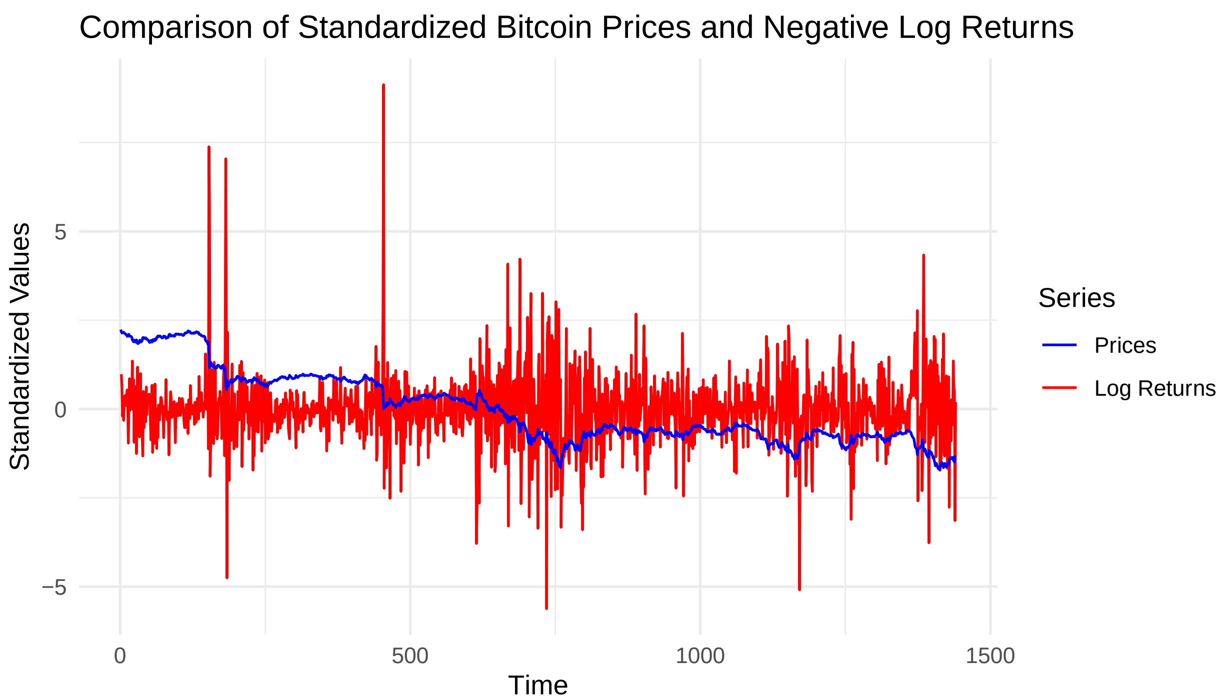


Figure 2: Bitcoin Prices Over Time

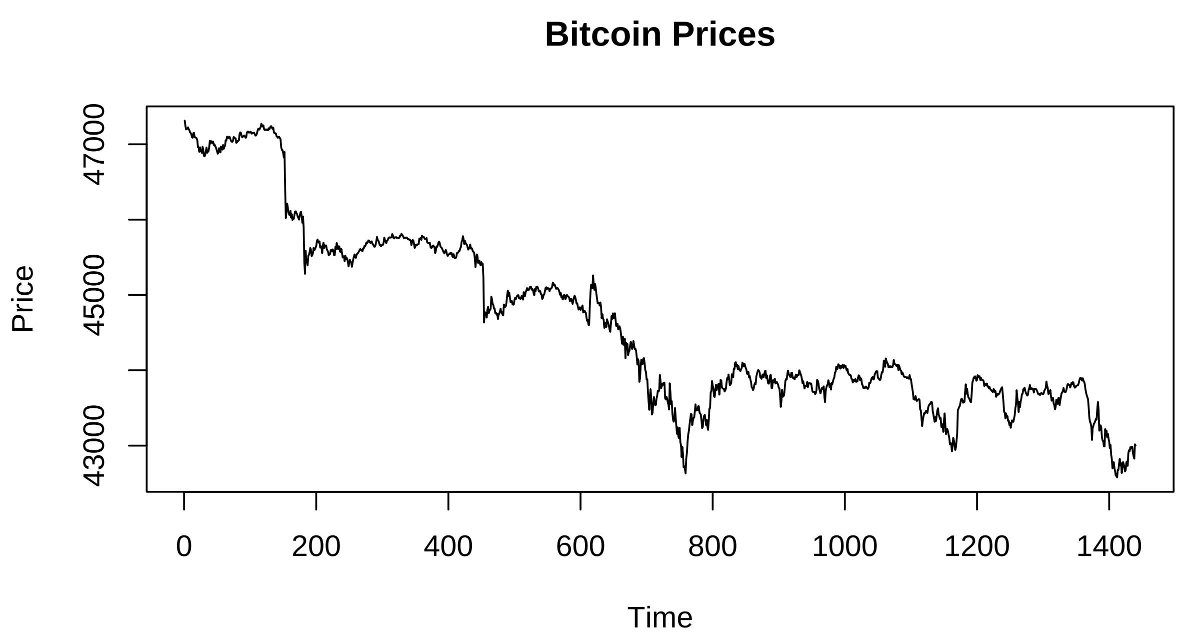


Figure 3: Negative Log Returns of Bitcoin Over Time

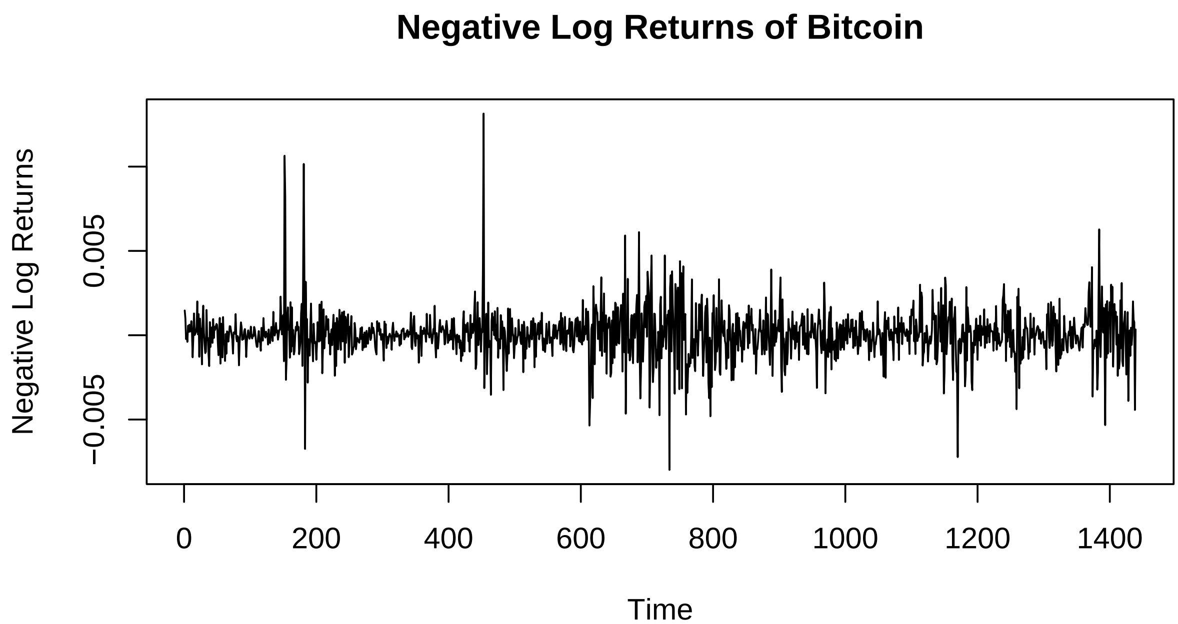


Figure 4: Histogram of Negative Log Returns

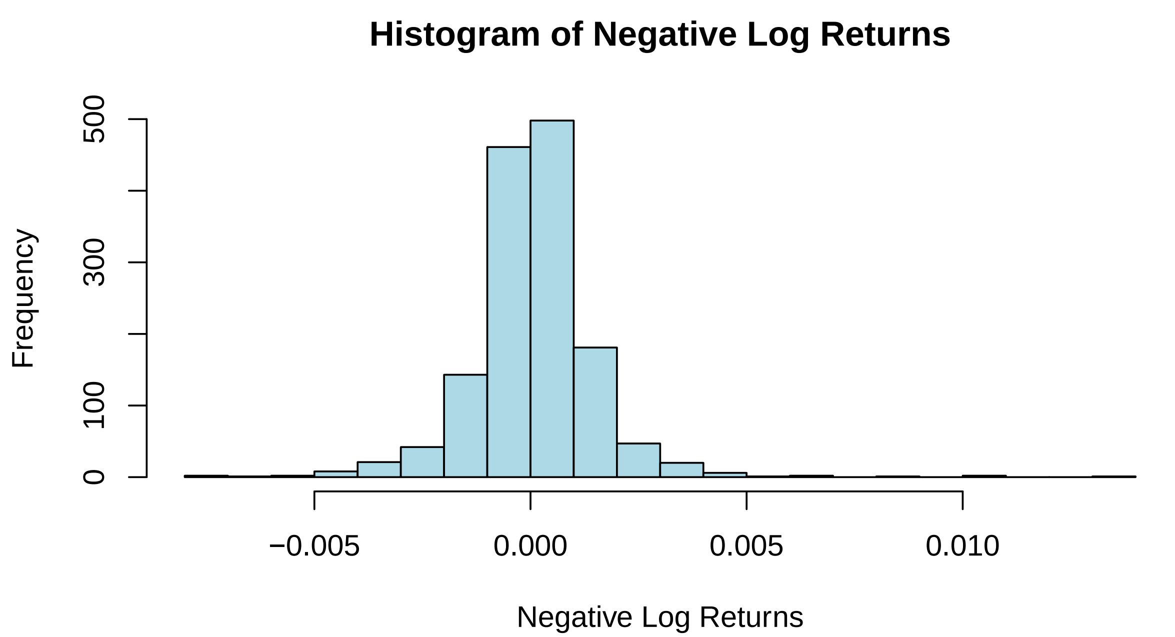


Figure 5: QQ-Plot of Negative Log Returns

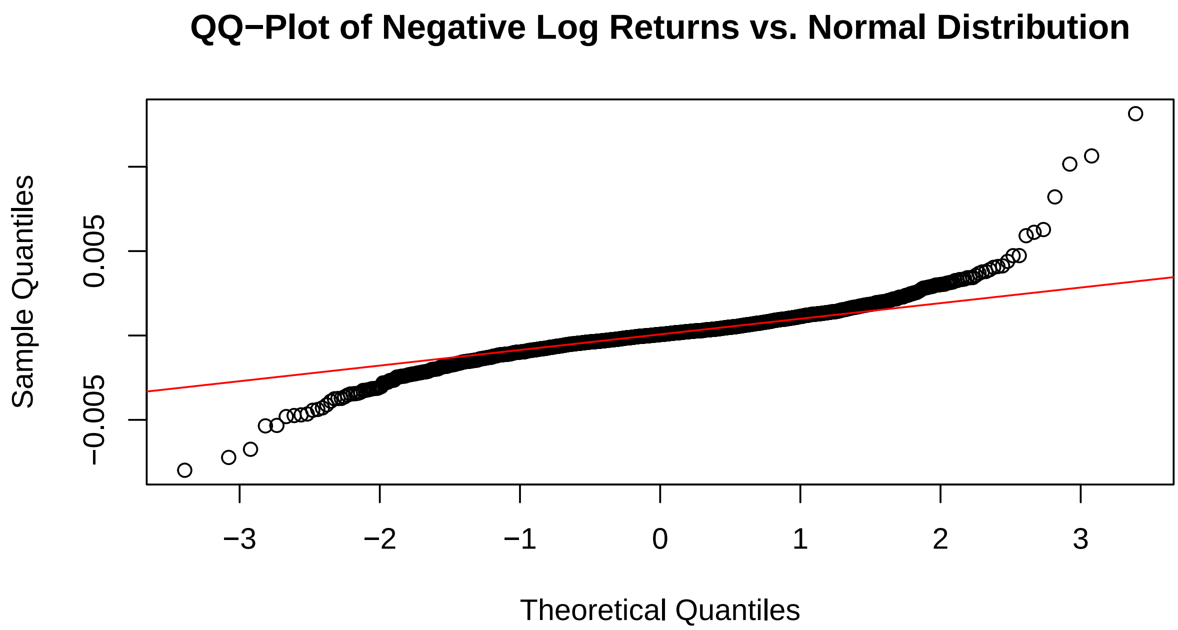


Figure 6: QQ-Plot of Negative Log Returns with t-Distribution

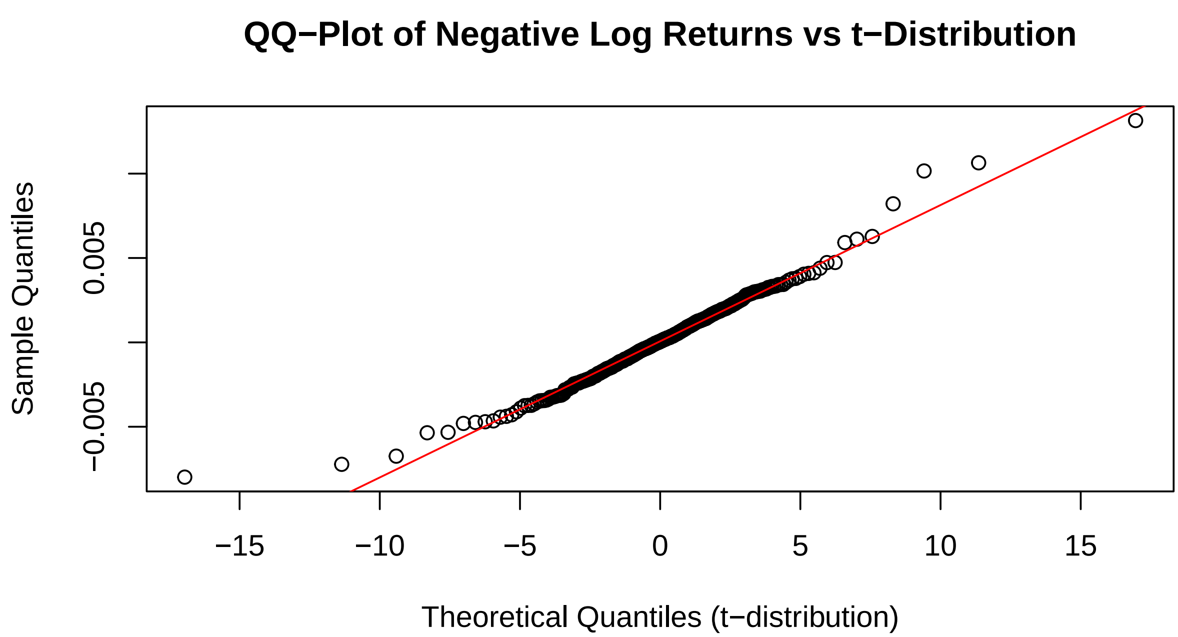


Figure 7: Histogram of Bitcoin Negative Log Returns with Fitted t and Normal Distribution

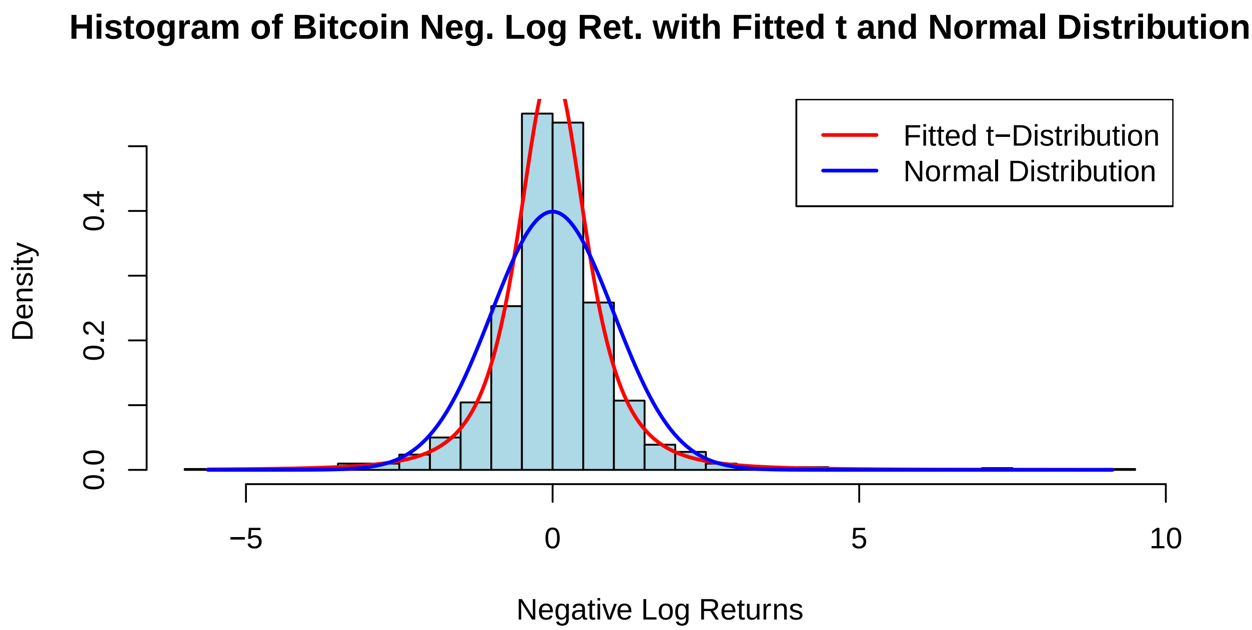
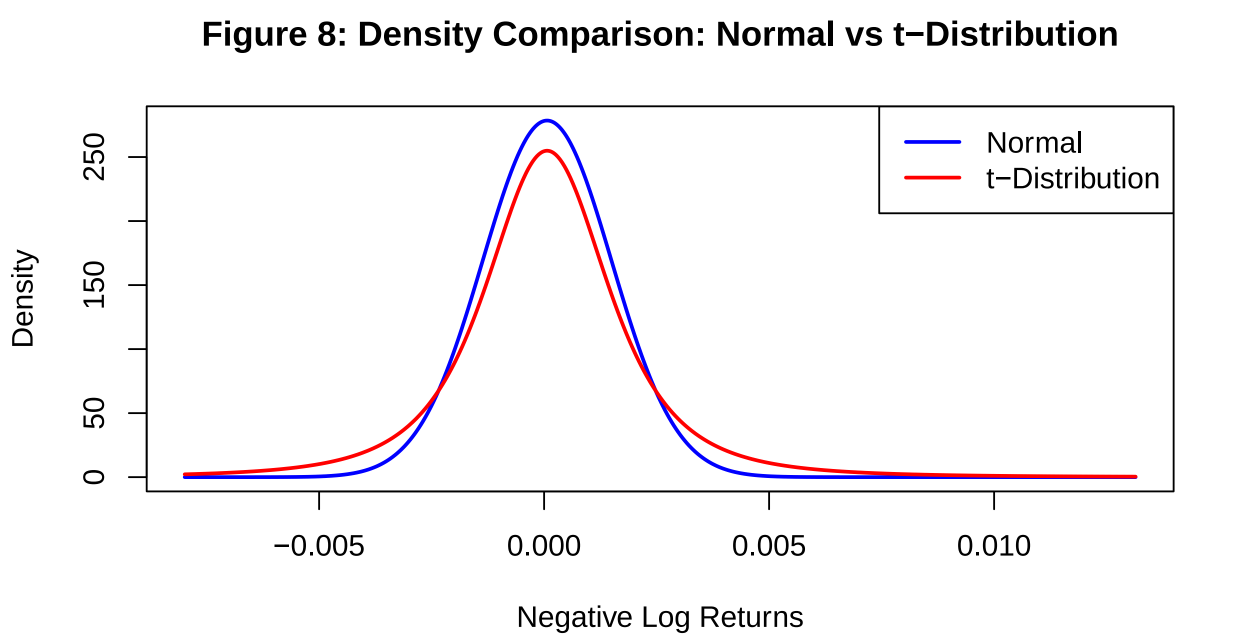


Figure 8: Density Comparison: Normal vs t-Distribution



Results tables

Table 1: Augmented Dickey-Fuller Test for Bitcoin Prices

```
##
##  Augmented Dickey-Fuller Test
##
## data:  bitcoin_prices
## Dickey-Fuller = -2.4484, Lag order = 11, p-value = 0.3885
## alternative hypothesis: stationary
```

Table 2: Augmented Dickey-Fuller Test for Negative Log Returns

```
##
##  Augmented Dickey-Fuller Test
##
## data:  bitcoin_negative_log_returns
## Dickey-Fuller = -11.035, Lag order = 11, p-value = 0.01
## alternative hypothesis: stationary
```

Table 3: Anderson-Darling Test for Normality of Negative Log Returns

```
##
##  Anderson-Darling normality test
##
## data:  bitcoin_negative_log_returns
## A = 26.277, p-value < 2.2e-16
```

Code Appendix